Developing Complex Unfamiliar Mathematics Questions: A Perspective

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This paper reports on the development of complex unfamiliar mathematics questions using conventional mathematics questions. The autoethnographic research data included retrospective reflections of two educators and review of literature. Data analysis resulted in a three-phase development process: identifying conventional questions and subject matter to be modified to enhance levels of understandings and skills, modifying conventional questions and subject matter to complex unfamiliar questions, and identifying enhanced understandings and skills in modified questions. The paper then discusses the educators' insights developing complex unfamiliar mathematics questions for the practice of teaching and learning.

In Australia, the focus on developing students' mathematics proficiencies of understanding, fluency, reasoning, and problem-solving skills is meant to help them develop the capacity to solve complex unfamiliar problems (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2023). Likewise, in states like Queensland and South Australia students are expected to engage with complex unfamiliar or non-routine problems as part of the mathematics assessments. Particularly in Queensland, it is expected that about 20% of the marks in any general mathematics subject examination at senior secondary level should be complex unfamiliar. The subjects are General mathematics, Mathematical Methods and Specialist Mathematics and they provide a pathway for different tertiary courses. However consistent curriculum and assessment reports (Queensland Curriculum and Assessment Authority [QCAA], 2021; 2022; 2023; 2024; South Australia Council of Education [SACE], 2021) have identified lack of consistency by senior secondary mathematics teachers in Queensland and South Australia when it comes to developing complex unfamiliar questions for internal examinations. The reports noted that teachers still need to improve in assessing students under this level of difficulty. As a result, there has been limited use of complex unfamiliar problems in mathematics teaching and learning (Lee & Kim, 2005; Nguyen et al., 2020). Senior secondary mathematics teachers in Queensland are expected to include complex unfamiliar problems when they set internal assessments that are moderated by Queensland Curriculum and Assessment Authority (QCAA). Thus, providing resources that can help develop understanding and consistency in developing complex unfamiliar questions is central in promoting quality mathematics teaching and learning. This paper employed an auto-ethnographic method to report on how complex unfamiliar mathematics questions can be developed using conventional (routine) mathematical questions.

Complex Unfamiliar Mathematics Questions

There is a general agreement that complex unfamiliar or non-routine mathematics problems require deeper understanding of concepts, analytical thinking, interpreting and creativity beyond standard algorithm strategies as there is no clear path to the solution (Asman & Markovits, 2009; Mullis et al., 2009; Polya, 1966; QCAA, 2018; Schoenfeld, 1992). Complex unfamiliar or non-routine questions can take different definitions and forms depending on the nature of how they are presented and are solved. Some researchers (see for example Robinson, 2016) view non-routine problems as open-ended and involving multiple solutions. Similarly, other researchers (e.g., Asman & Markovits, 2009; Kloosterman, 1992) identify complex unfamiliar problems as having a unique solution but the procedures to solve and the concept

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the problem is derived from are not readily available. Importantly, "a non-routine problem appears when an individual encounters a given situation, intends to reach a required situation, but does not know a direct way of accessing or fulfilling his or her goal" (Elia et al., 2009, p. 606). Similarly, complex unfamiliar problems require students to demonstrate knowledge and understanding of mathematics and application of skills in a situation where:

- Relationships and interactions have several elements, such that connections are made with subject matter within and/or across the domains of mathematics;
- All the information to solve the problem is not immediately identifiable; that is, the required procedure is not clear from the way the problem is posed, and in a context in which students have had limited prior experience (QCAA, 2018 p. 44).

Student engagement with complex unfamiliar or non-routine problems in mathematics is said to enhance their development of mathematical knowledge (see Russo, 2019; Sullivan et al., 2015). Complex unfamiliar problems are credited for the promotion of strategic thinking capacity, integration of concepts and motivating students towards learning (Lampert, 2001). Calls to include complex unfamiliar questions in mathematics assessments have been going for a long-time (see Clarke, 2011; Ergen, 2020). It is fundamentally important that any mathematics activity should provide students with the opportunity to engage with content at different levels of sophistication (Clarke, 2011). Clarke went further to posit that mathematics curricula across the world is now advocating for inclusion of non-routine (complex unfamiliar) mathematics problems as a key goal of school mathematics (Clarke, 2011).

