Pre-Service Teachers' Struggles With Core Numeracy Concepts

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In Ireland, Initial Teacher Education [ITE] standards require all teachers to possess an adequate level of numeracy themselves in order to teach for numeracy learning across the school curriculum. This paper reports on a study that investigated 204 pre-service post-primary teachers' numeracy capabilities. Analysis of questionnaire data from participants in three universities showed that pre-service teachers tend to struggle to complete the numeracy tasks correctly, especially tasks related to ratio and proportional reasoning. If pre-service teachers of all disciplines are not capable of completing core numeracy concepts, then they will struggle to teach for numeracy learning.

Internationally, educators and government stakeholders are advocating that all citizens should have gained the necessary literacy and numeracy competencies in school, to live and work in today's world (Goos & O' Sullivan 2023; Department of Education and Skills [DES] 2011; Norwegian Directorate for Education and Training 2012). The Irish government developed a strategy for literacy and numeracy and in this strategy, have stressed the importance that all teachers should be teaching for numeracy learning across all subject disciplines (DES 2011, 2015). More recently, and in a bid to support this initiative of a numerate society, The Teaching Council of Ireland (2020), which is the regulatory body for teachers, have agreed that teachers need to develop their personal numeracy knowledge and also specified that all universities involved in preparing pre-service teachers must:

> Ensure that student teachers are afforded opportunities to enhance their own literacy and numeracy and are required to demonstrate an acceptable level of proficiency in literacy and numeracy. Students shall be required to demonstrate their competence in teaching and assessing literacy and numeracy appropriate to their curricular/ subject area(s). (CÉIM, 2020, p. 14)

Similarly, in Australia, Hall and Forgasz (2020) argue that pre-service teachers need to possess a certain level of numeracy skills themselves, prior to teaching for numeracy learning. Supporting teachers in gaining the appropriate knowledge required to teach for numeracy learning in an effective manner should be a priority for educators. Researchers in the field of numeracy argue that possessing mathematical knowledge is not enough to teach for numeracy, nevertheless they recognise that mathematical knowledge is a core aspect of numeracy and therefore is important for teachers to possess (Venkat & Winter, 2015). Forgasz and Hall (2019) argue that in order for teachers to improve the numeracy capabilities of their students, teachers must first equip themselves with the necessary skills to develop their own understanding of how mathematical concepts and numeracy affect their own lives and their subject area. The following paper presents results from a study conducted in Ireland which investigated pre-service secondary teachers' abilities to complete numeracy questions. For the purpose of this paper, responses to 2 numeracy questions were analysed to address the following research question: How well are pre-service post-primary teachers able to complete numeracy tasks?

Research Design and Method

Pre-service secondary teachers enrolled in the two-year Professional Master of Education (PME) programme in three different universities, were invited to take part in this research study. Pre-service teachers were asked to complete a questionnaire at one of their general education lectures at the beginning of their second year on the PME. There were 204 pre-service teachers who completed this questionnaire. The questionnaire consisted of three sections, Section A:

(2024). In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 415–422). Gold Coast: MERGA.

Demographics, Section B:Pre-service Teachers perceptions of numeracy and preparedness to teach for numeracy, and Section C: Preservice Teachers assessment of numerate capabilities.

Section C of the questionnaire consisted of 7 numeracy tasks in total; however, the numeracy tasks were presented in 6 questions. The first two numeracy tasks were part of question 1. Table 1 presents a short explanation of each numeracy task in the questionnaire. Pre-service teachers were asked to display their workings for each task in a text box provided. Asking pre-service teachers to provide mathematical workings in the space provided enabled the authors to identify if the answer was correct or incorrect and furthermore allowed the authors to understand the pre-service teachers' mathematical thinking. Three of the numeracy tasks were published by the OECD as PISA test questions: Earthquake task (PISA, 2003), Car task (PISA, 2003) and the Salad dressing task (PISA, 2012). The other three numeracy tasks were developed specifically for this study. Pre-service teachers were allowed to use a calculator and they were asked to indicate at the end of the questionnaire if they had done so.

