Can a Short Online Test Diagnose Student Thinking?

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This paper provides evidence that a short, fully online, well-constructed diagnostic test thoroughly based on research can give teachers information about their students' thinking and the strategies they are able to use. The information is sufficiently accurate for teachers to plan future teaching. A test for students learning to solve equations is shown as an example. Its main goal is to inform their teacher of the solving strategies each student can use. This test is one of 66 pre-post pairs of 'SMART::tests' that we have developed for middle years.

There are three different strategies for solving equations, all of which significantly contribute to mathematical progress. In order of cognitive complexity, these are substituting, unwinding (sometimes called backtracking) then balancing. Substituting (relying on a guess and check process) is important to show students what a solution is. Unwinding works well provided the equation can be read as specifying a sequence of operations applied to an unknown number to give a result. Balancing ("do the same to both sides") is a powerful equation solving principle. Substituting only uses the given operations whereas unwinding and balancing require inverse operations. Substituting and unwinding only use operations on and with numbers, but balancing requires students to operate with and on unknowns. That is harder conceptually and needs better knowledge of algebraic rules. In our *Solving Linear Equations* test, students solve a very carefully designed set of equations requiring gradually harder methods. The system automatically reports to the teacher which strategies each student can use. Our online diagnostics also flag students showing common misconceptions or errors.

We used the responses from 3010 students mostly from Years 8 and 9 (looking at correctness and the errors they made) to verify that the test provides a very effective guide for future teaching of a class. Because it is a short test, the results for a small number of individuals may be wrong; teachers can follow them up individually and look at their responses.

Our close examination of the students' responses highlights some important messages for teaching. It is sensible to start with equations such as $3x + 8 = 23$, that can be solved by substituting. It has one occurrence of the unknown and a positive integer solution, and solving it emphasises implicit multiplication (e.g., that $3x$ means $3\times x$). However, it is important to move quickly to equations such as $5x + 7 = 15$ which is not easy to solve by substituting (non-integer solution). With one occurrence of the unknown, it can be solved by unwinding, emphasing inverse operations. Balancing builds on this, adding the new concept of operating on and with unknowns. To succeed, students should see equations with substantial variation of algebraic form, so that they do not learn false rules like "first subtract, then divide". For example, the success rate for a simple variation $3x + 6 + 2x = 7$ was surprisingly low. The data also showed that it is considerably easier for students to deal with subtraction of a number (e.g., $7x – 11 = 2x – 4$) than subtraction of a pronumeral (e.g., $12 – 11x = 5 – x$).

For more information, please refer to the following paper presented at the 46th Annual Conference of MERGA in July 2024.
Steinle, V., Stacey, K. & Price, B.. (2024). Can a Short Online Test Diagnose Student Thinking? In J. Višňovská, E. Ross, & S. Getenet (Eds.), Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia (pp. 495 - 502). Gold Coast: Australia: The Mathematics Education Research Group of Australasia Inc.