Student Engagement with Dynamic Digital Representations of Decimal Fractions to Prompt Conceptual Change

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Decimals and the related areas of ratio and proportion are recognised as one of the most challenging and complex areas of mathematics for young children to learn. In this study, four dynamic digital representations were used by Year 4 (9–10 years old) students in a set of video-recorded, individual task-based interviews to explore key concepts of decimals. Microgenetic analysis methods were used to detect conceptual changes through specific shifts in the learner's attention. The data collected revealed what features of each dynamic digital representation prompted attention shifts and how these translated to changes in conceptual understanding of decimal fractions.

Children have difficulty in understanding the abstract nature of decimals, and a plethora of misconceptions arise from incomplete understandings (Steinle & Stacey, 2004). Decimal fractions present complications as part of the number-relationship needed to make sense of the number, the denominator, is not visible. Developing conceptual understandings of decimal density, place value and relative magnitude of decimal fractions are considered key for consolidating number sense. The properties of density and place value unify all numbers and rational number magnitude knowledge is related to future mathematics achievement (Siegler et al., 2011). Minimal research has been conducted into what effective representations should be employed by mathematics educators to assist students in developing 'decimal sense' and managing 'cognitive conflict' in the context of decimal fractions (Swan, 1983). This study investigated a collection of dynamic digital representations that featured the three key concepts of decimals.

Representations are used in the mathematics classroom to assist students in understanding abstract concepts that are central to mathematics learning (Pape & Tchoshanov, 2001). An important feature of mathematics education is working with appropriate mathematical diagrammatic representations, but there is uncertainty around the best representations for decimals. Dynamic representations are potentially an effective support for student learning within mathematics because of their dynamic affordances. Unlike with static representations, students are able to actively interact with the mathematics content by making conjectures and testing them immediately to experience the concept from various perspectives and receive feedback on their thinking (Orrill & Polly, 2013). In particular, learner-centred computer tools that encourage interactive engagement with mathematics, allow students to 'see' notions that would otherwise be 'unseeable' (Orrill & Polly, 2013). Relatively little work has focused on the role of dynamic digital representations in teaching and learning decimals, although there is an abundance of research into the use of digital learning objects in mathematics education (Litster et al., 2018). The intention of this study was to establish a foundation for research in this topic by exploring how children's interactions with dynamic digital representations of decimal fractions may develop their conceptual understanding in this numerical area.

Theoretical Framework

In mathematics education, conceptual understanding involves being able to represent a variety of mathematical ideas in different ways, to see the connections between those representations, and to know how to use each representation for different purposes (Caldwell, 2020). The growth of students' conceptual understanding is synonymous with mathematics learning. Mason's (2008) theorisation of the notion of attention has been employed to account

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for students' learning of mathematics. The theory is underpinned by the idea that students must experience shifts of attention for learning to occur. As students interact with mathematical tasks and representations there are ongoing shifts in their focus and structure of attention. The learning of mathematics occurs through these attention shifts and students' developing awareness of them (Mason, 2008). At present, there is little conclusive research on how students are best supported to build a conceptual understanding of decimals. The theory of shifts of attention provides a scaffold for educators' understanding of teaching mathematics as well as proposing five observable attending stages that indicate what and how students are learning in mathematics lessons. Mason (2008) described the 'focus of attention' (what the learner attends to) and the form or 'structure of attention' (how the learner attends) whereby several shifts of attention occur in the process of thinking mathematically related to an object. Structure of attention refers to "holding wholes, discerning details, recognising relationships, perceiving properties, and reasoning on the basis of agreed properties" (Mason, 2008, p. 35). See Table 1 for examples. As attention shifts from one concept to another, its structure alters and connections are made, relationships are perceived, and new ideas are appreciated. Applying the theory of shifts of attention may provide a scaffold to plot a progression of understanding that students may take when learning about key concepts related to decimal fractions.

The focus of this paper is to explore how children in middle-primary grades engaged with dynamic digital representations of decimal fractions to develop their conceptual understanding. The overarching research question communicates the aim of the study:

- What features of dynamic digital representation do children attend to that prompt conceptual changes related to decimal fractions?