Table1

Explanation of Each Numeracy Task in the Questionnaire

Numeracy tasks	Explanation
Time task	Calculate the difference between two Olympic swimmers' finishing race times for a 100 metre butterfly race. The results were presented in a table and the pre-service teachers had to subtract one decimal number from another decimal number (51.14–50.39)
Distance task	Joseph Schooling had a result of 50.39 in the 100 metre race and if the race was 30 metres longer, given that he was travelling at the same average speed as he did in the first race, calculate the new time he would finish the 130 metre race
Earthquake task	A documentary about earthquakes and how often they occur is broadcast. A geologist stated "In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three". Pre-service teachers were asked to use mathematical knowledge and understanding of statistics to predict an event occurring in this specific context. Pre-service teachers were provided with 4 different scenarios and asked to choose which one best reflected the meaning of the geologist's statement
Pie Chart task	Given a pie-chart, calculate the proportion of the pie chart (as a percentage) that represented the participants who chose biology as a subject for the Leaving Certificate
Best Car task	Calculate the score of the "Best Car" given an equation. The "Best Car" is evaluated based on scores for safety features (S), fuel efficiency (F), external appearance (E) and internal fittings (T) and these were the variables in the given equation (Ca) = (3 x S) + F + E + T. Pre-service teachers had to substitute values into the equation and work out the final answer for the Best Car
Salad Dressing task	A recipe for 100mls of salad dressing has three ingredients which are Salad Oil (60mls), Vinegar (30mls) and Soy sauce (10mls). Pre-service teachers were asked to calculate how much salad oil is required to make 175mls of salad dressing
Mobile Phone task	David uses 500 minutes per month and 15GB of data. Recommend the best mobile phone plan for David, given price tariffs for 3 mobile phone companies

Pre-Service Teachers' Numeracy Capabilities

The results of the overall numeracy tasks are presented in Table 2, which shows the number of pre-service teachers who answered each task correctly, incorrectly or left it blank.

Table 2

Breakdown of Correct, Incorrect and Blank Answers for Each Numeracy Task

	Correct	Incorrect	Blank
Time	112 (54.9%)	89 (43.6%)	3 (1.5%)
Distance	99 (48.5%)	68 (33.4%)	37 (18.1%)
Earthquake	165 (80.9%)	36 (17.6%)	3 (1.5%)
Pie chart	135 (66.2%)	53 (25.9%)	16 (7.8%)
Best Car	160 (78.4%)	22 (10.8%)	22 (10.8%)
Salad Dressing	118 (57.8%)	59 (29%)	27 (13.2%)
Mobile Phone	48 (23.5%)	94 (46%)	62 (30.5%)

The question with the highest number of correct answers was the Earthquake task with 165 (80.9%) pre-service teachers answering this task correctly. The Earthquake task was taken from PISA (2003) released sample items. It was the only question for which the pre-service teachers were given multiple choice options for their answer.

As can be seen in Table 2, the Time task had nearly as high a proportion of incorrect answers as it had of correct answers, which is interesting as I had considered this task one of the easier tasks as it involves only subtracting two decimal numbers. Also, it was noted that over 40% of pre-service teachers either answered incorrectly or left the task blank in four out of seven of the numeracy tasks. The Mobile Phone task was the question with the lowest number of correct answers. This question was also the only question for which there were more pre-service teachers who answered the question incorrectly than those who answered it correctly. The Mobile Phone task was also the question for which there was a substantial number of pre-service teachers who left the question blank. This may be due to the nature of the question and the fact that it involved much more time and more mathematical calculations such as working out how much extra data was needed per month and working out the cost of the extra data. Pre-service teachers needed to consider the total cost was over the course of 24 months and not just the price for one month when initially purchasing the phone. This question was also the last question on the questionnaire which may also have contributed to the low response rate.

From the initial analysis, 8 (3.9%) pre-service teachers were able to answer all 7 numeracy tasks correctly. There were 156 (76.5%) pre-service teachers who were able to answer 3, 4, 5 or 6 numeracy tasks correctly. However, there were still a considerable number of pre-service teachers (32, 15.7%), who were only able to answer either 1 or 2 numeracy tasks correctly. Finally, 8 (3.9%) pre-service teachers were unable to answer any numeracy task correctly and only one of these eight pre-service teachers left each answer blank, which means that 7 pre-service teachers attempted to answer the numeracy tasks but answered them all incorrectly.

