Symposium: Effective Mathematics Teaching: Building Partnerships to Co-Develop Evidence-Based Capability

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Providing professional development at scale requires engaging diverse stakeholders to ensure support is based on research evidence and meets a range of teachers' needs. This symposium outlines research, partnerships and initiatives undertaken by a mathematics team in a state department of education to build a cohesive network of resources and professional learning to improve mathematics teaching and learning across the state.

Supporting teachers with relevant resources and professional learning is a priority to promote improvement in mathematics teaching and learning. At a systemic level, providing support at scale while recognising the highly diverse needs of teachers and schools is a well-documented challenge. A significantly revised mathematics curriculum has heightened the need for timeliness and range of expertise and perspectives. Collectively, the papers in this symposium tell a story of how a state department of education strategically partnered with mathematics education researchers, teachers and schools to design and implement a range of co-ordinated initiatives to support teachers and improve students' learning in mathematics.

In the first paper, Wood and her colleagues outline the history and background of ways that the Queensland Department of Education (the Department) have sought to support teachers to develop their mathematics pedagogy through a range of strategic partnerships across two decades. In *Building system-wide mathematics pedagogy through collaborative partnerships*, the authors discuss the impetus behind building teachers' pedagogical expertise in guided mathematical inquiry by working with mathematics education researchers as critical friends and developing resources at scale. In the second paper, *Designing curriculum resources to support teacher learning,* Goos details her theoretical analysis of the design of resources supporting teachers to "learn how to learn" to teach content that was new to them in the Queensland senior secondary mathematics syllabuses. Her paper exemplifies the Department's initiative to create a suite of professional learning materials for teachers designed by mathematics education researchers in a range of topics in mathematics curriculum, pedagogy, and classroom strategies. In the next paper, *Building capability: What to do when you don't know what to do*, school practitioners Moran and Lambie discuss how their school worked with a mathematics education researcher as a critical friend to address a problem of practice: improving students' performance on a new state assessment using complex, open-ended problems. They provide school-based evidence of how the using a research-based framework supported students to build confidence in addressing these tasks. Finally, in *Building capability for teachers of mathematics,* Horne and Hillman outline the partnership between the Department and an experienced teacher to develop resources that build teachers' capabilities in teaching mathematics. The 'How to Teach Mathematics Toolkit' seeks in particular to support beginning teachers and those teaching mathematics out-of-field in an online resource.

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Building System-Wide Mathematics Pedagogy Through Collaborative Partnerships

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Research shows that when students engage with mathematical inquiry their problem-solving skills are strengthened. Demands in the revised Australian curriculum raised problem-solving of new senior secondary mathematics assessment, specifically in Queensland Problem-solving and Modelling Tasks (PSMTs). The challenge for Queensland was to scale inquiry pedagogies through secondary state schools in a way that was age appropriate and curriculum-aligned. A system, researcher and teacher collaboration produced a suite of resources and capability materials to build inquiry pedagogies of secondary teachers and ultimately students' problem-solving skills.

When students engage with guided mathematical inquiry, their problem-solving skills are strengthened (Lazonder & Harmsen, 2016). In addition, students experience greater engagement, enjoyment, and achievement (Collie & Martin, 2017). However, research outlines the difficulties that teachers experience in learning to adopt and appropriately guide their students' learning to engage in mathematical inquiry (Makar, in press; Munter, 2014). Facilitating system-wide pedagogical change in mathematics classrooms requires a multi-faceted, scalable approach (Roesken-Winter et al., 2021; Spillane et al., 2018).

This paper outlines the direction, history and development that the Queensland Department of Education made over 15+ years in supporting teachers to engage in adopting guided mathematical inquiry and problem-solving state-wide across all levels of schooling. The implications of this journey can provide guidance for system-level change over time in other jurisdictions. The outcomes highlight the importance of vision, partnerships, resource development and time in seeing systemic improvement in mathematics pedagogy from primary through secondary.

