Australian Junior Secondary Students' Approaches to Solving Ratio Problems Prior to Formal Instruction and Their Misconceptions

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Progressing from additive to multiplicative thinking is a key outcome of school mathematics, making ratios an essential topic of study in junior secondary. In this study, 15 Australian Year 8 students were administered a ratio test followed by semi-structured interviews to explore their conceptions of ratio prior to formal instruction. In this paper, students' responses to one of the ratio questions are analysed in detail. Analysis of incorrect responses was conducted using a modified version of Radatz's (1979) framework. Analysis of correct responses revealed that some students worked proficiently with ratio without formal instruction.

Ratio is termed a "big idea" in mathematics, and appreciation of the multiplicative relationship between quantities is fundamental to developing proportional reasoning (Siemon et al., 2012). The capacity to reason proportionally and work with ratios is a key outcome of high school mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2022). Ratios are an important topic of study in the secondary years since they unify the content strands of number, algebra, measurement, geometry, and data analysis and probability (Siemon, 2013). Despite the importance of ratios, researchers have considered the topic "the most protracted in terms of development, the most difficult to teach, the most mathematically complex, [and] the most cognitively challenging" (Lamon, 2007, p. 629). Analysis of student performance on the 2019 Trends in International Mathematics and Science Study (TIMSS) test confirmed students' difficulties with ratios. Only 35% of students internationally and 40% of Australian students were able to solve a ratio problem involving the enlargement of a figure (Mullis et al., 2020).

Given the importance of ratios and the noted difficulties that students experience when learning ratios, this study aimed to investigate Australian students' approaches to solving ratio problems prior to the formal teaching of the topic. Analysing student responses before explicit instruction allowed for the observation of students' natural approaches to ratio problems. This provides teachers with insight into the challenges and misconceptions students may experience when first introduced to ratios. The research question was "What approaches do junior secondary students use when solving ratio problems prior to instruction, and what misconceptions do they hold?"

Ratio Misconceptions

Although ratios have been identified as an area of challenge for both students and teachers, research into students' misconceptions in this area is limited. International research on ratios has mostly focused on misconceptions and difficulties held by primary school students and those with mathematical difficulties (e.g., Dougherty et al., 2016). One study analysed secondary students' misconceptions and difficulties with ratios in South Africa (Mahlabela, 2012), but similar research has not yet been conducted in Australia. On a national level, research has focused on diagnostic approaches to assess primary school students' proportional reasoning

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(Hilton et al., 2013) and interventionist approaches to promote students' emerging proportional reasoning (Fielding-Wells et al., 2014). Siemon and colleagues have also extensively researched the development of students' multiplicative thinking, but their work was not focused on misconceptions (Siemon et al., 2006; Siemon, 2019). Key themes pertaining to students' misconceptions and difficulties with ratios included difficulties transitioning from additive to multiplicative thinking, and confusion between fraction and ratio representations when analysing the findings from this research.

Proficiency with ratios is dependent on students' ability to think multiplicatively as proportional reasoning is the most sophisticated form of multiplicative thinking (Callingham & Siemon, 2021). During the transition from primary to high school, students' progression from additive to multiplicative thinking is one of the major barriers to learning mathematics, including the topic of ratios (Siemon, 2019). Research has shown that 30–55% of Year 8 students do not think multiplicatively, and differences between students' overall mathematics achievement can be attributed to an inadequate understanding of multiplication, division, fractions, decimals, and proportion (Siemon et al., 2006).