Pre-Service Teachers' Struggles with Core Numeracy Concepts

This section presents the different types of answers pre-service teachers provided when completing the numeracy tasks. Initially, the numeracy tasks were coded as correct, incorrect or blank but then it became evident that many pre-service teachers had proceeded with answering the question incorrectly in the same way, thus suggesting there were many commonly held misconceptions. Therefore, I decided to code the types of answers which meant I was able to better understand the mathematical thinking and errors some pre-service teachers had made. For the purpose of this paper, I will focus on Question 1 which focussed on the Time and Distance tasks and I will also present errors in Question 5 which was the Salad Dressing Task. Both questions expect teachers to draw on their mathematical knowledge of ratio and proportional reasoning, which is fundamental and a core aspect of many numeracy tasks in the school curriculum and in the wider society that we live in.

The first numeracy task in Section C of the questionnaire asked pre-service teachers to calculate the winning margin in the 100 metre butterfly final in the 2016 Olympic Games, and then calculate the time to swim the race at this average speed over an additional 30 metres. The question and a model solution are shown in Figure 1.

Figure 1

Sample Correct Answer to Time and Distance Task

Section C: Numeracy problems

1. At the Olympics in Rio 2016, Joseph Schooling and Michael Phelps competed in the 100 metre Butterfly race. The results are shown in the table below:

Name	Result
Joseph Schooling	50.39
Michael Phelps	51.14
Chad Le Clos	51.14
Laszlo Cseh	51.14
Shuhao Li	51.26
Medhy Metella	51.58
Tom Shields	51.73
Aleksandr Sadovnikov	51.84

(i) By how much time did Joseph Schooling beat Michael Phelps?

51.14 – 50.39 = 0.75 of a second

(ii) How confident are you that your answer is correct?

Confident	Not Confident	Unsure

(iii) If the race was an extra 30 metres in distance, what would be the result time for Joseph Schooling given he was swimming at the same average speed that he swam in the 100m butterfly race?

100m ÷ 50.39s = 1.9845 m/s
130m ÷ 1.9845 m/s = 65.507s
65.5s

50.39s ÷ 100m = 0.5039 s/m
0.539s/m × 30m = 15.117s
50.39s + 15.11s = 65.5s

There were 89 pre-service teachers who answered this task incorrectly, and of this cohort, a considerable number of pre-service teachers arrived at the same incorrect answer. Different types of incorrect answers were observed and the mathematical thinking behind each error was also identified. As described in Table 3, there were three notable incorrect answers for the Time Task. There were 20 (22.5%) answers to this question which could not be allocated to either Type 1, Type 2 or Type 3 and the mathematical thinking of the error could not be drawn from the answer the pre-service teachers presented for these 20 answers. The incorrect answers for each type of answer are presented in the brackets in Table 3 along with a description of the error.