Queensland's Journey

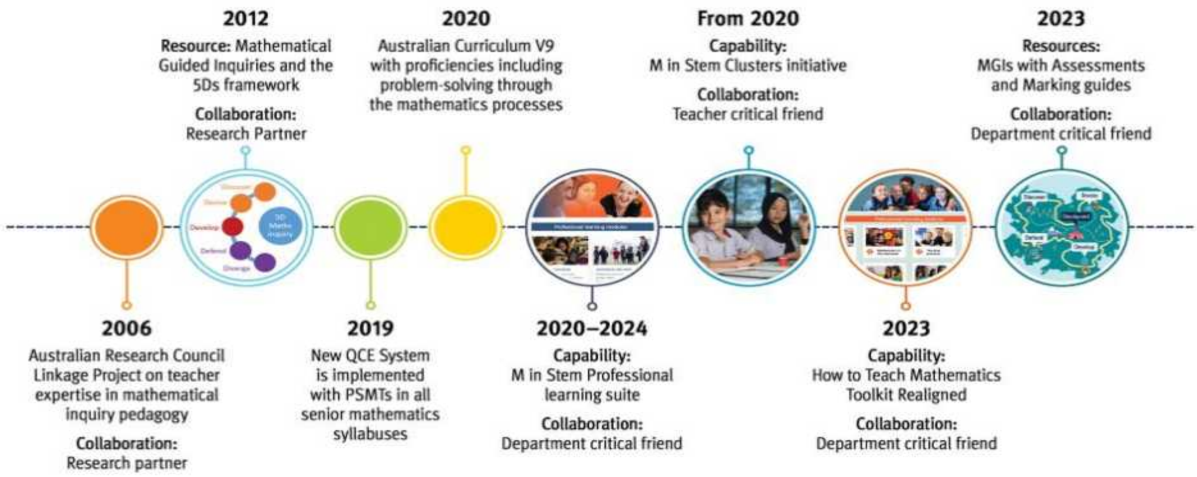
In 2023 the Queensland Department of Education (the Department) reaffirmed its commitment to prioritising achievement in mathematics in the education strategy, Equity and Excellence—a progressive, high performing education system realising the potential of every student—and continued a systematic approach to lifting outcomes for students in mathematics through building teacher capability. The Department has established a Prep to Year 12 approach to developing students' problem-solving skills with a focus on building teacher capability in inquiry pedagogies particularly in the secondary phase of schooling. Central to this has been collaborative partnerships with mathematics education researchers (Rosenquist et al., 2015). The Department's journey of valuing evidence-based practice through partnering with researchers has spanned almost two decades. In 2006, a formal partnership was initiated between the Department and a university in the form of an Australian Research Council (ARC) Linkage Project. The aim of the project was to study teachers' evolving experiences as they developed expertise in teaching mathematics through inquiry. The partnerships expanded to other universities over the years.

A Strategic Approach to Building Teacher Capability in Mathematics Pedagogy

The M-in-STEM initiative of the Queensland Department of Education was established in 2019 to strengthen age-appropriate and curriculum-aligned pedagogical practices of teachers of mathematics in Prep to Year 12. Through the STEM team, the Department engaged with fourteen mathematics education researchers from seven universities across Queensland and beyond to co-develop resources and capability programs to support teachers in adopting inquiry and problem solving pedagogies. The partnerships contributed to a range of co-ordinated and differentiated resources to build teacher capability (Figure 1), with many of these partnerships continuing.

Figure 1

Nature and Outcomes of Research Collaboration



Professional learning resources were developed for teachers across a range of experiences. The *How to Teach Mathematics Toolkit* was designed to support beginning, returning and out of field teachers of mathematics and is self-paced and online. Mathematical inquiry is addressed in two modules: Teaching practices—the pedagogy of inquiry, and Problem-solving—using the mathematical guided inquiries (MGIs). A partnership with a mathematics education researcher saw the adaptation of a framework for effective teaching of mathematics (National Council for Teachers of Mathematics [NCTM], 2014) supported with video content unpacking the framework and its implementation in classroom practice.

The M in STEM professional learning suite was developed to support experienced teachers of mathematics, particularly in secondary. Researchers co-developed professional learning resources with evidence-based pedagogical approaches across a range of topics supported with video content of classroom implementation. For example, in the inquiry module, the video case study *Effective inquiry strategies: Implementing a mathematical inquiry framework* documents how a secondary school adopted the 5Ds approach (Allmond et al., 2010) by implementing MGIs by collaborating with a mathematics education researcher. The partnership supported a consistent approach to inquiry across Years 7 to 10 in the school to ensure their students developed the age appropriate and curriculum aligned skills they needed to meet the demands of the PSMT into Years 11 and 12.

M in STEM Clusters Initiative

Building capabilities at scale required systemic partnerships between the Department, schools, and mathematics education researchers to support teachers with identified problems of practice (e.g., Koichu & Pinto, 2018). More direct collaboration between researchers and teachers was facilitated through the M in STEM clusters initiative. Researchers walked the journey of school improvement in mathematics with middle and aspiring leaders working in a cluster with a similar problem of practice. Researchers played the role of critical friend and provider of professional learning.

Case Study—M in STEM Collaborative Mathematics Inquiry

In 2021 a cluster of six state secondary schools undertook a collaborative inquiry to investigate how to improve implementation and outcomes in the PSMT requirements of the senior secondary mathematics syllabuses. PSMTs were prioritised as they contributed 20% of the final mathematics grade in Year 12. Pre-intervention data analysis across the six schools showed that students achieved lowest in the *Solve* and *Evaluate and Verify* criteria of the PSMT (compared with, for example, *Communicate*). The working hypothesis was that low performance in Solve was due to difficulties in the *Formulate* criterion, and, that low performance in Solve led to difficulties in the Evaluate and Verify criterion. Furthermore, both teachers and students were challenged by the language of the PSMT criteria.

In discussing the problem of practice with a content expert researcher as critical friend, the cluster agreed to use brainstorming in senior secondary (Years 11 and 12) to support student confidence in the Formulate and Evaluate and Verify stages of PSMTs. Taking an inquiry approach, the cluster backward mapped from the intended student outcomes and class behaviours, requisite teacher practice and professional learning, and the expected evidence of these anticipated changes, to understand how the cluster leaders needed to create the conditions for the change to occur. The M in STEM initiative provided the professional learning in instructional leadership and opportunities for intentional collaboration.

