Teacher Expectations of Student Strategies for Algebra Problems

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This paper investigates the expectations that a group of preservice secondary mathematics teachers had for how students would approach four particular algebra problems. Their responses to a set of open-ended items were content analysed. Findings show that the teachers ranged in the expectations that they held for their students. Some teachers expected their students would approach the problems with more sophisticated problem-solving strategies, but many of the teachers expected that the students would only use less desirable "guess-and-check" strategies. We believe teachers' expectations for student strategies for problem-solving is a topic that warrants further investigation.

For about a decade, we have worked with teachers (both preservice and practising) to understand the knowledge bases that are needed to teach their subject and to support their professional knowledge and practices. When analysing data about preservice secondary mathematics teachers' own problem-solving strategies, an investigation captured a thought-provoking issue. The strategies that teachers most frequently expected students to use were "guess-and-check" or "trial and error", despite these approaches being less desirable than more sophisticated problem-solving strategies such as systematic numerical approaches or algebraic methods. Possible strategies to the relevant problems were articulated in a manner that underestimated the students' potential success and creativity in mathematical problem-solving. This stimulated the following research question:

- What expectations for student strategies to algebra problems are evident in preservice secondary mathematics teacher responses?

Every student can solve mathematical problems, so mathematics should be taught with high teacher expectations (National Council of Teachers of Mathematics [NCTM], 2016). The Australian Institute for Teaching and School Leadership (AITSL) standards stipulate that teachers know their students, including the suitable expectations for task difficulty (AITSL, 2017). Knowledge frameworks for mathematical problem-solving provide the foundation for developing teacher education programs and guiding teacher learning about how students think and approach mathematical problem-solving. Some research has investigated teacher expectations for student mathematical capabilities, but little is known about teachers' expectations for student approaches to problem-solving. The following literature shows that research about teachers' expectations of students' strategies for algebraic problem-solving can be important because of the well-established association between teachers' expectations and student academic attainment.

Teachers' Expectations of Students

Broadly, teachers' expectations of students are the beliefs that teachers hold about if, when, and how much students will accomplish academically at school (see Johnston et al., 2024). The expectations that teachers hold for their students can be short term, such as how they will perform on a given learning task, or long term, such as which post-secondary pathways they will take when they graduate (Wang et al., 2019). The association between teachers' expectations and student outcomes is called the 'teacher expectation effect' (Szumski &

(2024). In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 271–278). Gold Coast: MERGA.

Karwowski, 2019). Research on teachers' expectations shows that high teacher expectations correspond with high student achievement, so students benefit from teachers who expect a lot from them academically (Rubie-Davies et al., 2015). Expectations can be accurate, but inaccurate low expectations hinder student results, while inaccurate high expectations can improve student results (Szumski & Karwowski, 2019) as students' responses to the teachers' differential treatment shapes their consequential educational attainment (Johnston et al., 2024).

Teachers' Expectations of Mathematical Problem-Solving

There has been very little research about teachers' expectations for students' mathematical problem-solving. Research in this area has usually been general to mathematics as a learning area, showing that teachers' mathematical expectations can be biased and based on student backgrounds, such as their ethnicity (Lorenz et al., 2016) or gender (Copur-Gencturk et al., 2023). Some teachers have higher expectations for boys' mathematical achievement than girls, but this is not always the case (Soto-Ardila et al., 2022). Classroom and school contextual factors, such as ability grouping and community engagement, can also influence teachers' expectations of students' mathematics abilities (Wang et al., 2019), as can a teacher's perception of their own ability (Copur-Gencturk et al., 2023).