According to QCAA (2018), complex unfamiliar questions that require more levels of cognitive skills should not be equated with elaborate problem-solving tasks and modelling questions only. A single-answer, conventional question such as an example from Goos (2014): Find the equation of the line passing through the points (2,1) and (1,3); can be adapted to a more open-ended question such as: Write the equations of at least five lines passing through the point (2,1). This revised question targets the identical subject matter and assesses more and advanced cognitive understanding and skills. This means that, development of complex unfamiliar questions can start with: (1) identifying conventional mathematical questions and subject matter that can be modified to enhance levels of cognitive understandings and skills, (2) modifying the conventional questions and subject matter to complex unfamiliar questions, and (3) identifying the diverse and enhanced cognitive understandings and skills in the modified questions, leading to the development of assessment rubrics or guides to making judgements. This study is guided by the process of modifying conventional questions to complex unfamiliar questions as highlighted above by QCAA (2018). The paper reports on two educators' perspectives on how complex unfamiliar questions can be developed using conventional questions.

Method

The paper employed an autoethnographic approach to examine perspectives of two mathematics educators on how complex unfamiliar mathematics questions can be developed using conventional mathematics questions. According to Adams and colleagues (2017) autoethnography is a research method that uses personal experience ("auto") to describe and interpret ("graphy") cultural texts, experiences, beliefs, and practices ("ethno"). Chang and colleagues (2016) highlight that some auto ethnographers focus more on self, while others adopt a more analytical stance, focused on understanding and interpreting events and experiences involving self. This investigation takes the form of an analytical approach to autoethnography. The two educators met fortnightly for an hour over two school terms to articulate how complex unfamiliar mathematics questions can be developed using conventional mathematics questions. The meetings were reflective interpretations of experience, practice, and review of literature, which resulted in a three-phase analysis of the development of complex unfamiliar mathematics questions (Adams et al., 2017). Phase one centred on conventional mathematics questions and subject matter that can be modified to enhance levels of cognitive understanding and skills. Phase two centred on modification of conventional questions and subject matter to complex unfamiliar questions. Phase three centred on diverse and enhanced cognitive understandings and skills in modified questions and the development of assessment rubrics or guides to making judgements. The next section highlights a summary of the three-phase analysis and retrospective reflections of the educators.

Reflections and Analysis

This section reports on phases one, two and three retrospective reflections and analyses of how complex unfamiliar questions can be developed using conventional mathematics questions. Our overall position was that for developed complex unfamiliar questions: students will be required to apply their knowledge of multiple concepts to solve each question; the method required to solve the question will not be obvious in its wording; the question will most likely be in a context that students have not come across before; no scaffolding (for example provision of a graph); and that interpretation, clarification, and analysis will be required to develop responses to the questions.

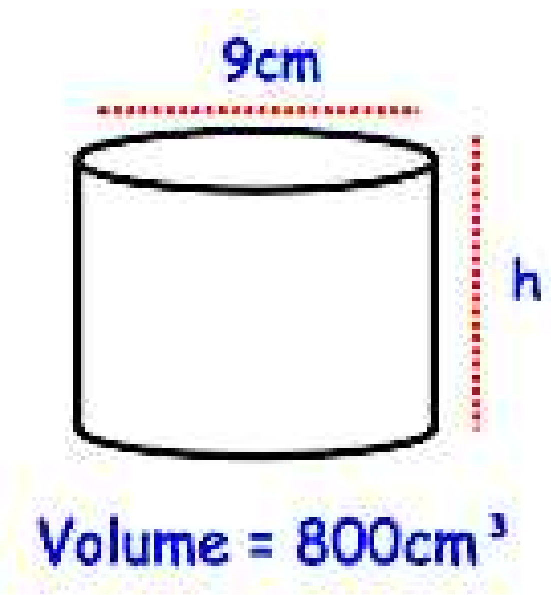
Phase one involved identifying the appropriate conventional mathematics questions and subject matter that can be modified to enhance levels of cognitive skills. We suggest starting with nominating a single-answer, conventional question; and then highlighting the subject matter, routine processes that may be involved, routine calculations that may be involved, and formulae and steps to aid the development of a solution that can be immediately applied to the solution of the given question.