Schools in the cluster introduced brainstorming in junior secondary to strengthen problem solving skills and a whole school approach to language and pedagogy for problem solving tasks. Co-developed lesson plans and teaching resources were developed for implementation across the schools that developed brainstorming skills. Brainstorming encourages people to think in a free and open way with no restrictions. As a result, they often generate more possibilities than they would using a structured approach (Dugosh et al., 2000). The shared lessons were implemented with classroom routines and norms established to build students' confidence in the process and a safe environment for sharing ideas. *Fermi* problems were also used to stimulate ideas to evaluate and make assumptions. Fermi problems are miniature modelling problems that emphasise estimation (Albarracín & Ärlebäck, 2019).

Monitoring and reviewing activities including classroom walk-throughs, feedback from teachers and students, and pre- and post-intervention data analysis, showed an increase in confidence in both brainstorming and PSMT processes for students and teachers, improved disposition towards PSMTs for teachers and students and improved assessment literacy. The school found evidence of its effectiveness not only in the initial criterion (Formulate), but gave students confidence to proceed across all four criteria (Formulate, Solve, Evaluate and Verify, Communicate). A video capturing the case study was provided to all schools as an example of a high quality strategy and benefit of engaging a critical friend. The benefits of working in a cluster were identified as access to critical friend, opportunities to collaborate with other schools, and professional learning around an inquiry approach to school improvement.

Next Steps

The quality assured and curriculum-aligned materials developed through system-researcher partnerships are provided to all Queensland state schools to lift mathematics outcomes for students. In addition system-researcher-teacher relationships have further strengthened the network of mathematics educators across the state. While we have significant case studies demonstrating impact (see for example, Moran & Lambie, 2024), next steps are to gather further evidence of reach and impact of these materials. This includes evidence of downloads and access to online resources and professional learning, localised case studies of impact in schools and clusters, opt-in surveys of teacher feedback after engaging with and implementing the materials, and monitoring trends in system-wide mathematics reporting data.

This paper outlined strategies that enact the Department's commitment to strengthening teaching and learning in mathematics at scale to ensure every student realises their potential. It exemplifies the Department's focus on fostering collaborative partnerships to build evidence-based, inquiry pedagogies in mathematics across the system to build problem-solving skills.

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Designing Curriculum Resources to Support Teacher Learning

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This paper presents an analysis of how resources were designed to support implementation of the new Queensland senior secondary mathematics syllabuses. The analysis draws on the concept of educative curriculum materials that build teachers' subject matter knowledge and pedagogical content knowledge. Such resources are intended to help teachers "learn how to learn" to teach mathematical content that is new or unfamiliar to them.

Introduction of the Australian Curriculum: Mathematics at F–10 and senior secondary mathematics levels has led to diverse initiatives by state and territory education jurisdictions to provide resources and support for teachers. This paper examines one aspect of the Queensland Department of Education's *M in STEM* initiative, which involves collaboration with university-based mathematics education researchers to develop a professional learning suite for secondary school mathematics teachers. The resources are intended to strengthen teachers' curriculum knowledge and pedagogical practices across six topics: productive dispositions, problem solving and inquiry, modelling, reasoning, new content in the senior mathematics syllabuses, and strategies for long term retention of knowledge and preparing students for assessment.

The aim of this paper is to analyse the process used to develop resources for one of the focus topics in the professional learning suite: learning to teach new content in the senior secondary mathematics syllabuses. The analysis draws on the concept of *educative curriculum materials*, that is, curriculum resources that are designed to promote teachers' learning of mathematical content and pedagogy as well as student learning (Ball & Cohen, 1996; Davis & Krajcik, 2005). The paper addresses the following research question:

- How can curriculum resources be designed to support mathematics teacher professional learning in the context of curriculum reform?

Curriculum Context

In 2019, the Queensland Curriculum and Assessment Authority (QCAA) introduced new syllabuses for senior secondary mathematics based on the subjects developed by the Australian Curriculum, Assessment and Reporting Authority (ACARA, n.d.): General Mathematics, Mathematical Methods, and Specialist Mathematics (version 8.4). The new syllabuses included mathematical content that was either new or at a higher level of difficulty than in the previous Queensland syllabuses for the equivalent subjects of Mathematics A, Mathematics B, and Mathematics C respectively. The new suite of subjects represented the most significant change to senior secondary mathematics curriculum in Queensland since the previous syllabuses were launched in 1992.

The support offered to teachers for implementing a new syllabus is often in the form of instructional materials that help teachers interpret the official curriculum and create their own personal plans for teaching specific groups of students (Remillard & Heck, 2014). One challenge in designing such resources is to find a realistic balance between pedagogical prescription and professional autonomy (Davis & Krajcik, 2005). Determining the appropriate amount of guidance needed by senior secondary teachers was a particular challenge for General Mathematics, the subject most likely to be taught by out-of-field teachers who have undertaken limited advanced studies of mathematical content and little or no formal preparation in teaching mathematics. Text-based and online curriculum materials are also more educative for teachers if combined with in-person social support (Robutti et al., 2016). However, delivering a state-wide professional learning program containing face-to-face elements is challenging in Queensland, the Australian state with the most decentralised population spread over a very large area (see Australian Bureau of Statistics, 2022; Geosciences Australia, 2023). Each of these constraints influenced the design of the *M in STEM* professional learning suite.