Very few studies have investigated teachers' expectations for students' problem-solving in mathematics. Previous studies of teacher expectations of student mathematics achievement have asked teachers questions about their beliefs about students' potential, showing that teachers communicate these beliefs in ways that shape student achievement in mathematics problem-solving (Fyfe & Brown, 2020; Szumski & Karwowski, 2019). For example, Soto-Ardila et al. (2022) asked prospective teachers whether their students would be able to solve an elementary arithmetic problem while they were practicing in schools. Their findings revealed high teacher expectations of students in private schools, which corresponded to students' eventual high performance. This study did not control for prior achievement or consider how teachers' expectations were developed. However, a controlled experimental study from Fyfe and Brown (2020) found that teachers' expectations for mathematical problem-solving shape students' ability to generalise learning from one problem to another after feedback. Negative expectations caused students to perform significantly worse on these transfer problems, while positive expectations helped students to do better in the face of negative feedback. Copur-Gencturk et al. (2023) determined that when a teacher views a student as having a low ability (as opposed to a low effort) a cause-and-effect feedback loop occurs. For example, the teachers' views of students' helplessness are communicated to the student as a perception that they have a lower ability. Tohir et al. (2020) also investigated beginning teachers' projected expectations for their future students' mathematical thinking processes. Their findings were that the teachers' own approaches to answering the question were not related to their expectations of students, but poor written articulation of the findings made their important work unclear and unconvincing. Copur-Gencturk et al. (2023) found links between a teacher's own mathematical disposition and how they perceived student ability. For example, if a teacher viewed their own difficulties as a lack of ability, rather than as a problem to be solved, it could have a negative effect on what was expected of students. To our knowledge, these are the only studies that have investigated teachers' expectations for students' mathematical thinking or problem-solving specifically.

Interventions designed to raise teacher expectations can improve mathematical achievement of students from migrant and low-SES backgrounds. When teachers with low expectations for student mathematical achievement are encouraged to develop higher specific performance goals for mathematics, student achievement can improve (Ritzema et al., 2016). Thus, further exploration of teachers' expectations about their students' approaches to mathematical problem-solving could be a useful starting point for addressing any deficit or strength-based views of students' mathematical abilities. This paper seeks to establish what the participant teachers' expectations of their students are because these views can shape their future students' academic outcomes when they are enacted.

Methods

The study used to underpin this paper was conducted by the first author with the voluntary participation of Bachelor of Education and Master of Teaching preservice teachers. Both groups of participants were specialising in junior secondary mathematics teaching (Years 7 to 10; aged 12 to 16), and all were enrolled at a Western Australian university. Of the participants, 11 were enrolled in the Bachelor of Education program—a four-year degree at the undergraduate level, and 12 were enrolled in the Master of Teaching degree—a two-year degree at postgraduate level. The participant group was comprised of 9 females and 14 males.

In the two units where data were generated, preservice teachers were supported to enhance their mathematics pedagogy knowledge for Number and Algebra and to develop an understanding of the Australian curriculum structure. Across four weeks there was an explicit focus on the proficiency strand: problem-solving, as a pedagogical technique that would enhance student learning. Each week one of the problems introduced below was explored. Participants were asked to solve the relevant problem and then explain the strategies they would use to solve it in teaching. They were provided with three prompts: the first prompt was to answer the problem in as many ways as possible. In the second prompt, the participants were asked to compare their preferred solution strategies, the strategies that they anticipated students would use, and the strategies they hoped their students would use. In the final prompt, the participants were asked to describe why these problems would be useful to use in teaching and which of the problems they would use. In this paper, we focus on participant responses to the question about how students might solve these problems.

At the time of data collection, all participants had completed at least five weeks' professional experience, during which they planned, taught, and assessed student learning in mathematics. This is of contextual importance as the participants had some practical experience teaching mathematics, using pedagogical approaches to enhance mathematics, and witnessing learning within the students.

Mathematical Problems Used in the Study

The four problems used in this study—adapted from published literature (see Hatisaru et al., 2024)—are referred to in this paper as *any to numbers*, *farmer*, *dice*, and *books*. These problems are typically used (or can be used) in everyday junior secondary mathematics classrooms, and the participants had experience with the style of problems:

- *Any two numbers*: If you are given the sum and difference of any two numbers, show that you can always find out what the numbers are;
- *Farmer*: A farmer had 19 animals on his farm—some chickens and some cows. He also knew that there was a total of 62 legs on the animals on the farm. How many of each kind of animal did he have?
- *Dice*: Die A and Die B have twelve sides each. Suppose that you roll die A and die B at the same time. When do the dice satisfy the following two conditions? The sum of 2 times A plus B equals 15. 3 times A minus B equals 5;
- *Books*: You have some teen and young adult books. You gave one-half of the books plus one to a friend, one-half of the remaining books plus one to another friend, and one-half of the remaining books plus one to another friend. If you have one book left for you, how many books did you have at the start?