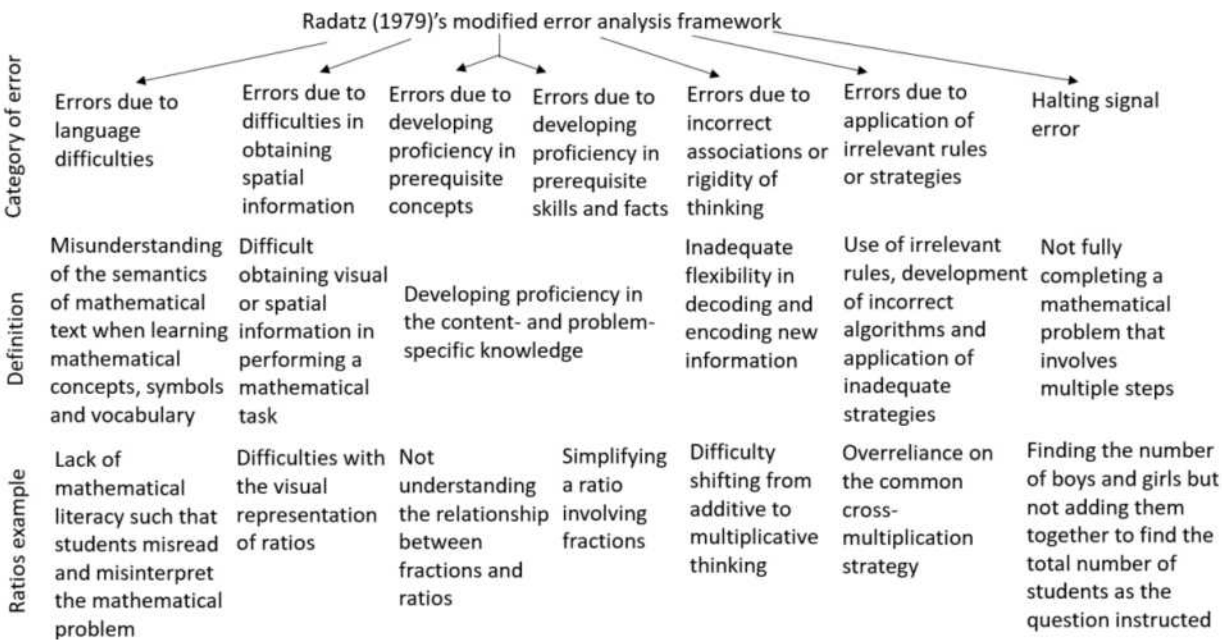
Misconceptions surrounding ratios have also been attributed to difficulties with prerequisite knowledge related to fractions (Dougherty et al., 2016). Students can have difficulties differentiating the part-part relationship of ratios from the part-whole relationship of fractions (Clark et al., 2003). Moseley (2005) also found that students' conceptual understanding of ratios is lacking as they focus on the numbers rather than the relations the numbers represent, as ratios are a multiplicative comparison.

Theoretical Framework

Categories of ratio errors are currently not well-established in the literature given the limited research on students' misconceptions and difficulties with ratios. This study drew from Radatz's (1979) error analysis framework that described five error categories: errors due to language difficulties; errors due to difficulties in obtaining spatial information; errors due to developing proficiency in prerequisite skills, facts, and concepts; errors due to incorrect associations or rigidity of thinking; and errors due to application of irrelevant rules or strategies.

Figure 1

Modified Version of Radatz's (1979) Error Analysis Framework



This framework uses an information-processing approach to classify errors because the "mechanisms used in obtaining, processing, retaining and reproducing the information in mathematical tasks" are examined (Radatz, 1979, p. 164). This study modified the original Radatz framework (Figure 1) through small language changes (removing reference to 'deficient' skills and knowledge to encourage a move away from a deficit view), adding the halting signal error (a partially complete response: Brodie & Bergie, 2010), and splitting the error category of 'errors due to the developing proficiency in prerequisite skills, facts, and concepts' into two for clarity (separating concepts and skills/facts).

Methodology

Participants were 15 Year 8 students from one class in an Australian independent school. The school's demographic was representative of a higher socioeconomic background, with a school ICSEA value of 1087 given the 1000 average. The study was conducted prior to the class formally learning ratios. A mixed methods approach was used, drawing on both qualitative and quantitative analysis of student error types on an 8-item ratio test. The test was developed from an analysis of the local mathematics syllabus (Stage 4 New South Wales Syllabus) which allowed for the identification of key ratio concepts and skills covered in Year 8. The test was completed in 20 minutes under exam conditions without calculators. In this paper, the findings from one of the test items are reported as they allow for a deep analysis of all students' attempts. Following the test, semi-structured interviews were conducted to clarify and confirm the researcher's interpretation of students' test responses. Interviews enhanced the validity of findings, since the different datasets elaborated, enhanced, and clarified each other (Greene et al., 1989). When coding students' incorrect responses, error analysis was conducted. All incorrect responses were collaboratively analysed and coded using the modified version of Radatz's framework (Figure 1).

