Partitive Division: The Numbers Matter to Young Children

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This paper contains an analysis of some early thinking of 94 young children aged 5 years 7 months to 6 years 5 months. These children were interviewed as part of a larger study of the multiplicative thinking of children who were midway through their first year of school in Australia. They had not been formally taught multiplication or division at school. Three task-based interview questions were posed using partitive contexts. The children attempted to solve 12 ÷ 3, 7 ÷ 2 and 22 ÷ 4. An analysis of the responses of these children reveals many children can solve division problems even when the numbers get larger and where the divisor is not a factor of the dividend.

The Multiplication and Division Investigations project sought to understand what intuitive multiplicative knowledge young children develop before they are introduced to ideas formally at school. Our published results indicate that some children aged 4- to 6-years old can think about multiplication and division prior to school (Cheeseman et al., 2022a; Cheeseman et al., 2022b). Our recent focus has centred on children's strategic thinking about division problems.

National test results show that division is poorly understood by many Years 5 and 7 Australian students (e.g., Roche & Clarke, 2013). Years 3 and 4 students were also found to have limited understanding of the relationship between the dividend, divisor, and quotient, and limited experience of contextual problems (Downton, 2009). We believe that if we understand how young children interpret division contexts meaningfully, we may be able to help them develop multiplicative reasoning. Decades ago, Davis and Pitkethly (1990) investigated young children's interpretation of division and found that children as young as 4 and 5 years old could solve division problems using a range of intuitive strategies. We were keen to investigate current division thinking by young children. The research question we sought to answer was: How do the numbers in partitive division tasks influence young children's strategy choices?

Much of the research involving young children's understanding of division was conducted in the 1980s and 1990s and has only recently received renewed attention (e.g., Barkai et al., 2023). Earlier research concluded that children's initial ideas of division often stem from their experiences with sharing (e.g., Davis & Pitkethly, 1990; Pepper & Hunting, 1998; Squire & Bryant, 2002). These studies have shown that children as young as 3 years and 6 months can make "fair shares" when dividing quantities like 12. Partitive division is often described as a "natural" way of thinking about division, using words like "sharing" and "dealing". In partitive division, the number of recipients is known, and the task involves dividing a collection into equal parts (Fischbein et al., 1985). A key feature is that the shares must be equal. Fischbein et al. argued that partitive division arises from children's action schemas. However, others maintain that division involves more than just sharing; it also requires understanding the relationship between the dividend, divisor, and quotient. For instance, children must understand that the more parts a quantity is divided into, the smaller each part becomes (Correa et al., 1998). In a sharing scenario, this would mean that the more people there are, the fewer candies each person would receive. While previous studies have shown that children can perform division in a sharing context (Squire & Bryant), it remains unclear whether the sharing scenario specifically, or social and cultural factors, facilitates performance (Hedefalk et al., 2022).

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Young Children's Intuitive Strategies

Several earlier studies found that most children commonly used one-to-one correspondence or "dealing out" objects to allocate equal shares (e.g., Carpenter et al., 1993; Davis & Pitkethly, 1990; Desforges & Desforges, 1980; Frydman & Bryant, 1988). Dealing, defined as cyclically distributing identical objects so each group receives the same amount until no items remain (Davis & Pitkethly), was a common strategy. While children as young as three and four could share 12 items equally between 2, 3, or 4 recipients, Frydman and Bryant found that only 41% of 4-year-olds could state the number in each group and recognise the equivalence of the shares without counting each set. Desforges and Desforges observed two alternative strategies for dealing: (i) dividing the whole set into equal portions and assigning one portion to each recipient, and (ii) sharing small groups of two or three items at a time. Although children as young as three can distribute items equally in a structured way, some failed to realise that simply engaging in the act of dealing is enough to establish fair shares (Davis & Pitkethly). For instance, 93% of children in Pepper and Hunting's (1998) study used dealing, but those who struggled often lost track, resulting in unequal shares. In summary, these studies suggest that young children can make equal shares, typically by systematic dealing by ones, though they are unaware that dealing itself is sufficient for ensuring fairness. Other research, involving children aged 5-7 with formal instruction, found that children often estimated the number of items per group and tested their estimates, or used trial and error, adjusting the groups until the sharing seemed fair (Carpenter et al., 1993; Mulligan & Mitchelmore, 1997).