Each of these problems can be solved in multiple ways, which reveals not only the extent to which the participants were able to generate different strategies, but also the strategies they anticipated that students would use. Algebraic approaches are suitable for solving these problems, but they can also be solved by using numerical approaches that follow a systematic or logical reasoning process. Table 1 presents each of these approaches along with their descriptions, and Figures 1 and 2 capture example participant solutions where some of these strategies were used.

The last group of strategies in Table 1 refers to those that are less desirable because they are unsystematic or random numerical trials. Whilst systematic numerical approaches can provide a sufficient basis for solving these problems, strategies such as random guess-and-check or similar concrete methods (e.g., rolling dice and checking combinations) are not as desirable as systematic numerical approaches or algebraic strategies because they demonstrate no insight into the question. To truly demonstrate understanding of a problem, teachers and students should use a range of mathematical thinking processes and choose the most viable strategy, or combination of strategies, for the problem to be solved. The more logical and integrated the mathematical processes, the more sophisticated the thinking.

Table 1

Example Strategies for Solving 'Any Two Numbers', 'Farmer', 'Dice', and 'Books' (Reported in Hatisaru et al., 2024)

Strategy	Description
Equations	
Symbolic solving	Write algebraic equations and solve using standard algebraic method
Numerical solving	Write algebraic equations and solve numerically
Graphical solving	Write algebraic equations and plot to find the intersection point.
Using parameters, symbolic solving	Write algebraic equations with two unknowns and with two parameters and solve using standard algebraic methods
Using parameters, numerical solving	Write algebraic equations with two unknowns and with two parameters and solve for a specific example
No parameters, symbolic solving	Write algebraic equations with two unknowns but with two specific numbers and solve using standard algebraic methods
Pattern	Write algebraic equations with two unknowns and with selected specific sums and differences, solve using any method and look for a pattern linking solutions to sum and difference.
Numerical	
Systematic	Use a numerical path in a systematic way such as guess-check-and-improve or guess-and-check with tables
Logical arithmetic reasoning	Think about the relations between the numbers/quantities involved and work from known numbers towards the solution
Other less desirable strategies	Guess-and-check; concrete approaches such as rolling dice or counting

Data Analysis Approach

There were four problems and 22 participants; a few participants were absent in the class when the relevant problem was posed. So, in total, there were 70 responses. Unsurprisingly, most participants generated more than one solution to the problems—because they were asked to solve the problem in different ways—and that means in total we had 128 solutions for the four problems to analyse. Solutions could be 'complete', 'incomplete', 'erroneous', or 'suggested' without implementation.

The data were analysed by the first author of this paper who has extensive experience in content analysis (e.g., Hatisaru et al., 2024). She first determined the success rate; that is, if at least one of the solutions to the problem was complete and a correct answer was given. Next, she examined these solutions to find out what strategies the participants used in solving the problems. She classified each of the 128 solutions into solution strategies presented in Table 1. To illustrate this categorisation, example solutions are presented in Figures 1 and 2; strategy types mapped to them are noted. Participants were assigned codes: P1, P2, P3, P4, and so on to protect their anonymity.

To uncover the participants' expectations for student strategies to these problems, the author analysed participant responses to the relevant prompt in the same fashion. She coded the strategies addressed by the participants as possible strategies that student may apply to each problem. Some participants named more than one strategy, and each was counted. All authors reviewed and agreed on the data analysis.

