Understanding Australian Teachers' Conceptualisation of Angle

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In geometry, the concept of angle is complex and multi-faceted. One way of understanding this complexity was provided by Mitchelmore & White (1998) when they argued that three different classes of angle definition are present in schools at years three to eight. These are: the dynamic concept of turning about a point between two lines; and the static concepts of a pair of rays with a common endpoint and a region formed by two intersecting half-planes. These definitions are referred to as *turn*, *geometric* and *region* concepts for angle.

Vinner's (1991) theory of concept image and *concept definition* provides a framework for understanding angle complexity. He argues that students learn geometry primarily through the formation of concept image that comprises mental pictures and their associated properties and processes, as well as word and symbol strings that students associate with a concept. Formally structured mathematics is deductive, being based on theorems and axioms. Although concept image is central to geometry learning, it is not sufficient. The mathematics teachers' role is to ensure that student concept image interacts with concept definitions so that difficulties and misconceptions can be overcome as students learn.

Mitchelmore & White (1998) argue that no single definition can completely define angle. To understand angle students need to operate the different concepts for it simultaneously. This understanding needs to be acquired through experiencing many different angle contexts and abstracting what is common between them. Eventually students develop a concept image for angle which Mitchelmore and White (1998) refer to as the *standard angle concept* of "two lines meeting at a point with some relation between them (p. 5)." This concept image can be used in all angle situations as a mental model to identify and measure angle magnitude.

Research has shown that teachers and students have a shallow and fragmented understanding of angle (Smith & Barrett, 2017). The author's PhD research studies how Australian teachers conceptualise angle and how enhancements to that understanding might impact teaching practice. The research involved a survey of 43 Australian teachers (primary and secondary), which establishes a baseline of understanding for the study of how teachers conceptualise angle, how they view problems with angle teaching referred to in extant research and their interpretation of aspects of the curriculum related to angle.

This presentation reports on findings from the survey of teachers related to their concept image and concept definition for angle. Although the majority (72%) of teachers in the survey gave a written definition for angle used in their teaching, only 33% defined angle in both static and dynamic terms, as conceptualised by Mitchelmore and White (1998). Teachers' concept image for angle was classified in the authors' survey. The presentation discusses some apparent disconnects that were found between the concept image and concept definitions for angle reported by survey participants. Discussion will include the potential for teaching a broader and deeper understanding of geometry through enhanced concept image of angle.

References

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