Preservice Primary Teacher Pedagogical Content Knowledge of Fractions Using the Refined Consensus Model

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Fractions remains a difficult topic for primary preservice teachers (PST). This paper analyses a PST teaching episode using the *Refined Consensus Model* (RCM) of Pedagogical Content Knowledge (PCK) to explore challenges faced by PST in acquiring specialised knowledge for effective fractions teaching. Data include semi-structured interviews highlighting relationships between collective and personal PCK with a videotaped teaching episode allowing an interrogation of her enacted PCK. Key findings illustrate the relationships among a PST collective, personal, and enacted PCK, and the importance of the reflection phase of enacted PCK.

Empirical work has shown the significant relationship between primary teachers' mathematical content knowledge and their students' mathematics achievement (Campbell et al., 2014). Teaching demands deep knowledge of "subject matter for teaching which consists of an understanding of how to represent specific subject matter topics and issues appropriate to the diverse abilities and interest of learners" (Shulman & Grossman, 1988, p. 9). This deep knowledge is what Shulman termed 'Pedagogical Content Knowledge' (PCK), a framework that has been taken up widely in order to better understand knowledge development for teaching with both inservice and preservice teachers (PST). However, there are differing conceptual viewpoints of PCK in the extant literature. Some scholars position PCK as the knowledge of expert teachers that preservice teachers begin to accumulate during initial teacher education (ITE), that is, knowledge *for* teachers. For example, Ball et al. (2008) included PCK as a standardised and measurable part of mathematical knowledge *for* teaching. Alternatively, knowledge *of* teachers describes a more responsive and adaptive kind of knowledge that is particular to each teacher. Whilst acknowledging the central role that the specific learning context plays alongside the unique composition of each individual teacher's knowledge, the collective canonical knowledge in the field as developed through empirical and theoretical work is also key to understanding a teacher's PCK. Thus, we consider both knowledge *for* and *of* teachers to be important to understand how PST knowledge develops.

Research calls for greater clarity and rigour in PCK models (e.g., Abell, 2007), recommending further interrogation and refinement of models of teachers' PCK to better illustrate the relationships between teachers' knowledge and experience and how these impact practice and student outcomes. Building on the 2012 Consensus Model of PCK (Berry et al., 2015), the *Refined Consensus Model* (RCM) of PCK now encompasses three distinct realms of a teacher's PCK (Carlson & Daehler, 2019). The three realms and their proposed interrelationships are summarised in Figure 1. Collective PCK (cPCK) is the professional knowledge held in the field by the full range of educators and educationists. Personal PCK (pPCK), as the name suggests, is more personalised, residing in the individual teacher. Enacted PCK (ePCK) describes the knowledge subset upon which the teacher draws to guide pedagogical reasoning in planning, teaching, and reflecting on lessons taught.