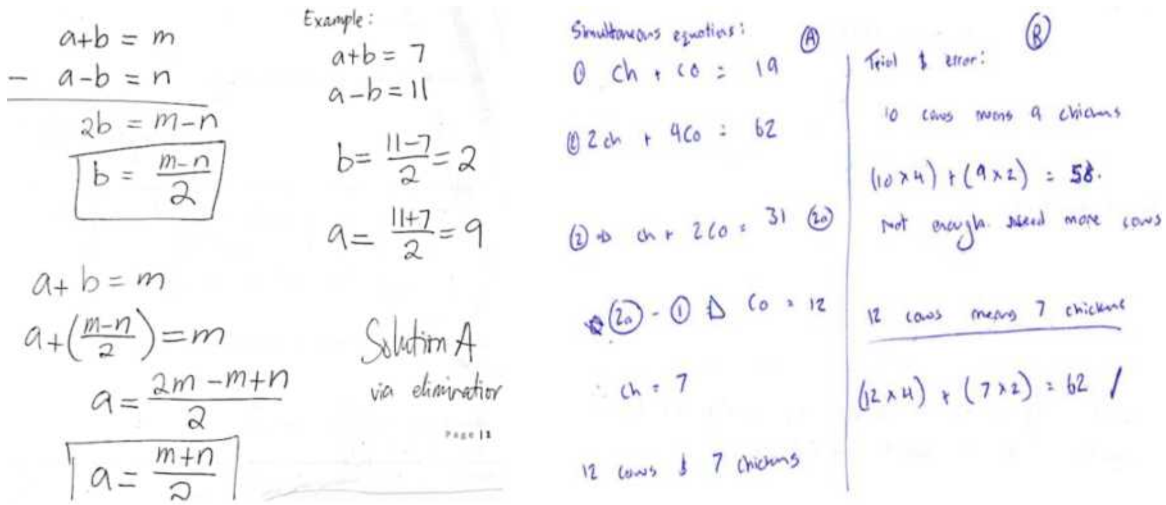
Results

Out of 70 solutions, 61 of them included a correct answer by at least one solution strategy (see Figures 1 and 2 for examples) and 4 responses were incorrect. Partially correct was only relevant for *any two numbers* when 5 participants solved the problem based on examples without generalising the solution to any sum and difference.

As noted earlier, a total of 128 solutions were identified for the four problems. Out of these, 13 were suggested solutions without showing any implementation, while 11 were erroneous responses where the participant had made incorrect assumptions about either the problem or the solution strategy (or both). Of the remaining 104 solutions, 7 were incomplete, while the remaining 97 were implemented correctly (76%). This means that most participants were able to solve the problems and also identify other possible ways to solve them.

Figure 1

Solutions of P16 and P12 to the 'Any Two Numbers' and 'Farmer' Problems



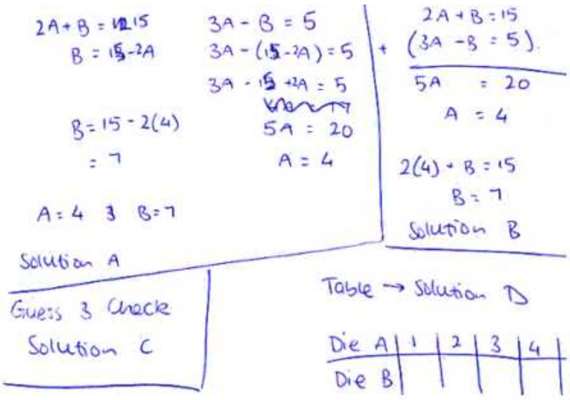
P16's solutions to any two numbers, including the equations, parameters, symbolic solving strategy.

P12's solutions to farmer, including the equations, symbolic solving (left) and numerical, systematic (right) strategies.

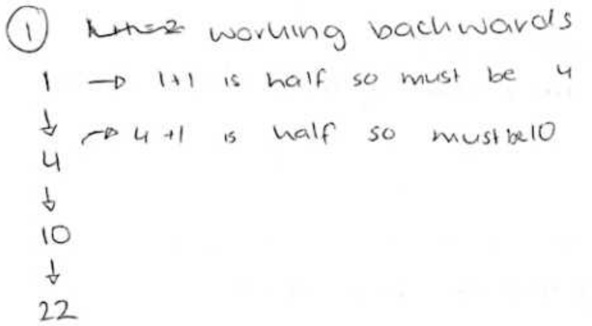
Focusing on participant responses where at least one of the identified—implemented or suggested—strategies to the relevant problem was correct (total of 114), we aimed to uncover the expectations of these participants with regard to how students would approach these problems. We excluded the other 14 responses as it was unlikely that the participants could anticipate sound student strategies where their own solution strategies were incorrect or erroneous. Table 2 summarises the number of strategies identified in the participants' solutions across all four problems, and in their responses where they anticipated student solutions.

