Changes in Year 11 Mathematics Students' Choices About the Use of a Computer Algebra System (CAS) to Solve Routine Problems

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This paper reports changes in the extent to which a group of Year 11 students used a Computer Algebra System (CAS), pen-and-paper (P&P), or a combination of both, when solving routine problems across seven months in four different topics. Comparing the frequency of CAS use across topics shows students made greater use of CAS in the topics of *Linear, quadratic, and cubic functions* and *Exponential and logarithmic functions* than in the topics of *Trigonometry and circular functions* and *Calculus*. These differences suggest that students either made different choices about the use of CAS in these topics or used CAS less frequently as they gained experience with CAS.

Students with access to a Computer Algebra System (CAS) need to make effective decisions about when to use CAS, pen-and-paper (P&P), or a combination of both, when solving mathematics problems (Pierce, 2001). These decisions could be based upon considerations such as their perceptions of the speed of CAS, the difficulty of the problem, and their P&P facility (e.g., Ball & Stacey, 2005). However, making effective choices about the use of CAS can be problematic for students who are new to working with CAS (Thomas et al., 2004). Despite this difficulty, few studies (e.g., Guin & Trouche, 1999; Orellana, 2016) have explored how students' choices about the use, or non-use, of CAS change as they gain experience.

For the students in this study, technology (i.e., a handheld CAS) was to be embedded into the teaching, learning, and assessment of mathematics (VCAA, 2015). Consequently, students needed to make decisions about when the use of CAS would be useful for supporting their learning of mathematics and solving of problems. This paper builds upon existing literature by exploring changes in students' use, or non-use, of CAS across seven months in one school year.

Literature Review

Artigue (2002) uses theory of instrumental genesis to explain how an artefact is transformed into an instrument through the dual processes of instrumentation and instrumentalisation. An artefact describes an object that can be used as a tool to complete tasks (e.g., a calculator, pencil, or algebraic symbol), while an instrument is "… a mixed entity, part artefact, part cognitive schemes" (p. 250). These cognitive schemes, formed through the instrumentation process, link an individuals' mathematical thinking and their gestures, where gestures describe the actions (i.e., functions used, buttons pressed) that are used to complete a given task (Trouche, 2005). The development of these schemes requires (i) technical understanding of how the technology can be used (e.g., knowing there is a *Solve* command that can solve equations); (ii) understanding of the affordances of the technology (e.g., CAS can perform routine calculations quickly and thus save time); and (iii) understanding of the constraints of the technology. Together, these understandings shape the use of an instrument.

The instrumental genesis of CAS has been described as a "time consuming and lengthy process" (Bukhove & Drijvers, 2010, p. 48), as students need to develop new cognitive schemes

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that allow them to complete tasks, while also considering how CAS should be used (Trouche, 2005). Through instrumentation, schemes are either developed personally or adopted from pre-existing social schemes, such as uses demonstrated by a teacher or peer (Artigue, 2002). While there can be a social element to instrumental genesis, the overall development of schemes and techniques is unique to an individual (e.g., van Dijke-Droogers et al., 2021). Consequently, different students may make different choices about whether to use CAS based on their cognitive schemes, knowledge of techniques, technical facility, and understanding of affordances and constraints of CAS.

When working with CAS, students can experience an "explosion of possible techniques" (Artigue, 2002, p. 260) that can be used to complete tasks due to the availability of a range of CAS techniques that can be added to existing P&P techniques. Where multiple techniques are available, students select one based on their evaluation of the pragmatic and epistemic value assigned to each technique (Artigue, 2002). In the early stages of instrumental genesis, some students tend to favour the use of CAS rather than P&P (Guin & Trouche, 1999). As students progress through instrumental genesis, they become more selective in their use of CAS and reintroduce P&P techniques (Guin & Trouche, 1999). The reintroduction of P&P techniques, and subsequent need to choose between CAS or P&P, requires judicious use of CAS which involves consideration of the effectiveness of CAS approaches for solving problems and making effective choices about the use of CAS based on these considerations (Pierce, 2001).

