Classroom Expectations: Listen to the Maths

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For students to engage in mathematical thinking, a greater emphasis is needed on the expectations of students, by themselves and others, as active listeners and explainers. As student responsibility for mathematical thinking and participation increases, the roles of the teacher and learner change. A qualitative study of changing classroom expectations in a Grade 2 class was conducted. The teacher saw her role shifting to that of facilitator and the students' expectations were as active listeners and explainers. The type of questions asked by the teacher was instrumental in whether justification and mathematical reasoning was elicited from all learners.

With a focus on classroom expectations of the *teacher and students* in primary classrooms, Wood (2002) argued that differences in teaching impact what mathematics students value and learn. From observing classrooms, Wood's research group were led to appreciate the importance of the quality of students' "explanations and justifications they were required to give for their solution processes. Furthermore, these differences were found to affect the quality of [students'] thinking and reasoning about mathematics" (Wood, 2002, p. 61). More recently, Dorier and Maass (2020) have reiterated the necessity for this focus on student-centred learning and the critical role of the teacher. They describe student-centred learning as being when students work in ways similar to mathematicians, including asking questions, focussing on mathematical ways of thinking about questions, discussing, interpreting, evaluating, and communicating solutions in effective ways. Following Wood (2002), the focus of this paper is on the expectations of teachers and students in such classrooms and what role the students perceive and enact in such a classroom.

A Mathematical Community of Inquiry

Drawing on extensive video-recordings of Grade 2 and 3 mathematics classes in the USA, Wood and Turner-Vorbeck (2001) closely examined *teacher questioning and students' mathematical reasoning*. Wood (2002) reported that differences in students' reasoning were correlated with the type of questions teachers asked, the cognitive demand these questions placed on students' mathematical reasoning, and the expectations for students' participation. Dorier and Maass (2020) note the role of the teacher in such classrooms, "includes making constructive use of students' prior knowledge, challenging students through effective, probing questions, managing small group and whole class discussions, encouraging the discussion of alternative viewpoints, and helping students to make connections between their ideas" (p. 385). According to Wood (2002), the key is not so much on the style of question, as it is on the expectations placed on students, as listeners and explainers, by themselves and others, in particular, the expectation that they be actively listening and explaining, that bring this change in student thinking and reasoning. Makar et al. (2015) report that the uptake of inquiry focused lessons in mathematics has been slow. They suggest this is a result of the challenges for both teachers and students.

In setting up a classroom culture fostering the development of mathematical thinking, the responsibility for thinking mathematically by students can be increased as the discussion context implemented by the teacher changes according to Wood (2002). Wood described three discussion contexts (above and beyond the traditional reporting answers context), as reporting strategies $\rightarrow$ inquiry $\rightarrow$ argument whereby student responsibility for thinking and participation

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increases. Similarly, responsibility for participation increases as the expectations of the role of the student changes from being a student explainer to an active student listener.

The first row of Table 1 is described as a conventional classroom culture (Wood, 2002). The following rows describe these increasingly sophisticated discussion contexts: strategy reporting, inquiry, and argumentation, described by Wood as reform classrooms. As the discussion context increases in complexity, the roles of explainers and listeners expand (e.g., an inquiry discussion context includes reporting of strategies and more). Furthermore, the *responsibility for mathematical thinking* and *responsibility for participation* increasingly shift from the teacher to the students. The role of the teacher does not diminish, rather it shifts to becoming a key member of the mathematical community of inquiry (Dorier & Maass, 2020). Expectations for the role of the student shift from being a passive to an active listener and having a vital part to play in discussions. Providing genuine opportunities for mathematical reasoning "requires a shift of role for both educators and students" (Hunter et al., 2020, p. 312).

Table 1

Theoretical Framework for Teaching and Learning Based on Wood (2002)

Discussion context	**Mathematical thinking**	**Role(s) of explainers (student)**	**Role(s) of listeners**	
			Teacher	**Student**
Reporting of correct answers	Recall answers Recall procedures	Tell answers Tell procedures	Evaluate Ask testing questions	Pay attention (passive)
		Responsibility for Thinking	**Responsibility for Participation**	
Reporting of Strategies	Comparing Contrasting	Tell different ways	Accept solutions Elaborate solutions	Compare/ contrast solutions
Inquiry	Comparing Contrasting Reasoning Questioning	Tell different ways Clarify solutions Give reasons	Accept solutions Elaborate solutions [Ask questions] [Provide answers]	Compare/ contrast solutions [Ask questions]
Argument	Comparing Contrasting Reasoning Questioning Justifying Challenging	Tell different ways Clarify solutions Give reasons Justify Defend solutions	Accept solutions Elaborate solutions [Ask questions] [Provide answers] Make challenges	Compare/ contrast solutions [Ask questions] Disagree Make challenges