Findings

This paper focuses on analysing students' correct and incorrect responses to one test question, "Can 10 people be divided into two groups with a ratio of 1:2? Explain your answer." Overall, the 15 students reported a 26.7% success rate on answering this item; 10 students answered incorrectly, and one student did not attempt the question. Students' challenges in answering the test question were expected since they had not yet been taught ratios. There was a diverse range of correct and incorrect answers provided by students.

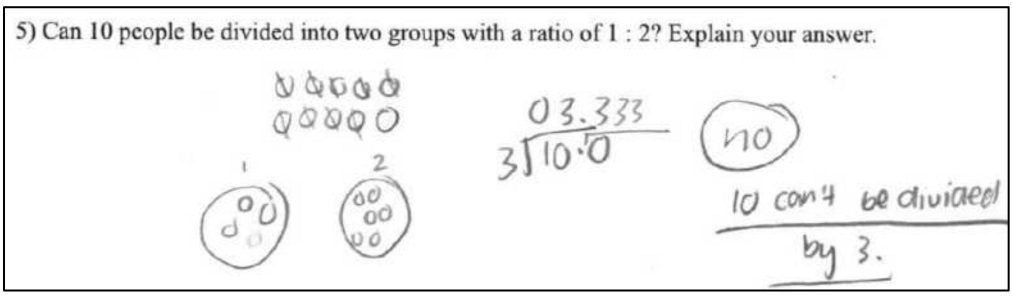
Correct Responses

Students' correct responses reflected varying levels of developing proficiency with ratios, although the semi-structured interviews after the ratios test revealed that some students still held some misconceptions on the topic.

In Figure 2, the student added the antecedent (i.e. 1) and consequent (i.e. 2) of the ratio together to reach 3.

Figure 2

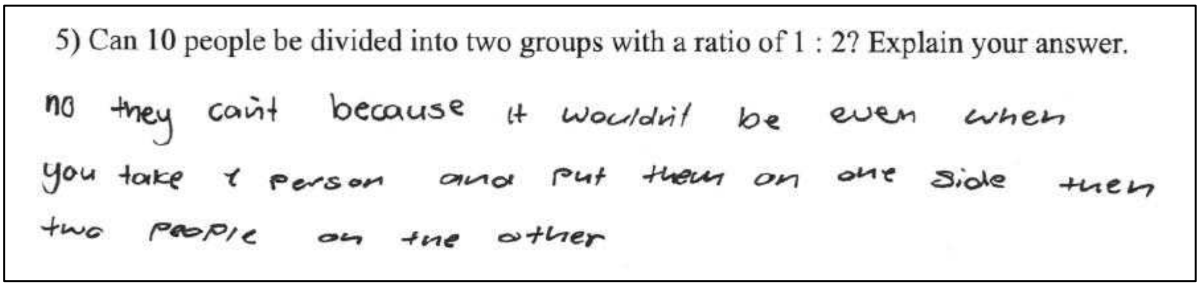
Example of Student's Correct Response Using Concepts of Division, Factors, and Multiples



To check if 10 people can be divided into two groups with a ratio of 1:2, this sum of the ratio parts (i.e., 3) must be a factor of the total number of people (i.e., 10), or conversely, the total number of people must be a multiple of the sum. The student also provided an accompanying diagram showing the first and second group having three and six people respectively, and their last uncrossed circle of 10 suggests the leftover person. This strategy for solving the question was also encapsulated in the working out of the student in Figure 3. Both students drew on key prerequisite knowledge of division and the concept of equal groups, which are foundational concepts of ratios.