Several studies have explored students' CAS use from a range of perspectives. Analysis of self-reported frequency of CAS use suggests that students believe they frequently use CAS in class (Kissane et al., 2015; Orellana, 2016). Analysis of actual CAS use, through asking students to indicate where CAS has been used to perform a calculation, identified different ways that students used CAS when solving calculus problems (Thomas & Hong, 2005) and analysis of written responses to selected examination problems suggested that Year 12 students frequently used CAS to complete problems involving routine procedures (Ball, 2015). While these studies provide important insights into how students use CAS, existing studies were conducted in the context of a single unit of work (e.g., Thomas & Hong, 2005), with a single cohort of students who were experienced with CAS (e.g., Ball, 2015), or from different cohorts with different levels of experience (e.g., Orellana, 2016). These studies provide insight into how students use CAS in a particular topic, or how their frequency of CAS use changes with experience. This study builds on this literature by investigating whether students make different choices about the use of CAS when completing problems in different topics.

The research question for this study is: For Year 11 Mathematics students, how does their frequency of CAS use differ across the topics of: (i) *Linear, quadratic, and cubic functions*; (ii) *Exponential and logarithmic functions*; (iii) *Trigonometry and circular functions;* and (iv) *Calculus*? In this paper, "CAS" refers only to the symbolic functionalities of a CAS calculator and "P&P" refers to the completion of a calculation using mental or pen-and-paper procedures.

Methodology and Research Design

The participants for this study were the students ($n = 13$) of a single Year 11 Mathematical Methods (VCAA, 2015) class in a suburban school in Victoria, Australia. The school was selected for the larger study as students were not required to use CAS prior to Year 11. This provided a suitable context for a larger study which investigated changes in CAS use in different topics (reported here) and as students gained experience with CAS (see Cameron, 2023). All students were novice CAS users with eight not using CAS prior to Year 11, and the remaining five making limited use of CAS prior to Year 11. Data were collected through four worksheets, completed across a period of approximately seven months in one school year, to determine how CAS use differed with experience and across the four major topics studied (see Table 1). All students used a Texas Instruments (TI) Nspire CX series CAS calculator. The use of CAS was supported by the teacher who had extensive experience in teaching mathematics with CAS.

Table 1

Worksheet Topics and Timing of Data Collection

Worksheet	Topic	Date of data collection
1	Linear, quadratic, and cubic functions	Week 5 of Term 2
2	Exponential and logarithmic functions	Week 5 of Term 3
3	Trigonometry and circular functions	Week 2 of Term 4
4	Calculus	Week 6 of Term 4

Development of Worksheets

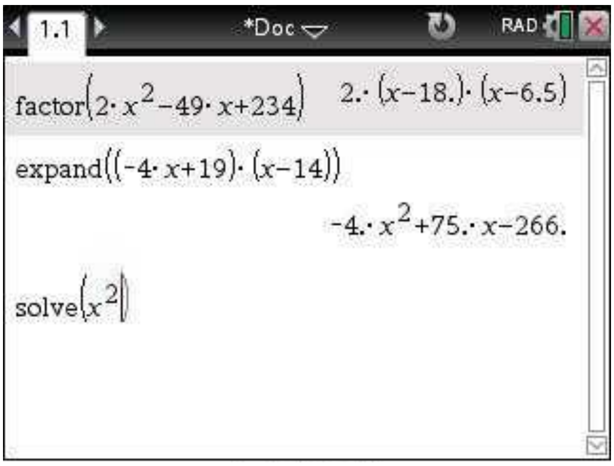
Four worksheets were developed (i.e., one per topic). The lead author developed worksheet items based on a review of curriculum documents and student textbooks; these items were reviewed by co-authors and the teacher. Items were predominantly routine and reflected the content that students had been learning in class. Worksheets included both problems where students could choose between CAS or P&P and others where it was expected that students could only solve the problem with CAS (i.e., where problems were outside the expected pen-and-paper range of students). The latter type of problems enabled investigation of the types of features of CAS used by students when solving problems within or outside the anticipated range of their P&P skills; this analysis is reported in Cameron (2023). Students completed the worksheets, under test conditions, at the conclusion of each topic. Students could choose to use CAS (or not) when completing items on each worksheet.