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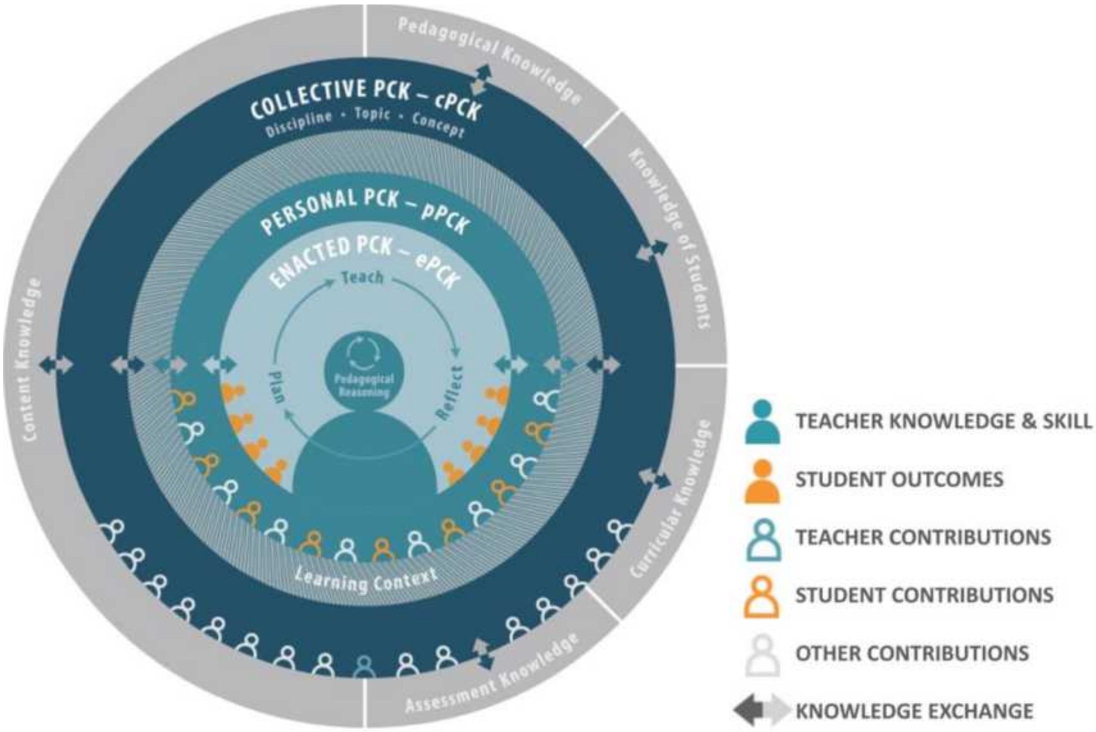
Figure 1

Refined Consensus Model of PCK (Carlson & Daehler, 2019, p. 82)



Also depicted in the RCM of PCK is the two-way knowledge exchange (⬅➡) that occurs between the knowledge realms. In this way, the model shows how teacher knowledge and skills are filtered or amplified by teacher attitudes and beliefs, thus shaping their pPCK over time. For example, a teacher's attitudes and beliefs about "students, the nature of content knowledge, or the role of the teacher" (Carlson & Daehler, 2019, p. 82) can amplify and/or filter the teacher's developing pPCK. The RCM of PCK has been taken up widely in science education research (Mientus et al., 2022), yet relatively few studies of mathematics teachers' PCK have adopted the RCM of PCK (e.g., Botha et al., 2023), and even fewer have used this model to interrogate *primary* school teachers' PCK (e.g., Amador et al., 2022). In the current paper, we use the RCM of PCK to explore its potential to conceptualise the relationships between PCK as both integrative (ePCK as knowledge *of* teachers) *and* transformative (cPCK as knowledge *for* teaching) as PST learn mathematics for teaching.

Knowledge for, and of, Pre-Service Teachers: Fractions

Often cited, Shulman's (1986) original description of PCK as "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9) makes clear that representations are key to effective teaching. In mathematics, this includes a collective knowledge base about effective ways of representing fractions and teaching about fraction representations. PST experience fraction instruction in school as children yet the content knowledge teachers need goes beyond that required by students. Fractions are one of the most complex areas of the primary school curriculum, both to learn and to teach. Whilst children's knowledge of fractions has been studied extensively (e.g Roesslein & Codding, 2019), primary PST have similar difficulties themselves (Vula & Kingji-Kastrati, 2018). This is concerning as limited teacher knowledge is related to children's difficulties in learning fractions (Van Steenbrugge et al., 2014) and indeed, may go some way in accounting for children's problems. The development of PST knowledge during ITE for teaching mathematics is complex. PST are learning the mathematical content and how to teach it simultaneously. In framing PST knowledge, it is important to conceptualise both the knowledge needed for teaching as well as PST emerging understandings, hence the value of a theoretical perspective, such as RCM, on teacher knowledge and its development. Additionally, PST bring with them prior experiences from their own education, including beliefs about the nature of mathematics and how to teach mathematics (Johnson & Olanoff, 2020).

