Learning Mathematics Through Sequences of Connected, Cumulative, and Challenging Tasks: A Self-Determination Theory Perspective

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The current paper overviews a nine-month PhD study that investigated the impact of learning mathematics through sequences of challenging tasks on the mathematical competence and attitudes of Year 2 students (n = 59). Adopting a Self-Determination Theory lens, a pragmatist paradigm and a mixed-method design, the study found that at all levels of investigation and analysis, the experience of learning through sequences of connected, cumulative, and challenging tasks had positive benefits for Year 2 students.

Substantial evidence suggests that teaching mathematics through problem-solving approaches improves student outcomes (Schoenfeld, 2007; Sinha & Kapur, 2021). However, debate exists as to how early in their schooling students should be exposed to and engage in problem-solving approaches as part of their learning. In Australia, this issue has received recent attention in terms of the overall decline in both academic performance and student engagement in mathematics (Thomson et al., 2021). One of the key arguments in delaying exposure to problem-solving approaches is based on the premise that students should first develop and be able to demonstrate aptitude with basic skills, facts and procedures, taught through traditional methods, before being introduced to more complex mathematics problems (Kirschner et al., 2006). This stance is founded on an assumption that without established foundational knowledge, students' disengagement in mathematics could be exacerbated as a consequence of the difficulties encountered through less direct instruction (Westwood, 2011).

Alternatively, others advocate that it is through problem-solving approaches that students not only construct strong foundational mathematics knowledge but also develop greater mathematics proficiency (Baroody et al., 2007; Schoenfeld, 2007). This perspective aligns with the stance that simply replicating mathematics procedures does not automatically transfer to an effective application of knowledge (Schoenfeld, 2007). Posing appropriately challenging tasks, orientated through student-centred inquiry pedagogies, has been shown to increase student engagement and motivation to learn mathematics with students of secondary school age (Boaler, 2016; Gresalfi et al., 2009). The active instructional moves used in structured inquiry pedagogies, such as guiding questions and the facilitation of rich classroom discussions, offered older students opportunities to develop reasoning skills, strategize beyond traditional methods, and collaborate with peers (Chan & Clarke, 2017). Given these benefits, there is a need to investigate whether these findings are replicated when younger students are engaged in problem-solving approaches implemented through structured inquiry pedagogies.

The current study was conducted as part of a larger research project led by Emeritus Professor Peter Sullivan and colleagues titled Exploring Mathematical Sequences of Connected Cumulative and Challenging Tasks (EMC^3). This project encouraged the introduction and implementation of sequences of challenging tasks to students in Foundation to Year 2 (5 to 8 years old) by supporting teachers to utilise the EMC^3 materials within their mathematics programs (Sullivan et al., 2020). The use of challenging tasks (Sullivan et al., 2015), is one approach to teaching mathematics that incorporates active engagement in problem-solving, explored through a structured inquiry pedagogy. Often posed as non-routine questions within authentic contexts, challenging tasks provide opportunities for students to build on their prior

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knowledge, demonstrate persistence, and develop mathematics connections by thinking flexibly about concepts (Sullivan et al., 2020). To date, much of the research on challenging tasks has focused on teacher professional development (Ingram et al., 2020; Sullivan et al., 2015) and student responses from the middle to high school years (Russo & Minas, 2020; Sullivan & Mornane, 2014). Less is known about the ways students in the early years of schooling (aged 5 to 8 years old) respond to the experiences of learning mathematics through challenging tasks. While Russo & Hopkins (2017) found that Year 1 and 2 students respond positively when self-reflecting on the use of challenging tasks in mathematics, overall research studies reporting on challenging tasks with children of this age is limited.

The aim of the current PhD study was to investigate how ongoing exposure to learning through sequences of challenging tasks influenced the mathematical competence and attitudes of Year 2 students; whilst the goal of the current paper is to overview the study and share key findings with the MERGA community. Readers whose interest is piqued by the contents of this necessarily short overview are encouraged to read the full dissertation (see Hubbard, 2024).