Table 3

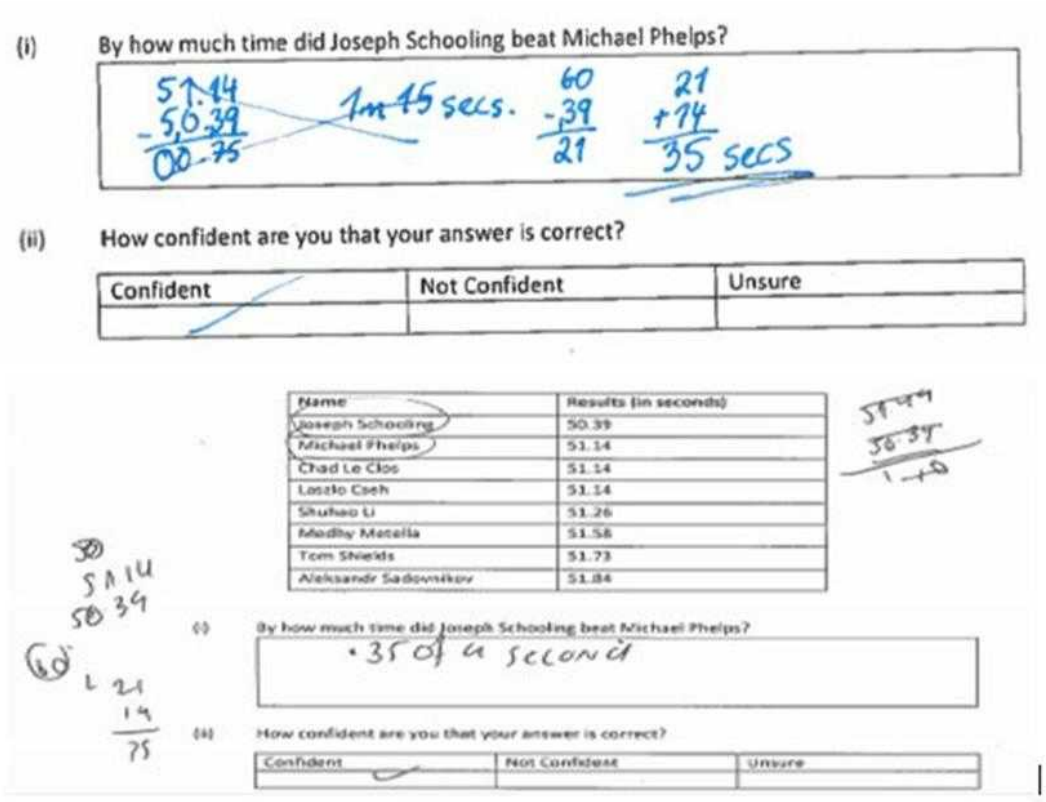
Types and Frequency of Occurrence of Incorrect Answers for Time Task

Types of answers	Description of misconception	*n* (%)
Type 1 (0.35 seconds or 35 seconds)	The number of centi-seconds in a second is the same as the number of seconds in a minute or number of minutes in an hour	52 (58.5%)
Type 2 (1 minute 15 seconds or 75 seconds)	Convert 0.75 to 1 minute and 15 seconds or 75 seconds whereby the respondents omitted the decimal place in their answer	9 (10.1%)
Type 3 (1.25 Seconds)	Subtract the smaller number from the larger number	8 (8.9%)

The main difficulties that led pre-service teachers to give a Type 1 answer (0.35 seconds or 35 seconds) involved the misconception that the number of centi-seconds in a second is the same as the number of seconds in a minute or number of minutes in an hour. Hence, 52 of the 89 pre-service teachers who gave a Type 1 answer incorrectly calculated the answer as either 0.35 seconds or 35 seconds. Examples of how some of these 52 participants calculated these answers are given in Figure 2. It seems that they have "carried over" 60 seconds after the decimal point to change the task from 51.14 − 50.39 to 50.74 − 50.39, which gives an answer of 0.35. However, some pre-service teachers who worked only with the numbers after the decimal point interpreted the answer as 35 seconds, which might suggest that students who answered 35 seconds do not understand the place of a decimal point.

