Mathematical Modelling for a Class Party: Challenges for Students in one Year 4 Classroom

Kym Fry
Griffith University
k.fry@griffith.edu.au

Judith Hillman
Department of Education
jhill98@eq.edu.au

Rhonda Horne
Department of Education
rhonda.horne@qed.qld.gov.au

Elizabeth Rasmussen
Griffith University
e.k.rasmussen@griffith.edu.au

The revised Australian curriculum presents a new emphasis in primary schools on the process of Mathematical Modelling. A modelling focus brings close attention to confidence and capability with a potentially new problem-solving process, associated language, and pedagogical processes. This paper presents a Year 4 classroom modelling experience that arose as students planned their end-of-year party through Guided Mathematical Inquiry. Classroom video data captured two students working on vertical whiteboards as they formulated and solved a problem involving carrot sticks and dip. Findings reflect the students as doers of mathematics, engaged in productive struggle.

Mathematical modelling is presented in the revised Australian Curriculum: Mathematics as a new process that extends through all year levels from Prep to Year 6. The new emphasis supports students through problem solving to make connections between mathematics and real-life situations, between mathematical topics, and to strengthen interdisciplinary links (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2023). Previously in primary classrooms, such connections and links may have struggled to surface, especially when the focus in mathematics lessons remains on replicating taught procedures to correctly answer closed problem situations. The revised Curriculum presents an opportunity for students to 'use mathematics to gain insights into, and make predictions about real-world phenomena' to 'inform judgements and make decisions in personal, civic and work life' (ACARA, 2023). Prior to the release of the revised Curriculum, mathematical modelling as a problem-solving process in the primary years was not explicit. For example, previously in Foundation to Year 3, content descriptions included students *modelling* addition and sharing, *modelling* large numbers, and *modelling* unit fractions (ACARA, 2010). This content did not explicitly link to the proficiency of problem solving. The change to surface this mathematical process presents a significant shift for primary classroom teachers and brings competency with mathematical modelling to the forefront of professional development to support these experiences in the classroom. Although much literature offers excellent classroom modelling examples in secondary contexts (Geiger et al., 2022) and in STEM in the primary years (English, 2021), these examples do not address the current mathematical goals, nor specifically do they meet the needs of classroom teachers seeking classroom examples exploring the potential of modelling in Australian primary classrooms.

This paper offers insights into one Year 4 classroom as students problem solved through the inquiry, *How much does a class party really cost?*, addressing revised Curriculum requirements. The researcher and classroom teacher-researcher (first and second authors) were keen to make mathematical modelling explicit in this mathematics classroom, having never taught the process before, and felt that the class party context would offer a real-life situation in which to support the modelling process. We wondered, (RQ1) what modelling opportunities could exist for Year 4 students planning their class party through Guided Mathematical Inquiry, and (RQ2) how might Year 4 students in one classroom approach mathematical modelling for the first time.

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Mathematical Modelling in the Primary Years

Mathematical modelling is a well-researched topic and many explanations of the modelling process exist. Blum's (2011) model is well accepted and presents 7 steps for structuring classroom mathematical modelling. Briefly, this involves starting with (1) a real situation to frame the construction of a problem which is then (2) simplified to create a *situation model*, and further structuring of the situation leads to a *real model* that can be (3) mathematised. A mathematical model of the problem (e.g., equations) allows the solver to (4) work mathematically to generate mathematical results. Problem solvers then return to the real-life context to (5) interpret their results to inform decision making. If results are (6) validated, the problem solver can decide whether further mathematising is needed to address other variables until the final solution is (7) exposed or communicated. A key tenet of modelling is the connection between mathematics and reality thereby reminding problem solvers of the usefulness of mathematics to their lives. This is especially important given the current school curricula focus on solving word problems that do not reflect the complexity of real-life problems (Maass et al., 2022). Real problems are messy and so in modelling, investigation progresses through multiple cycles of interpretation and explanation to inform decision making (Doerr & English, 2003). The design of modelling tasks attempts to capture real-life complexity through the inclusion of an open-ended approach (Geiger et al., 2022) and careful attention is paid to scaffolding, to 'unstick' blockages students may encounter when working to solve the problem (Park, 2023). To emphasise relevance, recent depictions of modelling processes integrate mathematics with socio-scientific issues and STEM knowledge, including the development of concrete materials that explore controversial issues, to progress the role of mathematics in STEM and to include a citizenship focus (English, 2021; Maass et al., 2019; Makar & Doerr, 2020). Student competence with mathematical modelling to address global challenge is important yet it can also be helpful for children with little modelling experience to initially explore and develop competence with closer contexts, as is the case in this paper.

