# An Analysis of Place Value Visual Representations in South African, Singaporean, and Australian Workbooks

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Place value competence in early years mathematics is a precursor for success in later grades. In this paper, we analyse the place value visual representations in workbooks from South Africa, Singapore and Australia. A cross-country comparison of curricula materials provides an opportunity to understand similarities and differences in use of visual representations in teaching place value. Drawing on an adapted Visual Representation Framework, we analyse the types of place value VRs in the Grades 1-3 workbooks across the three countries. The results show that there are significant differences in the number of VR and types of VR privileged across the three countries.

## Introduction

Place value is an important concept in primary school mathematics, particularly for understanding and computing with multi-digit numbers (Sari et al., 2021). Place value understanding is central to success as learners move up the schooling system. As Gebhardt et al., (2014) note, a conceptual understanding of place value supports not only whole number understanding, but also that of decimal fractions. Teachers play a central role in developing learners' understanding of place value, yet many only have a procedural understanding of the role of place value in multi-digit calculation (Ma, 1999). In such cases, teachers become more reliant on textbooks (including workbooks) to assist them in teaching mathematics. As Hoadley and Gallant (2016) note, workbooks can support the development of learners' understanding of mathematics and assist teachers in developing their mathematics and pedagogical content knowledge. Visual representations (VR) in mathematics workbooks often serve as a tool to assist learners in understanding abstract concepts (Mainali, 2021). Given this, it is important to ensure the quality and appropriateness of the VR in workbooks.

This paper forms part of a larger international research project on developing learners' and teachers' understanding of place value. To understand how the curriculum is enacted, the research team has shifted their focus to textbooks and workbooks. While shaped by differing socio-cultural and economic contexts, cross-country comparative studies offer valuable insights that potentially could lead to improvement (Huang, et al., 2022). In this paper, we focus specifically on the VRs in workbooks used by learners to support their understanding of PV. The guiding questions are: *What is the nature of visual representations of place value in the workbooks of three countries?* and *What variations exist within the type of place value visual representations in the three countries?*

## Literature Review

Developing an understanding of place value is complex as it requires an understanding of multiple "big ideas" (van der Walle, 2015). Ladel et al. (2023) demonstrate this complexity in

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 77–84). Canberra: MERGA.

developing learners' place value understanding in their knowledge map which highlights the progression and connections in developing the "big ideas". These include decimal part-whole, grouping and ungrouping, bundling and unbundling, and positional notation (value of the digit, value of the number and the role of zero) (Ladel et al., 2023), as well as the additive and multiplicative structure of place value (Larkin et al., 2019). Prior to developing the big ideas, learners need to become confident with counting, part-whole structure, grouping and ungrouping objects in tens, and unitising (understanding that ten objects can become one bundle of ten). Larkin et al. (2019) suggest that the development of these big ideas can be supported by resources, for example, manipulatives, digital tools, and iconic representations. As such, the use of VRs in workbooks is likely to be key in developing learners' understanding.

Workbooks are commonplace in primary mathematics classrooms (Hoadley & Gallant, 2016), although not mandatory in all countries. Workbooks are curriculum resources that seek to link the intended curriculum with the implemented curriculum by providing a structured sequence of activities that organise teaching and learning in the classroom, and ultimately teachers' and learners' knowledge, thinking, beliefs and attitudes about the subject (Hadar, 2017). There is broad agreement in the research on mathematics workbooks that indicates that the quality thereof has an influence on student learning (Hadar, 2017). The cross-country comparison, on which this paper is based, has the potential to engage with what well-developed workbooks may look like in the context of supporting learners' PV understanding.

Workbooks incorporate various modalities, for example, text and images (Papageorgiou et al., 2019) that, if appropriate to the content taught, assist learners in developing an understanding of mathematics. VRs are external images that assist the learner in constructing internal representations (Papageorgiou et al., 2019). Teachers need to understand which VRs effectively support the teaching of a particular concept (Mainali, 2021), their characteristics, and the multiple ways learners interpret them (Papageorgiou et al., 2019). Hence, the quality of the VRs is important as this impacts their potential to support meaning-making. Mainali (2019) suggests that learners require access to a range of VRs to develop an understanding of a concept from different perspectives. However, Papageorgiou et al. (2019) take a more cautious view, suggesting that multiple representations for a single concept can cause "cognitive overload" (p.189). Thus, designers of workbooks need to carefully consider which VRs are most likely to assist learners in developing their understanding of different mathematical concepts.

