Pre-Service Teachers' Use of Jump Strategy on the Empty Number Line When Recording Micro-Teaching Videos

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This paper is part of a broader study which explores South African pre-service teachers' use of the jump strategy on the empty number line for enhancing their confidence to do and teach mental mathematics computation strategies. The focus of this paper is the use of micro-teaching in the form of video recordings by pre-service teachers. Forty videos were analysed thematically indicating that most pre-service teachers could implement the strategy with some level of fidelity. Three micro-teaching videos are shared in this paper to exemplify the range of strong, medium, and poor implementation fidelity.

The South African Department of Basic Education (South Africa. DBE, 2020) rolled out resources developed from the national Mental Starters Assessment Project (MSAP), intended for improving learners' mental mathematics and number sense skills (Graven et al., 2020). An offshoot of this project is the Mental Mathematics Work-Integrated Learning (MM-WiL) project, directed at preparing pre-service teachers (PSTs) for using the MSAP materials to effectively teach mental mathematics, with the additional benefit of improving their own confidence and competence in performing mental mathematics calculations. The broader study explores how South African PSTs use the jump strategy on the empty number line. This paper specifically aims to answer the research question, 'How do PSTs use the jump strategy on the empty number line when recording micro-teaching videos?' As mental calculations and number sense are vital for understanding mathematics (Bobis, 2007), I posit that supporting PSTs in learning how to teach strategies such as the jump strategy on the empty number line through the supportive and engaging context of micro-teaching videos is of interest to mathematics teacher educators internationally.

Background and Rationale

Literature indicates that South African teachers lack the necessary mathematical and pedagogical knowledge, support, and confidence, and are subsequently hindered by poor morale (for example, Graven & Heyd-Metzuyanim, 2014; Hoadley, 2012; Jojo, 2019; Spaull, 2019; Venkat & Graven, 2017). Studies on South African PSTs further show that their mathematical knowledge is lacking, and that there is little improvement during the four years of completing their BEd degree (Bowie et al., 2019; Fonesca, 2018). Fourth year PSTs in Bowie's study were unable to achieve a mean score over 50% for higher cognitive demand items across all mathematics topic areas, and she claims these results mirror the knowledge and proficiency of in-service teachers. Bowie (2019) therefore argues that mathematics teacher educators "cannot take it for granted that student teachers will arrive with a command of the mathematics they will be expected to teach" (p. 295). In the Australian context, Beswick and Goos (2012) similarly found that it could not be taken for granted that students entering as PSTs have fundamental mathematics knowledge and skills. PSTs are shown, across international studies, to have low number sense (e.g., Aktaş & Özdemir, 2017; Bowie et al., 2019; Courtney-Clarke & Wessels, 2014; Şengül, 2013). According to Bobis (2007), well-developed number sense is vital to efficient computation and understanding of mathematics. These factors indicate the rationale for projects such as MSAP and MM-WiL to support PSTs developing number sense and mental mathematics knowledge and skills. In this paper, I explore the opportunities for micro-teaching videos to support PSTs in learning how to implement mental calculation strategies, specifically the jump strategy on the ENL.

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Literature Review

The MSAP and MM-WIL Resources

The MM-WiL project uses the MSAP material (Graven et al., 2020) consisting of resources in a teacher guide such as scripted lessons, diagrams, video-recorded demonstrations (see for example, https://youtu.be/6RkP5bSpINQ), notes, pre- and post-tests and learner worksheets (available from https://www.education.gov.za/MSAP2022.aspx). Six mental mathematics calculation strategies are taught in MSAP: bridging through ten, jump strategy, doubling and halving, rounding and adjusting, re-ordering, and linking addition and subtraction. These strategies were carefully designed into tasks to prepare learners for fluent and flexible mental computation to move them away from over-reliance on unit counting (Graven et al., 2013; Graven & Venkat, 2021; Schollar, 2008). The current paper focuses on the jump strategy, which uses an empty number line (and eventually a mental image thereof) to gesture and visually represent addition or subtraction of a number. Mathematics teacher educators (MTEs) at eight South African universities are actively using the MSAP resources with their PSTs, followed by opportunities for them to implement teaching them during their practical teaching experience. This paper, however, reports on the use of micro-teaching as a way for PSTs to explore using and teaching the jump strategy for addition and subtraction. Rather than expecting these first-year PSTs to teach to a class of children, they are first given the opportunity to micro-teach to their peers in small groups of two to three members. PSTs worked together to record their micro-teaching similar to the demonstration videos provided in the teacher resources.