Method

The methodology considered most appropriate for the study was structured, task-based interview because it complemented the theoretical framework from a constructivist, learner-centred view. Constructivist learning theory has been linked with structured task-based interviews in the work of Goldin (2003) who found the research method to be an effective means of understanding conceptual knowledge. In this study, task-based interviews were used to prompt students' interactions with mathematical tasks and representations and record ongoing shifts in their focus of attention and the structure of attention.

Data collection for this study consisted of four task-based interviews each focussing on a different dynamic digital representation: *Wishball-hundredths*, *Decimal Strips, Zooming in on Decimals,* and *Zoomable Number Line* (Figure 1). Four digital representations were selected from resources readily available to teachers on the internet, following an analysis of the affordances and constraints of each potential digital tool, and the focus on at least one of the three key decimal concepts. The number-line based, place-value symbolic, and area-model dynamic digital representations of decimals used were in the form of applets which are embedded into a webpage and displayed using a web-browser.

Four participants were from one Year 4 (9–10 years old) class situated in a school from the outer suburbs of Sydney, Australia. Although the students were not intended to be representative of the whole class, the researcher selected four that spanned a range of general mathematical achievement levels, based on results from a decimal comparison pre-test. This was in case there were differences in the ways in which students of varying abilities interacted with the dynamic digital representations. This grade level was selected because, according to the NSW syllabus (Board of Studies NSW, 2012), students are introduced to decimal notation for tenths and hundredths in Year 4. The decimal number concepts and skills were therefore still in the early stages of development at the time of the study.

Figure 1

Screenshots and Basic Information About the Digital Tools

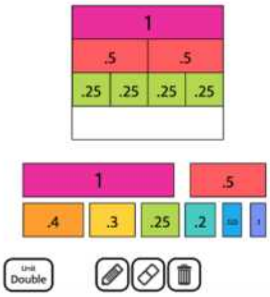
Wishball

Focus on place value.

Involves adding or subtracting to reach a target decimal fraction number.

(https://www.scootle.edu.au/ec/viewing/L8456/index.html)

Decimal Strips

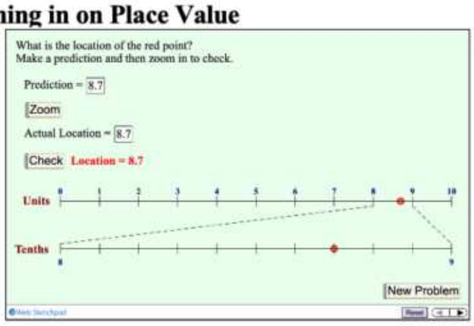


Focus on number magnitude.

Involves dragging coloured tiles representing different decimal values.

(https://toytheater.com/decimal-strips/)

Zooming in on Place Value

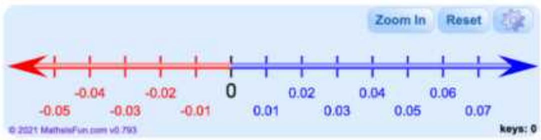


Focus on number magnitude.

Involves predicting the location of a decimal fraction on a number line, then self-checking with the embedded Zoom function.

(http://www.sineofthetimes.org/zooming-in-on-place-value/)

Zoomable Number Line



Focus on number density.

Involves using Zoom-in/out capability to respond to teacher-designed tasks.

(https://www.mathsisfun.com/numbers/number-line-zoom.html)

The researcher worked individually with each participant in four 30-minute sessions, over a period of four weeks. During the interviews, each digital resource (Figure 1) was introduced by inviting the student to freely investigate the tool for 5-minutes and them asking any clarifying questions. The tasks were then initiated by the researcher and completed by the student. In addition, the researcher, who was a qualified teacher, regularly asked probing questions to encourage further explanation by the student, for example 'Can you tell me what you are thinking?' but remained neutral regarding 'correctness' of responses. Video-recording the interviews was critical to collecting quality data as it was necessary to capture what was said and done by the child simultaneously, including gestures, expressions and emotional responses.

