Engaging Multilingual Students in Frequent and Supported Opportunities for Discourse to Strengthen Their Mathematical Thinking

Rachel Restani
University of California, Davis
rmrestani@ucdavis.edu

Margarita Jimenez-Silva
University of California, Davis
mjimenezsilva@ucdavis.edu

Tony Albano
University of California, Davis
adalbano@ucdavis.edu

Suzanne Abdelrahim
University of California, Davis
suzanneishere@gmail.com

Robin Martin
University of California, Davis
romartin@ucdavis.edu

Rebecca Ambrose
University of California, Davis
rcambrose@ucdavis.edu

Determining the most effective ways to support student-to-student talk requires some negotiating from a responsive teacher. This paper reports on a case study of two emergent multilingual students in Grade 3 (seven to eight-year-olds) who explained their strategies to each other. The transcript of their conversation was analysed using Chan and Sfard's (2020) *participation profile framework*. Findings indicate that the two students learned from each other during the interaction because they revised their work to be more precise. One implication of this study is that strategic pairing may be a useful practice to eliminate inequitable power dynamics.

The value in having learners share their verbal or nonverbal mathematical ideas with each other is of increasing interest to mathematics educators around the globe. A growing body of research has emphasised the importance and many ways educators can create learning environments that support all students, specifically multilingual students, with engaging productively in rich mathematical activities and classroom discussions (Turner, Dominguez, Maldonado, & Empson, 2013; Moschkovich, 2007, 2013; Khisty & Chval, 2002). Participating in discourse is also important for developing conceptual understanding (Moschkovich 2015; Bailey 2007). When students share solutions publicly, they strengthen their own comprehension, learn how peers make sense of the mathematics, and present opportunities for teachers to better understand student reasoning and mathematical thinking.

The more students talk about mathematics, the more they see themselves as doers of mathematics, therefore dismantling and restructuring the current negative stereotypes about multilingual learners. Furthermore, students benefit from responsive teaching (Richards & Robertson, 2015), where teachers promote peer interactions by getting to know students as individual learners, differentiating instruction with enabling or extending prompts, and seating students in heterogeneous groups (Bobis, et al., 2021).

Finally, student learning through sharing can also be facilitated by Cognitively Guided Instruction (CGI, Carpenter & Fennema, 1992), which encourages exploration of student-centred thinking and problem solving over the provision of teacher-structured solution strategies. CGI is a foundational element of our work with teachers, specifically, Empson and Levi's (2011) framework for building students' conceptual understanding of fractions and decimals through discussing and solving word problems. The framework also guides our support of teachers with understanding the progression of student learning trajectories for solving fraction word problems to further guide instructional practice. In CGI classrooms, students learn mathematics by engaging in problem solving, explaining their problem-solving strategies to the teacher and their peers, and by listening to various ways of solving problems (Carpenter et al., 1999; Carpenter & Franke, 2004).

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Although we know that responsive teaching and CGI can help teachers provide valuable opportunities for interaction and discussion during mathematics instruction, additional research is needed on how to plan for, scaffold, and facilitate peer interactions effectively while also maintaining a high level of rigor and genuine inquiry, especially in the context of mathematics instruction with multilingual learners (Tai & Wei, 2020). Facilitating effective mathematical discussions, such as supporting students to produce conceptual explanations and connections between ideas, may be more difficult and take longer for teachers to develop (Hufferd-Ackles et al., 2009), and is more challenging in classrooms with linguistically diverse learners at various stages of English development (Walshaw & Anthony, 2008). Our study, part of a larger project on developing linguistic repertoires and mathematical conceptual knowledge in tandem in multilingual classrooms in the United States (US), sheds light on these important issues.

Our research seeks to understand:

- What does it look like when multilingual students have conversations with their peers about their solution strategies for equal sharing problems?

We examine here a conversation between two emergent multilingual students discussing their solutions to an equal-sharing fractions problem. We summarise key findings, including how CGI and responsive teaching can address inequitable power dynamics through strategic pairing while also encouraging collaborative revision when problem solving.

Relevant Literature

Most documented peer-to-peer interactions consist of either a pair or a small group of students collaborating on a problem together or one student tutoring another. Both types of peer-to-peer interactions can lead to asymmetric power relations which could be disadvantageous to learners. For instance, Chan and Sfard (2020) identified a phenomenon where pairs of students did not benefit from learning afforded by the participation structure because one student led the problem-solving effort and the other followed along without comprehending what the partner was doing. Given the possibility that partner work can be unproductive, we assert that partner and small group conversations need to be strategically organised to be effective.