Collection of CAS Screenshots

While students were completing the worksheets, the TI Navigator system was used to record screenshots of the display of each student's CAS calculator (see Figure 1). The use of the Navigator system required the provision of a TI Nspire CX CAS calculator on each student's desk as the researcher needed to setup the Navigator system prior to students commencing the worksheet. While the provision of a calculator on the desk may have prompted CAS use, overall, this method was non-intrusive as it did not interfere with students' usual ways of working with CAS. Students were asked not to delete or clear any calculations so the researcher could download and review their CAS use following completion of the worksheet in the case where the Navigator system failed (as occurred when collecting data for Worksheets 2 and 3).

Figure 1

A Screenshot of a Student's CAS Display Collected With the Navigator System



Analysis of Worksheet and Screenshot Data

In the initial phase of analysis, written responses recorded on the worksheets and data collected from screenshots were analysed independently. Each student's written response for each worksheet item was analysed using Ball's (2015) indicators of CAS use. The indicators enabled CAS use to be inferred based on the appearance or absence of specific features in the written response (e.g., the appearance of CAS syntax, the absence of anticipated P&P working). This provided initial categorisation for each solution (Table 2). CAS screenshots were then reviewed for each student to identify evidence of a calculation being attempted with CAS (e.g., the use of CAS to solve in Figure 1). Following these independent analyses, evidence of CAS use in written responses and screenshots were compared for each student and for each problem to code the student's approach to the problem as one of five calculation methods (see Table 2). Figure 2 provides an example of this analysis. This comparison provided a form of methods triangulation which serves to support the credibility and trustworthiness of the findings reported here (Hastings, 2014). Following coding of each students' approach to each problem on each worksheet, the frequency, and percentage for each calculation method for all students on each worksheet were calculated; this enabled differences in use or non-use of CAS to be identified.

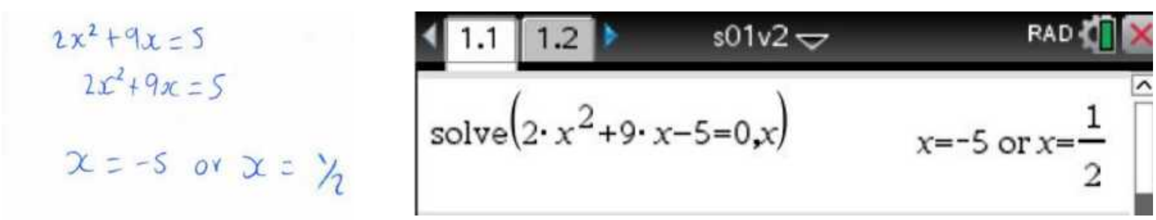
Table 2

Description of Calculation Methods

Calculation method	Description
CAS only	A written response was/was not provided There are indicators of CAS use in the written response and/or screenshots provide evidence of CAS use No intermediate P&P working was recorded
CAS with P&P	A written response was provided There are indicators of CAS use in the written response and/or screenshots provide evidence of CAS use Intermediate P&P working was recorded
P&P	A written response was provided There are no indicators of CAS use in the written response Screenshots do not provide evidence of CAS use Intermediate P&P working was recorded
Graphical	A written response was/was not provided There are no indicators of CAS use in the written response Screenshots provide evidence that graphical functionalities were used
No data	A written response was not provided Screenshots do not provide evidence of CAS use

Figure 2

An example of 'CAS only'. Although a Written Response is Provided, the Absence of Intermediate Working Suggests CAS was Used; This was Confirmed With the CAS Screenshot



Results and Discussion

Table 3 provides the frequency and percentage of the calculation methods evident in students' approaches to the problems on each worksheet. In the first worksheet, 190 instances of *CAS only* were identified, and this is 56% of the total 338. Combining the top two rows for Worksheet 1 gives a total of 75% for *CAS [Total],* as shown in the last row. Comparing the percentage of CAS [Total] across the four worksheets provides a broad understanding of how students' CAS use changed across the four worksheets. Despite the different topics, the frequency of CAS use was similar for problems from the topics of Linear, quadratic, and cubic functions and Logarithmic and exponential functions (75% cf. 73%). In contrast, CAS was used less frequently when completing problems from Trigonometry and circular functions and Calculus (19%, 27% respectively). The decreased percentage of CAS [Total] in the topics of Trigonometry and circular functions and Calculus is explained by an increase in the frequency of P&P only on these worksheets compared to the other two topics. Overall, this broad analysis suggests students less frequently used CAS as they gained experience, however, as discussed below, this change may be a result of the different topics. When comparing the frequency of calculation methods across worksheets, it is important to note the increased percentage of No data for Trigonometry and circular functions and Calculus. Many of these increases are attributable to student absences (rather than students not attempting problems) as two students did not complete Worksheet 3, and one did not complete Worksheet 4, so all calculation methods in these topics are underreported. Further, there was an unavoidable delay between students studying Trigonometry and circular functions and completing the worksheet; this delay may have resulted in an increased incidence of students not completing the more challenging problems due to a loss of proficiency.