Unequal Grouping Situations

Three studies included unequal grouping situations (Blevins-Knabe, 1988; Carpenter et al., 1993; Desforges & Desforges, 1980). Desforges and Desforges' study described strategies when children dealt with unequal sharing situations. Strategies included: asking for one or more to make the shares equal; removing the excess to make the shares equal; breaking the "left over" into smaller parts to equalise the shares; looking puzzled about what to do with the "left over"; one to one sharing; and ignoring the "left over" (3.6 - 4.6 -year-olds). This last strategy was consistent with those used by 4-year-olds in Blevins-Knabe's study. Other studies reported that children often adjusted their distributions when they recognised the shares were unequal (Davis & Pitkethly, 1990; Pepper & Hunting, 1998). Older children, (5 – 6-year-olds) were more likely to ask for an extra item to make the shares fair or remove the excess, indicating stronger concern for fairness (Desforges & Desforges). In summary, these studies suggest that young children approach division tasks with remainders in diverse ways, using a variety of strategies and with varying degrees of success.

Selection of Numbers

Many studies reviewed for this paper used familiar numbers for both the dividend (e.g., 8, 12) and divisor (2, 3, or 4). In contrast, Desforges and Desforges (1980) used less familiar dividends (e.g., 9, 10, 11, 15, 20, 30) and varied the divisors (2, 3, 5), as did Carpenter et al. (1993) with dividend and divisor combinations such as 15, 3; 20, 4; 19, 5. Sherin and Fuson (2005) argued that young children build strong knowledge of specific numbers (like 4 and 12) through computation, which they then apply to new contexts. Mulligan and Mitchelmore (1997) suggested that children first develop strategies for familiar situations where the number facts are well known. Downton (2009) further noted that the numbers used influenced solution strategies, with smaller, familiar numbers resulting in less sophisticated strategies.

In contrast, our research included partitive division tasks with increasingly complex numbers, including those where the dividend was not divisible by the divisor. Our aim was to explore strategic thinking children use when faced with more challenging division problems.

Method

A task-based interview was considered the most suitable instrument to elicit children's existing knowledge, ways of representing their mathematical ideas, structures, and reasoning. Six interview questions were developed that mirrored the calculations from an international pencil-and-paper test (Cheeseman et al., 2022b; Tumusiime & Peter-Koop, 2019). The questions were designed with simple language, increasingly complex numbers and everyday contexts familiar to children. Three worded problems were created for each partitive and quotative division contexts to explore children's thinking about: 12 ÷ 3 =; 7 ÷ 2 =; 22 ÷ 4 =. This paper focuses only on partitive thinking, analysing, and discussing children's responses and the strategic thinking they reveal. In Table 1 the partitive division questions as they appear in the interviewer's script and each question's intended increasing difficulty are shown.

Table 1

The Mathematical Content of the Interview Questions and the Increasing Difficulty Each Presents

Content	Question (Italics instruction to interviewer)	Features of the task
12 ÷ 3	*Place the candies (12 glass beads) on the table together with 3 jars.* Here are 12 candies. Share all of them out equally between the 3 jars. How many candies go in each jar?	• Small familiar numbers • Dividend is divisible by the divisor • Readily modelled
7 ÷ 2	*Place the "donuts" in front of the child.* *Place the 2 plates on the table*. Here are 7 donuts. Share all of them out equally between the 2 children. How many will each child get?	• Small familiar numbers • Divisor is not a factor of the dividend. • Not easy to model • Sharing the odd number is not straightforward
22 ÷ 4	*Place 4 dolls representing children onto the table place and count aloud 22 cards.* 4 children want to play cards. 22 playing cards are on the table. *Hand the 22 cards to the child.* Share them out equally between the children. How many cards will each child get?	• A larger, unfamiliar dividend • Divisor is not a factor of the dividend. • Not easy to model, even when dealing

