Dynamic Learning Artefacts: A Methodological Approach for Analysing and Interpreting Student Thinking in Mathematics

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Opportunities to closely monitor and evaluate students' mathematical thinking enables accurate interpretations of their conceptual understanding. This methodological paper reports on a novel and versatile analysis approach to extract student thinking during mathematical games through the generation of Dynamic Learning Artefacts (DLAs). Illustrative examples drawing upon lesson observation notes, interview transcripts and photographs describe the process of generating DLAs to better understand student thinking. The implications for adopting this method of analysis to support teachers' knowledge for instruction are presented and discussed.

There is wide recognition that gaining accurate insights into the thinking processes students adopt when engaged in mathematics is a key component to improving student learning outcomes (Cohors-Fresenborg et al., 2010). The ways in which students are able to access, combine and build upon their prior knowledge reflects their conceptual understanding and comprises more interconnectedness and depth than the mastery of facts and procedures (Lester & Cai, 2016). The multifaceted pathways students take in the space between delivered instruction and their output production can be difficult for teachers to navigate yet becomes critical in being able to plan instruction that will best meet learning requirements (Hiebert & Stigler, 2023). Many studies have reported on the ways in which examining extracts of student learning has supported teachers to become better attuned in noticing the mathematical thinking that occurs throughout problem-solving lessons (e.g., Franke et al., 2001; Jacobs et al., 2010). However, accurately capturing the fleeting, yet vivid mathematical thinking students exhibit during other rich mathematical contexts, such as when playing games, into translatable extracts for analysis beyond the classroom, is not straightforward (Goldsmith & Seago, 2010). The purpose of this methodological paper is to present illustrative examples of a developing analysis approach known as Dynamic Learning Artefacts (DLA) designed to document clear and coherent examples of the thinking processes students demonstrate when playing mathematics games.

Background Literature

Mathematical thinking is a broad term generally used to describe various cognitive processes students undertake when constructing or applying mathematical knowledge. For teachers, accurately monitoring student thinking is critical in being able to authentically respond to and appropriately guide the next steps of learning (Thanheiser & Melhuish, 2023). When teachers take genuine interest in what students say and do as part of the learning process, they are instigating cycles of constructive feedback and signalling to students that the public sharing of thinking will lead to more tailored support and guidance (Ryan et al., 2023). Moreover, these interactions offer instant evaluative feedback to teachers in terms of how effectively their instruction is being received (Matos et al., 2018). However, as Franke et al. (2001) recognised, teachers often require professional learning support to initiate and establish these cycles of noticing effectively as part of their instructional practice. Their longitudinal study showed that when teachers are supported to notice and interpret student thinking for conceptual understanding, their beliefs shifted from a perception that students are incapable of thinking without direct instruction towards an orientation in which students' thinking became the starting point for instruction (Franke et al., 2001). Framing the benefits of interpreting and evaluating

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student thinking in this way is distinct from traditional practices that focus on output derived from written assessments, narrowly reflecting learning experiences of students (Wiliam, 2007).

There is a myriad of methods reported within the literature that document and distil student thinking for analysis that extend beyond simple output production to include: work samples (Jacobs et al., 2010); video recording (van Es et al., 2014); transcripts of lesson dialogue (Sfard, 2001); and student interviews (Bobis et al., 2005). However, as Goldsmith and Seago (2010) caution, the artefacts themselves do not inherently magnify student thinking processes and care should be taken to consider *what* is being analysed and by *whom* (Hill, 2019). For example, Jacobs and Empson (2016) identified that when evaluating the transcripts of mathematical interviews conducted with early years students, experienced teachers were able to notice and expand on students' thinking more effectively than novice teachers who tended to accept incomplete or procedural explanations. Other studies concur that experienced teachers often have developed their noticing skills to engage with student thinking as a sense making process rather than an evaluation of procedural application (Thanheiser & Melhuish, 2023). As Mason et al. (2010) noted, it is prudent to consider how different levels of pedagogical content knowledge may lead to varying interpretations of the same artefact.