## Theoretical Framework

Vygotsky's (1978) three forms of mediation include tools, signs, and social interaction. This research focuses on the signs (i.e., visual representations) that mediate learners' reasoning. A VR is a symbolic tool that assists learners in making meaning of a particular concept, in this case, place value. While Kozulin (2003) insists that the facilitation and appropriation of symbolic tools is dependent on a person (more knowledgeable other), we suggest that a well-developed workbook could potentially perform the role of the more knowledgeable other. This is particularly important in situations where teachers do not have the required mathematical content knowledge (MCK) or pedagogical content knowledge (PCK).

In this research, we explore the different types of VRs in workbooks across three countries (Australia, Singapore and South Africa) to ascertain the VR used to develop learners' understanding of place value. In their comparison of textbook analyses, Huang et al. (2022) note that numerous frameworks for doing so exist. While their aim is to develop a framework for analysing workbooks across the disciplines, such a broad analysis is not the intention of this paper. We draw on the VR framework of Fotakoupoulou and Spiliotopoulou (2008), Papageorgiou et al. (2019), and Booysen (2023) in analysing the VR. The framework used here examines the VRs based on the type, relation to content, function, relation to reality, and dimensionality. In this paper, we focus on one aspect of the VR frameworks mentioned above, that is, the types of VR used to support PV understanding in three, Grade 1–3 mathematics workbooks.

## Methodology

This study adopts an interpretivist orientation to document analysis. Activities from the workbooks were included in the analysis if they had an explicit link to the knowledge of PV.

In Phase 1, the first author analysed the South African and Singapore workbooks from Grades 1 to 3. The analysis was captured on an Excel spreadsheet. In this initial analysis the first author was able to identify the pages that contained PV VRs.

In Phase 2, each author independently analysed a workbook of the same grade from South Africa and Singapore. The analysis was guided by the Excel spreadsheet used in Phase 1. The page numbers that contained VRs were included in the first column and the categories were in the first row. Each researcher wrote a '1' to indicate the presence of a VR from a specific category. During this phase, additional pages with PV VRs were identified. A Zoom meeting was held to go through the analysis and reach agreement.

In Phase 3 of the analysis process five authors re-looked at the analysis and came to the realisation that there were sub-types within each type of VR that has emerged. The authors noted that the VRs in the text went from concrete representations to being more abstract representations. The sub-types included specific variation within some of the types of PV VRs. For example, the variations in the Dienes block category were Dienes blocks (3 dimensional) and Dienes blocks (2 dimensional). It was in this phase that the Australian workbook was added to the analysis process as the data had not been sufficient for this paper. The Australian workbooks for Grade 1-3 were then analysed by the 3 authors and consensus was reached.

In Phase 4, three authors discussed and compared findings in a zoom meeting and the initial drafted paper was sent to all co-authors for commentary and further recommendations. The suggestions were taken into consideration in the final version.

## Data Analysis, Findings, and Discussion

This section presents the data analysis and findings of the research. Across the three workbooks (WB), South Africa (SA) has the greatest number of PV activities (n=191), followed by Australia (Aus) (n=93), and then Singapore (Sing) (n=30). The number of VRs across the three-workbook series differed substantially with the SA WB having 296 PV VRs, compared with 248 in the Sing WB and 87 in the Aus WB. The most common VRs in the SA WB were PV cards (25%) and Dienes blocks (22%). This stands in contrast to the Sing WB and Aus WB. Most VRs in the Sing WB series are part-whole diagrams (15%) followed by bead strings (13%). The Aus WB series does not show such an extreme situation where one type of representation dominates. The most common VR in the Aus WB series are PV chart (symbolic) (11%), followed by Dienes blocks (10%) (Table 1).