The Jump Strategy on the Empty Number Line

Şahin and Danaci (2020) suggest that one of the challenges learners face with mental computation is not visually showing their thinking. There is, however, a relationship between mental computation and number line representations, according to Ruiz and Balbi (2019). Vermeulen et al. (2020) further posit that the empty number line (ENL) has a "temporary value" (p. 233) in supporting learners with calculations by reducing their cognitive load in the process through visually showing what is happening mentally. The ENL is a number line with no predetermined numbers or unit markers, making it a useful tool to support addition and subtraction in any number range, and it can stimulate learners in explaining their thinking around calculation strategies (Bobis, 2007; Klein et al., 1998; van den Heuvel-Panhuizen, 2008). As learners develop understanding and fluency, they no longer need the visual representation, as it becomes a mental image supporting mental computation. The jump strategy on the ENL can assist learners in visually representing addition and subtraction on a linear representation, where one number is kept as a whole and the addend (or subtrahend) is jumped forwards or backwards in manageable parts, such as in tens (Blöte et al., 2000; Bobis & Bobis, 2005). The jump strategy should not be taught as a mandatory procedure, but rather as a meaningful, flexible strategy enabling learners to visualise and work with number relations (Bobis, 2007; van den Heuvel-Panhuizen, 2008; Vermeulen et al., 2020). Therefore, learning and practising how to use and teach the jump strategy on the ENL to support learners' mental computation can be a valuable strategy for PSTs, as encouraged in the MSAP and MM-WiL teacher resources.

Micro-Teaching for Enhancing Pre-Service Teachers' Skills

Micro-teaching is a widely used practice to prepare PSTs for professional classroom teaching and for education research (Cheng, 2017; O'Keeffe & White, 2022; Ünlü, 2018). O'Keeffe and White (2022) suggest that micro-teaching is an opportunity to move PSTs away from teaching the traditional way they were taught, which could result in negative views towards mathematics, to a more reflective and engaging way of teaching. Micro-teaching involves creating a controlled, supportive environment made up of smaller class sizes, normally

of peers, and for a shorter duration of a standard lesson (Ünlü, 2018), allowing PSTs the opportunity to practise a specific teaching skill without the complexity of a typical classroom (Basturk & Tastepe, 2015). MTEs can use this opportunity to connect theory with practice, before sending PSTs out into schools for practicum. Literature reports benefits of micro-teaching for mathematics PSTs, including developing confidence with time and classroom management, planning, task design, use of questioning, self-efficacy, and precision (Cheng, 2017; Ünlü, 2018). Cheng (2017) highlights a "special kind of micro-teaching" (p. 281) where the university micro-teaching experience is linked to the mentor teacher experience at schools, which helped PSTs improve their precision in verbal (language use, reasoning) and non-verbal (accurate calculations, exact formulas) skills. Cheng (2017) however noted in their study that while the combined micro-teaching experience had value, some lack of precision was still observed in PSTs' lessons. Basturk and Tastepe (2015) caution against the "artificial nature" (p. 1) of the micro-teaching setting during a university course lecture. Nevertheless, probably the most beneficial of the micro-teaching experience is the opportunity for PSTs to be guided in reflecting on and self-evaluating their recorded lessons with peers and MTEs, stimulating collaborative discussion and development, thus improving their content and pedagogical knowledge (O'Keeffe & White, 2022). While an explicit focus on reflection was not guided by the MTE in the study reported on in this paper, it was clear that working in pairs, the PSTs used the opportunity to self- and peer-evaluate their micro-teaching of the jump strategy on the ENL. The next section describes the methodological process of the study.