One strategy teachers can use to address the power relationships during partner talk is to provide protocols for students to follow. In the UK, Amodia-Bidakowska et al., (2023) identified as one of their design principles that establishing various mathematics language routines may be beneficial to children as they interact with one another when sharing strategy explanations. The students in their study engaged in peer-to-peer dialogue that supported *elaboration* and *querying* of each other's ideas. Zwiers et al. (2014) suggest providing sentence frames that support development of certain conversation skills to help students build on each other's ideas. These skills include creating, clarifying, fortifying, and negotiating. To create an idea, students are provided with multiple opportunities to express original ideas about the content. Clarifying involves both partners figuring out ways to represent their idea through elaboration, paraphrasing and explanation. Fortifying ideas requires students to support or justify their ideas/problem solving with logic or models. Sometimes ideas are challenged with opposing ideas or strategies which requires the need to negotiate ideas. It may result in coming to a compromise, agreeing to disagree, or conceding to a new idea. All ideas are valued by both partners. Teaching students how to use conversation skills can prepare and support children for more effective interactions.

There are additional participation structures, tasks, and roles that teachers can leverage to facilitate productive interactions between students. In New Zealand, Hunter and Miller (2022) worked alongside a primary classroom teacher that used translanguaging and cultural artefacts in problem contexts to connect the mathematical situations to students' cultural backgrounds. Specifically, *waka* was a commonly used term for boat, *whanau* was a well-known term for family, *tamariki* for children, and *iwi* for tribe/community, etc. Discussing indigenous words and cultural situations during the launch of the problem supported students in making sense of the mathematical situation, identifying with classroom mathematics, and learning about the Indigenous and Pacific Island cultures in New Zealand. In Australia, Muir (2023) reported on a case study of a Grade 3–4 teacher who implemented some of Liljedahl's (2021) *Thinking Classroom Strategies*. The teacher noticed her students were more likely to engage in collaboration and persevere on tasks when they were given challenging tasks, when they believed they were grouped randomly, and when there was one person holding the pen for the group and that person was only allowed to write down other people's ideas. Researchers in the Netherlands found that groups of children, including multilingual students, discussed the meaning of unfamiliar words in story problems before they set out to solve the problems (Elbers & de Haan, 2005). They focused on the aspects of the words that were relevant to the mathematics at hand rather than the more general meaning of a word. For example, when asked what "rye bread" is, the children pointed to a picture in the textbook. Hence, peer-to-peer conversations have the potential to help or hinder student learning depending on how problems are launched and discussions are structured. Formal protocols such as sentence frames, random grouping, and one person holding the pen combined with rigorous, culturally-sustaining contexts are a few ways educators have eliminated inequitable power dynamics, leading to productive conversation between peers.

Theoretical Framework

We draw upon Chan and Sfard's (2020) *participation profile framework*, based on their commognitive framework, to evaluate the effectiveness of dyadic learning. They posited that conversations can be multi-modal, multi-channel, and not exclusively verbal. Chan and Sfard described one type of learning as a change in the command of mathematical discourse. They distinguished between mathematising talk and subjectifying talk. When learners use mathematising language they describe the mathematical features of the topic at hand (e.g., "intercept is negative 5"). Subjectifying occurs when learners navigate their moves out loud (e.g., "I was thinking", "oh then that makes sense to me"). Being able to assess what they've done so far and what their next steps are is the crux of learning. On the other hand, stating subjective evaluations of one's identity is not helpful for learning (e.g., "I'm not good at maths"). Chan and Sfard theorised that the proportion of mathematising and subjectifying utterances make up a learner's thematic profile and demonstrate the level of their command of mathematical discourse.

Methods

Context

This study was conducted in a large urban area in the western US from 2021 to 2024. Students came from linguistically, ethnically, and economically diverse backgrounds. Participants were involved in a design-based research project in which researchers met with teachers and coaches. Researchers met once a month with twelve participants during the first year and once every two to three months with the remaining eight participants during the second year. Teachers mostly taught Grades 3 to 5 students (seven to eleven-year-olds), with one teacher who switched to second grade and one who switched to transitional kindergarten during the second year. Each year of the study, participants identified four students who were classified as Emergent Bilingual because they spoke a language other than English at home. Following a CGI method of lesson planning during the integrated English Language Development mathematics period, the teachers asked pairs of students to share answers with each other after solving on their own. In addition, the pairs used a partner interview protocol (script) with Zwiers et al.'s (2014) conversation skills included as sentence frames. The pairs took turns explaining their strategies to each other using the protocol and then following up by asking each other questions about their individual strategies.