Interview kits consisting of a script, record sheets and manipulatives, were provided to 10 interviewers together with video cameras. These interviewers were experienced teachers who were knowledgeable observers of young children. They were trained to conduct individual task-based interviews with children. The wording of the script could be re-read if a child required it, but not paraphrased. Interviewers were instructed to video-record the results with approximately 10 children from each Foundation class in 12 schools from whom written parental consent had been received. A total of 94 video interviews and matching interviewer written record sheets were collected for analysis. The authors created an initial coding scheme to categorise student responses. The third author independently viewed the videos and record sheets and coded children's responses on a spreadsheet for review and analysis. Inter-rater reliability measures of 85% were achieved; the codes were then refined and validated. After these processes the findings were examined.

Findings

The facility of young children with the chosen division tasks is shown in Table 2. As can be seen the tasks became considerably more difficult as the interview progressed.

Table 2

The Percentage of Correct Responses for Each of the Three Tasks

Interview questions	Number = 94 (Percent)
12 candies shared between 3 jars	84 (89%)
7 donuts shared between 2 children	68 (72%)
22 cards shared equally between 4 children	41 (43%)

The first question was designed to be accessible for 5- to 6-year-old children. It seems that the small familiar numbers (12 ÷ 3), straightforward wording, and materials that made sense to children, made the task easy to model and 89% of the children solved the problem correctly. The thinking strategies children used for this task are described elsewhere (Cheeseman, Downton & Driscoll, submitted). The main error in thinking for this task was children's (n = 10) lack of awareness of the need for equal groups as the dividends. Dealing by ones was the technique used by most children (37%) and a further 21% dealt by groups to arrive at a correct solution. An *estimate and adjust* technique was used by 26% of the children (see Table 3). This thinking is characterised by children dealing multiples rather than single objects. We hypothesise that the child may think that, with plenty of objects to share, it will be quicker to deal in twos (or another multiple). Some of these children kept dealing in twos for 12 ÷ 3 and some switched to dealing by ones after a cycle was completed to adjust their division and to take into account the remaining objects to be shared. In Table 3 it can be seen that 24 (26%) children grouped by 3's then by 1's. For this task, approximately half (26%, 21% and 5%) of the children used a grouping strategy to find a correct solution.

Table 3

Frequency of Strategies Children Used to Solve 12 Candies Shared Equally between 3 Jars

Category	Solution strategy	Number = 94 (Percent)
Correct		84 (89%)
Dealt consistently	Shared by 1s	35 (37%)
Estimated groups and adjusted	e.g., Grouped by 3s then 1s	24 (26%)
Dealt in multiples	Shared by groups (2 & 4)	20 (21%)
Used a known number fact	e.g., said, "4 + 4 + 4 is 12"	5 (5%)
Incorrect		10 (11%)

The second interview task involved a dividend that was not a factor of the divisor (7 ÷ 2) and it provided some challenges for young children. However, 72% of children revealed knowledge of division. The correct ideas of division are detailed in Table 4.

Only 5% of interviewees solved the task perfectly correctly – with a fractional share correctly named. A further 6% halved one model donut but could not name the solution. Another 11% divided the 7 donuts correctly but when asked, "How many will each child get?" said, "Four" as their solution. We reason that these children could divide into equal quotients but did not understand fractions; therefore, they counted the number of pieces on each plate.

Table 4

The Categories of Response and Percentage in Each Category for (7 ÷ 2)

Category		Solution strategy	Number = 94 (Percent)
Correct			68 (72%)
	Said 3 ½ - perfectly correct	These children broke the last playdough donut into two, gave each piece to one plate and correctly labelled the result	5 (5%)
	Said split in ½	These children could divide and understood a fraction could be a solution but could not name the shares.	6 (6%)
	3½ said 4	This response showed good division but no knowledge of naming fractional pieces – they saw 4 pieces in each share.	11 (12%)
	Left over or awareness of uneven shares	Children placed 3 on each plate then put the last donut back, saying "No-one can have this one." Or "This one is left over." Or "I need an extra one."	46 (49%)
Incorrect		Most commonly children shared 4 donuts on one plate and 3 donuts on another	26 (28%)