Questioning

Teacher questioning is thus an important aspect of shifting responsibility for mathematical thinking in the classroom from the teacher to the students. The NCTM (2014, p. 35) elaborates that effective teaching is associated with purposeful teacher questions that both advance and assess reasoning and sense by students. Bauserfeld (1992) argued that to generate the flexibility of thinking necessary for problem solving, "it would seem more promising to develop the art of mathematical interpreting, rather than the usual funnel-like limiting to prepared, unequivocal reactions" (p. 469). Furthermore, to develop such flexibility students need to reflect on their own mathematical thinking. Hence, teachers need to provide opportunities whereby students are expected to make explicit their ways of thinking as they engage with complex tasks.

Wood (1998) noted that although well intentioned, many teachers—once a student offers a method representative of what they intend students to use—emphasise their preferred method thus inadvertently communicating to students a single approach to obtaining a desired solution. Bauersfeld (1992) described this as *funnelling*. The need to become more purposeful in mathematical classroom discourse remains an issue (Hunter et al., 2020). In contrast to the funnelling approach, Wood (1998) argued for a more dialogical approach whereby “a variety of solutions are accepted and valued … [with the intention being] to help students notice an idea” (p. 168). In this *focussing* pattern of discourse, students participate more equally in the discussion. Focusing occurs when “the teacher expects the students to think about the mathematics. To figure things out for themselves, and to discuss their ideas with others” (p. 176) and the students are aware of this expectation. A study by Hagenah et al. (2018) in Year 9 science classrooms found focussing (rather than funnelling) question use to be aligned to increased student learning outcomes and confidence.

Reasoning

Mathematical content knowledge (MCK) contains what Kilpatrick et al. (2001) describe as five strands that are the basis of students’ mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. In the Australian (and Victorian) Curriculum: Mathematics (ACARA, 2022) these are modified to understanding, fluency, problem solving, and reasoning. One important point is that the ‘content’ of MCK extends beyond number, algebra, geometry, measurement, probability, and statistics to ways of thinking mathematically. The proficiencies describe what students ‘do’ including compare, contrast, explain, identify, justify, and reflect. Kilpatrick et al. (2001) stressed the proficiency strands “are not independent; they represent different aspects of a complex whole … [they are] interwoven and interdependent” (p. 115).

The interwoven nature of the proficiencies in the enacted Australian curriculum and emphasis on each has been challenged by Australasian researchers, as has student capacity to reason. Analysis of the Australian Curriculum: Mathematics by McCluskey et al. (2016) suggests the intentions of the curriculum writers for the proficiencies to be integrated with conceptual understanding has not been met, and reasoning is the least emphasised proficiency. Given that Ball and Bass (2003) argue that understanding without reasoning is “meaningless” (p. 28), this apparent lack of emphasis on reasoning is a concern. International studies consistently show that Australian Year 4–9 students have difficulty explaining and justifying their mathematical thinking.

The Study

The data for the analysis of the primary mathematics class presented here were collected as part of the *Teacher as Learner Research* (TALR) project that examined teaching and learning in one Victorian state primary school. The project objectives were for teachers to promote the understanding of mathematical concepts through student discussion and reflection following Wood (2002). The development of *mathematical content knowledge* and mathematical *pedagogical content knowledge* (building on the work of Shulman, 1986) by the teachers, and hence classroom practices, including establishing classroom expectations of teachers to enhance student learning, were the focus. Tasks and approaches were developed collaboratively emphasising both the content and processes of mathematics, including ways of working mathematically such as conjecturing, convincing, and justifying that underpin mathematical thinking. Both questioning style and eliciting reasoning were of high importance in the project.

Key elements of the project included: *demonstration days* (each term the researcher taught several lessons observed by some teachers and discussed by all teachers at the end of the school day, following Clarke et al., 2013), *professional learning* (a two hour session each term focusing on elements of MCK and PCK including task design and development, questioning, and reasoning), *teacher lesson diaries* (TLD) documenting two focus lessons per term taught by the teacher), and *reflection.* The lesson diary included a record of the lesson intention, the lesson enacted, and a reflective component focussing on what the teacher learnt from the lesson.