Despite the wealth of research into primary PST knowledge of fractions, few studies illustrate what this knowledge looks like as teachers draw on it while teaching (Thanheiser et al., 2014). Additionally, most research has focused on what these teachers do or do not know (Olanoff et al., 2014) rather than how they apply the knowledge to teaching (Mewborn, 2001). In the current paper, we use the constructs of the RCM to explore the relationships among cPCK, pPCK, and ePCK for teaching fractions. Thus, the following research question was posed:

- How can the *Refined Consensus Model of Pedagogical Content Knowledge* be used to examine *PST Pedagogical Content Knowledge* of fractions?

Methods

The research took place in a post-graduate primary ITE program at a regional university in Australia. As part of a larger study, this instrumental case study (Stake, 1995) enables particular attention to key elements of the RCM. We chose Fiona, a female in her late 20s, as a single case in this study because (1) she taught fractions on her Professional Experience (PEx) and (2) her prior mathematics knowledge was typical of many PST in this program. Analysis through the RCM thus offers opportunity to unpack the relationships between layers of PST PCK during an ITE program. The instrumental case also has the potential to refine theory, which we address later in the paper. Fiona had completed a Bachelor of Arts in the year prior to enrolling in the ITE program. In high school, she had not studied any mathematics subjects in her final two years and thus was required to complete a 14-week pre-university access program that included fundamental mathematics skills. Also, prior to enrolling in the ITE program, Fiona had worked at an early childhood centre, which inspired her to pursue a career in primary school teaching.

In the ITE program, PST took two sequential mathematics subjects both of which included numeracy content and pedagogy for teaching Kindergarten to Year 6 children (ages 5–12). Fraction content was addressed in three 2-hour lectures and four 1-hour tutorials. The instruction followed a *Representational Reasoning Teaching and Learning* approach (see Thurtell et al., 2019 for an elaboration of this approach). Key representations of fractions included area models, linear models (such as number lines), and discrete models that were used to develop PST understanding of different fraction ideas. To assist PST in constructing knowledge of fraction concepts and operations, discussions of PST representations and explanations, including the misconceptions evident, were scaffolded. These discussions also addressed the difficulties children experience with an explicit focus on fraction representations. The ITE program also included three PEx opportunities of varied duration for PST to teach in primary school classrooms: PEx 1 (3 weeks); PEx 2 (3 weeks); and PEx 3 (5 weeks).

Data Collection and Analysis

Data for the study include 30–60 minute semi-structured interviews held at four points of the teacher education program: before and after PEx 1 (Int. 1 and 2) and after PEx 2 and PEx 3 (Int. 3 and 4). Additionally, a segment of Fiona's teaching on PEx 2 was videotaped and transcribed alongside photographs of the fraction representations constructed by Fiona and her students. Interviews focused on general perceptions of mathematics, fractions, and experiences in the subjects and/or on PEx. As the study progressed, interviews explored how Fiona's fraction knowledge developed, and Int. 3 and 4 included explicit questions interrogating the teaching episodes. The interviews were analysed using the RCM of PCK framework to identify the 'realms' of knowledge (ePCK, pPCK, and cPCK) as *a priori* categories. Working definitions of each realm were developed from Carlson and Daehler (2019) and then used to identify examples of these layers of Fiona's PCK.

Since PST fraction representations serve as indicators of ePCK (Thurtell et al., 2019), we observed how Fiona tailored her teaching to specific children and their mathematical work and selected the teaching action of *responding to children about their representations* as the unit of analysis for the current case study. There were 12 such instances in the videotaped teaching episode; eight of these involved co-constructed representations. Only four responses addressed a child's own fraction representation and all of these came in the final lesson segment. Further analysis of Int. 2, 3, and 4 sought additional indicators of Fiona's ePCK.

In the current paper, pPCK is represented by Fiona's perceptions of her knowledge of mathematics, fractions, and fractions representations. Thus, Int. 1–4 were analysed to identify Fiona's PCK related more broadly to teaching mathematics and fractions. Further, in alignment with the RCM of PCK (Carlson & Daehler, 2019), potential filters/amplifiers were identified as Fiona's beliefs about and attitudes towards teaching fractions and herself as a learner and teacher of mathematics. Finally, cPCK is the specialised professional knowledge independent of a specific learning context generated and held by multiple professionals. The present paper focused on the *concept-specific* idea (Carlson & Daehler, 2019) of *equality of parts* of fractions. In identifying this concept in the data, we also drew on research about learning and teaching fractions, fractions as represented in curriculum documents and teaching handbooks, and the fraction content of the ITE program to identify illustrative examples of Fiona's cPCK.

