Student Problem-Posing During Open Mathematical Inquiry

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Problems presented in mathematics classrooms often focus on routine tasks with students practicing mathematical techniques demonstrated by the teacher. However, this does not reflect the problem-solving process in the real world, and students often find it difficult to connect school mathematics with authentic contexts. This paper discusses findings from a study of Year 5 student perceptions of problem-posing during a two-week open mathematical inquiry. While the semi-structured interviews indicated the students perceived themselves to be skilled at problem-posing, triangulation of the video observations and work samples told a different story.

Inquiry-based learning (IBL) in mathematics has gained prominence in Australian educational policy and curriculum as policy makers and educators seek to increase student engagement and achievement in mathematics. For example, an increase in available educator resources such as The Australian Academy of Science's Mathematics ReSolve project (Australian Academy of Science, 2023), coupled with a growing number of presentations focusing on inquiry approaches at Mathematics Education Research Group of Australasia (MERGA) conferences (e.g., Fielding et al., 2023; Wadham et al., 2023), suggests an increased interest in applying this approach in classrooms. The need to provide students with opportunities to transfer their mathematics learning to authentic situations and to connect the 'real world' to the 'mathematics classroom' is imperative and has been noted in the current version of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2023a). One way to achieve this is through open mathematical inquiry, which engages students in problem posing; indeed, an aim of the Australian curriculum is for students to:

> Develop proficiency with mathematical concepts, skills, procedures and processes, and use them to demonstrate mastery in mathematics as they pose and solve problems, and reason with number, algebra, measurement, space, statistics and probability. (ACARA 2023b, para. 2)

Inquiry Approaches and The Australian Curriculum

Bruder and Prescott (2013) discuss different studies that examined IBL in both science and mathematics, highlighting three types of inquiry:

> *Structured Inquiry:* The teacher gives the students a problem or question to be solved as well as the appropriate method and materials to solve it.
>
> *Guided Inquiry:* The teacher provides the students with the problems or questions and the necessary materials. The students' task is to find the appropriate problem-solving strategies and methods. The teacher guides students through the problem-solving process.
>
> *Open Inquiry:* The students' task is to find problems or questions they would like to solve and answer. They also decide upon the methods and materials they would like to use. (p. 812)

Makar's (2012) approach to guided mathematical inquiry is slightly different to what is described by Bruder and Prescott (2013) as, in her approach, students are presented with a well-considered, ill-structured question, and are supported by the teacher to identify and define important parts of the question and to determine what evidence is needed for them to answer the question. Much of the more recent research in Australia regarding inquiry-based pedagogy

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in mathematics has focussed on guided mathematical inquiry (e.g., Fielding et al., 2023; Fry & English, 2023), with little classroom-based research related to open mathematical inquiry, which was the approach that the students in this study undertook.

During open inquiry, students must devise questions based on a stimulus, locate additional information to answer their questions, and make decisions regarding the methods or materials they want to use. While having students formulate and pose their own problems is not a new phenomenon, it is less commonly used by teachers than other, non-inquiry based pedagogical approaches. Problems presented in mathematics classrooms are often presented by the teacher, and commonly focus on students finding a particular answer in a prescribed way, or practicing a mathematical technique demonstrated by the teacher through routine exercises (Schoenfeld, 2017).

In addition to being noted in the aims of curriculum, connections to problem-posing are also evident in the proficiency strands (ACARA, 2023c) and under Mathematical Processes: Mathematical modelling (ACARA, 2023a). The curriculum requires students from Year 3 onwards to use mathematical modelling to solve practical problems, formulate problems, and choose operations and strategies, as well as interpret and communicate solutions (ACARA, 2023d). We argue that open mathematical inquiry meets these curriculum requirements as, during open mathematical inquiry, the students must identify practical problems in situations, mathematise their ideas, solve the problem, and share their solutions.