Links made between mathematical modelling and inquiry-related processes articulate how both problem-solving approaches involve a collaborative approach between teacher and learner, that values creating questions, asking probing questions, using mathematical representations, iterations of refinement and improvement, and a sense of discovery (Maass et al., 2019; Maass et al., 2022; Makar & Doerr, 2020). Specific to Guided Mathematics Inquiry, is the use of an ill-structured question to support an extended nature, with opportunities for students to engage in productive struggle, and where the classroom culture aims to challenge and advance a student's conceptual development (Fielding & Makar, 2022). In the classroom depicted here, students were engaged in the inquiry *How much does a class party really cost?* to encourage thinking about and modelling the cost of foods, even when that food is made with products in the pantry. Solutions would present the amounts of money spent in each household of students who contribute a plate of food on party day. Due to the open nature of the question, many mathematical topics could be addressed, including measurement (e.g., cups of flour in one packet), additive and multiplicative operations (e.g., each carrot can be cut into 8–10 sticks for dipping. How many sticks can I get from 3–4 carrots?), and financial mathematics (e.g., the total cost of making chocolate brownies for 28 people). The modelling experience depicted in this paper centres focus on one aspect of the investigation; how much it might cost to provide carrot sticks and dip for the class to share. This was a simplified situation model that could be mathematised, where results could inform decision making. If dishes cost a family more than $10 to prepare, would it make better sense to order takeaway food for each person to be delivered to the classroom for the same price?

Key to both Guided Mathematics Inquiry and mathematical modelling, is a focus on problem solving. To clarify, the problem solving depicted in this paper illustrates students working mathematically on solutions where they did not know the solution process. The boundaries and constraints of this task offered students the freedom to explore how the problem could be approached, through solution pathways that appeared to be non-routine. For the students, the problem solving was challenging and, in this instance, a collaborative experience that promoted mathematical discussion and reasoning. The task outlined in this paper involved the use of Liljedahl and colleagues' (2021) vertical whiteboards approach to encourage students to discuss their thinking, to persist through mathematical challenge. The pedagogical aim was for students to learn *through* problem solving and to be engaged in thinking mathematically.

Research Design

A wider program of design research (Cobb et al., 2003) frames this case study, focused on understanding and improving the mathematical process of modelling in inquiry settings. The classroom teacher was experienced with Guided Mathematical Inquiry and a participant in the ongoing research. The aim of the larger study was to conceptualise and operationalise terms that contribute to ontological innovation (di Sessa & Cobb, 2004), with refinement and extension of developing theory. In this case specifically, we were interested in the modelling contexts that naturally arose from the inquiry and instead of teaching modelling, we wanted to know what the students could do when modelling, including the design of their solutions. Guided Mathematics Inquiry presented a suitable pedagogy in this instance to support the teacher with introducing mathematical content knowledge and processes using teaching *through* problem solving approach that is student-centred. The classroom episode reported on here as a case study, took place in the final term of the school year and the classroom teacher had already established a classroom culture that valued mathematical thinking and reasoning, including socio-mathematical norms of *building on the ideas of others* and *active listening.*

The Year 4 (8–9-year-olds) classroom depicted here was situated in a large Queensland metropolitan school. There were 30 students (co-educational) in the class, and only two of these students chose not to participate in the study. Consent was fluid in that students could opt-in or out to suit their level of comfortability once parent consent was received. The students were finding out through inquiry, how much having their end-of-year class party really cost, and to capture the messy nature of realistic classroom processes, the researcher used a video camera on a tripod to film lessons (6 sessions over 6 weeks, > 6 hours), accompanied by an iPad to capture student discussion in groupwork. An additional invite by the children meant the researcher also attended the party although this was not filmed.

Analysis

Videotape methodology (Powel et al., 2003) has been established as suitable for this kind of observational research in mathematics education. Analysis involves the research team frequently and flexibly viewing excerpts. To identify the case presented here, the researcher and teacher, who had been involved in all classroom lessons, reflected on and identified four instances in the inquiry that they felt were mathematical modelling experiences. These involved students formulating a problem from the real-life situation using number sentences and involving multiplicative thinking, to meet curriculum goals. To distinguish from the inquiry, student solutions to modelling problems were based on assumptions (that 3 serves of a menu item is a suitable amount for a student to eat, for instance) rather than being refined to suit the specific context (my data showed that one person wants 15 serves of this item so three won't be enough!). In an inquiry, assumptions often become the line of focus and interrogation. For example, rather than assume that three carrot sticks per person will cater for the class, a classroom inquiry involving statistics might find out more closely whether this is a reasonable serving size for Year 4 classmates.