One overarching research question guided the study, focused on the holistic nature of student learning: *How do sequences of connected, cumulative and challenging tasks shape Year 2 students' experience of learning mathematics?* However, recognising the distinctive attributes unique to each of mathematical competence and attitudes towards challenge, the following two additional subsidiary questions further supported the inquiry:

- How does learning through the EMC^3 project approach support the development of Year 2 students' competence in mathematics?
- How does learning through the EMC^3 project approach influence Year 2 students' attitudes towards challenging tasks?

Theoretical Framework

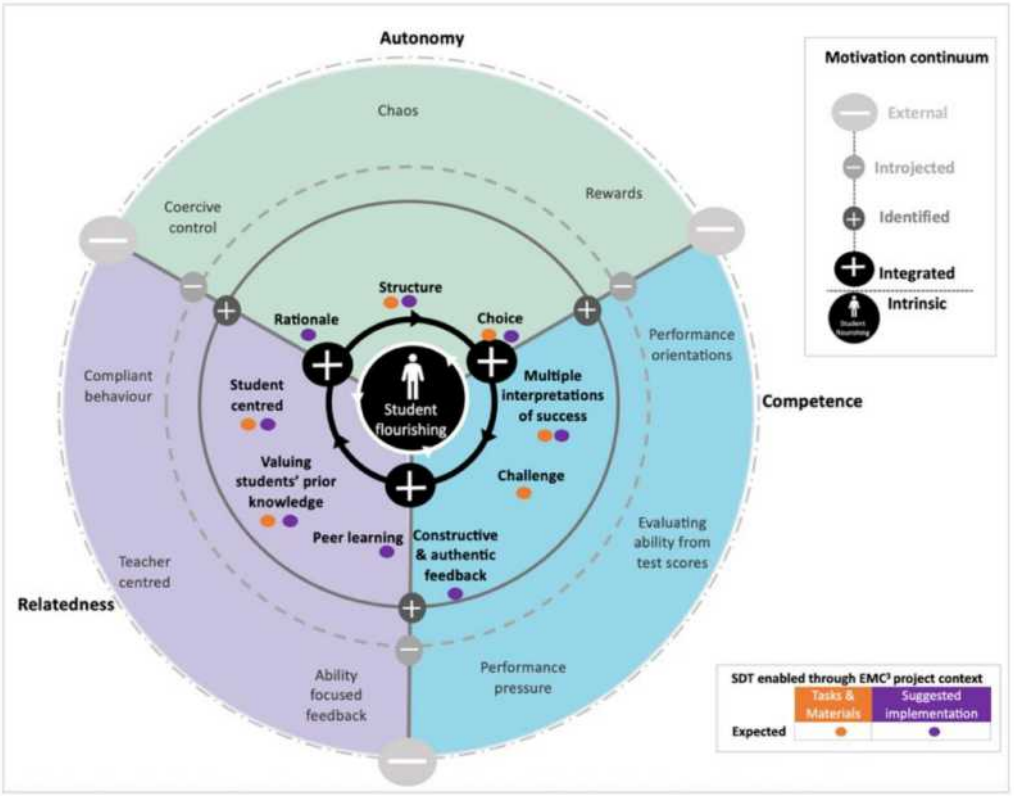
The theoretical framework Self-Determination Theory (SDT) (Ryan & Deci, 2017, 2020) underpinned the current study. SDT is grounded in the belief that learning outcomes are likely to be positive when the basic psychological needs of autonomy, competence and relatedness are satisfied. These needs are dynamically interconnected whereby the satisfaction of one can in turn satisfy another. Autonomy is generally interpreted as having a desire to carry out particular actions with a sense of ownership. In a learning context, autonomy is satisfied when experiences align with interests or values and can be impeded when actions are externally controlled. Competence is satisfied through associated feelings of mastery, success, and growth. From this perspective, a sense of competence is framed as a process of ongoing development and effectiveness, rather than the attainment of a particular static outcome. Finally, the need for relatedness is underpinned by the desire to connect with others through feeling valued and accepted within social environments (Ryan & Deci, 2017, 2020).