For example, in a year 9 measurement question below, the original mathematics concept is on volume of a cylinder. The information provided includes the volume in cm^3, diameter in cm and the unknown height (h) as illustrated in the labelled diagram (Figure 1) below. The question also requires students' knowledge of one decimal place. This is a single-answer conventional question, and all students need to do is recall and evaluate the formula of volume of a cylinder:

> Conventional question: Figure 1 is a diagram of a cylinder with a volume of 800 cm^3, a diameter of 9 cm and a height (h). Calculate the height of the cylinder (to one decimal place).

Figure 1

Diagram of a Cylinder



Phase two involved modifying the conventional questions and subject matter to complex unfamiliar questions. We suggest starting with situating the conventional mathematics question in a real-world context/story or a context that the students have not seen before. The next step will be to remove any implied routine calculations and processes, then removing any labelled diagrams, formulae, and recognisable steps that aid the development of a solution. The aim is to de-identify routine procedures (no standard algorithm strategies, no clear path to solutions). The next step involves making connections of subject matter within and/or across the domains of mathematics. This makes the method and procedure to solve the question not immediately identifiable from the way the question is now posed. Using the previous year 9 measurement question stated above, it can be modified to the following:

> Complex unfamiliar question: A teacher proposes to the school administration to build a drum-shaped pool with no deep end to minimise the risk of drowning for prep to year 2 students. The pool must be bounded by the boundaries of a squared block measuring 3,600 m^2. How deep will be the pool (to two decimal places) for it to contain 2.5 million litres of water? Comment on the reasonableness of the teacher's plan.

The modified question is now more situated in a real-world context and the labelled cylinder diagram has been removed. The question is now multi-stepped and focuses on more than four aspects of mathematics, which include identifying 3-dimensional shapes, knowledge of calculating different areas of two-dimensional shapes (surface of the pool base) in m^2 (which can be any shape-from a circle, square, rectangle, etc.), different volumes of three-dimensional shapes in m^3, conversion of volume units to capacity and knowledge of rounding off decimals. The question also seeks a contextual mathematical explanation that justifies the reasonableness of the answer.

Phase three involved identifying the diverse and enhanced cognitive understandings and skills in modified questions. We suggest identifying the multiple contexts, including diagrams and models, procedures and skills involved in solving the modified questions. Importantly, this phase hypothesises how diverse responses to the question can be developed. This process leads to the development of the assessment rubric or guide to making judgement. In the example above, this involves identifying the different areas and volumes of shapes and conversion of units. Additionally, having more than one correct answer means students have opportunities to justify the reasonableness of their answers and then have something unique to contribute to discussions with other students. The question can provide opportunities of generalisations as students explore a diverse of possible answers. Moreover, the question can be further modified for students to find just one answer, for example in an examination, the second last statement in the question can be: 'How deep can be the pool if it is to cover the maximum possible area of the available block?' Thus, depending on the objective and purpose of the assessment the modified complex unfamiliar question can adapt flexibly with very minor changes but still catering for the same content and cognitive demands.