Methodology

The MSAP resources and training were delivered by the author, also the MTE, to 190 first-year PSTs during their mathematics didactics course. One hundred and two PSTs agreed to participate in the broader study, including pre- and post-tests, questionnaires, and interviews. After completing the pre-test, the MTE shared the hardcopy and electronic MSAP resources with the PSTs. She also demonstrated the designed lesson starters and jump strategy on the ENL to the PSTs during the first 20 minutes of six lecture sessions spread across three weeks (with the intention of modelling how the PSTs would use the resources in their own classrooms). This was followed by the post-test. For the purpose of this paper, I focus on 40 PSTs' submitted micro-teaching videos for which consent to enter them into the data set was given. PSTs had the right to withdraw from the study at any stage; only their hands are shown in the videos, and pseudonyms were used to uphold their anonymity.

Powell et al.'s (2003) analytical model was used to guide the thematic analysis of the videos: the author (1) watched and re-watched the videos; (2) made detailed notes describing the data; (3) identified critical moments in each of the 40 videos; (4) transcribed them indicating both PSTs' actions/gestures and dialogue; (5) coded the data according to identified patterns; (6) constructed a storyline and (7) wrote a narrative description referring to screenshots of the critical moments' dialogue and gestures. This was a cyclical and iterative process. The author noticed four common key elements that contributed to the implementation fidelity of the MSAP resources in the PSTs' micro-teaching videos, i.e., use of (i) the ENL, (ii) jump strategy, (iii) gestures and (iv) key phrases. These four key elements were further used to analyse the 40 micro-teaching videos. It was noticed that for each of the four key elements the PSTs' videos either faithfully followed the scripted MSAP resources with strong implementation fidelity or they departed from the resources.

Findings

In Lovemore and Graven (in press) it was found that most PSTs followed the MSAP resources with some level of implementation fidelity in their micro-teaching videos. Over 70% of the PSTs used the ENL to teach the jump strategy in a highly similar way to the scripted resources in at least three of the four key elements (i)–(iv) listed above. Micro-teaching videos

that had all four of the key elements present were categorised as having strong implementation fidelity. Videos with two or three key elements were categorised as medium implementation fidelity, and those with one or no key elements as poor implementation fidelity. In this paper, I move on from this global perspective of the broad data and focus on three examples of micro-teaching videos across the spectrum of implementation fidelity. This allows me to demonstrate specific examples of the PSTs' use of the jump strategy on the ENL when recording micro-teaching videos and to consider its implications for MTEs.

PSTs could select various calculations, taken from the MSAP resources, to demonstrate the use of the jump strategy on the ENL to create their micro-teaching video. For this paper, I selected the calculation 62 − 30 = __. This subtraction problem involves a known minuend and subtrahend, and the missing difference. It could be solved by placing the minuend (62) towards the end of the ENL and jumping backwards either by breaking the subtrahend (30) into three jumps of ten or by a whole jump of 30. Eight of the 40 PSTs selected this calculation, seemingly because it appears relatively straight-forward. Our analysis, however, shows that for this calculation two PSTs implemented the resources faithfully (all four key elements present); five PSTs implemented the strategy with medium implementation fidelity (three PSTs met three key elements, and two PSTs met two key elements); and one PST met only one key element. I selected this calculation to demonstrate the varying extent to which three PSTs faithfully implemented the key elements in their micro-teaching videos.