Theoretical Background

The design of resources for one of the *M in STEM* focus topics is analysed by reference to five high-level guidelines for educative curriculum materials set out by Davis and Krajcik (2005). They proposed that educative curriculum materials should:

- Develop teachers' capacity to anticipate and interpret student thinking during instructional activities, as well as how to respond to student thinking (e.g., by using appropriate examples or instructional representations);
- Support teachers' learning of the subject matter and related disciplinary practices;
- Help teachers recognise how a learning objective, instructional activity, or lesson Sequence is related to the curriculum as a whole;
- Make visible the resource developer's pedagogical reasoning, thus enabling teachers to integrate this knowledge into their own repertoire;
- Promote teachers' pedagogical design capacity so they are able to make principled adaptations to the original curriculum materials.

In these ways, educative curriculum materials build teachers' subject matter knowledge and pedagogical content knowledge.

Designing Curriculum Resources for General Mathematics

In the curriculum context outlined above, consultation with the Queensland Department of Education led to a decision to focus on teaching new content in the General Mathematics syllabus (QCAA, 2019). It was not feasible to design curriculum resources for every topic in the syllabus that was likely to be new or unfamiliar to teachers. Instead, three syllabus topics considered to be most demanding for inexperienced or out-of-field teachers were selected: linear equations and their graphs; geometric sequences; and planar graphs, paths, and cycles.

Teachers are also time poor and not always willing to engage with extensive materials. Thus, the resources needed to concisely address key ideas for teaching while simultaneously illustrating how teachers could "learn how to learn" to teach other new topics in the syllabus. This was done by creating, for each topic, a series of three recorded PowerPoint presentations outlining evidence-based pedagogical strategies (1 hour total) and a placemat that defined the topic together with planning and teaching principles. These static resources were supplemented by an interactive 40-minute online professional discussion with teachers from around the state.

The intention was to develop a consistent structure for the recorded presentations that would expose the pedagogical decision-making underpinning the design. The rationale for these decisions was also made explicit in the placemat representing the design process. The design process moves through three stages: (a) interrogating the senior syllabus to identify and understand the subject matter; (b) mapping connections backwards, forwards, and across the Australian curriculum; and (c) designing pedagogy by selecting appropriate representations and real-life examples, and addressing common misconceptions that hinder student learning. The design process for the geometric sequences PowerPoint presentation is illustrated in Table 1 and mapped against Davis and Krajcik's (2005) guidelines for educative curriculum materials.

The principles underpinning the design process illustrated in Table 1 were articulated in the topic placemat, which is presented in abbreviated form in Figure 1. The placemat highlights the teacher's role in bringing the curriculum to life for students, by moving back and forth between the curriculum world, real world, and classroom world.

Table 1

Design Process for Geometric Sequences PowerPoint Presentation

(a) Interrogate the syllabus

Educative curriculum materials guideline 2: Support teachers' learning of subject matter

What subject matter is included?

- Generating sequences using recursion or rule for the n^{th} term
- Displaying the terms of a sequence in tabular or graphical form
- Using geometric sequences to model and analyse (numerically or graphically only) practical problems involving geometric growth and decay

How are key terms defined?

- Syllabus glossary definition of a geometric sequence, "a sequence of numbers where each term after the first is found by multiplying the previous term by a fixed non-zero number (excluding ±1) called the common ratio" (QCAA, 2019, p. 59)
- Common ratio > 1 → exponential growth; Common ratio < 1 → exponential decay
- Illustrate two methods of generating the geometric sequence 2, 6, 18, 54, …:
- Recursion relation $t_1 = 2, t_{n+1} = 3t_n$ for $n \geq 1$
- Rule for the n^{th} term $t_n = 2 \times 3^{n-1}$ for $n \geq 1$

Why is the topic important?

- Geometric sequences are used to understand real life situations and solve real life problems involving exponential growth and decay

(b) Map the curriculum

Educative curriculum materials guideline 3: Relate the topic to the curriculum as a whole

What prior learning have students experienced from the F–10 mathematics curriculum?

- Understand the connection between algebraic and graphical representations
- Solve basic problems involving simple and compound interest

What other topics in the General Mathematics syllabus connect to this topic?

- Loans, investments, and annuities: Use a spreadsheet to investigate the effect of interest rate on the future value of an investment

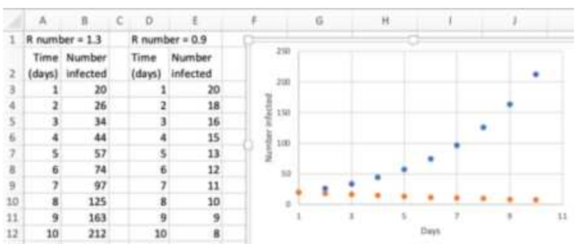
What other curriculum areas connect with this topic?

- Physics and ancient history: radioactive decay, carbon dating
- Biology: population growth, bacterial growth, spread of infectious diseases such as COVID-19

(c) Design pedagogy

Educative curriculum materials guideline 1: Anticipate, interpret, and respond to student thinking

- Select appropriate representations and link to real life examples.