Table 3

Summary of Calculation Methods for Four Worksheets

Calculation method	1. Linear, quadratic, and cubic functions ($n_1 = 338$)	2. Logarithmic and exponential functions ($n_2 = 182$)	3. Trigonometry and circular functions ($n_3 = 260$)	4. Calculus ($n_4 = 195$)
CAS only	190 (56%)	101 (55%)	50 (19%)	19 (10%)
CAS with P&P	64 (19%)	33 (18%)	0 (0%)	33 (17%)
P&P only	53 (16%)	41 (23%)	116 (45%)	105 (54%)
Graphical	0 (0%)	4 (2%)	2 (1%)	1 (0%)
No data	31 (9%)	3 (2%)	92 (35%)	38 (19%)
Total	338 (100%)	182 (100%)	260 (100%)	195 (100%)
CAS [Total]	254 (75%)	134 (73%)	50 (19%)	52 (27%)

Note. n is calculated by multiplying the number of items on a worksheet by 13. *CAS [Total]* is the sum of *CAS only* and *CAS with P&P*. Shading indicates the modal calculation method.

There are two different perspectives which may explain the variation in frequency of CAS use evident across the four worksheets, either (i) the mathematical content of each worksheet, or (ii) increased experience with CAS.

Potential Influence of the Mathematical Content of Each Worksheet

Differences in the frequency of CAS use across the four worksheets suggested that the topics Linear, quadratic, and cubic functions and Exponential and logarithmic functions (i.e., Worksheets 1 and 2) may have had more opportunities to use CAS than Trigonometry and circular functions and Calculus (i.e., Worksheets 3 and 4). All worksheets were designed to allow students to choose between CAS and P&P for all items, so the reduced frequency of CAS use on Worksheets 3 and 4 did not stem from the design of the worksheets.

A teacher's choices about the use of CAS or P&P, including the bounds placed around student CAS use, can impact student CAS use (Kendal & Stacey, 2001). Therefore, if the teacher made different choices about how they and the students should use CAS in different topics, it could be expected that students would make different choices about how and when to use CAS on each of the four worksheets. Although data were not collected from the teacher, students described the teacher demonstrating CAS features and supporting CAS use throughout the study (Cameron, 2023), so it is unlikely that different choices made by the teacher caused the differences observed on the worksheets.

Each of the topics studied by students required the learning and use of a range of different CAS commands and syntax. Learning how to use CAS commands and syntax can present barriers that impact students' CAS use. For example, Meagher (2012) reported students who avoided the use of CAS due to anxiety caused by difficulty in navigating and using CAS commands. Different topics may present different difficulties for students, with Pierce (2001) reporting that university students more frequently experienced difficulties with CAS when studying trigonometry than for other topics. Considering Meagher's and Pierce's findings, it is possible that the students in this study found the use of CAS more difficult in the topics of Trigonometry and circular function and Calculus (two topics that were new to students), and thus used CAS less frequently on these worksheets to avoid these difficulties.

Cameron et al. (2023) reported the students' beliefs about useful features of CAS (as part of a larger study). These beliefs reflected their overall experiences of working with CAS as a tool for the everyday learning of mathematics and many students believed that CAS was useful in mathematics across the study. Approximately half of the students believed that it was easier to use CAS than P&P to solve problems (5 of 11 at the start of the study cf. 7 of 11 at the end), so it is unlikely that students experienced greater difficulties when completing problems with CAS in Calculus compared to Linear, quadratic, and cubic functions. Overall, this suggests that the mathematical content of the worksheets did not influence students to make different choices about the use of CAS when solving problems from different topics.