**Table 1**

*Types of PV VR*

| Types of VR | | South Africa (%) (n=296) | Singapore (%) (n=248) | Australia (%) (n= 87) |
|---|---|---|---|---|
| **Dienes Blocks** | Dienes Blocks (3D) | 22 | 6 | 10 |
| | Dienes Blocks like (2D) | 1 | 11 | 0 |
| **PV cards** | Flard card | 25 | 0 | 0 |
| | Flard card like | 5 | 0 | 2 |

**Table 1**

*Types of PV VR (continued)*

| Types of VR | | South Africa (%) (n=296) | Singapore (%) (n=248) | Australia (%) (n= 87) |
|---|---|---|---|---|
| **Numberline** | Numberline | 4 | 0 | 9 |
| | Numberline like | 1 | 0 | 6 |
| | Empty numberline | 1 | 0 | 6 |
| **PV chart** | PV chart | 2 | 4 | 2 |
| | PV chart (symbolic) | 0 | 0 | 11 |
| | PV chart money | 1 | 0 | 0 |
| **Ten frame** | Ten frame | 3 | 0 | 6 |
| | Ten frame like | 0 | 0 | 0 |
| **Number chart** | Number chart | 7 | 0 | 6 |
| | Partial number chart | 5 | 1 | 9 |
| **Money** | Money (base ten focus) | 1 | 1 | 0 |
| **Beads** | Bead string | 0 | 13 | 1 |
| | Bead like | 6 | 0 | 0 |
| **Counters** | Counters (structured) | 2 | 0 | 0 |
| | Counters (unstructured) | 1 | 0 | 2 |
| **Ice cream sticks** | Ice cream sticks (structured) | 0 | 0 | 7 |
| | Ice cream sticks (unstructured) | 0 | 0 | 0 |
| **Abacus** | Abacus | 1 | 0 | 3 |
| **Part-whole diagrams** | PW diagram (a) | 0 | 15 | 0 |
| | PW diagram (b) | 1 | 0 | 1 |
| **Arrays** | Array representation (structured) | 9 | 0 | 0 |
| | **Total** | **98** | **51** | **84** |

## Dienes Blocks: Dienes Blocks (3D) and Dienes Blocks (2D)

All of the workbooks had Dienes blocks as a type of VR for developing PV understanding. While analysing the Dienes blocks VRs, it became clear that there were two sub-types, those that looked like Dienes blocks (3D), and those that were a 2D representation of Dienes blocks (Dienes blocks-like). In SA WB and Aus WB, there were more examples of Dienes blocks (3D) (SA: 22%, Aus: 10%) and in Sing WB (6%), there were more Dienes block-like (2D) (SA:1%, Sing: 11%) VRs (Table 1).

## PV Cards: Flard Bards, Flard Card Like, and Numeral Expander

Visual representations of PV cards were found in the workbooks from the different countries. The PV cards assisted learners in the decomposition of numbers in PV. The analysis found that the PV chart category had three sub-types namely flard cards (SA: 25%), flard card like (SA:5%, Aus: 2%) and numeral expanders (Aus: 3%). The flard card like VR can be explained as a VR which has the same formatting of a flard card where they are placed on top of one another to compose the number. There were more examples of numeral expanders in the Aus WB (Figure 2). These are similar to flard cards that show the number in expanded notation.

**Figure 1**

*Flard Card Like (SA, Gr2)*

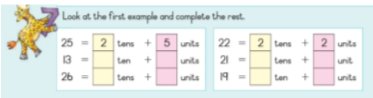



**Figure 2**

*Numeral Expander (Aus, Gr2)*

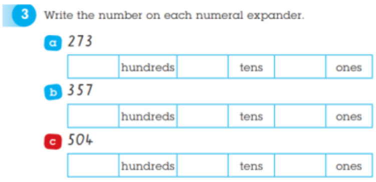
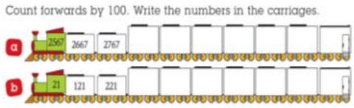



**Figure 3**

*Numberline Like (Aus, Gr3)*




## Numberline: Numberline, Numberline Like, and Empty Numberline

The SA WB and Aus WB included numberlines as a type of VR for the development of PV understanding. While analysing the numberlines in the workbooks, it became clear that there were three sub-types, the numberline (SA: 4 %, Aus: 9%), those that looked numberline like (SA: 1%, Aus: 6%) and an empty numberline (SA: 1%, Aus: 6%) (Table 1). A VR that is numberline like has a similar formatting of a numberline (i.e., it is linear and has consecutive numbers with equal spaces in between). In Figure 3 the learner is required to count forwards by 100 using the carriages on the train.