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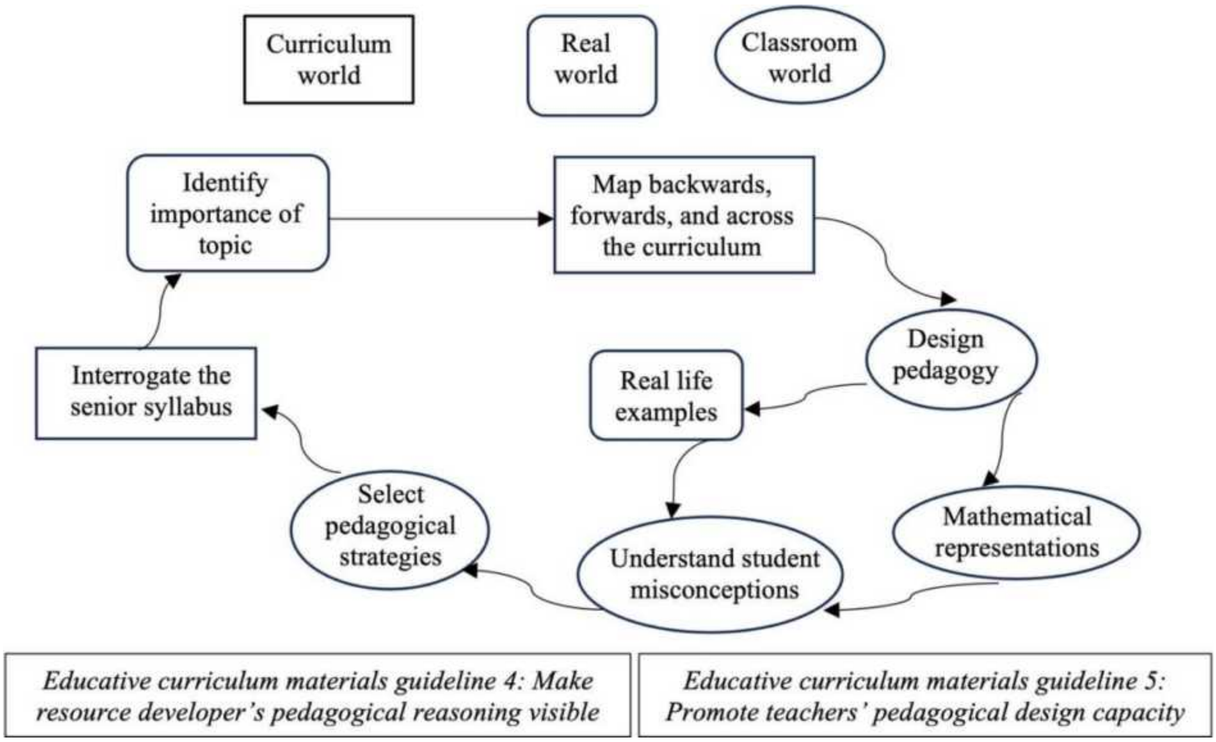
Time (days)	Number infected	Time (days)	Number infected
1	20	1	20
2	26	2	18
3	34	3	16
4	44	4	15
5	57	5	13
6	74	6	12
7	97	7	11
8	125	8	10
9	163	9	9
10	212	10	8

- Understand and respond to students' thinking
- Recognise misconceptions about working with numerical expressions in exponential form.
- Encourage students to explore both recursion relationships and the rule for n^{th} term to define sequences
- Provide experiences for students to explore and compare additive and multiplicative patterns that model arithmetic and geometric sequences respectively
- Use technology so students can investigate patterns of growth and decay in sequences, generating both tables of values and graphs

Figure 1

Placemat for Teaching New Content in Senior Mathematics



Concluding Remarks

This paper illustrates one approach to designing educative curriculum materials that support teacher learning as well as student learning. In principle, the resources developed for Queensland senior secondary mathematics teachers align with the guidelines proposed by Davis and Krajcik (2005). However, little is known about teachers' uptake of these resources and what difference this makes to their professional knowledge and classroom practice. These promise to be fruitful areas for future research on teachers' learning in times of curriculum reform.

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Building Capability: What to do When You Don't Know What to do

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The Queensland Certificate of Education (QCE) system is Queensland's senior school qualification. To support the introduction of the system in 2019, existing senior syllabuses were redeveloped and a new senior assessment model was established, this included the implementation of a mandatory high-stakes assessment task, the Problem-solving and Modelling Task (PSMT) in all four mathematics syllabi. The PSMT required new skills from both students and teachers to manage complex, open-ended investigations. In this paper, we reflect on our school's approach to build teacher capability in designing PSMTs and supporting student engagement with PSMTs.

In 2019, the new Queensland Certificate of Education (QCE) system was introduced. To support the introduction of the system, existing senior syllabi were redeveloped and a new senior assessment model was established to strengthen the quality and comparability of school-based assessment. These changes included the introduction of a mandatory high-stakes assessment—the Problem-solving and Modelling Task (PSMT)—to four senior mathematics syllabi: Essential Mathematics, General Mathematics, Mathematical Methods and Specialist Mathematics. PSMTs are designed by schools and then must be approved by the state assessment authority. Designing these tasks and supporting students to successfully engage with them required new skills. The PSMT was designed to evaluate a student's ability to respond to an investigative mathematical scenario or stimulus in relation to the mathematical concepts they have learned against four assessment criteria: Formulate, Solve, Evaluate and verify, and Communicate (QCAA, 2021). In most cases, the key feature of this task has been to provide for a response that addresses a real-life application of mathematics. Indeed, rich mathematical understanding goes beyond being able to correctly complete mathematical exercises, but also to make connections and transfer learning to unfamiliar problems (Skemp, 1978; Sullivan, 2011). As Peter Sullivan (2011) advised, "One of the major constraints that teachers experience when utilising such tasks is that many students avoid risk taking and do not persist with the challenges that are required in order to complete the task." (p. 38).