Figure 3

Example of Student's Correct Response Using an Evenness Argument

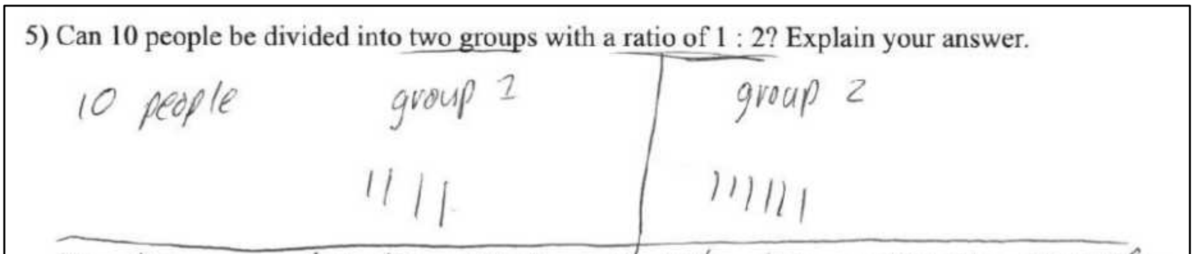

5) Can 10 people be divided into two groups with a ratio of 1 : 2? Explain your answer.

no they can't because it wouldn't be even when you take 1 person and put them on one side then two people on the other

Two students provided an alternative explanation that demonstrated a developing conceptual understanding of ratios. Instead of providing an explanation involving visuals or relating the problem to division, factors and multiples, their answer was that the group division would not be possible because the two groups would not be even in number. Although even groups were not a criterion of the question, these students identified that when ten is grouped in a ratio of 1:2, there would be one person leftover who has not joined a group. The response from the first of these two students, shown in Figure 4, shows emerging proficiency with ratios but some misconceptions are still held. Although their explanation aligns with the student's explanation in Figure 3, their diagram shows that they have not recognised that 4:6 = 2:3 is not equal to the desired 1:2 ratio.

Figure 4

Example of Student's Correct Response Using an Evenness Argument but With Misconceptions


5) Can 10 people be divided into two groups with a ratio of 1 : 2? Explain your answer.

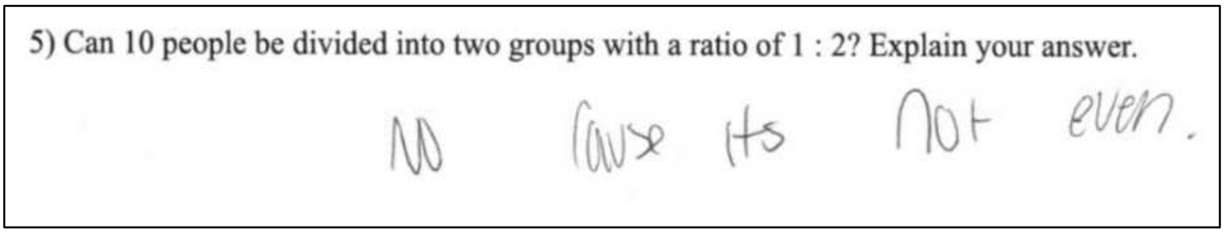
10 people | group 1 | group 2

Yes they can but the groups will not be even. The reason for this is everytime one is added to group 1 another two is added for group 2

The second student similarly identified that the groups would not be even (Figure 5), and when questioned on what they meant by 'even', they stated that "if there's one person on one side, for example, and there are two on the other", the number of people in each group would not be even. When the student was probed further, their misconceptions on ratios were revealed. After asking them their next steps in their working to ensure no halting signal error "Would you have added more people on either side or would you have left it as one person on one side and two people on the other?", the student responded "I would add an extra one to the other side" to make it even. Their thought process behind this was that "[the question] says 'to be divided into two groups' so it has to probably be even".