Singer et al. (2015) argue that support of problem-posing in school mathematics can be considered from at least two vantage points, the first historical, demonstrating that problem-posing is an agent of change, and the second futuristic, with the knowledge economy and increasing societal pressure on educational systems. While both perspectives are beneficial, and the policy and curriculum interest in problem-posing is strong, researchers are still developing an understanding of the effects of problem-posing activities on engagement and achievement, as well as conceptualising the instructional strategies that support their effective implementation (Cai et al., 2015). Although the understanding of these processes may be somewhat limited, researchers have found that “problem-posing can enhance learners’ mathematics conceptual understanding, dispositions on mathematics, and mathematical creative thinking” (Xie & Masingila, 2017, p. 102). Furthermore, research has shown that open mathematical tasks promote engagement in mathematical thinking, communicating, problematising, and creativity (Attard, 2013; Xie & Masingila, 2017). These claims suggest that research on open mathematical inquiry is needed to support teachers in providing students with authentic problem posing and problem-solving opportunities.

This paper reports on one aspect of a study (Zorn et al., 2022) that sought to understand the ability of a group of Year 5 students to problem-pose by investigating their perceptions of engagement and ability to create their own mathematics investigations based on a video stimulus. The research question for this paper is How do Year 5 students perceive their ability to problem-pose using a video prompt as stimulus during an IBL mathematical problem-posing investigation?

Method

The qualitative findings reported in this paper are taken from a single instrumental case-study. Case-study methodology provides a framework to investigate various participant perspectives and discover patterns, relationships, and themes (Holosko & Thyer, 2011). A single instrumental case-study was appropriate as this study was conducted with one class (n = 17) and findings were developed based on the student perspectives from that one ‘bounded’ case. The first author led a two-week open mathematical investigation that required students to develop their own investigation questions based on a video stimulus centred on a tennis theme.

Seventeen students worked in collaborative pairs or groups of three to investigate their own questions and present their findings.

Data Collection and Analysis

Semi-structured interviews that were held with students to identify their perceptions of their own problem-posing ability after the open mathematical inquiry were triangulated with video observations and student work samples. The semi-structured interviews were recorded, transcribed, and analysed by the researcher, and this process allowed for multiple exposures to the data. Thematic analysis was used to identify, analyse, and interpret patterns as it provided an accessible and systematic way for generating codes and themes from the qualitative data (Clarke & Braun, 2017). Following thematic analysis, the video observations, student work samples, and literature related to the phenomenon, were used to triangulate and substantiate the findings.

Participants

The research was conducted at a co-education, independent school in South-East Queensland, Australia. The participating class consisted of 17 students and one teacher. Due to student absence, only 16 out of 17 students participated in a semi-structured interview. The school delivered the Australian curriculum through individualised pathways and students were engaged in a range of pedagogies throughout each day. From Year 5 the students learnt mathematics through 'Maths Pathways' as the school leadership believed it supported teachers to deliver personalised learning. Students engaged in online tutorials and questions based on their readiness and took fortnightly paper tests to identify growth and areas for improvement. As part of the program students also engaged in rich tasks and projects; however, the regularity of this was determined by each classroom teacher. Students had some experience with IBL pedagogy, and regularly posed questions for personal investigations in other curriculum areas; however, this was not done in mathematics.

Findings and Discussion

The data from the semi-structured interviews revealed that the students felt confident to problem-pose and develop their own mathematical investigations. When asked whether they found it challenging to create their own investigation and questions based on the video stimulus, 12 students reported that they did not find it challenging, two reported that it was slightly challenging, one student did not respond to the question, and one student's response was inaudible due to the student mumbling. Many of the student's responses were simple and direct, with minimal or no elaboration to explain whether it was initially challenging for them to pose questions to investigate. For example, responses included, 'no, not really', 'nah, not actually', 'a bit'.

However, triangulation of the interview data, video observations, and student work samples indicated that the students' perceptions of their ability were not aligned with their actual ability to independently problem-pose. The video observations and work samples revealed that the students had difficulty problem-posing and that they required teacher support to make connections between the video and mathematics and to mathematise their ideas (Zorn, 2022).

In the beginning of the unit, after watching the video stimulus, the students engaged in the Visible Thinking Routine—'see, think, wonder' (Lowe et al., 2013). The students were specifically encouraged to pose mathematical questions during the 'wonder' phase and were required to post their questions to a shared Padlet. While the students recorded 54 questions, only four were linked to mathematical concepts (e.g., area, perimeter, and money) for investigation in the classroom. This revealed how challenging it was for the students to connect mathematics to the stimulus. Most of the questions focussed on the emotions of the people in the video or were closed, subjective questions (see Table 1).