Potential Influence of Experience on Frequency of CAS Use

An alternative explanation for the decreased frequency of CAS use on Worksheets 3 and 4 compared to Worksheets 1 and 2 is that students less frequently used CAS as they gained experience with CAS. It was expected that the P&P facility of these students would increase throughout the study as the development of P&P skills features explicitly in the outcomes of MM, with several key skills for Units 1 and 2 specifying the use of 'by hand' approaches (VCAA, 2015). Hence, a decrease in the frequency of CAS use could be expected as students develop facility with P&P skills. We also anticipated an increase in CAS facility throughout the study as students learnt more about CAS and how it could be used. Through the process of instrumental genesis, students are expected to become more selective in their choice of techniques (Artigue, 2002). As students gain experience with CAS and progress through the instrumentation process, they reintroduce P&P techniques despite displaying a propensity for working with CAS in the initial phase of the instrumentation process (Guin & Trouche, 1999). The reintroduction of P&P techniques may result in a decrease in the frequency of CAS use. The differences reported here suggest that, for many students, this change occurred approximately six months after commencing learning mathematics with CAS.

Students' Limited Use of CAS with P&P

Another trend event in the data was the comparatively infrequent use of CAS with P&P when compared to CAS only or P&P only in all topics except Calculus. The CAS with P&P

calculation method involved the use of both CAS and P&P when completing a problem (see Figure 2). These approaches are reported to be less common than the use of only CAS or P&P methods (Thomas & Hong, 2005; Weigand & Weller, 2001). Results from Worksheets 2 and 3 are consistent with the findings of Thomas and Hong (2005) and Weigand and Weller (2001), as CAS with P&P was evident in a smaller percentage of approaches than CAS only and P&P only (18% cf. 55% and 23% for Worksheet 2; 0% cf. 19% and 45% for Worksheet 3). However, results from Worksheets 1 and 4 provide a contrast with both CAS with P&P and P&P only evident in a similar percentage of approaches on Worksheet 1 (19% cf. 16%), and with CAS with P&P evident in a greater percentage of approaches than CAS only on Worksheet 4 (17% cf. 10%). It is important to note that few items, except for some on Worksheets 1 and 4, required the application of more than one mathematical procedure, so it was difficult for students to concurrently use CAS and P&P when completing a problem. Overall, when excluding Worksheet 3, CAS with P&P accounted for approximately one-fifth of approaches to problems on each worksheet, which contrasts with Weigand and Weller's finding that use of an "integrated working style" is "rare" (p. 99). In many cases, responses categorised as CAS with P&P reflect students completing a problem with P&P and using CAS to check their answer (Cameron, 2023), so CAS and P&P were used independently as students completed a problem.

Conclusion

In summary, *CAS only* was the most frequently identified method in students' responses to problems on Linear, quadratic and cubic functions and Exponential and logarithmic functions, while P&P only was the most frequently identified method for Trigonometry and circular functions and Calculus. This change corresponded with a reduction in the frequency of CAS use (as evidenced by a reduction in the percentage of CAS [Total] on Worksheets 3 and 4 when compared to Worksheets 1 and 2) and suggested a change from preferencing the use of CAS to the use of P&P. The most likely explanation for the differences in the frequency with which CAS was used on the worksheets was that students were becoming more selective in their CAS use as they progressed through the instrumentation process rather than due to the topic being studied. Guin and Trouche (1999) reported that students in the first phase of instrumental genesis often demonstrate a dependence on CAS, prior to the second phase where P&P methods are reintroduced and there is less reliance on CAS. In contrast to what was expected, students did not demonstrate a prolonged dependence on CAS, with the use of P&P calculation methods being more common than CAS within approximately six months of commencing to learn mathematics with CAS.

Prior studies have reported either increases in frequency of CAS use with experience (Weigand & Bichler, 2010) or no relationship between frequency of CAS use and experience (Orellana, 2016). In contrast, this study found a decrease in CAS use as students gained experience. Although there might be a concern that students working with CAS defer to CAS for solving all problems, rather than make choices about use of CAS or P&P, the results presented here show that for the small sample of 11 students in this study this concern would be unwarranted. Further research could investigate whether the results presented here, where use of CAS decreased as students developed P&P facility, was a wider phenomenon and applicable to other cohorts.

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