Video extracts and work samples are two prominent methods used to evaluate student thinking beyond the classroom, as they often simultaneously detail the mathematical content of the task as well as offer insights as to how students may have solved the tasks (Santagata, 2010). However, as Goldsmith and Seago (2010) noted "video has multiple channels of information but also multiple channels of distraction" (p. 186). This was evident in one study utilising video footage to discern effective ways to leverage student thinking throughout a lesson, reported by Thanheiser and Melhuish (2023). In this particular instance, viewers inadvertently attended to the procedure written on the board by the teacher rather than the audio detailing the student's explanation. The authors concluded that this misdirection was a result of the visual being more readily perceived than the audio (Thanheiser & Melhuish, 2023). Similar challenges are apparent in the evaluation of written work samples where what students say may differ from what they actually do (Sfard, 2001). When this occurs, it is difficult to accurately discern the thinking students actually applied during the task itself (Jacobs & Empson, 2016), and it is common for teachers to then revert back towards expected performance standards rather than try to discern nuanced solutions or unpack the evident misconceptions (Hill, 2019). Therefore, it is necessary to ensure that the source and selection of artefact are closely aligned to the intended focus of analysis from the outset (Santagata, 2010).

Much of the literature reporting on student thinking processes is contextualised through problem-solving approaches to mathematics (e.g., Franke et al., 2001; Mason et al., 2010; Lester and Cai, 2016; Schoenfeld, 2016). One justification for this co-occurrence is that problem-solving tasks offer ample scope for students to adapt their existing knowledge to new situations and to think creatively rather than procedurally (Schoenfeld, 2016). However, an alternative context in which students are also encouraged to think flexibly and demonstrate mental agility is through engagement with mathematical games (Russo & Russo, 2023). Often requiring no written output at all, mathematical games are considered rich and engaging contexts for students to readily apply critical and creative thinking skills (Applebaum, 2025). In an analysis of questionnaire data from 248 Australian teachers, Russo et al. (2021) reported that the majority of participants readily identify the value of mathematical games to support the development of mathematical proficiency, recognising the value of games to develop reasoning, problem solving and conceptual understanding in addition to fluency. Yet, there is limited literature reporting on the thinking processes students undertake when playing mathematical games, with no evident studies analysing student thinking through artefacts extracted from the learning itself. Therefore, exploring the potential of suitable analysis processes to support the noticing of student thinking when engaged in mathematical games is warranted, and forms the basis of this methodological paper.

Dynamic Learning Artefacts (DLAs)

DLAs, originally developed for capturing student behaviours when working on challenging tasks (Hubbard, 2023; 2025), offer a novel approach for extracting and analysing student thinking in mathematics as they combine multiple data sources into a single and coherent graphical representation of learning. DLAs are generated by taking a work sample and overlaying it with time-related annotations to provide greater detail of what occurred, and when, over a selected lesson excerpt. In addition to the observational data, where relevant, the student's reasoning–derived from interview transcripts–is also weaved into the annotations to portray the experience from a holistic perspective. Essentially, the core aim of documenting accounts of student thinking in this way is to simultaneously communicate what is seen and heard from both the student and observer's perspective into a single unit for analysis.

The notion of triangulating multiple sources of learning to generate the DLAs was informed by Schoenfeld's (2016) study that used video data to document when and how effective problem-solvers used various types of thinking when completing tasks. In his study, over 100 college students were video recorded, with changes in behaviour tracked according to time points throughout the lesson. The findings reported that students who readily and efficiently solved the tasks demonstrated more frequent shifts in strategy selection over the timeframe than those students who persisted, unsuccessfully, in the application of a single rehearsed strategy (Schoenfeld, 2016).

The impetus for the current iteration of DLA design reported next was in response to the limitations associated with relying on either video data or written work samples reported throughout the literature (e.g., Goldsmith & Seago, 2010; Jacobs & Empson, 2016; Thanheiser & Melhuish, 2023). Hence, the intention was to draw upon the strengths of video data by underpinning the DLA with a visual image, whilst also ensuring that student explanations were contextualised through time-bound annotations. As the thinking in mathematical games is often not visible, determining how this could clearly be communicated through a visual graphic to underpin the DLA posed further challenges that had not been addressed elsewhere. Critical to the design was being able to maximise focus on student thinking and minimise unnecessary information that could divert attention away from the strategies students used in each turn. This malleable construction process offers a versatility to the DLA method that is not afforded through video extracts or work samples.