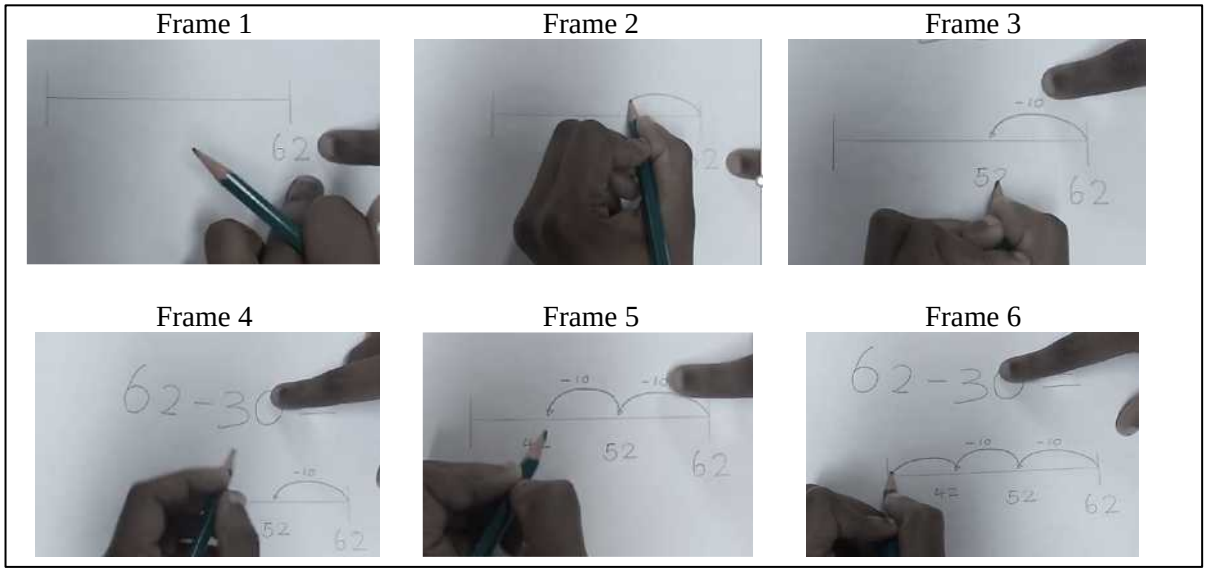
Strong Implementation Fidelity

Somila demonstrated an example of implementing the use of the jump strategy on the ENL in a way that is highly similar to the scripted MSAP lessons. An example from the MSAP scripted lesson for a subtraction calculation encourages the teacher to make the steps explicit to the learners, i.e., "Remind learners of the steps they learnt previously: **plot** [on the ENL], **break down** [into tens and ones], **jump** backwards, and **answer**" (Graven et al., 2020, p. 41). A demonstration video was also part of the teacher resource (see for example, https://youtu.be/JQq2zL6pwCM). Figure 1 below shows screenshots from key moments in her micro-teaching video, illustrate a transcript of her micro-lesson:

> You've got a sum of minusing 30 from 62, and we're going to be using the number strategy. The first step is to firstly draw your number line and place 62 on the end of your number line (*points to end of ENL–Frame 1*) since you're going backwards (*gestures backwards on the ENL*). So you need to take away three tens (*points to 30*) from 62 (*points to 62*). So you place your 62 at the end of the number line and you move backwards (*emphasises drawing jump arc – Frame 2*) by minusing 10. Then 62 minus 10 will give you 52 (*Frame 3 – points to 62 on ENL and then -10 jump*). Since you're not yet by 30 (*points – Frame 3*), you move back another 10 (*draws jump of 10*), and you minus 10 once again and you've got 42 (*keeps her righthand finger on the first ten jump while gesturing and drawing the next jump to 42*). So since we see that it's already two tens make up twenty (*points to jumps of ten – Frame 5*), that's why it takes one more ten to make 30, therefore we move back for the last time (*points with righthand finger on 30 in the calculation while drawing jumping the final 10 – Frame 6*), and you've got 32. These tens (*points to tens jumps*) make up 30, therefore you know you've moved back enough times.