Figure 5

Example of Student's Correct Response but Their Interview Reveals Misconceptions

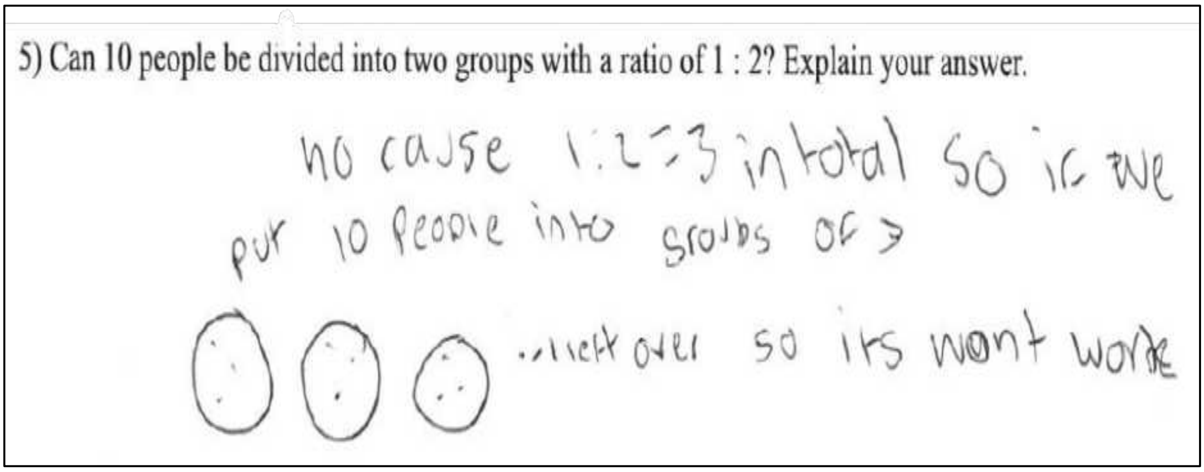


Incorrect Responses

While some students correctly solved the ratio problems, some students were not as successful. Like the student in Figure 2, the student response in Figure 6 demonstrated that they added the ratio parts together to help them solve this problem.

Figure 6

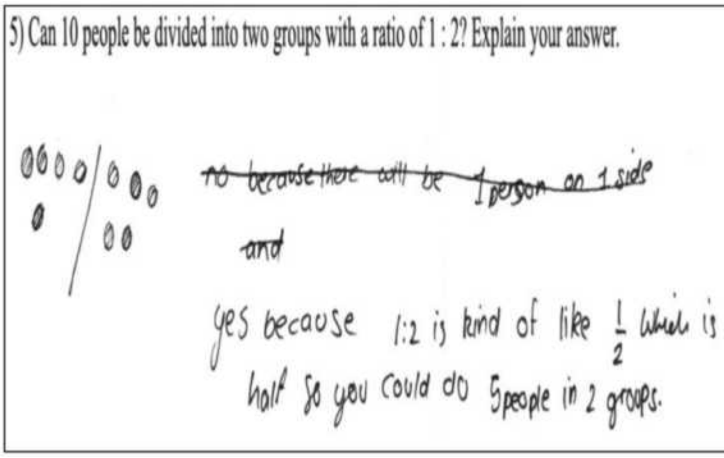
Example of Student's Correct Response but Their Working Reveals Misconceptions



However, instead of using this sum of three to conceptualise putting one person in one group and two people in the other group every round, they misinterpreted this as putting 10 people into groups of three. They were, however, able to identify that one person would be leftover so this division into three groups would be unsuccessful. Although the working here does not reflect the expected understanding of splitting a quantity into a ratio, it does indicate some developing conceptual understanding of ratios, based on the idea of division into equal groups. The root error cause was coded to be the developing proficiency in concepts and language difficulties, leading to procedural errors.

Figure 7

Example of a Student Misinterpreting 1:2 to Mean $\frac{1}{2}$



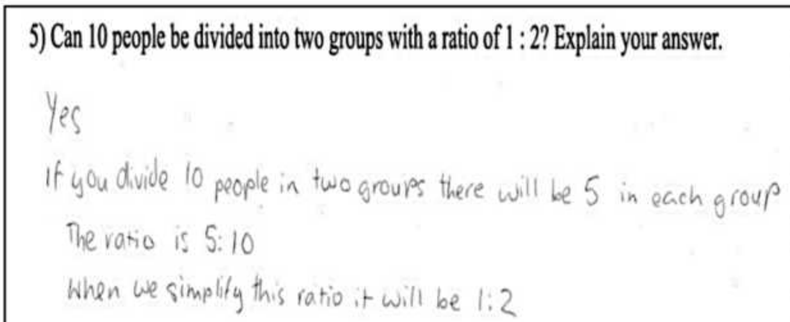
Error categorisation: Developing proficiency in ratio concepts, seen in students incorrectly associating ratios with fractions (occurrence: 5 out of 15 students).
Instead of interpreting 1:2 as 1 and 2 parts out of 3, five students interpreted it to mean $\frac{1}{2}$ or half, demonstrating difficulty with mathematical symbols (language difficulty). Hence, students split the group of 10 into two groups of 5, which would incorrectly form a ratio of 1:1.