## PV Chart, Place Value Chart (Symbolic), and Place Value Chart (Money)

All the workbooks provided examples of PV charts as a type of VR. While analysing the PV chart VRs, it became clear that there were three sub-types, those that looked at a physical PV chart, PV chart (symbolic) and a PV chart (money) (Table 1). Most of the PV charts were found in the Sing WB (4%), while the SA and Aus WB contained a few less (2%). In the Australian workbook, there were more examples of PV chart (symbolic) (11%). This is a VR that looks like a PV chart with hundreds, tens and units but does not include a VR of Dienes blocks or counters in each column because the number serves as the symbolic representation (Fig 4). The SA WB contained PV chart (money) VRs (1%). A PV chart money is when the R100, R10 and R1 notes are used as the hundreds, tens and units in a normal PV chart. In Figure 5 the learners are required to master the concept of bundling (swapping ten R1 coins for a R10 note).

**Figure 4**

*PV Chart (Symbolic) (SA, Gr3)*

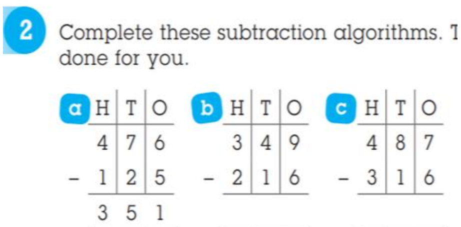



**Figure 5**

*PV Chart Money (SA, Gr3)*


## Money (Base Ten Focus)

The SA and Sing WB also included money (base ten focus*)* as a type of VR, with an equal number of examples (SA: 1%, Sing: 1%).

## Ten Frame and Ten Frame Like

The Aus and Sing WB contained ten frames as a type of VR. The Aus WB contained the most ten frames (6%), while the SA WB contained 3%. There were no examples of the second sub-type namely, ten frame like across the workbooks.

## Number Chart and Partial Number Chart

All the workbooks had number chart as a type of VR for the development of PV understanding. While analysing the number chart VR sub-types were identified, the actual number chart (SA: 7%, Aus: 6%), and a partial number chart (SA: 5%, Sing: 1%, Aus: 9%) (Table 1). A partial number chart is when a section of the number chart is extracted for learners to complete.

**Figure 6**

*Partial Number Chart (Sing, Gr1)*

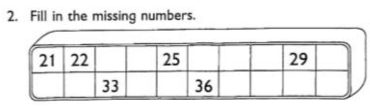



**Figure 7**

*Bead Like (SA, Gr2)*

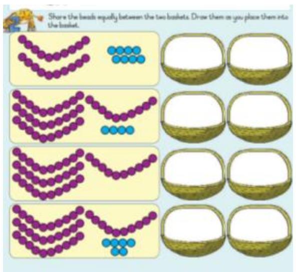
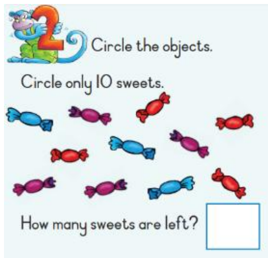


**Figure 8**

*Unstructured Counters (SA, Gr 2)*




## Bead String and Bead String Like

All the workbooks had beads as a type of VR. In analysing the beads, it became clear that there were two sub-types: the beads on a string (bead string) and examples of bead-like VRs (Table 1). In the Sing WB, there were more examples of bead strings (Sing: 13%) and the SA WB that contained more bead like examples (SA: 6%). A bead string like VR can be found in Figure 7 where the beads are arranged as a necklace without the string connecting them.

## Structured Counters and Unstructured Counters

The SA WB and Aus WB contained counters as a type of VR for the development of PV understanding. During the analysis process it became clear that there were two sub-types, namely structured counters and unstructured counters (Table 1). The only examples of structured counters were found in the SA WB (2%). Both the Aus WB (2%) and SA WB (1%) contained examples of unstructured counters. Figure 8 is an example of unstructured counters where the sweets are arranged randomly for learners to count and circle.

## Structured and Unstructured Ice-Cream Sticks

The Sing WB had examples of ice-cream sticks as a type of VR. While analysing the ice-cream sticks, two sub-types emerged, structured ice-cream sticks (Table 1), and unstructured ice-cream sticks (i.e. the sticks are arranged randomly). Singapore contained the only examples of ice-cream sticks structured (7%).

## Abacus

In the analysis process it became apparent that the Sing WB (3%) and SA WB (1%) had examples of an abacus as a type of VR (Table 1).

## Diagrams: Part-Whole Diagram (a) and Part-Whole Diagram (b)

All of the workbooks had part-whole (PW) diagrams as a type of VR. While analysing the PW diagram VRs, it became clear that there were two sub-types, those were PW diagram (a) and PW diagram (b) (Table 1). Figure 9 is an example of an PW diagram (a) where learners need to decompose numbers to be able to do the subtraction sum. While in Figure 10 the learners are expected to work out what the missing part of the PW diagram (b) is.