In this paper, we reflect on how our department in a state secondary school, in which the two authors were teacher leaders at the time, sought to facilitate this process for teachers and students. We drew on research and partnered with a university researcher, using a framework and seeking strategies to support students to build problem solving competence on complex, unfamiliar problems to allow us to reflect on, improve, and evaluate our progress.

Initial Practice of PSMTs

With the introduction of the new senior syllabi, our mathematics department wanted to ensure students were given every opportunity to perform well in the PSMT. A decision was made to introduce a practice PSMT as part of the problem-solving proficiency of the Australian curriculum at Year 10 to strengthen transition from junior to senior secondary mathematics courses. While we gave plenty of scaffolding on how to set out the response and developed understanding of the valued features outlined in the marking criteria, many students struggled to complete the task and wanted to be shown how to find the answer (Sullivan, 2011). This expectation was in line with the students' experiences of class work, but could not be provided in a PSMT setting where students are required to develop their own unique response to the set problem.

We noticed that our initial PSMT results in 2019 indicated that our high achieving students, who were preparing to study Mathematical Methods (Cohort A), generally performed well, with

88% passing (grade of C or better); whereas students who were preparing to study General Mathematics (Cohort B) did not have the same level of success, with only 63% passing. To better understand where our students were struggling, we followed up with some students to seek their feedback. Many told us that they simply did not know what to do and the teachers could not tell them, so they gave up. Essentially, they did not know how to start, and if they did attempt the problem and got stuck, they struggled to look for alternative approaches. We wondered how to help students with the challenge, 'What do you do when you don't know what to do?', so that they would have the confidence and skills to solve the unfamiliar problems posed by PSMTs.

A New Approach

We realised that it was necessary to employ a different pedagogical approach to improve student success and disposition. We were teaching mathematics using direct instruction with a gradual release of responsibility and a move from simple to complex questions. We also had an established problem-solving approach. However, these approaches were not helping students address unfamiliar, open-ended tasks like PSMTs. That is, our pedagogies were based on students being modelled how to do the mathematics first, before moving to a related application or a question they were less familiar with; however, even in these instances, the problems we gave students were fairly well-defined and typically had a single correct answer. The difference the PSMT posed was that students needed to come up with a way of responding to the task that was not directly modelled first by the teacher; and to find a solution to a task that did not come with an answer page to reassure them of their accuracy.

We decided to explore different pedagogical approaches that may help address students' unwillingness to make a start on the PSMT and complete it without teacher guidance. We conducted a search for solutions and identified research on the pedagogy of mathematical inquiry. Mathematical inquiry is an approach to solving complex, open-ended problems that relied on mathematical evidence (Makar, 2012). (Excerpts are from a video case study created about our journey by the Queensland Department of Education for the M-in-STEM Professional Learning Suite.)

> Guided inquiry [is] responding to the question, 'What do you do when you don't know what to do?' Complex open-ended problems like problem-solving and modelling tasks ask students to negotiate, adapt and revise their solution. This can be a real challenge for students. (K. Makar, quoted in Queensland Department of Education, 2022, 02:56)

We contacted the researcher, who agreed to discuss their research with us and act as a critical friend. We were particularly interested in their 5D model (Discover, Devise, Develop, Defend, Diverge; Allmond et al., 2010), which seemed to align well with the criteria already used with PSMTs. The Discover phase in the 5D model extends the QCAA modelling criteria by providing students with extra scaffolding to get started with the problem. It also engaged students with the problem context in a low-stakes setting before starting to plan a possible solution. This seemed particularly helpful when students did not know how to start the PSMT.

Together with the researcher, we (second author) designed an inquiry approach to support students in approaching the PSMT confidently and appropriately, using the 5D model as a guiding framework. We observed how students responded and plans to evaluate student data on improvement:

> In the past, our students have struggled to engage because of the size of the [PSMT] task. What we are seeing now, is that by using the 5Ds, students are less stressed and more engaged with the problem-solving and modelling tasks, and in turn, more willing to engage with [the] assessment. Next, would be a checkpoint at the halfway mark to see how much of the task students have completed. We can compare this directly with previous assessment pieces. Thirdly, we'll be comparing student achievement data. (K. Lambie, quoted in Queensland Department of Education, 2022, 05:54)

From 2020, we also began giving our Year 9 students inquiry tasks similar to PSMTs to give them additional practice, each time using the 5D framework as a scaffold. In particular, five lessons were developed to assist Year 9 students in unpacking a PSMT-like task using the 5D inquiry model. This approach sought to:

- Give students an opportunity to experience risk in a low stakes environment;
- Reduce the size of the task;
- Reduce student stress;
- Include a ‘checkpoint’ opportunity, where students shared their interim progress and discussed difficulties they were encountering with peers to generate possible ideas.