Figure 2

Solutions of P9 and P3 to the 'Dice' and 'Books' Problems



P9's solutions to *dice*, including the *equations, symbolic solving* strategy.



P3's solution to *books*, including the *logical arithmetic reasoning* strategy.

Table 2

Distribution of Strategies in Participants' own and Anticipated Student Solutions

Strategy	In participant own solutions (114)	In anticipated student solutions (99)
Equations		
Symbolic solving	Farmer (20); Dice (19); Books (10)	Farmer (9); Dice (6); Books (6)
Numerical solving	Farmer (1); Dice (7)	Farmer (1); Dice (2);
Graphical solving	Dice (1)	Dice (1)
Using parameters, symbolic solving	Any two numbers (17)	Any two numbers (1)
Using parameters, numerical solving; no parameters, symbolic solving; pattern	Any two numbers (8)	-
Numerical		
Systematic	Farmer (9); Dice (4); Books (1)	Any two numbers (3); Farmer (7); Dice (1)
Logical arithmetic reasoning	Farmer (3); Books (4)	Books (3)
Other less desirable strategies	Any two numbers (2); Farmer (1); Dice (6); Books (1)	Any two numbers (18); Farmer (18); Dice (12); Books (11)

Out of 114 identified strategies for solving these four problems, many of them are viable or desirable strategies in the sense that they would give the correct answer, and only 10 of them (9%) classified as "other less desirable strategies". As opposed to this, out of 99 participant anticipations of student approaches, the majority (59/99 or 60%) are less desirable strategies for these problems, while only 26 are viable strategies.

Our analysis shows that for each of the four problems, and across the distribution of strategies, the participants repeatedly anticipated that the students would use less viable strategies. This highlights that the participants expected students to use strategies which would give less effective, complete, or correct solutions.

Discussion and Conclusion

Our findings from this study demonstrate that many of the participants held low expectations for their future students' mathematical strategies for algebra problem-solving. Low expectations of students are problematic, because previous research has shown that teachers who have low expectations of students' mathematical ability manifest low achievement through the teacher expectation effect (Szumski & Karwowski, 2019). Educators working in initial teacher education might consider strategies to raise teachers' expectations for students' mathematical problem-solving.

Previous research has shown that teachers with low expectations for students can treat their students in ways that shape the students' respective achievements in mathematics (Ritzema et al., 2016). This literature highlights how teachers frame mathematical problems for students in the ways that reflect their expectations, describing some problems as achievable and others as less likely to be achieved by students (Fyfe & Brown, 2020). Teachers' expectations can be conceptualised in terms of lower or higher order anticipated strategies that students will use to solve prospective mathematical problems (Tohir et al., 2020). Using this conceptualisation of students' problem-solving approaches, we determined that the expectations of the participants in this study were low. The strategies that they anticipated the students would use were categorised according to their sophistication and suitability. We found that many of the participants expected less desirable strategies to be employed by their students. These expectations about less desirable approaches to problem-solving need to be addressed so that students are set up for success in problem-solving.

Assessing the expectations that preservice (and practising) mathematics teachers hold for their own and their students' approaches to mathematical problem-solving could be an important first step in addressing any deficit views these teachers may hold. Previous literature about teachers' mathematical expectations has suggested that high teacher expectations can improve students' academic outcomes and encourage successful approaches to problem-solving (Ritzema et al., 2016). Our findings add to this literature by showing that teachers may bring pre-existing ideas about students' low mathematical problem-solving abilities to initial teacher education courses. If academics working in initial teacher education institutions know that their preservice teachers do not have high expectations for their future students, intervention strategies might be developed and employed to address this problem. Existing literature highlights the capacity to raise teacher expectations through interventions that create awareness about expectations and the practices of high expectation teachers (de Boer et al., 2018).

The limitations of this study should be noted, including that exploring the participants' expectations for student mathematical problem-solving in algebra was not the original intent of the research that informed data collection. The participants wrote responses on a written test and had not been interviewed, so rich data with explanation of their expectations was not available for analysis. Thus, questions about how these views were developed and more in-depth understanding of the expectations is not provided here. Interviews explaining the preservice teacher responses could be a potential avenue for further research. Interviews could also uncover whether participants' responses were based on students' actual performance in algebra problems while they were on placements, and if/how this was related to any characteristics of the schools and classes they had been based in.