The extent that these basic psychological needs are considered satisfied is dependent on a range of motivational tendencies that derive from positive or negative influences present within various learning conditions (Ryan & Deci, 2020). Interpreting student learning through this multifaceted lens aligns with key concerns of engagement and proficiency that are deemed critical in supporting mathematics development (Watt et al., 2017). The EMC^3 approach offered many prospective opportunities for students to develop a sense of competence, autonomy, and relatedness. In particular, the use of non-routine tasks where a solution is not immediately apparent challenged students to transfer prior knowledge to new situations, deepening conceptual understanding and supporting mathematical competence. Similarly, a sense of autonomy within the project approach manifested through, for example, students being offered choices in terms of how they would like to solve tasks and represent their thinking, while many of the features that constituted relatedness were addressed through the suggested lesson structure and proposed classroom norms. Therefore, aligning SDT and EMC^3 served the

complementary purpose of developing holistic understandings of mathematics learning in order to better inform the implementation of challenging tasks in the early years of schooling. Drawing upon the SDT literature, Figure 1 was developed to show how the design features of the EMC^3 project are closely aligned to classroom conditions that are deemed integral for inducing positive motivational tendencies to enable student flourishing. It was proposed that using an SDT lens to investigate students' experiences as participants in the EMC^3 project creates an opportunity to develop greater insights into the ways learning through sequences of challenging tasks supports student flourishing in mathematics.

Figure 1

Explicit Theoretical Links Between the EMC^3 Project and SDT



Study Design

This PhD study was based on a pragmatist paradigm and utilised a parallel mixed-methods design to investigate the holistic experiences of Year 2 students as they learned mathematics through sequences of challenging tasks over a nine-month period. The quantitative component focused on the shifts in mathematical competence and attitudes towards challenge of the Year 2 cohort (n = 59) collected through a written mathematical assessment and attitude questionnaire. The process of analysing the written assessment responses resulted in the creation of a unique seven-point marking-key that not only identified different levels of content knowledge progression, but also showcased the problem-solving skills students utilised as they worked through the sequenced non-routine assessment items. The mathematical competence and attitude data from Phase 1 was combined to establish learning profile charts, representative of the overall influences that learning through sequences of challenging tasks had on the cohort.

Phase 2 comprised six focus students (n = 6) that were selected from the wider Year 2 cohort as participants for the qualitative component of this investigation. Monitoring the shifts in the focus students' mathematical competence throughout the study was determined in two ways. The first consisted of a one-on-one assessment context where observations about

behavioural tendencies were documented noting the ways students worked through non-routine assessment items. The second interpretation of mathematical competence derived from classroom settings. Here, data collected from lesson observations, work samples, and interview responses were triangulated into learning artefacts that provided time bound insights into the dispositional behaviours students exhibited at different stages of challenging tasks lessons. Together, these insights enabled holistic interpretations of student progress that would not have been possible to ascertain from the analysis of single written responses or traditional assessment processes.

Throughout Phase 2, students were given a variety of opportunities to self-report their attitudes in relation to their experiences of learning mathematics within the EMC^3 context. The qualitative data collected included one-on-one response tasks, interviews, and lesson evaluation reflections, as well as behavioural observations documented throughout lessons and assessments. In adopting an iterative reflexive thematic analysis approach, it was possible to deduce clear interpretations about the ways students' attitudes towards challenging tasks were influenced over the nine-month study.

Further to this, additional in-depth case studies (n = 2) were investigated and reported based on the findings from the initial Phase 2 qualitative analysis. These students were selected according to their Phase 1 learning trajectory, developed by comparing their learning profiles to the Year 2 median. One of the affordances in compiling and reporting on these individual case studies in such depth was obtaining insights into the reflexive relationships that emerged between mathematical competence and attitudes that had not been possible to conclude through the other analysis processes conducted initially.

Summary of the Findings

Mathematical Competence

Overall, the reported findings concluded that at all levels of investigation and analysis, the experience of learning through sequences of connected, cumulative, and challenging tasks had positive benefits for Year 2 students. In Phase 1, the quantitative findings reported substantial evidence that students developed basic mathematical knowledge and facts. Furthermore, the majority of students demonstrated improved problem-solving skills, and were able to demonstrate flexible thinking and enhanced reasoning skills. The process of creating a written assessment instrument that simulated the actual learning experiences of students generated insights into broader interpretations of mathematical competence that would not have been possible to achieve through the use of traditional style assessments. The quantitative analysis that derived from the seven-point marking-key codes not only enabled specific learning progressions to be identified, but also offered clear interpretations about student efficacy, cumulative progress, as well as future learning needs relating to higher-order thinking skills (see Hubbard et al., 2022).

