Comprehending and Applying the First Isomorphism Theorem

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First Isomorphism Theorem (FIT) is usually the culminating point of an introductory course in Group Theory (Ioannou, 2012). FIT is a particularly important mathematical result in such educational context, since it is essential in Group Theory, and it is rather complex in relating numerous concepts in Abstract Algebra in general. Several studies have reported that this complexity is due to the difficulty novice mathematics students have in comprehending concepts such as the quotient group, which is reportedly the most challenging concept to grasp in a first course in Group Theory, the cosets, which novice students are challenged with due to their inability to visualise them (Ioannou & Iannone, 2011), and the group isomorphisms, together with the kernel and image.

This qualitative study investigates novice undergraduate mathematics students' first encounter with FIT. For the analysis of student responses, there has been used the Commognitive Theoretical Framework (Sfard, 2008). Presmeg (2016, p. 423) suggests it is a theoretical framework of unrealised potential, designed to consider not only issues of teaching and learning of mathematics per se, but to investigate "the entire fabric of human development and what it means to be human." It proves to be an astute tool for the comprehension of diverse aspects of mathematical learning, which although grounded on discrete foundational assumptions, can be integrated to give a more holistic view of the students' learning experience.

This study suggests that there have emerged three major challenges in the learning, and application of FIT. The first is due to the problematic comprehension of the notions of kernel, image and quotient groups. The second is related to the initially problematic application of the FIT as a routine in the context of proofs. Finally, the third challenge is related to the comprehension of the notion of isomorphism, which emerges from the incomplete learning of the notion of homomorphism, and the special characteristics that make homomorphism to be an isomorphism, namely being injective and surjective.

References

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