When given the ratio 1:2, it is speculated that some students misinterpreted the colon sign of ratios to mean the same as the vinculum sign of fractions. This is because students lacked conceptual understanding of what a ratio is, which led to the language difficulty of misinterpreting the colon as a vinculum due to incorrect associations with fractions. An example of this misconception is shown in the student response in Figure 7.

Students also demonstrated conceptual difficulties with working with a fixed number of total people (i.e., 10) despite correctly identifying equivalent ratios (Figure 8).

Figure 8

Example of Student's Developing Proficiency in Concepts



Error categorisation: Developing proficiency in concepts leading to use of incorrect procedures (occurrence: 1 out of 15 students)
One student thought that 1:2 is equivalent to 5:10, which is correct. However, the new equivalent ratio involves 15 people instead of the required 10.

Students' responses also demonstrated that a flow-on error resulting from developing proficiency in concepts was the halting signal error, as students only partially completed the question (Figure 9). This can be attributed to students not having formally learnt ratios, although it reveals their approaches to solving ratio problems.

Figure 9

Examples of Two Students' Halting Signal Errors

Error categorisation: Developing proficiency in concepts, resulting in a halting signal error (occurrence: 2 out of 15 students)	
5) Can 10 people be divided into two groups with a ratio of 1 : 2? Explain your answer. 1:2 1:2 No because the ratio shows only six people	The student knew that 1:2 meant 3 people and how they should check whether 3 is a factor of 10. However, they did not continue up to the highest multiple of 9 and instead stopped at 6. It is possible that the student stopped at 6 because the question says, "two groups".
5) Can 10 people be divided into two groups with a ratio of 1 : 2? Explain your answer. no, because there would be 7 people left over.	The student only considered the first group of three people. This suggests that the student did not conceptually understand what the question was asking.

Discussion and Conclusion

This study aimed to investigate Australian students' approaches to solving ratio problems prior to the formal teaching of the topic and address the research question. For the analysed question, it was found that the main root error cause was developing proficiency in ratio concepts, leading to other error types including incorrect associations with fractions, application of irrelevant procedures and halting signal errors. This was not unexpected as the students had not learnt ratios yet. What was particularly interesting for teachers to observe was that several students successfully reasoned through the ratio problem despite not having learnt the topic. Students' prior experience with fractions (Dougherty et al., 2016) and emerging multiplicative thinking (Siemon et al., 2006) set the foundation for success when learning ratios. The implication of this finding is that it is beneficial for teachers to help students connect ratios with this prerequisite knowledge, rather than viewing it as a distinct and entirely new topic when first introducing it. Moreover, since students may resort to fraction strategies and concepts when first approaching ratios, another teaching recommendation is to clarify the similarities and differences between fractions and ratios in terms of vocabulary, concepts, and procedures.

The findings from this study have implications for research investigating student errors in mathematics. Through the error analysis conducted in this study, the modified version of Radatz's (1979) framework was shown to be a viable and useful way of coding student errors. Potential directions for future research include ascertaining whether the same errors exist before and after formal instruction on ratios, and testing whether Radatz's (1979) error analysis framework can be used to hierarchically classify errors for other mathematical topics.

Although it was anticipated that one class of students would provide adequate data to gain insight into students' understanding of ratios and corresponding errors, generalisability may be limited beyond this study because of its dependence on only one class of students. Despite this, the demographics of the school and participants provide context for the findings and support the reader to make judgements concerning the generalisability of the study to other contexts.

Acknowledgments

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