**Figure 9**
*PW Diagram (a) (Sing, Gr1)*

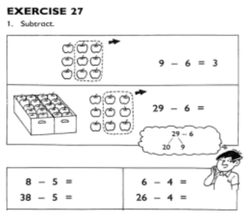
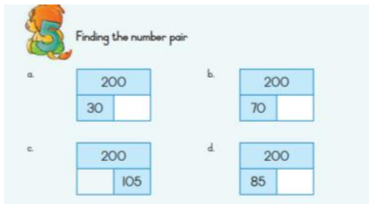



**Figure 10**
*PW Diagram (b) (SA, Gr3)*




## Array Representation

The SA and Sing WB contained array representations that assisted in supporting place value understanding. Most array representations were found in the SA WB (9%) (Table 1).

The above section provides insight into the sub-types of the different types of PV VR found across the workbooks of the three countries. The sub-types identified above provides a methodological contribution of adapting a framework to expose the existing variations of the different types of PV VRs found in the workbooks.

## Discussion

Workbooks are replete with symbolic tools that have the potential to mediate learners' understanding of place value (Mainali, 2021). There was a substantial difference in the number of PV activities and PV VRs in the workbooks from the three countries. The SA WB had the most PV VRs and the Sing WB the least. The Sing WB had almost half of the number of PV VR when compared with the SA WB. The SA and Aus WB had examples of all the types of VRs (n=15), whereas the Sing WB only had 6 different types of VRs. Not evident in the Sing WB series, which were included in both the SA and Aus WB series, were PV cards, numberline, ten frames, counters, ice-cream sticks, abacus and array (Table 1).

The research highlights that there are different variations of each type of PV VR. For example, Dienes blocks were categorised as Dienes blocks (3D) and Dienes blocks (2D). Dienes blocks (3D) bore direct resemblance to the actual Dienes blocks, whereas Dienes blocks (2D) were a drawing of the front of the Dienes blocks only (no depth). All the WBs included Dienes blocks; however, these were privileged in the Sing WB series, and in particular, Dienes blocks (2D). By contrast, the dominant PV VR was the PV chart in the Aus WB series and the PV card in the SA WB series. In the Aus WB series, most of the PV chart were place value chart (symbolic) and in the SA WB series, the majority of PV card were flard cards (Table 1).

The SA WB do not provide opportunities for learners to work with PV chart (symbolic) and ten frame like, Dienes blocks (2D and 3D), ice cream sticks (structured), ice cream sticks (unstructured) and numeral expanders. Learners who make use of the Sing WB are not exposed to flard cards, numberlines, partial numberlines or empty numberlines, PV chart (symbolic), PV chart (money), ten frame, ten frame like objects (unstructured), bead like, counters (structured and unstructured), decimal blocks (3D and 2D), ice cream sticks (structured and unstructured) and numeral expander when it comes to PV VR in the WBs. The Aus WB does not expose learners to Dienes blocks like, flard cards, PV chart (money), ten frame like, money (base ten focus), bead like, counters (structured), photos, ice cream sticks (unstructured), part-whole diagram and array representation.

Given Singapore's dominance in the Grades 4 and 8 Trends in Mathematics and Science Study, and their use of fewer PV VR raises a question of whether it is beneficial for learners have access to a wide range of VR. Mainali (2019) argues that learners should be exposed to a wide variety of VR. However, this raises concern for Papageorgiou et al. (2019), who argue that multiple representations could lead to cognitive overload.

The significance of this study lies in its recognition of the variations within each type of PV VR. For example, for teachers, it is important to select the most appropriate variation based on the concept being taught, which is dependent on whether the learners are at a concrete, semi-concrete, or symbolic stage of their learning. The variations in types of VR reveal the complexity and richness embedded in each classification, offering both teachers and learners a deeper understanding of how different representations can be used in the classroom.

## Conclusion

This research sought to examine the type and variations of PV VR across the WB of three countries. However, the analysis only focused on one category (i.e., types of VR) and not the complete VR framework. Further research is required to ascertain how each VR promotes PV understanding and the extent to which the use of multiple types of VR supports learners' PV understanding. This research has prompted the research team to focus on examining how the VR supports place value understanding, particularly with regard to the sequencing and progression of the "big ideas" in PV.

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