Of the four instances identified, the problem involving carrot sticks and hummus was selected as a strong modelling example. The researcher, teacher, and the research assistant attentively viewed the selected video excerpt (from the third classroom session) independently, aided by a transcript of this section of the lesson, to identify critical events. The researcher and teacher then flexibly viewed the video again together, and narrowed focus to the critical event presented here, involving two girls working together on a vertical whiteboard surface, to work out how much it will cost to purchase enough carrots for the class to enjoy carrot sticks dipped in hummus at the end of year party. To respond to the second research question, coding focused on the content of the critical event, how Paris and Cristina approached modelling, related to the difficulties these students faced with mathematising the problem situation, and the feedback they received to support them towards a successful solution. The identified collection of events within the student-to-student discursive interaction offers an emerging narrative about the data and the storyline is presented in the following section.

Findings and Discussion

Modelling Task

By the third session of the inquiry, children had confirmed with their families and nominated a menu item that they would bring to the party. Each child was tasked with finding out the cost of making the dish that would be prepared at home. When asked for pricing for ingredients, Hayden revealed his research efforts: a bag of carrots cost $1.99, and he would only need 3 or 4 carrots to cut into sticks for his carrot-stick and hummus-dip dish. He also shared that he could cut 8–10 sticks from each carrot. In response, and in an impromptu manner, the teacher added that when she last bought a bag of carrots, there were 8 carrots in the bag. She rounded the $1.99 to $2 and posed the following questions:

(17m 8s) Teacher: So you guys now, can you work out how much it's going to cost for the carrots that we need? Now think about if there are eight to ten carrot sticks from each carrot, consider how many you might want for the class (pauses). You've said three or four. Work out how many carrot sticks that's gonna give us.

Whereas the inquiry was focused on finding out the costs and designs of dishes to share with 28 people at a party, based on their tastes and dietary requirements, this problem task reduced the focus on personal preference.

Table 1

Characteristics of Carrot Stick Modelling Task, Informed by Blum's Model (2011)

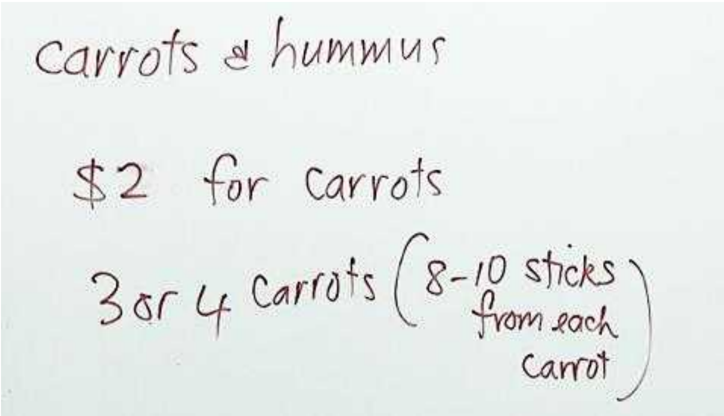
Modelling phase	Elements of carrot stick task
Real situation	Inquiry: How much does a class party really cost?
Situation model	Simplified to: number of sticks per carrot, number of carrots per bag, and cost of one bag of carrots
Real model	Mathematised as a series of equations involving multiplicative thinking
Working mathematically	Solving equations
Interpret results	Does the solution sound reasonable? What might this look like?
Validation	Through drawing, peer and teacher feedback, checking interpretations (5 iterations).
Communication	Solutions are displayed on vertical whiteboards and peers explain solution pathways.

To solve the carrot problem, students would work on the following assumptions: each carrot would be large enough in size to cut into 8–10 sticks, the carrot sticks would be large enough for dipping, everyone was going to choose to eat carrot sticks on party day and the total number of carrot sticks would be shared equally (the same number of sticks per person). Put simply, the problem now seemed to meet the characteristics of a modelling task (Table 1) that would align with curriculum goals. By the end of Year 4, students use mathematical modelling to solve financial and other practical problems, formulating the problem using number sentences, solving the problem choosing efficient strategies and interpreting results in terms of the situation (ACARA, 2022).