The data collected from the task-based interviews were analysed using a microgenetic approach that involved closely examining intensive data of what learners do and say over several task-based interviews to capture a record of moment-by-moment learning processes. This close examination allows researchers to draw conclusions about what prompts knowledge change, how learning occurs, and other aspects of the processes of acquiring conceptual understanding (Voutsina et al., 2019). Data analysis began with repeated viewings of the video-audio recordings of each task-based interview to deduce what new understandings the participants had gathered during the task-based interview. The initial analysis period occurred over several weeks as the researcher transcribed and documented all observed changes in decimal understanding. Four evidence excerpts were selected for this paper from the larger data-set. These examples showed key tasks that provoked the most change in conceptual thinking as a result of the student's interaction with a dynamic digital representation of decimals. A framework to support analysis was designed by the researcher as a reference tool when analysing the verbal reasoning and manipulative actions made by the participants (Table 1). The 'Shifting Attention with Decimal Concepts Framework' correlated Mason's five components of structures for attending with the developmental progression of decimal fraction concepts outlined in the National Numeracy Learning Progression (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020). The framework was developed prior to data collection by anticipating the specific decimal concepts students should attend to when developing their conceptual understanding of decimal place value, magnitude and density. This was then modified after the interviews based on observations of student understandings. The value of this extended theory of shifts of attention enabled the researcher to understand what each participant experienced in the learning process and identify what elements of the dynamic digital representations prompted shifts of attention and changes in understanding. This analysis tool was used alongside a microgenetic approach to data analysis when the researcher discerned observable shifts in how and what the participants were attending to as they engaged with the four dynamic digital representations of decimal fractions.

Table 1

Shifting Attention with Decimal Concepts Framework

Structure and microqualities of attention (Mason, 2008)	**Connecting Mason's structures with decimal fraction learning progression (ACARA, 2020)**	**Key concepts of decimal fractions**
Holding wholes	Names decimal fractions without an understanding of place value	Place value
Discerning details	Discern mathematical details of decimal fractions with an understanding of place value and reforming decimal numbers with value	Place value Decimal magnitude
Recognising relationships	Understands the relationship between decimal fractions and the division operation, as the decimal is parts of a unit resulting from dividing the whole into equal parts. Understands the positional value of decimals as well as the scaling effect as digits move to the right and left in place value	Place value Decimal density
Perceiving properties	Perceives properties of decimal fractions as they demonstrate knowledge of the relative size of decimals when comparing and applies this knowledge to the relationship with measurement and position when plotting values on a number line	Decimal magnitude
Reasoning on the basis of agreed properties	Relates their understanding of positional value number knowledge to perform operations with decimals.	Place value Decimal density

Findings

Four scenarios were selected to illustrate how each of the students interacted with one of the four dynamic representations. The evidence and accompanying interpretation reveal the specific dynamic affordances that facilitated shifts in attention which indicated changes in conceptual understanding of decimals. Pseudonyms are used in replacement of students' names throughout.

Example One: Annelise Wishball-Hundredths

In the interview examining the affordances of Wishball-hundredths, Annelise experienced shifts in attention where she was able to generalise mathematical principles pertinent to adding and subtracting decimals, including the concept of place value when reforming decimals. During the task-based interview, Annelise needed to reach the target number by adding or subtracting in fewer than 20 moves, the current starting number was 29.67 and the spinner displayed 9. Annelise attempted to calculate the subtraction $29.67 - 0.09$ before the screen displayed the answer, to decide whether it was an ideal move within the game. This was a key moment of conceptual learning as she exclaimed "which will actually make it closer to the target number!" Through her engagement with the dynamic affordances embedded in Wishball-hundredths, namely the dynamic counting frame, as well as the feedback she received when the starting number changed based on her own carefully considered mathematical moves, the student's thinking progressed to *reasoning on the basis of agreed properties* (Table 1) as she related her understanding of positional place value knowledge to perform a subtraction operation with decimals. However, her approximated answer suggested that she was consolidating her knowledge across multiple structures of attention. Annelise acknowledged her change in conceptual understanding was the result of new learnings that occurred during Wishball-hundredths when she explained, "I knew that one of the tenths would need to be traded to the hundredths to subtract from, because I had seen the counting frame move before." The powerful partnering of observed dynamic affordances and Annelise's knowledge of theoretical concepts related to decimals allowed a transition of attention to generalising principles and therefore being able to *reason on the basis of agreed properties* when calculating a subtraction problem with decimal numbers.