Table 1

Student Question Examples

Mathematical questions	Examples of non-mathematical questions
What is the total perimeter of an average tennis court?	I wonder if they play it professionally?
How big is a tennis court?	How are they feeling?
How much does it cost to buy a tennis court?	Do they get worried before a tournament?
I wonder how much money they earn?	Is there a certain technique to playing tennis?

In addition to the 54 questions, two students explicitly questioned the connection to mathematics. For example, Nick wrote, "How does tennis relate to math?" A later comment from Nick in the semi-structured interview suggested a shift in his thinking during the two-week investigation:

> Well, it was kind of out of the blue when you showed us the tennis video. Also, I thought how can this relate to maths. And when we actually go into it, I realise how like, how much maths is involved in everything.

Other comments described the impact of scaffolding during the investigation. The scaffolding helped the students to connect mathematics to the 'real-life' situation presented in the video, and opened their eyes to a world that is full of mathematics:

> They [teachers] should have taught us that pretty much everything can be math. (Matt)

> Yeah, how you wouldn't think it would be that involved in math, but it is and you have to use your brain in a way that you wouldn't think to use it. And yeah, that was just really cool to experience. (Milly)

These findings are consistent with the findings of Singer et al. (2015) who found that it was challenging for students to problem-pose, and that students need scaffolding to mathematise questions and ideas. In response to their problem-posing efforts, and to help the students in this study to see the mathematics in their daily life and in the mathematics in the video stimulus, Math Curse, a picture book by Jon Scieszka and Lane Smith (1995), was read to the class. The book follows a boy through his week and, as the story unfolds, he realises that he is surrounded by mathematics problems everywhere he goes. Table 2 illustrates a sample of the ideas as they developed through the two weeks.

Table 2

Developing Ideas

Initial question/idea	Developing questions
Are they doing [sic] a tournament?	Are there winners, how much prize money?
	How many courts and how much space is needed?
	Hitting the ball—what angles are involved?
	How many seats on each court and price per seat?
Is Roger Federer the best tennis player of all time?	How long has he played for?
	What are his scores and points?
	Who is the 1st, 2nd, 3rd and how do people vote?
I wonder if we could have a tennis court at school?	What area would we need and how long to build?
	How much money would it cost?
How is a tennis racket made?	What is the cost of the materials?
	The size—measurements, different sized rackets

Although their ideas developed during the two weeks, and their questions became more mathematically focussed, the work samples and video observations demonstrated that some groups still found it difficult to mathematise their investigation. For example, one group decided to work on 'ticket prices' and started to create a list with different types of tickets, randomly selecting a dollar figure that they felt was appropriate. After conferencing with the teacher, these students changed their approach and began researching the costs of hiring a local tennis court, umpires, and commentators for their tournament. They then used the information to calculate total costs for the tournament, and then worked out ticket prices based on covering expenses and making a profit. In doing this they also made decisions about the number of players, the number of games, and the number of courts that would be needed. During their class presentation, at the end of the two weeks, these students noted that they had started to create ticket prices but "it didn't work out so well, so we had to restart." This indicated that they were aware of the challenges they faced in formulating questions and solving the problem, and contradicted their perceptions shared during the interview that problem-posing was easy.

It was beyond the scope of this project to fully understand why some students felt confident to problem-pose even though the evidence indicated they needed scaffolding and support. However, the kind of teacher support provided may have contributed to their confidence. It was observed that the students were able to maintain a sense of autonomy and competence, resulting in high self-efficacy. Self-efficacy has been found to increase during IBL (Dunlap, 2005); however, the degree to which it is influenced by teacher involvement is unclear (Tawfik et al., 2020). The student-centred environment created in this study encouraged students to be active participants in their learning, with teachers often adopting more of a facilitative role, using skilful questioning, scaffolding thinking, and avoiding the temptation to quickly impart knowledge. To illustrate this point, the following student comments, shared during semi-structured interviews, articulate the level of support provided by the teacher:

> You and Miss E checked around and looked if we had any troubles or not. And if we did, you guys would sit down and help us, and if we didn't, you guys would just walk off, and be like, they don't need any help, they're fine and go see if anybody else needs help. (Koby)

> Just looking over us just to make sure that we've got the right criteria of what we needed to accomplish. But other than that, we were mainly doing it ourselves and that's what I like. (Freya)

The idea of the teacher as a facilitator is often associated with student-centred learning (Goodyear & Dudley, 2015), and is not a new phenomenon; however, it remains a common misconception of open mathematical inquiry, and other constructivist, student-centred approaches, that the teacher just creates a task and leaves the students to work together to learn (Goodyear & Dudley, 2015). That said, there does need to be a balance between active teacher involvement and student-directed learning, and it is essential that teachers provide support in autonomy-supportive ways.

Cheon et al. (2020) used self-determination theory principles to develop a teacher intervention, which supports teachers in creating an autonomy-supportive classroom environment. They found that when teachers allow students to work in their own way, at their own pace, provide explanatory rationales, and use invitational language, that these actions support students to meet their autonomy need satisfaction. This was further supported by providing structure through clearly communicating expectations, scaffolding progress, offering help, and providing constructive feedback that allowed students to experience self-confidence and competence need satisfaction.

In the semi-structured interviews, when the students were asked if they could pose an additional problem to investigate, all the students responded positively with the belief that they could do so. However, most of the proposed ideas reflected the ideas of their peers that were shared at the end of the two-week period. For example, Matt thought they would "probably [plan] a tournament", and Nick agreed that "scheduling a tournament and building it up" was a good idea. While their ideas were investigable topics, they were all previously presented to the class by their peers, lacked originality and were not mathematical questions, suggesting that the students may not be able to problem-pose as well as they perceived they could. It would have been beneficial to provide the students with a second video stimulus to develop insights into their evolving ability to problem-pose as it was difficult for them to move beyond their peers' ideas. Other ideas suggested by the students also indicated a contradiction between the students' perceived and actual ability to problem-pose. At the end of the inquiry, the students were asked if they could pose an additional problem related to the original prompt. Gemma and Eric's additional problems demonstrated a limited understanding of mathematical concepts and processes, and/or a limited understanding of the idea of mathematical problem-posing:

> I'd probably come up with something like, what top 10 players would like, put into an even tennis match or something. But it's like, it's not like you're going to have someone like, David Smith or something against like Roger Federer, because that'd be extremely unbalanced. (Eric)

> Writing a letter to the school to try and build one. Once we are done that, if they let us build it, we could host a tournament. (Gemma)

Eric's idea lacked logic or an understanding of the task, and did not relate to mathematics, rather it focussed on something that Eric found interesting. Gemma's response focussed on what she'd like to do next in relation to her initial investigation rather than an additional mathematics question.

Conclusion

This paper focussed on problem-posing during an open mathematical inquiry. The findings indicated that while the students perceived themselves to be able to problem-pose effectively, the student work samples and video observations contradicted this. During the study the students reported experiencing high levels of autonomy and competence need satisfaction (Zorn et al., 2022) and this may have contributed to high levels of self-reported efficacy, confidence, and ability to problem-pose (Zorn, 2022). Confidence and self-efficacy have been linked to student growth and positive academic outcomes. Students experiencing these positive emotions are more likely to take academic risks, increasing the likelihood of improving their problem-posing abilities. This reinforces the need to create an environment that ensures students feel supported and safe to work autonomously and feel competent in doing so.

The challenge with problem-posing in this study possibly reflects the fact that this was the first time the students had been exposed to the idea of problem-posing or creating their own mathematical investigations. The support they were given assisted the students to develop their initial investigation topics and questions, and it is unknown whether they would be able to work more independently to pose problems that could be solved using mathematics with a new video stimulus. It was beyond the scope of this study to provide the students with an additional stimulus to evaluate whether this initial experience of problem-posing would support them to develop problems based on new video stimulus.

Although some suggestions have been made regarding why the students may have self-reported feeling confident to problem-pose, future research could involve a longer-term study investigating what teacher practices support students to problem-pose effectively, and how teacher practices change over time to support student engagement and achievement during problem-posing.

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