Data Collection and Analysis

Each month, the participants brought video, audio, written transcripts, and copies of students' solution strategies of one of the four focal students explaining to a partner how he/she/they independently solved an equal-sharing fraction problem. To code the transcripts, the research team used inductive and deductive codes to analyse the mathematical discourse between dyads. Following Chan and Sfard's (2020) framework, we differentiated between mathematising language and subjectifying language. When children described the mathematics that they or their peers did to solve the problem it was coded as *mathematising*. Comments about the mathematisers' actions were coded as *subjectifying*. We also noted intrapersonal and interpersonal talk between peers. Additionally, we used Amodia-Bidakowska et al.'s (2023) two core dialogue features, *elaboration* (including clarification/building) and *querying* (i.e. "doubting, full/partial disagreement, challenging, or rejecting a statement").

During deductive open-coding, we found that we also needed to include other codes such as *justification* (because reasoning is different than clarifying) and *equal-sharing language* ('what was your whole?' 'how many parts did you partition it into?') that were specific to the problems the students solved. Utterances were double-coded using both the umbrella codes, *mathematising* or *subjectifying*, and using the subcodes under each umbrella code. We used these codes to analyse the proportion of mathematising and subjectifying the pairs of students engaged in during each recorded session.

Findings

We highlight a case where two Grade 3 (seven and eight-year-olds) multilingual students individually solved an equal-sharing fraction problem using similar strategies. Conceptually, both student solutions were correct. The two peers, Student 1 (S1), male, and Student 2 (S2), female, started the conversation by each taking a turn explaining how they solved this problem, 'Four children share ten cookies. How much will they each get? Explain how you know your solution is correct'. Following the "partner interview protocol" (Zwiers et al., 2014), they each took turns sharing their explanations. Then, S1 probed his partner S2 for more justification. We found that the querying that occurred during the second part of the conversation fortified their understanding of unit fractions ("thirds").

Figure 1

Written Work from Student 1 Male (Left) and Student 2 Female (Right)

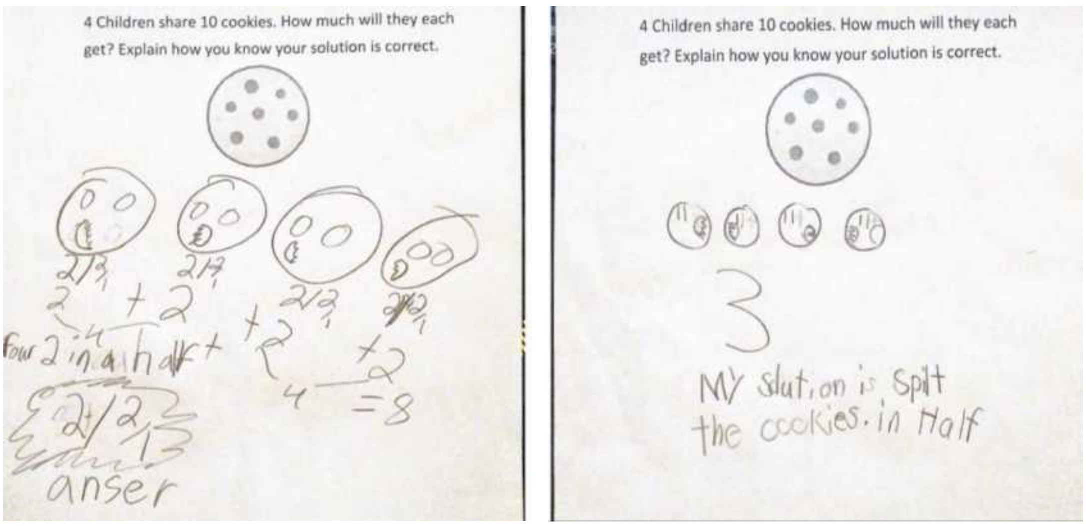

Table 1

Transcript (First 1 Minute, 30 Seconds) of a Dyad Conversation and Associated Codes

Transcript		Codes
S1:	How I solved it, S2, is that I wanted to skip count by twos. But when I did, I actually saw that if I add one here and another one here.	Subjectifying ("How I solved it … ") Subjectifying ("I actually saw … ")
S2:	Hmmm.	
S1:	I was just, it will be three, three, and two. So I decided to make little things like that they're cracked.	Subjectifying ("I decided … ")
S2:	Yeah.	
S1:	To put in each all of them. So then I put two and then half, two and half again, and two and a half like, and I put the same thing. So I put four and ah … four in … four twos and a half. So I did two plus two plus two plus two because I see one, two, three, four, so all of these equal to four. Then four plus four equals eight. So, the answer is, um, two and a half. So, and what did you do?	Mathematising/equal-sharing (distributing and halving cookies) Asked peer to share her explanation
S2:	First, I saw that there were four kids, right?	
S1:	Mmhmm.	
S2:	There was a partition ten cookies. And then what I did was do like line first and then like in each until I got to ten.	Mathematising/equal-sharing (distributing and halving cookies)
S1:	Mmhmm.	
S2:	But then I saw like two, these two didn't have, so, one I broke it in half, because there were two and there were supposed to be, so I broke it. And I just put that it's broken. And then like, I split them in half.	Subjectifying ("I saw") Mathematising/justifying ("because … ") Mathematising/equal-sharing ("I split them in half")