Before these lessons were implemented, teachers selected three ‘typical’ students to document their thinking. The students selected were not the ‘best’ three students but from those expected to have difficulties with the learning focus. The diary included specific details regarding these students’ contributions to classroom discussions and their progress on tasks. To facilitate articulation of student thinking and provide enhanced opportunities for teacher reflection, teachers used the iPad *ShowMe* app. This app allowed a photograph of student work to be taken and then used to facilitate students’ explaining their thinking. The photograph was annotated during this reflective discussion with the teacher, which the app also audio-recorded. Post-lesson viewing facilitated the teacher’s reflective process—a key part of the TLD.

A qualitative approach was followed to provide a comprehensive picture of what was occurring in the classroom. Following Stake (1995), a case study using an instrumental approach was undertaken, as the interest is what we can learn from this case about other classrooms. Data were coded from the perspective of teacher and student expectations for roles in discussion from Wood’s Framework and mathematical thinking as specified via the proficiencies. The analysis was similar to that of Strauss and Corbin (1990) in developing categories from the data. A rich description was essential for the expectations of the classroom to be evident. The research questions addressed were: (1) *From participation in a ‘teacher and researcher’ community of inquiry, what expectations does a teacher develop in a classroom community of mathematical inquiry*? (2) *From these expectations, what do students perceive as their role in the mathematics classroom and to what extent is this realised*?

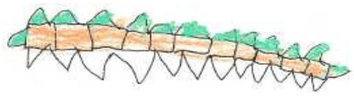
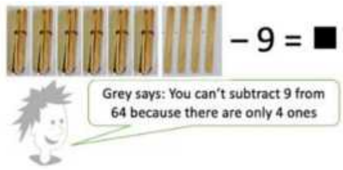
At the end of year, as part of the reflective aspect of the TALR project, all teachers were asked to implement a short survey with their class. Students responded to the question, “What do you think your role or job in maths is?” with the prompt, if needed, “What things are you expected to do in maths?” From these responses, one Year 2 class was chosen for analysis based on the response of one student. Muridi wrote, “My job is to listen to the maths, … listen to the shape figures and non-examples and the maths”. Not only had 8-year-old Muridi (all names are pseudonyms) recognised the importance of listening in his learning of mathematics, but also, he used the geometric language of figure and identified the importance of non-examples in his developing understanding. This response prompted an analysis of the classroom experiences and expectations that led him to articulate this view. Thus the case is the classroom expectations of the teacher and students in Muridi’s mathematics class.

Muridi, along with his classmates, attended three demonstration day lessons (taught by the researcher) during the year. Over the year, seven TLD lessons were taught and documented by Muridi’s teacher. Eleven of the 23 students in the class were selected as the focus student over this time for her diary. Muridi was a focus student on three occasions. To add to the richness of the data, two additional students, Ada and Madi, were selected as this meant every TLD included at least one of these students. In describing her “role or job in maths”, Ada nominated solving problems, figuring it out, understanding, show you know, and checking. She also noted “you could get it wrong” and suggested using “the maths equipment if it is hard or you get confused”. Madi included answering hard questions and solving challenging activities. Hence, three Grade 2 students, Muridi, Ada, and Madi are the main focus of the analysis reported. Whilst Muridi’s words prompted this class’s selection, it is critical to remember he is one student in a mathematical community of inquiry with other students and his teacher.

The data used were collected over four consecutive school terms and drew mainly on the TLDs. Table 2 summarises the lesson focus. Teacher reflection during and after demonstration days, and teacher reflection after professional learning sessions were used to substantiate the analysis as needed. For example, an analysis of student work collected by teachers during the year from the TLD lessons and the demonstration day lessons confirmed the three selected students were not the ‘best three’ in the grade.

Table 2

Overview of Teacher Lesson Dairies Lessons and Selected Focus Students

Lesson diary[α]	Focus students			Task title and brief details
	Muridi	Ada	Madi	
1		✓		*One is a snail, ten is a crab*: I saw 10 feet, what might I have seen …
2a		✓		*One change train*: Use attribute blocks to create a ‘train’ where adjacent pieces differ by one attribute
2b		✓		*Caterpillar Problem*: Investigate caterpillar growth over time  
3a			✓	*The Rubbish Bin Problem1 Convince Me*: Some digits from a rubbish bin number were found. What house number could the bin have come from? 
3a	✓			*Equality*: ■ = 64 − 9 Reasoning Cartoon 
4a	✓			Minibeasts What’s the favourite minibeast in our class?
4b	✓	✓	✓	Which is the Odd One Out? Why? 39 37 79

[α]The number in column 1 indicates the term (e.g., 3b is the 2nd Lesson Diary in term 3).