The benefit of providing students with an age-appropriate and curriculum-aligned inquiry task in Year 9 was that the students became much more confident by Year 10 because they had gone through a similar process the year before. As students were initially unfamiliar with this assessment style, the Year 9 task was designed to give students confidence moving forward and from the beginning, the Year 9 cohort demonstrated a high passing rate (93%). They also transitioned well into the more challenging Year 10 practice PSMT. We looked at the practice PSMTs again to see whether our Year 10 students felt more confident. Indeed, within these two years, students’ practice PSMTs significantly improved (Table 1), particularly the Year 10 lower-performing students (Cohort B) who increased their passing rate from 63% in 2019 to 96% in 2021.

Table 1

Practice PSMT Aggregated Results From our Year 10 Cohort

	PSMT percentage of students passing (grade C or better)	
Subject	**2019**	**2021**
Cohort A	88%	99%
Cohort B	63%	96%

Building Thinking Classrooms

Following our work above, we have continued to seek ways to improve students’ learning and their confidence to tackle complex, unfamiliar tasks. We recently attended a two-day professional development workshop provided by the Department on how to build thinking classrooms in mathematics (Liljedahl, 2020). The *Building Thinking Classrooms* workshop, delivered by Professor Peter Liljedahl represented the next step in our journey to increase problem-solving opportunities and success for students in our classrooms. Liljedahl’s approach to mathematics teaching emphasised a combination of group work, open-ended questions and opportunities to explore and test understanding in a low-risk environment. The combination of random grouping, the use of vertical whiteboards and the collaborative solving of problems that had not been previously modelled provided a fun-filled and engaging opportunity to solve mathematics problems in a safe and supportive environment. One of the difficulties we continue to have is getting students to make a start on solving problems when they are not sure if their selected methods will lead them to the answer. Liljedahl’s work provided us with further strategies to address this challenge with our students.

Following the workshop, we shared two different activities with the other teachers at our mathematics meetings and we have written a number of *Building Thinking Classrooms* activities into our mathematics plan. As we move towards the implementation of Australian Curriculum: Mathematics (Version 9), we have identified opportunities to embed these practices across all year levels, and our teachers are on board with this new approach, already incorporating the activities in their classrooms across all year levels. Teachers are motivated to learn more about the approach and how they can continue to support student engagement and success in mathematical problem-solving.

Conclusion

As a school, we are always interested in improving student learning. In particular, we were wanting to support students to respond to the challenge of "What do you do when you don't know what to do?" The inclusion of mathematical processes in the newly revised Australian Curriculum: Mathematics (v9.0) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2022) provides increased opportunities for students to address this challenge. "The mathematical processes ... are mathematical problem-solving and investigation processes that students learn to use in mathematics, and that draw upon students' mathematical process skills and proficiency in mathematics in an interconnected way" (ACARA, 2022, Mathematics, Key Considerations, Mathematical Processes section). We have been encouraged by the Year 10 practice Problem-solving and Modelling Tasks (PSMTs) in their capacity to increase student confidence, persistence, and skills in addressing complex, unfamiliar problems. Three approaches have assisted us to support students in this way: Drawing on research and working with a researcher as critical friend to guide the direction of our improvements to be evidence-based; engaging with the 5D framework (Allmond et al., 2010) and Thinking Classrooms material (Liljedahl, 2020) to improve our problem-solving pedagogy; and using data to reflect on, improve and evaluate our progress.

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Building Capability for Teachers of Mathematics

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Teaching resources and professional development based on mathematics education research have the potential to support teachers to develop and sustain improved pedagogies. The Queensland Department of Education provided online professional learning modules for teachers of Prep (Foundation) to Year 10 mathematics . To support implementation of the Australian Curriculum: Mathematics this evidence-based resource would assist teachers in understanding the curriculum and providing support for quality teaching and learning. This resource exemplifies the partnerships between the department, researchers and teachers in building capability in mathematics teaching.

The notion of curriculum is not static, with distinctions made between what is intended by curriculum writers and how curriculum is enacted in the classroom (Remillard & Heck, 2014). Teacher resources can greatly influence how teachers interpret the curriculum, and innovative resource materials have long been used to support teachers adopt more effective pedagogies (Ball & Cohen, 1996; Pepin, 2018). "The characteristics of these innovative materials ultimately influence teachers' instructional practices, including their use of curriculum materials" (Choppin, 2011, p. 332). Because of their potential impact on teachers' pedagogy, it is essential for resource materials draw on evidence of effective practice and based on contemporary research (Irgens et al., 2023; Munter, 2014; Roesken-Winter et al., 2021; Sullivan, 2011).

In this paper, we outline how a state education department developed an evidence-based online teacher capability to support beginning, returning and out of field teachers in diverse school and community contexts to implement Australian Curriculum: Mathematics by partnering with university researchers and teachers as co-constructors.