Table 1 displays the initial transcription where each student took turns explaining their individual strategies to each other. Both explanations resembled what could be described as simply reporting or recounting of procedural steps as neither engaged in elaborating or querying into each other's strategies. Both children used a similar strategy in which they distributed two whole cookies to each of the four children in the problem. Then S2 partitioned the last two cookies into halves to share equally between the four children. Although the method is conceptually accurate, S2 drew three similar-sized lines on their paper. She also wrote, "3," on her paper, implying that she knew the people in the problem each received three pieces of cookies without differentiating between wholes and halves. What occurred next is when the crux of the learning happened.

Table 2

Transcript (1:30–2:30) of a Dyad Conversation and Associated Codes

Transcript		Codes
S1:	But S2, how does this one have two and this one have three? I thought it was supposed to be some little lines to make it like half and half.	Querying
S2:	This one is half. And then like this one is half.	Mathematising/elaborating
S1:	Yeah but why are they all in three little rows? Like, for example like, this one, why would you put it in half? If you put it in half it would be two and a half, too. And this one would be two and a half too, also, because it's the same as this one. But this one would just be, um, two, and this one would be just three. So like I believe that that answer is not correct, so. But that does kind of make sense because you wanted to put them in half.	Querying Mathematising Subjectifying (correctness) Elaborating (clarifying)
S2:	Like, split them.	
S1:	Yeah, split them in half.	

In the second part of the conversation, the two children genuinely engaged in a productive dialogue as S1 argued that S2's written work did not align with her verbal explanation (Table 2). S1 stated that S2 initially drew "three little rows," even though S2 said aloud that the cookies were, "half." S2's written work (Figure 1) displays her revised solution after talking with S1. S2 erased the third tally mark and replaced it with a semicircle, similar to how S1 drew his diagram. S2 also wrote, "my solution is split the cookies in half", as if to clarify her solution strategy. Although S2 conceptually understood how to equally partition the cookies among the four people in the problem, her written and verbal explanations were fortified after engaging in a conversation with her peer. The student sample highlighted in this study, further demonstrates the importance of providing frequent and supported opportunities for students to interact. Engaging in mathematical discourse practices, such as explaining one's thinking to others can promote learning by encouraging learners to strengthen or restructure their own knowledge and understandings as well as acquire new strategies and knowledge (Webb et al., 2019; Erath 2017).

Discussion

In this case study, two students helped each other deepen their understanding of fractions via interpersonal dialogue. They both used similar strategies after they had solved it individually. The two students in this example were able to correctly partition the cookies even though they still needed to learn how to name the parts. This aligns with the learning trajectory put forth by Empson and Levi (2011). During the peer dialogue, S1 helped S2 clarify her thinking by asking why she drew three lines of equal length. After explaining her reasoning, S2 realised she needed to revise her picture to match her strategy. S1 also deepened his understanding of fractions when formulating his argument as to why S2's picture was inaccurate.

Unlike Chan and Sfard's (2020) observations, there was not an unequal power struggle between the two students because they had solved the problem individually and used similar strategies. S1's picture was more accurate than S2's, even though her verbal explanation implied that she "split the cookies in half". Rather than one student "tutoring" the other, S1 and S2 shared and compared their written and verbal explanations. They genuinely explored each other's reasoning and S1 helped S2 and himself accurately name the size of the parts. S1's querying helped S2 revise her work to reflect the mathematising she shared in her verbal description. This example demonstrates the ways S1 and S2 deepened their understanding of an equal-sharing fraction problem through dialogue.

The partner conversations that we are concerned with belong to a specific category of peer interaction that distinguish them from collaboration. Because CGI frameworks draw attention to students' development along a concrete to abstract trajectory, we do not advocate for students to collaborate in solving problems. Instead, we see value in independent problem solving followed by peer interaction. In this case, students explained their own approach to each other and the expectation was that they strove to understand what their partner had done.

Conclusion

This case study illustrates the importance of pairing students who have individually solved the problem with different, or in this case, slightly different strategies, and providing language scaffolds or protocols to extend student talk and meaning making. The two students in this example modelled conversation skills using a partner interview protocol. They also extended the script to query each other's ideas. Valuing students' solution strategies and asking their peers to listen to and respond to their explanations is not only beneficial for their mathematical and language development, it also elevates students' discipline-specific dispositions towards mathematics (Gresalfi & Cobb, 2006). This has implications for in-service and preservice teachers around the globe who understand the value of peer-to-peer discussions but need support with structuring those conversations, in-the-moment, in ways that lead to productive conversations between peers.

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