Figure 2

Examples of Type 1 and 2 Incorrect Answers to Time Task



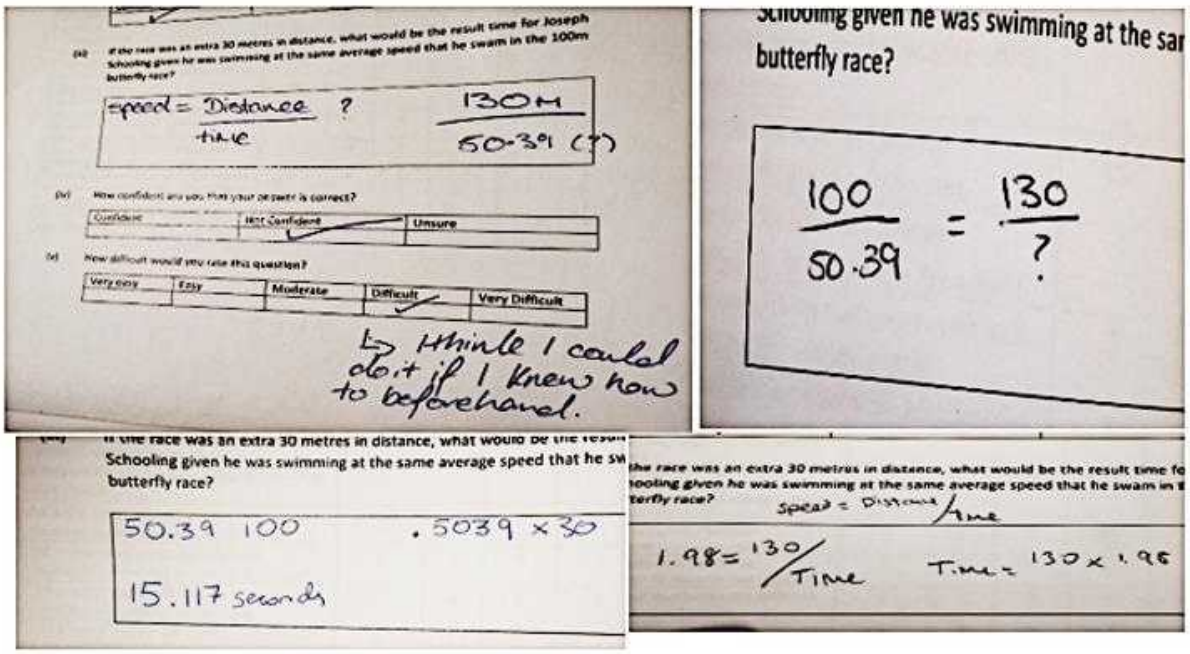
The second numeracy task presented in Question 1 was based on distance, average speed and time. This numeracy task asked pre-service teachers to calculate the result time of the butterfly race, if it was an extra 30 metres in distance. Here the pre-service teachers had to use

the results table provided in the first part of the question to calculate the new finishing time for Joseph Schooling, given that he was swimming at the same average speed as when he completed the 100 metre butterfly race. Two possible correct responses to this numeracy task are presented in Figure 3.

Fewer pre-service teachers responded to this numeracy task in comparison to the first one, with only 167 (81.9%) completing this part. Of the 167 pre-service teachers who answered the Distance task, 101 (60.5%) answered it correctly. There were 10 (6%) pre-service teachers who had partially completed workings for this task. For example, some had calculated how many more seconds it would take for the extra 30 metres, but never added it to the time for 100 metres which can be seen in an example in Figure 3, while others recalled the formula to calculate the distance, average speed and time but didn't complete the task. One pre-service teacher stated that they would be able to complete the task if they knew how to beforehand.

Figure 3

Pre-Service Teachers Partial Working for the Distance Task



The next task was Salad Dressing and 177 (86.8%) pre-service teachers answered this task. This was a question adapted from PISA (2012) that involved applying their mathematical knowledge of ratio and proportion to make a salad dressing of 175millilitres, when given a recipe for 100 millilitres. Two-thirds of the pre-service teachers (118, 66.6%) who answered this question gave a correct response, answered correctly. Figure 4 shows the correct answer to this numeracy task.

Seven (4%) pre-service teachers stated that they understood what they were being asked to do but didn't understand how to answer the numeracy task. These pre-service teachers stated that the numeracy task was to do with "ratio", "fractions", and "working out the correct proportion", but stated they didn't know how to "get the answer" and also stated, "it would be easier if it was double the amount because you just double each portion". Additionally, there were 52 (29.4%) pre-service teachers who answered the task incorrectly. While there were two different types of incorrect answers, I was unable to decipher how they arrived at the answer provided. However, I did manage to categorise the answers and the different types are presented in Table 4, along with pre-service teachers' sample answers in Figure 5.