Promoting System-Wide Capability Development in Mathematics Pedagogy

The Queensland Department of Education (the Department) outlines its commitment to realizing the potential of every student, including the prioritising achievement in mathematics, in the education strategy, Equity and Excellence. Version 9.0 of the Australian Curriculum highlighted the need for state-wide capability-building for teachers of mathematics. The challenge was also identified in providing systematic and contextualised capability development in mathematics pedagogy for a range of teacher needs that draws on contemporary research. These challenges have been identified in research beyond Australia as well:

> Mathematics teachers face challenges in modifying their teaching to incorporate effective pedagogical practices, technology tools, and new curricula resources. They also face challenges in making changes to address updated standards and expectations for mathematics. … Teachers often have limited resources to support professional development to learn how to make these changes. Many teachers are seeking out online professional development opportunities. … Evidence suggests online professional development (PD) that is accessible, meaningful, collaborative, and addresses varied needs and abilities of participants can lead to changes in teachers' instructional practices. (Hollebrands & Lee, 2020, pp. 859–860)

A universal, online resource would provide foundational instruction tailored to beginning, returning, and out of field teachers of mathematics, and accessible to all state school teachers in Queensland. This was realised through the redevelopment of the 'How to Teach Mathematics Toolkit' (the toolkit). To maximise relevance and engagement for teachers, the resource offers online, self-directed modules with research-validated information and advice to build teacher knowledge, skills, and understanding of mathematics curriculum and pedagogy embedded in the context of teachers' classrooms and supported with peers (Kleiman et al., 2015; Powell & Bodur, 2019). Teachers' engagement in substantial professional learning resources such as this has been shown across multiple studies to have substantial improvement in student learning (Yoon et al., 2007).

The Department has a long history of creating and sustaining partnerships with researchers on the improvement journey of mathematics curriculum and pedagogy (Horne & Makar, 2013). Existing and new research partnerships were activated to ensure the resource was informed by leading edge evidence of effective pedagogy in mathematics (cf. Berger & Baker, 2008; Lillejord & Børte, 2016). Collaborative research partnerships were instrumental to this resource in three ways: knowledge translation, critical friends, and content co-developers (Irgens et al., 2023). Collaborative teacher partnerships were equally important providing lesson plans and videos demonstrating examples of content, mathematical guided inquires and related assessment.

Designing a Capability Resource Using Evidence-Based Practices

The toolkit is focused on evidence-based research and is organised over eight modules. The modules address current teaching and learning, the structure of the Australian Curriculum: Mathematics—including the mathematical proficiencies (Understanding, Fluency, Problem solving, Reasoning), and the importance of ongoing teacher personalised learning (Figure 1).

Figure 1

How to Teach Mathematics Toolkit Professional Development Modules

Research on teacher professional learning has highlighted that teachers value professional development that includes a focus on content, active learning, alignment with curriculum, and engagement over time (Haug & Mork, 2021). In the Teaching and Learning modules (1–3), teachers will explore:

- *Teaching Mathematics*: Mathematical opportunities, knowing and planning the curriculum and mathematical content, knowing how to plan a lesson;
- *Mathematics in the classroom*: Addressing you own self-efficacy, understanding your school's context, knowing your students with a focus on assessment and differentiation and how to support mathematical language within the classroom;

- *Teaching practices*: Understanding effective teaching of mathematics, focusing on positive dispositions, orchestrating classroom discourse, supporting student engagement and pedagogy in the mathematics classroom.

Modules 4–7 model the structure of the Australian Curriculum: Mathematics. By unpacking the mathematical proficiencies (*Understanding*, *Fluency*, *Problem solving*, *Reasoning*). Within each proficiency, the modules use content strands in teaching, learning, and assessing the proficiency with lessons and assessment ideas for each phase of schooling (Prep to Year 2, Year 3 to 6 and Years 7 to 10).

Finally, *Personalised learning* supports teachers in their understanding of continued learning in teaching and learning mathematics and reflect on their own self-efficacy.

The toolkit modules are self-paced and combine online learning and offline self-reflection and practical application. Importantly, there are opportunities for participants to consolidate and extend their learning through collaborative activities with a mentor, thus allowing a contextualised approach (Fantilli & McDougall, 2009). The toolkit encourages further reading and engagement with research through the resources lists included at the end of each module.

The toolkit modules are designed to align to the Australian Institute for Professional Standards (AITSL) so that completion of the course contributes to teachers' continuing professional development requirement for registration. This validates participants' investment of time in completing the toolkit. Teachers have the opportunity to collect evidence of their participation by recording reflections, mentoring discussions, implementation trials in the classroom, peer observations and student observations and work samples.

Next Steps—Supporting Mentoring and Scaling up Effective Practice

In the context of widespread teacher shortages the Department recognises the critical importance of universal access to high quality professional learning in mathematics, scaling up effective practice, sharing expertise through clusters and attending to teacher wellbeing (Haug & Mork, 2021; Irgens et al., 2023; Powell & Bodur, 2019). There are opportunities to support mentoring partnerships through:

- Mobilising suitable expertise as mentors within/across the department and in research organisations;
- Facilitate clusters to share expertise—strengthen and expand the network of mathematics educators by supporting partnerships between teachers and researchers as mentors.

There are opportunities to embed toolkit modules in initial teacher education programs to support transition of beginning teachers into mathematics classrooms in Queensland state schools.

Conclusion

The *How to Teach Mathematics Toolkit* is an evidence-based resource to promote system-wide capability development in mathematics pedagogy in Queensland state schools. It supports the implementation of Australian Curriculum: Mathematics in F–10, and combines both universal access and contextualised implementation to maximise reach and impact. It builds and shares expertise in mathematics pedagogy through collaborative partnerships to build capability of teachers to deliver quality teaching and learning in mathematics.

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