Example Two: Valerie Decimal Strips

Valerie was asked to compare a series of two decimal numbers to determine the larger decimal value whilst completing the Decimal Strips task-based interview. One comparison challenge required the student to decide whether 0.2 or 0.25 was a larger decimal. Valerie acknowledged her insufficient decimal knowledge to answer the comparison question. Without hesitation she stated, "I'm not sure I'll have to check." Valerie was aware of the affordances of the dynamic digital representation to display the relative size of decimal fractions as was needed in this decimal comparison challenge. She dragged the 0.2 and 0.25 coloured tiles onto the blank fraction wall. The researcher used a probe question, "why do you think they are so close in value?" to elicit further student thinking. Valerie considered the question before responding, "maybe the zero-point-two has something else on it you just can't see it." A second probe question was asked, "what would be on the end?" Valerie replied, "it could be twenty-something and then this one [pointed to yellow 0.3 strip] could be thirty-something."

The experience of Valerie unpacking this particular decimal comparison question is an example of how the abstract nature of decimals present significant computation difficulties for students. Connecting her pre-existing decimal knowledge with how she manipulated the decimal strips to compare relative sizes, Valerie posited that 0.2 and 0.3 were actually "twenty-something" and "thirty-something". She independently uncovered the concept of decimal place value and accurately described 0.2 and 0.3 as having a 'hidden' value of tenths. Valerie's existing misconception thinking "longer-is-larger" (Steinle & Stacey, 2004) was being challenged through her engagement with the virtual manipulative as she dynamically compared the decimal lengths in this area-model representation. A significant first step to overcoming her erroneous thinking.

Example Three: Yehali Zooming in on Place Value

Yehali's default numerical thinking was focused on whole numbers only. She did not hold an understanding of decimal density and therefore initially couldn't predict decimals located between whole numbers in the Zooming in on Place Value interview. The interactive activity displayed the decimal 0.9 with the question, "what is the location of the red point? Can you make a prediction?" Yehali's first guess was "one", so she typed the predicted location in as 1.0 and watched the 'zooming in' animation reveal the location between zero and one with tenths integers marked as dashes on the number line. The researcher probed, "what did you see happen just then?" Yehali replied, "the numbers stretched out." The screen displayed a second question, "what is the actual location of the red point?" The student answered "nine" and typed her second predicted location in as 9.0 and then saw the actual location revealed as 0.9. The researcher checked for understanding, "what is the actual location written as?" Yehali read, "zero-and-nine." A follow-up question was posed, "why is that the actual answer for the location of the red point on the number line?" Yehali reasoned, "maybe because there is no number here it's just zero [pointed to zero integer marking on number line] and it hasn't gone to one yet [pointed to one integer marking on number line] so it's in the middle somewhere."

Yehali's current thinking involved *holding wholes* as her understanding was constrained by the whole number details presented in the first-step of the interactive activity. When Yehali observed the animated number line dynamically zoom-in to show ten-tenths between zero and one, she used her whole-number thinking to change her prediction to "nine" as she was viewing the integer markings having ones value rather than tenths. The feedback Yehali received from the interactive activity prompted her to reconsider her dominant whole-number thinking. When she noticed that her second prediction of "nine" was incorrect she needed to produce a mathematical reason why the actual location was zero-point-nine. By closely viewing the dynamic number line she grasped a more accurate understanding that included some decimal knowledge, "maybe because there is no number here it's just zero and it hasn't gone to one yet so it's in the middle somewhere." Yehali demonstrated a shift in attention to *recognising relationships* when she understood that between two whole numbers, decimal values are located, the concept of decimal density.

Example Four: Aarav Zoomable Number Line

Aarav displayed significant conceptual confusion around decimal fractions across a number of tasks in the Zoomable Number Line interview. He was asked to locate the decimal 0.9 on the zoomable number line and describe its location. Aarav immediately, zoomed-in to show hundredths on the number line and scrolled left from 0.12, he paused when 0.09 was shown on screen display exclaiming, "here is the number." The researcher questioned, "is that the same number as shown on the card?" The student considered and then answered, "no it's even littler" before he continued to scroll left from 0.09 and stopped when 0 was displayed. The researcher asked, "what are you thinking?" Aarav replied, "I zoomed in too much because the numbers are hundredths but this one [pointed to 0.9 decimal card] is tenths". He tapped the 'zoom out' button and adjusted the number line to show tenths only, he then scrolled back to the right along the number line and stopped upon 0.9 being shown, "I found it … zero-point-nine is between zero-point-eight and one-whole." Aarav's actions indicated that his understanding of tenths and hundredths place value in decimals was developed through his exploration of the zoomable number line. Initially, the student did not understand the concept of place value in decimals as suggested by his consideration of 0.9 to be the same value as 0.09. Using the 'zoom out' function of the zoomable number line helped Aarav correctly locate 0.9. Aarav was originally operating with a mathematical mindset of whole-number thinking and thus an incomplete ability to *discern details* related to decimal place value. However, through his interactions with the features of the zoomable number line, Aarav's attention shifted to *recognising relationships* by stating the place value of digits in the decimal number with an understanding of positional value.