Results and Discussion

The lesson in focus for the first teacher lesson diary (TLD1) is presented in sufficient detail for the reader to gain a good sense of the classroom expectations that are the focus of the research. Student comments are presented as recorded by the classroom teacher. The lesson for the Grade 2 class occurred in week six (of eight) of term 1. The lesson focused on problem posing and solving based on the picture book *One is a snail, ten is a crab* (Sayre & Sayre, n.d.). The focus task was ‘I saw a total of ten feet. What animals might I have seen? How many possible combinations of animals are there? How are you going to know when you have them all?’ The teacher expected students would use different ways of recording and documenting their understandings—drawings, words, numbers, equations, or a combination; make use of a range of strategies to solve the problem; apply thinking strategies; and articulate their learning approach for the problem using partner sharing and the *ShowMe* app if selected as a focus student. These expectations remained the same throughout term two as well.

When the storybook cover was presented at the start of the whole class discussion, the students recognised the type of task to come which they said was “Jill maths” drawing on their participation in a lesson on Demonstration Day 1 earlier in the term. When asked to explain, comments included “Jill gives us lots of maths things to do” (Madi), “challenges and questions” (Ada), “she gives us all sorts of new things that we don’t know yet” and “gives us a problem and we try and work it out”. According to her diaries the teacher used this connection regularly to how the students’ experienced the demonstration lessons to reinforce her classroom environment expectations for students and to convey that their role included explaining and engaging with mathematical thinking as they worked on similar tasks.

After reading the book, checking for understanding (“a crab has 10 feet and a snail only one foot”), and discussing a simpler version of the task for 6 legs, the students set to work individually for 30 minutes, followed by ten minutes where they partnered to share their work, and then returned to individual work on the task. The teacher noted (TLD1) that throughout the lesson she frequently asked students, “Have you found all the possibilities? How do you know?” and implored them to “Convince us! Check for duplicates”.

Muridi wrote, “I had eight combinations. I putted numbers on everything I drawed.” During the lesson his teacher also recorded some of Muridi’s thoughts on his sheet. These included “I counted by twos so that is 10 altogether” in explaining his solution using 10 people. For his solution of 2 people, 2 snails and a dog, his teacher recorded “I did 2 people for 2, 4 then I counted 2 more, then I did 4 then 8. 2 people and 2 snails equals 6 then I added the dog, 6 + 4 = 10.” Muridi was the only student who recorded the number of combinations he found. His comment regarding putting numbers with his drawings is inferred as an attempt to explain his solutions were correct rather than to argue he had found all possibilities. Although it is impossible to tell the order in which Muridi recorded his solutions, his work showed evidence of a systematic approach. He clearly was also able to keep in mind a focus on how many possibilities there were and that this required him as the task solver to make a note of this.

Five students appreciated the need to argue they had found all combinations. None stated how many they found and their argumentation skills were still developing. Madi, for example, found eight solutions and wrote “I think there are know (sic) more combinations”. Ada noted she “counted by twos because ten is even”, checked for duplicates and “there were only two”, and used her facts to ten. She claimed, “I know I have them all. I tried thinking and thinking and I couldn’t find anything more. I looked at the animals I looked [at the] legs and feet and added on numbers”. She recorded drawings with numerical equations to represent her findings. She thus fulfilled her teacher’s expectation that multiple representations would be used.

Data presented from this term 1 lesson (TLD1) clearly indicated that Muridi, Ada, and Madi were in a classroom where students were expected to ‘have a go’ and to try different approaches, in this case different representations—drawings, numerals, words, equations, to look for duplicates and find more than one solution to a problem. As has been shown these expectations were clearly discerned and students were seeing their role as doers of mathematics, expressers of mathematical thinking, and documenters of that thinking even if at this point in the school year they were still developing their skills to do so.

Given space restrictions, the remaining teacher lesson diaries (two per term for terms 2–4) will not be detailed in full. In each lesson diary there was evidence of students realising their role included explaining both task and solution. After making and drawing a “train” with his partner for the *One Change Train Task* (noted in TLD2a), Muridi recorded his reasoning as “it is a one change train, one was skinny and one was fat, one was big and one was short” as he described his train of alternating rhombuses using pattern block pieces. He and his partner were one of three pairs that recognised a simple train could be made using an ABAB pattern. When asked about task difficulty, he said the task was “easy because I used the same shape”. Ada articulated her approach clearly, “I am not changing anything except the colour—not the size, the length or anything except the colour”. Madi also described the task as easy clearly believing that identifying mistakes and revisiting the task were part of learning, “because I worked with a partner, and we started it with getting some blocks and made it into a train and figured out it wasn’t right and then made another one that was right”.

