An Analysis of Multiplicative Thinking Development in Years 3 to 6

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Many primary school students experience transition barriers between additive and multiplicative thinking. This paper analysed responses from 253 Years 3 to 6 students to a diagnostic assessment which consists of whole number multiplication and division word problems involving equal groups, arrays, multiplicative comparison and Cartesian product situations. Based on the Rasch analysis, item responses were differentiated into five developmental Stages indicating a wide range of understanding and pointing to different transition barriers that students experience. The Multiplicative Diagnostic Assessment test effectively explored these barriers and transition points.

This study highlights the importance of multiplicative contexts, size of numbers and operations during students' development for multiplicative thinking. It shows that recognising equal grouping structure (that is groups of equal size and the number groups) at the early stage of development for equal groups and arrays situations is a barrier for students who rely on *one-to-one correspondence*. Recognising equal grouping structure becomes even more challenging for students while solving problems involving division operation, and this barrier can also remain at the later stage of development for problems involving Cartesian product situation.

The acquisition of the concept of *many-to-one correspondence* is one of the key hurdles for many students as it entails using numbers to represent not the cardinal value of a single set but mapping relations between two sets. Students had difficulty in understanding the notion of "times as many" in multiplicative comparison situation, showing overreliance of additive thinking and being confused between "times as many" and "times more". Various misconceptions and inappropriate generalisations were evident involving two-digit by two-digit multiplication problems, where students apply place value partitioning incorrectly for multiplication problems, e.g., $12\times15 = 10\times10+2\times5$ or $12\times15 = 10\times17$ or $12\times15 = 10\times2\times15$. At a later stage, students still have trouble in recognising the multiplicative relationship between two related pairs of quantities, where knowing $13\times18 = 234$, some wrote $13\times19 = 234+1$ since 19 is 1 more than 18. Students appeared to have limited encounters with Cartesian product situations which involve repeated equal sets, unlike equal groups and arrays.

Understanding of distributive and associative properties of multiplication remains a barrier for middle and upper primary students where reliance on procedural based methods may provide correct answers but limit students' ability to see underlying multiplicative relationships, including properties of multiplication between pairs of quantities involved in the operation. It is important to assist students to see the structural properties of multiplication and to be able to identify and explain correct and incorrect solutions to multiplicative situations. Equally important is that students can use known results to arrive at a correct answer without having to do a full calculation, which remains a challenge for many students.

For more information, please refer to the following paper presented at the 46th Annual Conference of MERGA in July 2024.
Bao, L. and Stephens, M. (2024). An analysis of multiplicative thinking development in Years 3 to 6. In J. Višňovská, E. Ross, & S. Getenet (Eds.), Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia (pp. 95-102). Gold Coast: Australia: The Mathematics Education Research Group of Australasia Inc.