Examining Fiona's Pedagogical Content Knowledge through the RCM

Reflecting on the *content knowledge* aspect of her pPCK prior to the ITE program, Fiona was not confident in the quality or depth of mathematics in general. Fiona stated her knowledge "wasn't that great" (Int. 3) and she made the "mistakes that sometimes kids make" (Int. 4). The knowledge of fraction content and instruction Fiona had carried from her own schooling was limited. When faced with fractions in school, she stated she "just tried to learn the rules, like when to flip, when not to flip. We just learned that [and] all that stuff just makes me go *err*" (Int. 4). Although, she did remember "doing more of the basic stuff, halves, quarters" she felt that if the fraction content "got any more difficult than that I would be struggling" (Int. 1). In fact, Fiona continued to find the *content knowledge* aspect of pPCK required for teaching fractions difficult, "worrying about fractions, well worrying about maths in general" (Int. 1). She felt her procedural skill was the most important aspect of pPCK needing improvement, stating she "still just need[s] a lot of practice, just going over things" needing to learn "the actual maths of it … like doing fractions—actually adding and subtracting. Sometimes I just need to remember, like go back over it and do more of them" (Int. 2). This was also what she felt was needed to teach (i.e. for ePCK), "if I was to teach it to, say, a Year 6 or something, I wouldn't be confident. I'd need to just prepare, go over it again" (Int. 2). Her lack of confidence was consistent with her beliefs about the nature of fractions as synonymous with fraction notation, typically believing that "you can still use other stuff as well to help it, but I guess you've got to be able to do the symbolic to do the maths" (Int. 1).

The Teaching Episode and Fiona's ePCK

Because ePCK is the knowledge an individual teacher draws on in a particular setting with a particular child or children to support a specific concept, we now contextualise the teaching episode as an example of Fiona's ePCK. This episode occurred during PEx 2 in the final week of a 3-week block of teaching and was the second of two lessons about fractions. The class was a Year 1/2 composite class comprised of sixteen children (aged 6–8). The focus of the lesson was fractions needing equal parts using scenarios of sharing food. The lesson began with a 10-

minute, whole class introduction where individual children were invited to partition images on the interactive whiteboard, sharing a rectangular chocolate bar among two, four, and eight people, then sharing three circular lollipops between two people. The main segment of the lesson was completed at desks with children working individually. Children were given playdough with which to roll out and cut circles, partitioning to "share a cake" first between two then four people. Fiona showed two quarters "equals" one half, asking children to place the half next to two-quarters of the circle. We focus on the final moment in the lesson where children were invited to share a new 'cake' among eight people.

The Teaching Moment

There were four strategies used by children to partition their circles (see Table 1, one child did not produce a model) and Fiona responded directly to three children (Strategies 1–3).

Table 1

Children's Strategies for Partitioning Circles into Eighths and Fiona's Responses

Strategy	No. of children$^{\alpha}$	Work sample	Fiona's response
1. Three partitions along the diameter to cut the circle into halves, then quarters, then eighths	5 e.g., from Kai		"Great work Kai, good"
2. Individual cuts along the circle's radii with unequal parts	5 e.g., from Li		"I like it, Li. Well done"
3. One partition along the diameter and three perpendicular partitions, creating a grid-like pattern	3 e.g., from Jo		"That's a good try, Jo. Did you see Li's? Try and make it look like that. Okay? Maybe roll it back together and start again. But good try"
4. A combination of strategies that did not result in eighths	2 e.g., from Sam		Fiona did not respond

$^{\alpha}$Pseudonyms used for all children.