Figure 1

Screenshots From Somila's Micro-Teaching Video



Somila first drew a straight line and then included markers for the first (minuend) and last (difference) numbers. The ENL is a straight continuous line with no unit markers other than the markers learners need and chooses to use to visually demonstrate their thinking (Bobis, 2007; van den Heuvel-Panhuizen, 2008; Vermeulen et al., 2020). She breaks 30 down into three groups of 10, which she uses for the jumps. As shown in Figure 1, her jumps are visually proportionate in size. While Bobis (2007) explained that learners are not expected to draw their jumps on the ENL proportionately, we suggest that the teacher should draw the jumps in a way that is visually proportionate, to indicate that groups of 10 are the same. Following the key phrases of the teacher resources, Somila indicates that 62 should be placed at the end of the line and gestures to demonstrate that 'minus' means we need to jump backwards. Her pointing shows the process linking the written calculation (specifically the subtrahend) to the jumps on the ENL, thus showing the fictitious learners exactly what is happening in each step. Other useful phrases include referring to "tens" and a strategy for learners to check if they are correct, "These tens make up 30, therefore you know you've moved back enough times". In this example, Somila effectively used the jump strategy on the ENL to solve the subtraction calculation, including key phrases from the teacher resources and gesturing to demonstrate the process. While Somila's micro-teaching video had all four key elements present, there are still areas where the MTE can guide her towards greater accuracy. This resonates with Cheng's (2017) study where micro-teaching was used to improve PSTs' precision in verbal and non-verbal skills. Somila, for example, referred to "a sum of minusing" and the "number strategy". On reflection of her micro-teaching video, her MTE could point out that 'sum' refers to addition, whereas 'difference' refers to a subtraction calculation. Similarly, the name of the strategy being taught is the *jump strategy* on the *ENL*. Somila also plotted her endpoint on the ENL, making it a line segment rather than a true *empty* number line. By practising teaching these strategies in a safe, conducive micro-teaching context, Somila has the opportunity to gain pedagogical and content knowledge as well as confidence before going to teach in a real school classroom.

Medium Implementation Fidelity

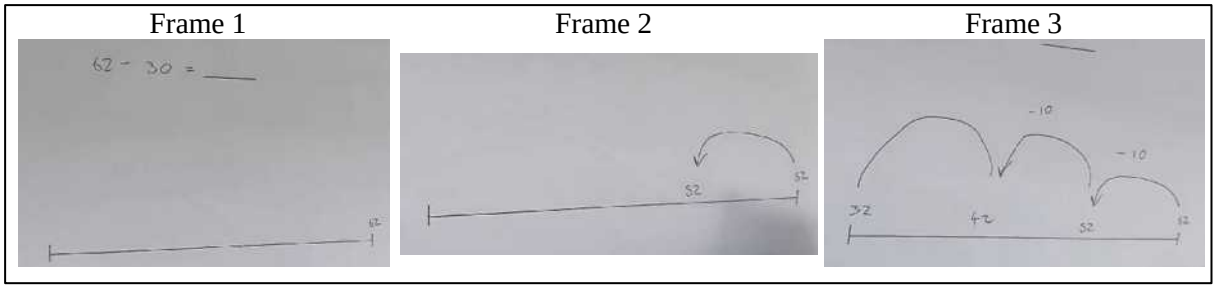
Johannes demonstrated a medium similarity to the MSAP scripted teacher resources, by making use of the ENL, somewhat proportionate jumps and some of the key phrases. His micro-

teaching video did not, however, include the gesturing encouraged in the teacher resources. His video transcript is below, followed by screenshots in Figure 2:

> 62 minus 30 is equal to (*62 plotted on ENL before starting – Frame 1*). To make it easier we are going to minus 10. 62 minus 10 is equal to 52 (*draws jump arc – Frame 2*). Minus 10 (*labels jump*). Minus another 10 gives 42 (*draws jump arc*). Minus another ten (*draws jump arc – Frame 3*) is your final answer, 32.