The Phase 2 qualitative analysis expanded on the Phase 1 findings, by including the dispositional behaviours students exhibited throughout the EMC^3 learning experiences. This provided critical insights into the development of students' problem-solving proficiency and reiterated there is much to interpret in-situ rather than relying on the evaluation of written output. Observing these behaviours throughout both individual assessment settings and lesson contexts generated holistic interpretations of mathematical competence that better explained how students utilised their mathematical knowledge and skills, rather than simply what they knew. The most informative of these processes occurred as a result of establishing learning artefacts which reflected different elements of mathematical competence that students demonstrated at various stages of learning. Such findings showed that when learning through structured inquiry approaches, students were cognisant of using particular strategies and skills

at different times, demonstrating how their thinking shifted as the lesson progressed. It became clear that there were distinct patterns between the dispositional behaviours students demonstrated and the comprehensiveness of their written responses when problem-solving. Accounting for these behaviours has not traditionally formed part of previous mathematics assessment processes (Hubbard, 2023).

Self-Reported Attitudes Towards Challenging Tasks

Through both phases of the study, the Year 2 students' self-reported mostly positive attitudes towards challenging mathematics experiences. The quantitative analysis showed there was some evidence of improved attitudes towards challenging tasks, with the majority of students consistently reporting positive attitudes throughout each data cycle. However, the limited nature of the data collection process (student questionnaire) in this phase prohibited further conclusions to be drawn.

The qualitative analysis revealed that while the focus students' attitudes towards challenging tasks remained positive throughout the study, there were distinct shifts in the motivational origins that influenced these changes. Students reported satisfaction in learning to identify and overcome mistakes and could articulate how such experiences improved their mathematical understanding over time. The learning conditions created through ongoing and consistent implementation of the EMC^3 project approach supported students to broaden their parameters as to what worthwhile and positive learning experiences entailed. As such, by the end of the study the focus students recognised that positive experiences for learning encapsulated more than superficial interpretations of success and fun.

The Interconnectedness of Learning Pathways When Problem-Solving

Synthesising the findings through the lens of SDT provided a means to authentically combine mathematical competence with student attitudes enabling the learning pathways of students to be interpreted holistically. Using this framework, it was possible to identify specific turning points in students' learning experiences that explained the trajectory of their learning over the nine-month study. This process reinforced the notion that there were multiple pathways for students' mathematical improvement which were contingent on more than the evaluation of mastery of content knowledge.

The in-depth case studies reported in Phase 2 of the study demonstrated that even when students presented as having similar learning needs according to written assessments, the conditions that proved influential in their eventual progress varied significantly. Using the SDT framework it was possible to track not only the ways students' basic psychological needs were being satisfied through their learning experiences, but also to identify the motivational tendencies that influenced students' initial engagement. Especially noteworthy was that for both of the in-depth cases, the catalyst for overall mathematical improvement emphasised the enabling conditions that acknowledged and validated their unique preferences as learners. These insights are invaluable in better understanding how mathematics instruction can be accurately tailored to effectively support all learners to improve their capacity as problem solvers in ways that are holistic and student-centred. Collectively the findings summarised from this research raise several implications for future research and practice that are presented next.

Conclusions, Future Research and Implication for Practice

This current paper has served to overview a nine-month PhD study that investigated the experiences of Year 2 students as they learned mathematics through sequences of challenging tasks (see Hubbard, 2024). While the conclusions derived from this dissertation contribute to an understanding of the ways problem-solving approaches can be effectively implemented within the early years of schooling, it is also recognised they are representative of only a single

study conducted within a single context. As such, suggestions for further research that expand on the findings from this study are offered.

Aligning the SDT literature (see Ryan & Deci, 2017, 2020) with the EMC^3 project (Sullivan et al., 2020) to explicitly highlight theoretical links (see Figure 1) offered a unique means of tracking student learning pathways that accounted for their internalised motivation, attitudes towards challenge, and mathematical competence. There is considerable scope to conduct further research using this proposed framework by attending to students that present with alternative learning profiles to the ones presented in this study. Such analysis would enable greater insights into the learning conditions that support students' self-determination satisfaction and enable teachers to more appropriately meet the diverse needs students present when they are learning mathematics through problem-solving approaches.