Teachers' self-concept about their own mathematical ability and their mindset (either growth or fixed) can impact the level of the expectation they hold for their students (Copur-Gencturk et al., 2023). While the participants in this study had high mathematical knowledge themselves, they had limited teaching experience. We would be interested to know more about how these preservice teachers have developed varying expectations of their students, and how these varying expectations would shape their practice. Future research might also consider further exploration of methods for evaluating preservice teachers' expectations for their students' mathematical problem-solving in algebra and might consider other expectations that they hold of their future students. Such research might explore how these expectations can be raised through interventions, when necessary. Interventions that raise preservice teacher expectations could lead to students' benefitting from targeted high teacher expectations in mathematics teaching contexts.

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References

Australian Institute for Teaching and School Leadership. (2017). *Australian professional standards for teachers*. AITSL. https://www.aitsl.edu.au/teach/standards

Copur-Gencturk, Y., Thacker, I., & Cimpian, J.R. (2023). Teachers' race and gender biases and the moderating effects of their beliefs and dispositions. *International Journal of STEM Education*, *10*, 31.

de Boer, H., Timmermans, A. C., & van der Werf, M. P. C. (2018). The effects of teacher expectation interventions on teachers' expectations and student achievement: Narrative review and meta-analysis. *Educational Research and Evaluation*, *24*(3–5), 180–200. https://doi.org/10.1080/13803611.2018.1550834

Fyfe, E. R., & Brown, S. A. (2020). This is easy, you can do it! Feedback during mathematics problem-solving is more beneficial when students expect to succeed. *Instructional Science*, *48*(1), 23–44.

Hatisaru, V., Stacey, K., & Star, J. (2024). Mathematical connections in preservice teachers' solutions to algebra problems. *Avances de Investigacion en Educación Matemática (AEIM)*, *25*, 33–55. https://doi.org/10.35763/aiem25.6354

Johnston, O., Wildy, H., & Shand, J. (2024). A grounded theory about how teachers communicated high expectations to their secondary school students. *European Journal of Psychology Education*, *39*, 211–235.

Lorenz, G., Gentrup, S., Kristen, C., Stanat, P., & Kogan, I. (2016). Stereotype bei lehrkräften? Eine untersuchung systematisch verzerrter lehrererwartungen [Stereotypes among teachers? A study of systematic bias in teacher expectations]. *KZfSS Kölner Zeitschrift für Soziologie und Sozialpsychologie*, *68*(1), 89–111.

National Council of Teachers of Mathematics. (2016). *High expectations in mathematics education. A position of the National Council of Teachers of Mathematics*. NCTM.

Ritzema, E. S., Deunk, M. I., Bosker, R. J., & van Kuijk, M. F. (2016). The relation between teacher-set performance goals and students' mathematical achievement. *Studies in Educational Evaluation*, *51*, 17–28.

Rubie-Davies, C., Peterson, E., Sibley, C., & Rosenthal, R. (2015). A teacher expectation intervention: Modelling the practices of high expectation teachers. *Contemporary Educational Psychology*, *40*(C), 72–85.

Soto-Ardila, L. M., Caballero-Carrasco, A., & Casas-García, L. M. (2022). Teacher expectations and students' achievement in solving elementary arithmetic problems. *Heliyon*, *8*(5). https://doi.org/10.1016/j.heliyon.2022.e09447

Szumski, G., & Karwowski, M. (2019). Exploring the Pygmalion Effect: The role of teacher expectations, academic self-concept, and class context in students' math achievement. *Contemporary Educational Psychology*, *59*, 101787. https://doi.org/10.1016/j.cedpsych.2019.101787

Tohir, M., Maswar, M., Atikurrahman, M., Saiful, S., & Pradita, D.A.R. (2020). Prospective teachers' expectations of students' mathematical thinking processes in solving problems. *European Journal of Educational Research*, *9*(4), 1735–1748.

Wang, S., Rubie-Davies, C. M., & Meissel, K. (2019). Instructional practices and classroom interactions of high and low expectation teachers in China. *Social Psychology of Education*, *22*(4), 841–866.