Figure 2

Screenshots from Johannes' Micro-Teaching Video



While Johannes made use of an ENL without unit markers, he had placed the minuend (62) at the end of the line without showing or explaining why the subtraction calculation would 'start' at the end of the line (see Frame 1). This omits an important pedagogical moment highlighted in the MSAP teacher resources, to give learners the opportunity to strategically represent and visualise the type of calculation (addition or subtraction). He proceeded to draw jumps, which are somewhat visually proportionate, but because he had placed the endpoints on the line, he had to ensure that his jumps landed on the market, thus resulting in his last jump being slightly longer than his first two. In this micro-teaching video, one key phrase from the MSAP resources is present, i.e., 'to make it easier', when indicating the breaking of 30 up into tens. He however does not refer to phrases such as 'at the end of the line' or 'jump backwards'. Instead, Johannes only uses the word 'minus' to indicate the subtraction of tens. Furthermore, his gestures are limited to the drawing of the jump arc. For the purpose of this study, I recognise the drawing of the jumps as a form of gesture, but also the emphasis of the jump through non-drawing hand movements, including pointing to the original calculation (such as in Somila's example), and indicating the jump direction (forwards or backwards) based on the calculation. While Johannes' micro-teaching video demonstrated the use of the jump strategy on the ENL to solve the calculation, there were some key elements missing that would enhance the pedagogical value of his demonstration. Using reflection on the micro-teaching video, the MTE could indicate the pedagogical value of using key phrases from the scripted lessons, and to scaffold learners with each step of the process.

Poor Implementation Fidelity

Wonga's micro-teaching video is an example of poorly implementing the MSAP teacher resources. In this video demonstration, he departed from the scripted teacher resources by breaking the subtrahend 30 into two groups of 15, rather than the suggested tens. The micro-teaching video is filmed at an angle which is difficult to clearly see, and as such, no screenshots are included. However, a transcript of his short video is below:

> (Draws ENL and plots 62). We have 62 minus (draws jump arc backwards) … 62 minus … 15 … this gives us … (is unsure and puts pencil down).

Wonga's 22-second micro-teaching video indicates that he is not yet confident with using the jump strategy on the ENL. He was able to draw the ENL, plot 62 and draw a jump arc backwards. However, he opted to break 30 up into two groups of 15, rather than using the suggested and scripted 'friendly numbers' such as groups of ten, in the MSAP resources. He

only drew one jump arc and did not label it as '–15'. The delay indicated by the ellipsis in the transcript further indicates his uncertainty. When reaching the point of having to subtract 15 from 62 he stops the attempt. This suggests that he has not gained confidence in using the strategy for flexible and mental calculations. Wonga's micro-teaching video suggests that he needs further support in how to use the jump strategy on the ENL as well as explicit instruction on how to teach it.

Discussion and Conclusion

This paper sought to answer the research question, 'How do PSTs use the jump strategy on the empty number line when recording micro-teaching videos?'. From exploring the three examples of Somila, Johannes and Wonga, it is evident that PSTs are able to implement the jump strategy on the ENL with some (varying) level of fidelity. The micro-teaching videos were an opportunity for PSTs to work in pairs, to engage with the MSAP resources and find ways to enact them. The micro-teaching allowed opportunities for reflection (although not the focus of this paper) for PSTs, and their MTE, to establish where they are confident and competent, and which key elements needed further attention.

The three micro-teaching videos showed examples of how PSTs implemented the teacher resources with strong, medium, and poor implementation fidelity. This can be useful to researchers and MTEs to take note of the importance of explicit instruction and guidance for PSTs when learning how to teach the jump strategy on the ENL, and other mental calculation strategies. Just as Beswick and Goos (2012) and Bowie (2019) state that it cannot be taken for granted that PSTs arrive at the didactic courses with content knowledge, so too we cannot take it for granted that by sharing teacher resources and demonstrations on calculation strategies, PSTs have the knowledge, skills, and confidence to teach them. In this study, the MTE shared hardcopy and electronic scripted teacher guides, video-recorded examples, and demonstrated the jump strategy over a three-week period in six lectures. PSTs still did not have full implementation fidelity in their micro-teaching videos. It is promising though to note that PSTs were able to implement most of the key elements of appropriate jumps on the ENL, using some gestures and some key phrases. Creating micro-teaching videos in a safe space, and reflecting with peers, is a useful opportunity for PSTs to practise their teaching before going to teach in schools.

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