Similarly, the proposed framework enabled students' attitudes towards challenging tasks to be interpreted through the theoretical lens of the motivation continuum. Identifying if students' internalised motivation for learning mathematics was positively or negatively orientated provided an additional perspective through which their experiences within the EMC^3 project could be contextualised. Given this was reported in-depth only for the two student case studies, further research is recommended to verify:

- Whether such analysis processes are feasible at a larger scale and applicable in other contexts (e.g., upper primary school or secondary school);
- The extent that this process offers insights into students' attitudes towards mathematics that would otherwise remain undiscernible.

There were many findings that became apparent in only the latter stages of the study, such as the emergence of students' higher order thinking skills and the noticeable demonstrations of their productive behaviours for problem solving. Replicating this study but extending the timeframe to include longitudinal data would offer the potential to gain greater insights into these two aspects of student learning through problem-solving approaches. Additional advantages of designing a longitudinal version of this study would be in the inclusion of system wide assessment data for comparative purposes.

Finally, the assessment processes described in this thesis were constructed and refined as part of this particular study design and analysis approach. Therefore, they are considered novel and emergent in terms of their overall utility beyond the context of this investigation. Further research focusing on the assessment processes and structures generated through this study should endeavour to:

- Determine the suitability of adapting the written assessment instrument and marking-key codes beyond a Year 2 context;
- Evaluate the utility of these instruments and processes beyond the EMC^3 context;
- Verify and refine the dispositional competence elements that are derived from the classroom context in order to develop practical observation guides with the aim to support teachers in developing and using holistic formative assessment practices.

This research also raises important implications for both policy and practice when considering the affordances and constraints in introducing and sustaining effective problem-solving approaches in the early years of schooling. Although it is beyond the scope of the current paper to detail all such recommendations, two key sets of implications relevant for education systems, schools, and teachers are outlined below. The first set pertains to the nature of student learning experiences in mathematics, while the second set concerns corresponding assessment processes.

First, to establish and maintain supportive environments conducive to meeting students' needs at various points throughout their learning trajectory, it is recommended that an ongoing commitment is made to regularly and consistently provide opportunities for students to

experience authentic problem-solving tasks as part of a balanced mathematics instruction. It is suggested that these problem-solving tasks are implemented through structured inquiry pedagogies that enable teachers to scaffold the learning experience in response to students' prior knowledge, responses to feedback, and collaborative class discussions. When interacting with students within such structures, teachers should strive to be constructive rather than evaluative when responding to students' thinking. Moreover, given that it was the ongoing and consistent exposure to learning through the EMC^3 approach that appeared to underpin the recalibration of students' attitudes towards challenging tasks, it is recommended that students of all abilities are provided with appropriate levels of mathematical challenge. This should include all students being given adequate time and space to adjust to working through a challenging task without direct instruction from the teacher. Doing so facilitates authentic opportunities to develop persistence and resilience as part of mathematics learning. These first-hand experiences offer scope for students to develop a better appreciation of what concepts such as growth mindset actually entail.

Second, to move away from outcome focused measures of achievement and towards more holistic interpretations of student progress, it is recommended that tiered assessment approaches are adopted that better reflect the holistic nature of problem-solving learning processes. In particular, when written assessments are used, they should preference the use of open-ended and non-routine items inclusive of problem-solving strategies to ascertain an accurate interpretation of the broad skills students apply and demonstrate as part of comprehensive written responses. When designed in this way, the instruments offer increased longevity and relevance to student learning that expands beyond evaluations of content knowledge attainment. As an alternative to written assessments, teachers should adopt lesson observation protocols that detail the nuanced but important productive behavioural tendencies students are likely to exhibit throughout problem-solving experiences. Utilising these observation protocols regularly will attune teachers into better recognising the range of behaviours students demonstrate at different stages of their learning process. It is important that such observation protocols aim to represent the variety of ways students may behave when solving problems to include both individual and collaborative experiences that occur frequently throughout structured inquiry pedagogies. Having a greater sense of what behaviours are productive and when serves to better inform the instructional choices of teachers in-situ, resulting in teaching practise that is more responsive to students' learning needs.

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