Figure 4

Sample Correct Answer to Salad Dressing Task

5. You are making your own salad dressing. Below is the recipe for 100 millilitres (mls) of salad dressing.

Salad oil	60ml
Vinegar	30ml
Soy sauce	10ml

(i) How many millilitres of salad oil do you need to make 175 mls of the salad dressing?

$$\frac{60mls}{100mls} \times 175mls = 105\,mls$$

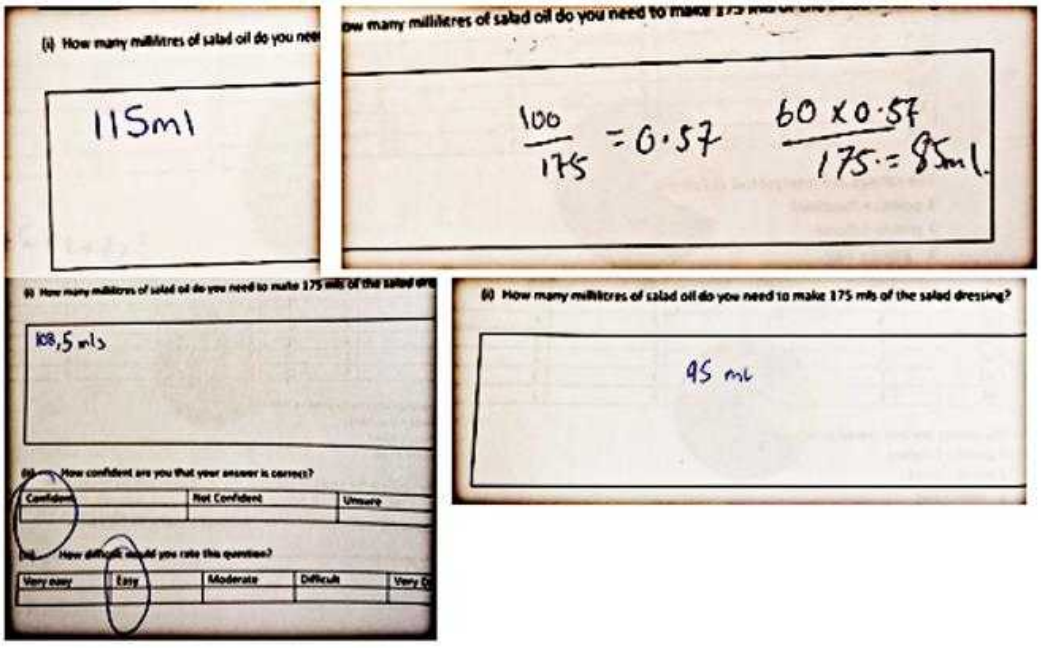
Table 4

Different Types of Incorrect Answers for the Salad Dressing Task

Types of answers	**Description of misconception**	***n* (%)**
Type 1 (Random Number)	This type of answer saw pre-service teacher write down a random, incorrect number but they did not provide any evidence of mathematical workings	35 (19.8%)
Type 2 (Obscure working out)	This type of answer showed pre-service teachers demonstrate some mathematical workings but the mathematical reasoning did not make sense	17 (9.6%)

Figure 5

Examples of Type 1 and Type 2 Incorrect Answers to Numeracy Task



Discussion

Researchers in Australia have advocated that in order for teachers to teach for numeracy learning in the classroom, they must also be proficient in their own numeracy capabilities (Forgasz & Hall, 2020; Goos et al., 2019). The findings presented in this paper showed that overall, two-thirds of the pre-service teachers were able to answer four or more numeracy tasks correctly and approximately one-third of the pre-service teachers answered fewer than half of the numeracy tasks correctly. Nevertheless, it did not deter them from completing the tasks and it was obvious that the pre-service teachers demonstrated a high level of engagement by attempting the numeracy tasks. The error analysis revealing the struggles of pre-service teachers that are presented in this paper is informative, in that identifiable errors and common misconceptions in the numeracy tasks were ascertained. This is interesting to note as these pre-service teachers had come through an undergraduate degree but were presenting with common errors that students in primary school settings may also make.

Furthermore, these results also highlight the gaps in pre-service teachers' competency which need to be addressed. It is important for pre-service teachers to identify the common misconceptions students may have, with a specific focus on misconceptions relating to numeracy within their specific subject discipline. I argue that pre-service teachers could discuss and identify the different numeracy misconceptions that their students may encounter when teaching a certain topic. This would be in line with the current mandate from The Teaching Council of Ireland (2020), who have stated that pre-service teachers need to be given ample opportunity to develop their own personal numeracy skills which in turn will have an impact on the way in which they teach for numeracy learning.

Acknowledgements

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