In her term 2 professional learning reflection, the teacher noted her intention for an increased focus on “continually encourage[ing] students to unpack what they already know and discuss/elaborate/convince peers of their thinking”. By term 3, the teacher had added additional expectations to those she expressed in TLD1 and specifically noted her own changed role. In her first lesson diary for term 3 (TLD3a), she was explicit about her classroom expectations progressing to “setting up community of learners (justifying, reasoning, questioning, learning from each other’s [questioning]—changed teacher role as facilitator)”. In addition to the other expectations, she had identified previously, she expected students to be articulating the way they solved the problem. In the second lesson diary (TLD3b) she reiterated, she expected students to be actively listening to, and to be open to, the thought processes of peers, articulation of their thinking—orally and written, and being able to write mathematical understandings.

Challenging other students was also part of the classroom culture. Reasoning cartoons were often used in demonstration lessons to support this, where challenging an unknown student (e.g., Grey TLD3b) supported subsequently challenging peers and explaining the thinking of others. During the *Caterpillar Task* (TLD2b), at one stage after making the Day 6 caterpillar, students were discussing the Day 5 caterpillar. They were asked to explain their thinking to a partner "to justify their reasoning". Muridi's partner, Hugo, proposed there were 11 triangles and 5 squares, but he was unable to explain his thinking. Muridi immediately challenged this thinking, demonstrating his preparedness to disagree, claiming, "It is not 11, it is 12."

In some tasks, students chose the level of challenge. In the *Caterpillar Task* (TLD2b), after considering the Day 6 caterpillar, Muridi chose the Day 11 caterpillar. The colouring of his drawing (shading triangles only on one side—see Table 2 row 3) suggests he recognised a doubling pattern as part of his reasoning, but he did not articulate this. Madi working with the Day 10 caterpillar was better able to explain how, if not why. She wrote, "The rule is <u>10 + 10</u> = <u>20</u> so when you add the <u>2</u> more all together there is 22 [triangles]". She also showed evidence of checking using an alternative approach (i.e., count all). Ada was able to articulate her reasoning including relating her rule to the day (15), "The rule is you double the amount of days the caterpillar is … and add two", and correctly identified the number of triangles required.

In the *Minibeasts Task* (TLD4a), 17 of the 23 students contributed to a discussion in response to *What do you think the task is about*? Madi suggested, "data on most popular out of lots of minibeasts". Muridi pointed out, "[we] need an axis so we could write about what kind of minibeasts so we could go around and see what is the most". Post-lesson Show-me questions to Muridi (and another boy), focused on telling type questions (e.g., What did you do? Tell/show me), sufficed to allow articulation of the way the problem was solved but elicited no evidence of reasoning although Muridi did use comparative language. For the third student, the teacher took on a facilitator role deftly asking questions that elicited reasoning and justification.

Whether in a discussion context with the whole class, talking to a partner, or responding to a task using concrete materials, diagrams, words or symbols, the students in this class, as evidenced by the experiences of Muridi, Ada, and Madi seemed to view *their role* in the mathematics classroom was to actively listen, tell different ways, clarify, reason and to convince others that their mathematical thinking was correct as expected by Wood (2002) and others (e.g., Hunter et al., 2020; Kilpatrick et al., 2001; McCluskey et al., 2016). This was true even when they found a task challenging and/or struggled to explain their thinking.

Concluding Remarks

In this paper, a qualitative study has reported on one classroom where the teacher articulated her role in developing a community of mathematical inquiry. The data indicated she saw her role had to change to facilitator rather than evaluator of student learning and the learners should be increasingly responsible for active participation in their learning. The expectations of Muridi and his classmates, both by the teacher and of each other, were as active listeners and explainers with responsibility for mathematical thinking and participation, although this was not always realised. Teachers can bring about change in students' mathematical thinking by increasing expectations of learners as active listeners and explainers. Students in such environments perceive themselves as having responsibility for participation and mathematical thinking. To sustain these expectations, teachers need to focus on asking *inquiry questions* (that require explainers to clarify their solution and provide reasons for their decisions and actions) that position learners as mathematical reasoners and *argumentation questions* (that disagree with, or challenge, the explainer) that position explainers as justifiers and defenders of their solution to all students. The type of question asked positions the explainers in the different roles (Wood, 2002). Questions requiring explainers only to tell answers and procedures do not encourage mathematical reasoning nor indicate expectations for this. However, "powerful and innovative pedagogies such as rich and robust dialogical interaction are challenging for teachers to develop in their classroom" (p. 296). The role of the teacher in asking questions of all students that require clarification, reasoning, and justification cannot be underestimated, nor the challenge for teachers in sustaining such practice. This study contributes to a greater understanding of the challenges for both teachers and students in describing and sustaining classroom expectations for mathematical thinking.

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