Discussion

Representations are used in the mathematics classroom to assist students in understanding abstract concepts that are central to mathematics learning (Pape & Tchoshanov, 2001). The affordances of four dynamic digital representations were employed by the students in this study to develop conceptual understanding of decimal place value, magnitude and density. The facility of digital tools to adapt, change, and provide various interactive features and capabilities that traditional static representations may not offer were pivotal in enabling the students to successfully complete decimal fractions tasks such as the decimal location task in the Zoomable Number Line interview. By actively "scrolling backwards" Aarav could mathematically reason that the decimal 0.09 must be smaller in value than 0.9 as it was located closer to the whole number 0 on the number line. In addition, the dynamic digital representation Zooming in on Place Value helped Yehali 'see' notions that would otherwise be 'unseeable' through its unique 'zooming in' animation between two whole numbers to show a magnified view divided into tenths (Orrill & Polly, 2013). She initially had difficulty predicting what tenths decimal was located on the number line, however by engaging with the dynamic nature of the Zooming in on Place Value web-app, she successfully solved later questions. This showed that within the timeframe of the 30-minute interview, the student was comfortable and confident enough to use the highly effective learning tool to eliminate the abstractness of decimals.

All students involved in this study experienced moments of cognitive confusion generated by the dynamic affordances embedded within the digital tools which interrupted the students' flow of attention, to guide their conceptual learning journey of decimal fractions. Seminal work by Swan (1983) found that students who experienced 'cognitive conflict' when learning gained better outcomes overall than students who learnt through 'positive only teacher'. Decimal strips generated cognitive conflict as it challenged Valerie's current misconception thinking about decimals. It was not until she used the functions of the Decimal Strips web-app and dragged the 0.2 and 0.25 coloured tiles onto the blank fraction wall that she recognised the value of five-hundredths. Valerie was able to self-correct her thinking by interacting with this dynamic representation as it challenged her prior understanding of decimals and allowed her to examine her errors in thinking and misconceptions.

Dynamic representations of decimal fractions are capable of offering direction to learners' shifts of attention. The dynamic nature of digital manipulatives force shifts in the object of children's attention when they are actively interacting with the mathematics content as well as the structure of their attention in how they attend to features of the mathematics representation and the embedded mathematics concepts. Conceptual development occurred when the learners experienced a shift between attending to relationships within and between elements of current experience and perceiving relationships as properties that might be applicable in other situations (Mason, 2008). For example, Annelise acknowledged that her thinking had changed from her limited prior understanding of performing operations with decimal fractions. The shift from *discerning details* of the place values within a decimal to then being able to *recognise relationships, perceive properties,* and *reason on the basis of agreed properties* by performing operations with decimals with an understanding of positional value number knowledge, was the result of new learnings that occurred during the Wishball-hundredths game. These changes in decimal understanding were prompted by unique affordances embedded within the web-app including the dynamic counting frame. By being aware of shifts occurring, teachers can be more alert to the possibility that learners are not making appropriate shifts and therefore implement teaching strategies and learning tools such as dynamic digital representations, to prompt these conceptual changes.

Conclusion

Teachers need to use various strategies, including visual representations, manipulatives, real-world examples, and interactive activities to make decimals more tangible and understandable. Examining a group of students completing four task-based interviews revealed the full scope of conceptual understanding of decimal fractions inclusive of decimal place value, relative magnitude and decimal density, which emerged from the students' interactions with four dynamic digital tools. At times, the students' experienced productive cognitive conflict, as the dynamic representation of decimal fractions challenged their existing understandings and misconceptions. Outside of the research-interview situation, these would have been ideal 'teaching moments'. While keeping in mind the inferential limitations of a small-scale study, the findings clearly indicate the potential of dynamic, digital representations to be used as effective learning and teaching tools to develop conceptual understanding of decimal fractions, and to address previously established misconceptions.

Acknowledgements

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