During analysis, focus turned to how the carrot stick problem was posed to the students as this framed the open nature of the task and how the students might approach the modelling situation. The transcript above outlines the moment the problem is identified and shared by the teacher, and when students are invited to help their peer, Hayden, work out a solution. Figure 1 is a photo taken of the board displaying the key information for students, recorded by the teachers as she posed the task. This is not displayed as a formal question.

Figure 1

Key Components of the Problem Recorded on the Board



Three key components are included in the excerpt (task proposal) and were recorded as key information for students to refer to (Figure 1). The first component is the first question posed by the teacher, *how much it's going to cost for the carrots that (the class) need*. To solve this part of the problem, two pieces of information are required: students would need to determine how many carrots are needed, then use this amount to determine a fraction of the cost of one bag. The second component is a consideration for the students, reminding them that they will need to determine *how many you might want (carrot sticks) for the class*. A solution might include: the number of carrots needed (3 or 4, as advised by Hayden), divided by 8 carrots (number of carrots in one bag), multiplied by the cost of one bag (rounded to $2). Such a problem is complex, and it is not obvious to students where to focus their attention first. To further complicate this task, the number of carrots would be determined by the third component; a question posed which is to work out *how many carrot sticks that's gonna give us*. This part of the problem opens the task in that one carrot may be cut into 8, 9, or 10 sticks (depending on the size of the carrot). Students could rely on Hayden's advice that 3 or 4 carrots would be needed, but it wasn't yet clear whether this would produce a reasonable number of carrot sticks for the class to share at a party (number of students = 28). How Paris and Cristina approach this problem is elaborated on in the following section.

Problem Solving

In less than one minute after introducing the task, students were moving to find a partner to problem solve with and a vertical surface on which to record their solutions. Paris and Cristina commenced by recording key information on their vertical surface, "each carrots $2, 8 carrots" followed by the equation, 2 x 8 = 16 (examples of students' written efforts on vertical surfaces includes student spelling and grammatical errors). Clarifying their progress, Paris notes to her partner that this should be sixteen *dollars* (the cost of 8 carrots) and they rewrite this as $2 x $8 = 16. Paris rubs the dollar sign off the board next to the 8, then rubs the entire equation of the board. Eventually they settle on the equation $2 x 8 = $16. In this brief start, we see the students are attempting to formulate the problem as an equation involving the two numbers they have already recorded on the board.

Following this first attempt, the two students check the board beside them where the teacher has recorded the components of the problem posed (Figure 1). The researcher with the camera is standing nearby and prompts the students by asking them to explain their thinking. They restate the number sentence they have recorded on the board, for the camera.

1 Researcher: You mean each carrot is $2?

2 Paris: I mean each carrot bag (rubs out $2 next to carrots and writes ‘bag’)

3 Cristina: So now we do 2 times 8

4 Researcher: So that’s 2 times 8? (points to the board)

5 Cristina: Because there’s 8 carrots.

6 Researcher: Yeah. So what are you finding out?

7 Paris: (Muffled. She looks to the key information written by the teacher which is beside them. She points 8 to each piece of information as she reads it.) Oh no wait! We did it wrong! We only need 3 or 4!

9 Researcher: Right!

10 Paris: (To Cristina) We need 3 or 4. (She rubs off the 8 on the board and replaces it with the number 4.)

11 Cristina: Ok.

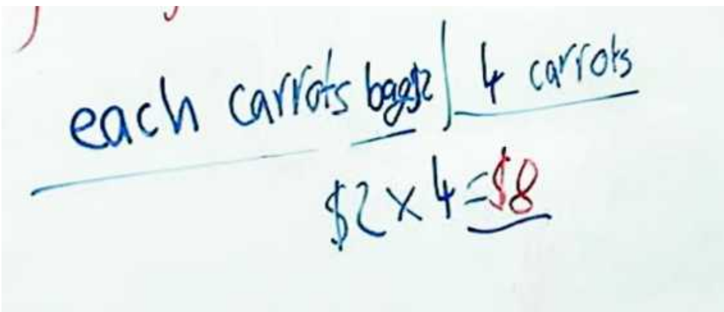
12 Paris: That means we’re wrong. (Rubs off the answer ‘$16’) Now it’s (muffled-reads the equation)

13 Cristina: (records answer on the board, ‘$8’)

Figure 2 displays the recording done on the vertical whiteboard by Paris and Cristina up to this point.