Both of the categories of response with mis-named fractions could be interpreted as having a partial understanding of a remainder as a fraction and at least a partial understanding of partitive division. Almost half of the children interviewed (49%) understood that with 7 objects it was not possible to divide the collection into two. The comments made by the children showed their thinking: "This one is left over." And, "I need an extra donut." These comments echo the earlier results of Pepper and Hunting (1998) where young children understood that the number could be made divisible with adjustment. This type of response shows some understanding of division – that the resulting partitions must be equal. In coding for correctness, we said that these children could divide and get the solution of three with a remainder of one for 7 ÷ 2 therefore their response was correct. Overall, only 28% of the 94 interviewed children were incorrect in their thinking about the task. The most common error was to place 3 donuts on one plate and 4 donuts on the other - to divide the objects into unequal groups.

The final partition task asked for 22 cards to be shared equally between four children. The difficulties in this task were with its more complex numbers and a more difficult division with a remainder. The wording was kept plain and we hoped that card games and dealing would be a familiar context for children. The results for this task are presented in Table 5.

Most noticeable in Table 5 is the proportion of incorrect responses to sharing 22 cards between 4 children (57%). We conjecture that the cultural context of playing cards may have overridden the mathematics of this task (Hedefalk et al., 2022). An examination of the error patterns in children's responses may help to understand children's thinking (Table 6). Nineteen children did not understand the task, made random shares, or only partially shared the dividend.

The most common error was in faulty dealing by one (17 children) where the children kept dealing a card to each of the four "players" until the cards were exhausted, thereby ignoring the unequal quotients forming groups of 6, 6, 5, 5 cards. The remaining two categories of errors

Table 5

The Categories of Response and Percentage in Each Category for (22 ÷ 4)

Category	Solution strategy	Number = 94 (Percent)
Correct		41 (43%)
	Dealt by ones and put 2 away	24 (25%)
	Dealt in groups (multiples) using estimate and adjust strategies	15 (16%)
	Used number knowledge	2 (2%)
Incorrect	e.g. gave 4 cards to each and did not share the other cards	53 (57%)

involved the idea of having "left over" cards. Some children (9) estimated equal group sizes and tried several options but found none of the trials shared without remainders and said it could not be done. Eight children said that they either needed two more cards or that they would "put two cards back". This echoes Desforges and Desforges (1980) findings. These children knew they were trying to create equal groups by sharing. Examining responses of children who correctly solved 22 ÷ 4 (Table 5) the largest proportion of successful children (25%) dealt the cards by one accurately and noted that 2 cards were "left over". However, it can be seen by comparing the proportion of the same children who could successfully deal 12 ÷ 3 in Table 1 by ones (37%) that the larger number of cards was more difficult for children. Some of the complexity of the model is immediately apparent, such as when dealing cards, they tend to overlap becoming difficult to see and count.

Table 6

The Error Patterns in Response to 22 ÷ 4

Error	Examples of a child's erroneous actions	Number (n =53)
Concept division unclear	Made random groups/Made equal groups of four/ Only partially used the dividend	19 (36%)
Faulty dealing by one	Dealt by 1s until all the cards were used	17 (32%)
Faulty "estimate and adjust" thinking	Tried several groupings unsuccessfully	9 (17%)
Problems with remainders	Needed two more or two fewer cards Knew 4 x 5 = 20 so 22 cards cannot be shared	8 (15%)

In addition, the dealer needs to visualise the "starting place" and track one cycle of dealt cards back to the start. The dealer also needs to consider whether each share has the same number of cards when the dealing is finished and to consider that it is possible to have some cards remaining "unshared" when the dealing is finished. Taking all of these understandings into account, it is notable that one quarter of children (24 responses in Table 5) have built their knowledge of partitive division through experiences prior to school and could solve a problem which used relatively large numbers and involved remainders.