A teacher's ePCK might prompt a response to the unequally sized pieces, however Fiona made no mention of this important issue of partitioning with Li. The one instance of Fiona prompting a child to correct their representation directly was in response to Jo who used Strategy 3. Here, Fiona directs them to copy Li's model, despite its unequal partitions. Interpreting Fiona's ePCK in this episode, her own understanding of partitioning is poor (e.g., endorsing Li's model as appropriate). Fiona later addressed the idea of equality of parts with the whole class but made no mention of the children's models. Fiona partitioned a circle on the interactive whiteboard into eight roughly equal parts (using Strategy 1) and asked children to confirm that their models resembled the displayed image, noting she "saw some people did it another way, but we need to make sure that they're getting the same amount of cake, don't we?" She demonstrated the equality of the slices of cake but did not capitalise on specific examples of unequal partitioning that more than half the children produced (Strategies 2–4) and thus showed limited ePCK of the common misconception(s) around equivalency. Strategy 2 highlights the difficulty of using the central angle to preserve the equality of its parts, where Strategy 3, the grid pattern, could be demonstrated to show that these horizontal and vertical partitions did not represent eighths as the pieces produced were not equal. Modelling an example of Strategy 2 and/or 3 demands pPCK of the problems associated with these methods of partitioning a circular area model.

Fiona's ePCK in Lesson Reflections During Interview 3

Fiona described the teaching as a "real[ly] basic lesson" whose motivation was to "find out how much they knew". Although Fiona reasoned that the children were already "down with sort of halves, and some of them could kind of understand the concept of quarters", the concept of eighths was new to them and she had not "necessarily planned to do the eighth part" but spontaneously included it "because some of them were really getting it". Citing the tutorials and assignments as inspiring the implementation of the area models chosen for her lesson, Fiona's pPCK had clearly been informed by the ITE program. Those teaching and learning activities "helped because I learned sort of all this stuff and why [the fractions] make sense, and why … that helps before you're doing the symbolic stuff". When asked to provide a rationale for her choice of fraction representations, Fiona drew on more *general pedagogical* knowledge rather than PCK, stating the playdough was "fun", "concrete", and "hands-on", wanting children to "see sort of the number of people so that they had a reference … so there was a visual to look at". Further, Fiona did not believe teaching this lesson had developed her own PCK as it was "so simple anyway, I kind of got it anyway". However, she added "maybe if I had had to do something more difficult I wouldn't have been like 'oh my god'. And I would have had to really make sure I was clear on what I was doing before I went into those lessons". Fiona is hinting at her awareness of the link between pPCK as developed in the ITE program and enacting the knowledge during teaching episodes.

Knowledge for Teaching Fractions: Children's Misconceptions as cPCK

Fiona noted some specific aspects of cPCK that enabled her to feel more confident for teaching fractions such as being "prepared before the lesson and know[ing] all the types of areas that kids might go wrong" (Int. 4), citing the most helpful source of this knowledge as "the assignments about different misconceptions, like conceptual things" (Int. 4). The ITE program explored cPCK of fractions specifically including research about the potential difficulties that circular fraction models present for children (Gould, 2005). Drawing on this *concept-specific* cPCK, Fiona claimed awareness of the problems in partitioning circular area models and asserted that course content outlined children might partition circles in a grid-like pattern:

> It was funny to see because we did use circles, it sounds weird, but it's just like what they said in my maths textbook about how they, rather than cutting like a pizza, they would go to do it like that [a grid] into those weird square shapes out of the circle. … In the lecture or something, they said if you used a circle that would happen. (Int. 3)

However, being aware of difficulties that children were likely to experience with the circular area model (cPCK) did not prevent Fiona from employing it in her lesson (ePCK). When asked later in the same interview whether the children had difficulties with any of the representations, Fiona responded "Not really. … I think because I used circles and those basic kind of shapes" (Int. 3). Fiona believed she had addressed the children who had used the grid strategy to partition circles, "because it was playdough we could pick up and move the pieces, so it was kind of like we need to look at this piece and that piece and when you cut it that way, they're not the same size, are they? So yeah, from there they just rolled it and cut it out again" (Int. 3). In the *reflect* phase of ePCK, Fiona identifies the problem with the grid strategy as not representing eighths as equivalent pieces. Yet in the teaching episode, Fiona's ePCK was visible in directing a child holding the misconception to copy another inappropriate representation. Here, Fiona's reasoning in the teaching moment, *reflection in action*, is not as robust as her reasoning about the episode afterwards, *reflection on action*, despite being part of her ePCK. Fiona's case shows the complexity of ePCK and the relationships between the reasoning that occurs during the lesson, reflection *in* action, and the reasoning before (*plan* phase) and after (*reflection* phase) the lesson, reflection *on* action (Carlson & Daehler, 2019). Fiona had only planned to address the concept of quarters with her class (*plan* phase) yet adapted the lesson while teaching for children who she considered to be ready for eighths (*teach* phase). Here, there is potential for the *plan* and *teach* phases of Fiona's ePCK to be more strongly integrated.