Figure 2

Vertical Surface Attempts by Paris and Cristina



In this next stage of problem solving, Paris has identified an error: $2 is the cost of a bag of carrots (Line 2). The students were prompted twice by the researcher to explain their efforts (Lines 1 and 4), and when the students checked the key components of the problem recorded on the board beside them (Lines 7 and 8, see Figure 1 also), they acknowledged another error in their efforts. The mistakes by Paris and Cristina take place during the very early stages of problem solving through modelling. In their initial attempts, we see the two students have devoted time to doing mathematics involving the quantities from the real-life situation. The students illustrate correcting each other when a mistake is made, using self-checking strategies such as checking key components of the problem written on the board, and respond when prompted by the researcher to interrogate their efforts by relating attempts to the real situation.

The students continued working on the problem for close to 14 minutes, exploring the different components of the situation, or variables, to solve the problem. Only the initial attempts have been presented here to illustrate what a teacher might expect to see in a Year 4 classroom when students approach mathematical modelling for the first time. Without direction on how to approach the problem, or teacher guidance on efficient strategies to use, we see two students at the end of Year 4 struggle with formulating the problem using number sentences. This highlights difficulties some students might face when simplifying and mathematising a real situation to create a situation model (Steps 1, 2, and 3 (Blum, 2011)) even when embedding the problem in an inquiry context. However, the inquiry framing the modelling experience (*How much does a class party really cost?*) may have supported students at this early stage of problem solving to interpret their results as invalid (steps 5 and 6 (Blum, 2011), generating feedback about how to progress with solving the problem of how much it would cost to buy the carrots they needed for the class.

Conclusion

In this classroom episode, we see all students working collaboratively on the same ill-structured problem, focussed on helping Hayden work out how much it will cost to purchase enough carrots for 28 students to enjoy at their end-of-year class party. The problem is framed by a Guided Mathematical Inquiry, and posed by the teacher in response to a real situation for one student. The teacher identified the carrot-stick problem as an opportunity to simplify a real situation as a modelling task for the class to solve that is aligned with Curriculum goals and is presented as a series of key components (variables, Figure 1). The task was open-ended (solutions would differ depending on whether each carrot was cut into 8, 9, or 10 sticks) and had open solution pathways (it was not clear to students which component needed to be addressed first). The problem involved a financial context, and the teacher has presented the key components of the problem in a way that challenges students to formulate the problem using number sentences, to choose their own strategies, and to interpret the results in terms of the situation (ACARA, 2022). The carrot-stick problem involved no 'correct' solution method and is combined with vertical surfaces to foster confidence through a collaborative approach. The task seems challenging to the two students presented in this paper, yet the struggle they face is productive when they employ their own ideas and strategies to mathematise a real-life situation and begin to identify themselves as *doers* of mathematics (Van de Walle et al., 2019).

We focus on the efforts of Paris and Cristina as they attempt to mathematise a real-life situation involving carrot sticks. Their efforts have been selected here to show the difficulties for students in completing this task, and how collaborative approaches can support students in the early stages of modelling. Blum and colleagues (2011) note *constructing,* as the first step in modelling, as difficult for students. Similarly, we see these students ignoring the context as they initially choose to multiply the values. Working on a vertical surface (Liljedahl et al., 2021) promoted collaboration between the girls as they sought to understand the problem through conversation. Researcher prompts for students to explain their efforts (So what are you finding out? (Line 6)) reminded students to check key components of the problem and progress through the modelling process. This prompt could be used by teachers when students construct situation models in their first attempts at modelling (mathematising the situation), to scaffold student progress.

We hope the problem is a useful example for teachers who are beginning to design their own modelling tasks to meet Curriculum requirements. We acknowledge that the context is not a universal issue, nor relevant to children everywhere. However, this example demonstrates how a teacher might capitalise on problems from students' lifeworlds that can connect to mathematics. We also hope that we have made a distinction through presentation of the carrot-stick problem, between inquiry in the mathematics classroom and mathematical modelling. In the inquiry, problem solving was complex and focused on the specific class of students with specific dietary requests and preferences. In the modelling task, assumptions generalised aspects of the problem-solving focus, simplifying the problem situation. However, inquiry and the process of modelling emphasise designing solution strategies that are purposeful rather than a focus solely on computation.

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