The strategic thinking used by 18% of children interviewed demonstrated their willingness to use their existing knowledge: (2%) used their number knowledge to think abstractly about the relationship between the numbers 22 and 4. One child said he knew that "four fives are 20" so he thought each person would get five cards. The strategic approaches used by a further 16% of children could be considered as a sort of trial and error or estimate and adjust thinking. In these cases, children tried dealing in equal groups of cards to see whether that would "work". Some children started with dealing out multiples of four and adjusted to groups of five when they had a handful of cards left over. Finding two cards still in his hand one child said, “these two we will put in the middle of the table for the children.” He was clearly thinking of the context and the numbers and the fact that some cards would remain not shared. We noted that this trial-and-error approach was taxing for some children and some (9) who attempted to deal in equal groups were not successful (Table 6).

Discussion and Conclusion

The aim of our study was to explore how numbers matter in the context of early division. While previous research examined how numbers influence young children’s mathematical thinking (Downton, 2009; Mulligan & Mitchelmore, 1997; Sherin & Fusion, 2005), we introduced unfamiliar numbers that are not easily divisible. In Australia, children are introduced to the concept of division as “sharing” in their first year of schooling, through everyday context and chiefly with numbers up to 20 (ACARA, 2023). However, our findings reveal that 43% of children correctly solved the sharing of 22 cards task, and 72% solved the donuts task. In both cases, the children demonstrated an ability to interpret remainders, showing they could consider more complex division. A subsequent finding relating to the sharing 22 cards was the children’s thinking about the size of the group and using estimation. For example, one 5-year-old estimated the group size saying, “Five might be good”, dealing out four groups of five and looked at what is in her hand, said, “only two left – I can’t use these.” Other children estimated four and then adjusted their thinking. The analysis of errors in the card task uncovered some unexpected insights into children’s strategic thinking and use of number knowledge. While they may not have fully grasped the formal division concept, we argue they showed partial understanding and sense-making of partitive division.

An additional finding was that some children’s interpretation of “sharing” was influenced by its everyday cultural meaning, which differs from the mathematical concept of division (Hedefalk et al., 2022). In everyday use, “sharing” simply means giving everyone a portion (e.g., sharing pencils), whereas mathematical division means dividing equally. This distinction needs to be made clear to children. In the card task, some children applied the everyday meaning of sharing, where it is common to deal a limited number of cards, rather than dividing the cards equally, which likely contributed to some of their errors.

Some research advice has been to consider sharing as an intuitive underpinning of division thinking (Squire & Bryant, 2002). In practice many young children are taught in school to share procedurally by dealing by ones with thinking characterised by “one for you and one for me”. However, the results of our study led us to question whether dealing by ones is straightforward as teachers often assume. While dealing by ones a person needs to consider: the total quantity to be shared, how many shares to make, giving one object to each person, tracking the cycle of sharing, watching - if not counting - the developing shares, noting the objects still to be shared, discontinuing dealing if insufficient objects remain for another cycle, and treating extras as “left-overs”. This list of things to notice is quite demanding. An unthinking use of sharing by ones leads children to the most common error we found - dealing until the dividend is exhausted. Further, these children often had no idea of the size of the quotient or whether the final shares they allocated were equal (Davis & Pitkethly, 1990).

While we acknowledge the limitations of the scope of the interview, the findings offer some insights into young children’s partitive thinking and extends earlier research. The present study found that the numbers matter when young children solve division problems. When numbers and contexts are familiar to children, the strategies they use are either: deal by ones, deal by groups, estimate and adjust or use known facts to solve the problem. Additionally, when numbers are small and contexts are familiar and the situation involves a remainder (e.g., donuts problem), most children (72%) could interpret the remainder mathematically. However, when the numbers are outside the children’s experience and the division was more complex and involved a remainder only 43% found a correct solution. It was interesting to discover that children used similar thinking strategies with more unwieldy numbers to those they used with familiar numbers. These findings offer fresh insights into the strategies young children use to solve partitive division problems, challenging the reliance on familiar numbers and situations where the dividend is divisible by the divisor.

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