Discussion and Implications

Fiona's case illustrates the utility of the RCM of PCK to show the complexities in PST knowledge *of* and *for* teaching fractions. The case showed that even when there is some evidence of cPCK as part of a teacher's pPCK (in this case, being aware of the difficulties a particular fraction representation might cause children), this may not be held in ways that adequately inform the *teach* phase of ePCK to enable a robust response to children's misconceptions. Knowing *about* children's potential difficulties can be insufficient as knowledge to *address* the misconceptions. This highlights the importance of PST having pPCK not just of likely difficulties of children but also the strategies to address these, probably best drawn from cPCK. Although anticipating children's responses should mean teachers notice and attend to children's responses when teaching (Vale et al., 2019), teachers also need to develop pPCK of ways *to address* children's misconceptions. The case of Fiona suggests there could be filters and/or amplifiers at work between a teacher's pPCK and their ePCK in a specific teaching moment. Behling et al. (2022) found that "the worse preservice teachers' knowledge-based reasoning, the smaller the transformation from pPCK to ePCK" (p. 592). Many PST do not emphasise the equality of the parts when explaining the part-whole concept in fractions (Magdaş et al., 2023) and their representations are often limited to rectangular and circular area models when demonstrating this relationship (Castro-Rodríguez et al., 2016). Responding to children's mathematics demands teacher pPCK comprising robust *conceptual knowledge* (Kahan et al., 2003) but disconnections in the *plan-teach-reflect* phases of ePCK were highlighted when Fiona spontaneously introduced eighths with circle models. It is possible that further developing Fiona's pPCK with more specific cPCK about children's misconceptions about fraction representations could inform the *plan* phase of ePCK, allowing her to prepare teaching moves to address children's understanding of the concept of equal parts more directly in the moment. In the *reflect* phase of ePCK, Fiona drew from the *concept-specific* cPCK to subsequently identify an appropriate response to the child regarding the need for size equivalency. Her post-lesson reflection was thus a critical element of her developing pPCK consistent with Carlson and Daehler (2019), "the insight a teacher takes away from each interaction with students further informs the teacher's pPCK" (p. 85). Analysis in this paper of the interactions among pPCK, cPCK, and ePCK as illustrated in the teaching episode in the case of Fiona demonstrate the value of such analysis and the utility of the RCM of PCK as a framework for unpacking the complexity of teaching and learning. We argue particularly that this small case study illustrates the importance of the *reflect* phase of ePCK during ITE and points to the need in ITE programs to enable this reflective work at key points of the program.

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References

Abell, S. K. (2007). Research on science teacher knowledge. In S. K. Abell & N. G. Lederman (Eds.), *Handbook of research on science education* (pp. 1105–1149). Lawrence Erlbaum.

Amador, J. M., Rogers, M. A. P., Hudson, R., Phillips, A., Carter, I., Galindo, E., & Akerson, V. L. (2022). Novice teachers' pedagogical content knowledge for planning and implementing mathematics and science lessons. *Teaching and Teacher Education*, *115*, 103736.

Ball, D. L., Thames, M. H., & Phelps, G. C. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education*, *59*(5), 389–407.

Behling, F., Förtsch, C., & Neuhaus, B. J. (2022). The refined consensus model of pedagogical content knowledge (PCK): Detecting filters between the realms of PCK. *Education Sciences*, *12*(9), 1–21.