Discussion

This section discusses our insights developing complex unfamiliar mathematics questions. During our reflective exchanges, a major challenge that we encountered was modifying complex unfamiliar questions that de-identify context and procedures and at the same time develop mathematics proficiencies of understanding, fluency, reasoning, and problem-solving skills, which are central to helping students to develop the capacity to solve complex unfamiliar problems (ACARA, 2023). Phase one agenda of identifying the conventional question that requires at least a three-step solution; highlighting the subject matter, context, diagram(s), routine processes that may be involved, routine calculations that may be involved, and formulae and steps to aid the development of a solution that can be immediately applied to the solution of the given question was the least challenging phase. We found it easy to reach consensus most of the times. Similarly, it was easy to reach consensus in phase three identifying the multiple contexts, including diagrams and models, procedures and skills involved in solving the modified questions. However, phase two agenda of modifying the conventional questions into real-world contexts/stories or unusual ways that students have very limited experience, removing implied routine calculations and processes, and making connections of subject matter within and/or across the domains of mathematics proved challenging and complex. Lampert (2001) suggests that this phase of modifying the conventional questions is central to promotion of students' strategic thinking capacity, integration of concepts and motivation towards learning. The challenges arose from finding the appropriate balance between removing appropriate levels of implied routine calculations and processes, making appropriate connections of subject matter within and/or across the domains of mathematics, and at the same time focusing on the mathematics proficiencies. As our reflective exchanges unfolded, it was evident that one purpose of our interactions which was to encourage collaboration and consensus on the modified products was proving elusive most of the times in phase two. To modify such conventional mathematics questions into complex unfamiliar questions requires collaborative effort among educators, as highlighted previously, curriculum reports (QCAA, 2021; 2022; 2023; 2024; SACE, 2021) have identified lack of consistency by mathematics teachers when it comes to developing complex unfamiliar questions for internal examinations. However, having such consistency and at the same time focus on the mathematics proficiencies proved to be very challenging.

The reflective exchanges and analyses of how complex unfamiliar questions can be developed using conventional mathematics questions, was particularly useful in helping us focus more on subject matter that can be modified to enhance levels of cognitive understandings and skills as well as making connections of subject matter within and/or across the domains of mathematics. The aim was to improve our theoretical and practical understanding of the nature of complex unfamiliar mathematics questions and the adaptive expertise developing them. In particular, developing deeper understanding of not only subject matter and concepts, but the required analytical thinking, interpreting and creativity beyond standard algorithm strategies (Asman & Markovits, 2009; Mullis et al., 2009; QCAA, 2018) was fundamental. During our reflective exchanges, we observed that the modification process might not be straight forward from the way the problem is posed and putting it in a context in which students have had limited prior experience. For the modification of complex unfamiliar questions, we tended to discuss which subject matter was better placed as the starting point for creating complex unfamiliar questions and then identifying routine procedures and standard algorithms that were evident in the question. We came to a position that a casualty in the whole process was making connections of subject matter within and/or across the domains of mathematics if the main focus is making the solution not immediately identifiable as suggested by Asman and Markovits (2009). Similarly, QCAA (2018) emphasise that the required procedure to solve the question should not be clear from the way the problem is posed, and it should be in a context in which students have had limited prior experience. Consequently, developing complex unfamiliar mathematics questions that connect subject matter within and/or across the domains of mathematics can be a challenging process.

The reflective exchanges and analyses played an important role in building our strategies on how to modify conventional mathematics questions to complex unfamiliar. We came to the view point that a good complex unfamiliar question tends to have a few of the following features but not necessarily all of them:

- Provides real world context/story; provides information in an unusual way (a context that the students haven't seen before).
- Can be worked backwards.
- Combines subject matter.

- Does not have a clear path to solutions (no implied routine calculations and processes, no labelled diagrams, formulae, and recognisable steps that aid the development of a solution).
- Is multi-stepped and creates different levels of sophistication.
- Depending on the assessment objectives, the question might have more than one correct answer.
- Asks to evaluate and justify the reasonableness of answers.

This perspective is not exhaustive; however, it can provide students with the opportunity to engage with content at different levels of sophistication, a position postulated by Clarke (2011) and Ergen (2020). Such a perspective can help develop understanding and consistency in developing complex unfamiliar questions and promote quality mathematics teaching and learning.

Conclusion

The autoethnography process has enabled us to reflect more deeply on how complex unfamiliar questions can be developed using conventional mathematics questions. The approach we report resulted in a three-phase analysis of the development of complex unfamiliar mathematics questions. Phase one focused on the identification of single-answer, conventional mathematics questions and the embedded subject matter that can be modified to enhance levels of cognitive understanding and skills. Phase two focused on modification of conventional questions and subject matter to complex unfamiliar questions. Phase three focused on the enhanced cognitive understandings and skills in modified questions, leading to the development of assessment rubrics and guides to making judgements. The autoethnographic process highlighted a perspective and the importance of consistency in teacher developed complex unfamiliar questions. Our hope is to encourage further research on developing complex unfamiliar mathematics questions using different types of mathematics questions to promote quality mathematics teaching and learning.

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