Berry, A., Friedrichsen, P. J., & Loughran, J. (2015). *Re-examining pedagogical content knowledge in science education*. Routledge.

Botha, H., Coetzee, C., & Zweers, L. (2023). Teaching measurement: The role of mathematics teachers' enacted PCK on gain in learner outcomes. *Journal of Mathematics Education*, *12*(2), 307–322.

Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., Neumayer DePiper, J., Frank, T. J., Griffin, M. J., & Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education*, *45*(4), 419–459. https://doi.org/10.5951/jresematheduc.45.4.0419

Carlson, J., & Daehler, K. (2019). The refined consensus model of pedagogical content knowledge in science education. In A. Hume, R. Cooper, & A. Borowski (Eds.), *Repositioning pedagogical content knowledge in teachers' professional knowledge for teaching science* (pp. 77–94). Springer.

Castro-Rodríguez, E., Pitta-Pantazi, D., Rico, L., & Gómez, P. (2016). Prospective teachers' understanding of the multiplicative part-whole relationship of fraction. *Educational Studies in Mathematics*, *92*(1), 129–146. https://doi.org/10.1007/s10649-015-9673-4

Gould, P. (2005). Really broken numbers. *Australian Primary Mathematics Classroom*, *10*(3), 4–10.

Johnson, K., & Olanoff, D. (2020). Using transformative learning theory to help prospective teachers learn mathematics that they already "know". *Mathematics Enthusiast*, *17*(2–3), 725–769.

Kahan, J. A., Cooper, D. A., & Bethea, K. A. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*, *6*(3), 223–252.

Magdaş, I., Henry, J., & Magdaş, A. (2023). A concept inventory to identify fractions misconceptions among prospective primary and preschool teachers in Romania. *Acta Didactica Napocensia*, *16*(1), 1–11.

Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers in the US. *Mathematics Teacher Education and Development*, *3*(1), 28–36.

Mientus, L., Hume, A., Wulff, P., Meiners, A., & Borowski, A. (2022). Modelling STEM teachers' pedagogical content knowledge in the framework of the refined consensus model: A systematic literature review. *Education Sciences*, *12*(6), 1–25.

Olanoff, D., Lo, J.-J., & Tobias, J. M. (2014). Mathematical content knowledge for teaching elementary mathematics: A focus on fractions. *The Mathematics Enthusiast*, *11*(2), 267–310.

Roesslein, R. I., & Codding, R. S. (2019). Fraction interventions for struggling elementary math learners: A review of the literature. *Psychology in the Schools*, *56*(3), 413–432. https://doi.org/10.1002/pits.22196

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14. https://doi.org/10.2307/1175860

Shulman, L. S., & Grossman, P. (1988). *Knowledge growth in teaching: A final report to the Spencer Foundation.* Stanford University.

Stake, R. E. (1995). *The art of case study research*. Sage.

Thanheiser, E., Browning, C., Edson, A. J., Lo, J. J., Whitacre, I., Olanoff, D., & Morton, C. (2014). Prospective elementary mathematics teacher content knowledge: What do we know, what do we not know, and where do we go? *Mathematics Enthusiast*, *11*(2), 433–448.

Thurtell, E., Forrester, T., & Chinnappan, M. (2019). Building conceptual knowledge of fraction operations among pre-service teachers: Effect of a representation-based teaching approach within a teacher education program. *Mathematics Teacher Education and Development*, *21*(1), 100–124.

Vale, C., Widjaja, W., Doig, B., & Groves, S. (2019). Anticipating students' reasoning and planning prompts in structured problem-solving lessons. *Mathematics Education Research Journal*, *31*(1), 1–25.

Van Steenbrugge, H., Lesage, E., Valcke, M., & Desoete, A. (2014). Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge? *Journal of Curriculum Studies*, *46*(1), 138–161.

Vula, E., & Kingji-Kastrati, J. (2018). Pre-service teacher procedural and conceptual knowledge of fractions. In G. Stylianides & K. Hino (Eds.), *Research advances in the mathematical education of pre-service elementary teachers. ICME-13 Monograph*s (pp. 111–123). Cham: Springer.