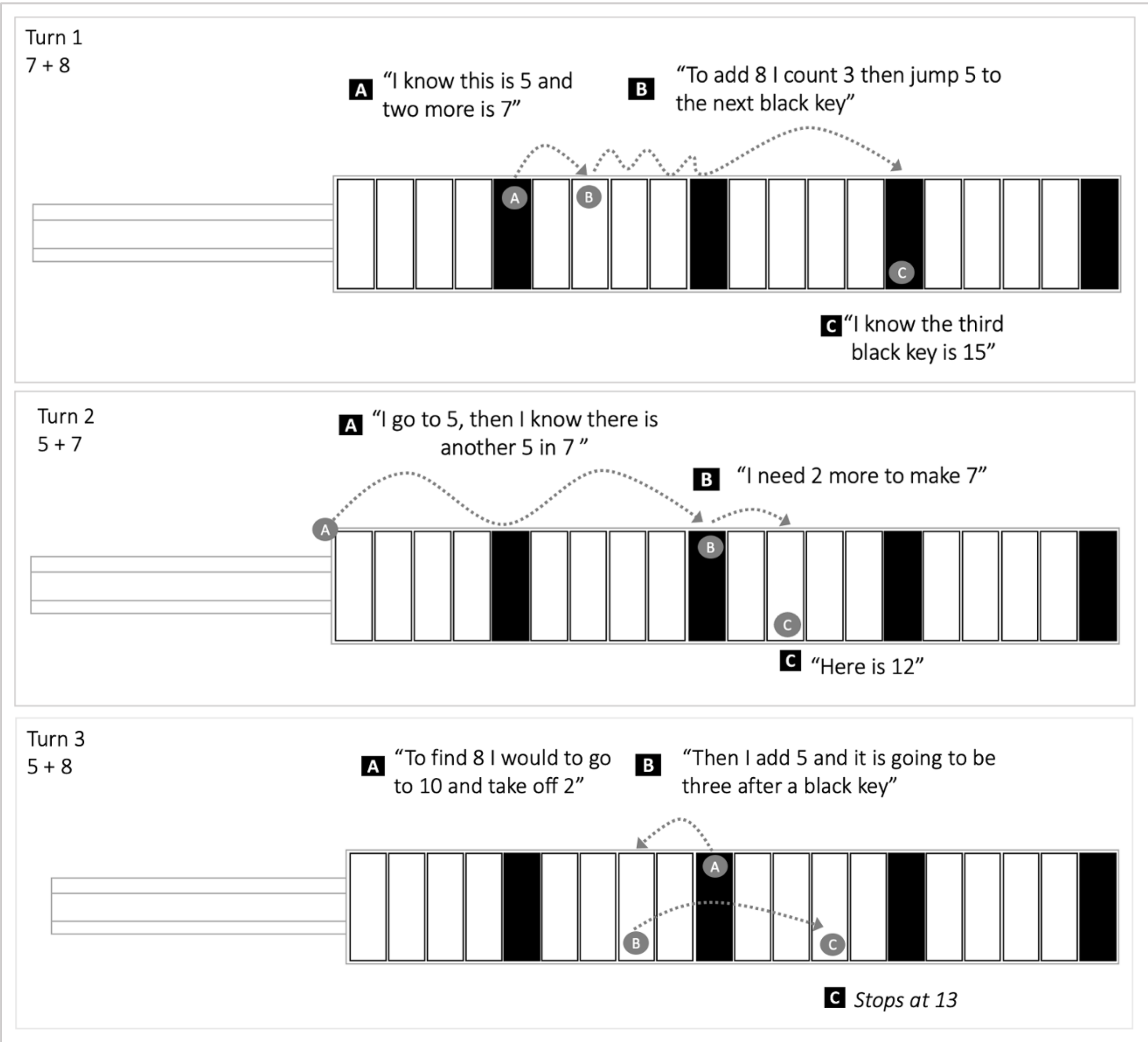
An Illustrative Example

The illustrative examples of DLAs reported in this paper were generated as part of a larger research project focused on how Year 2 students utilised and applied their number sense knowledge when playing mathematical games supported by a manipulative known as The Keyboard (see Hopkins et al., 2025). This project consisted of teaching a series of lessons over a two-week period, where students were introduced to The Keyboard representation and used it in various ways to play of a range of mathematical games to focus on their mental computation strategies for addition and subtraction. Given this context, The Keyboard formed the basis of the visual prompt in which the DLAs were generated. Further information from the lesson such as time bound observation notes, photographs and student explanations were layered onto this image to convey the subsequent game moves students performed throughout a series of game turns. To make explicit the strategies students demonstrated with each turn, a template of The Keyboard underpinned each observation. From there, grey dotted arrows were used to depict the sequential moves observed for each game turn, and these were labelled A, B, and C to correspond to the respective verbal explanations offered by students as they played the game, or retrospectively, once prompted in the post-lesson interview. Two exemplar DLAs, representing the game plays of two different students (i.e., Student 1 and Student 2) are described next, demonstrating potential affordances of adopting this methodology for analysing student thinking when playing mathematical games.

The two DLAs presented next make visible the thinking strategies that would otherwise be indiscernible as students took turns in playing a mathematics game involving single-digit addition. Figure 1 shows a DLA from focus Student 1, comprising three consecutive game turns observed over a 5-minute period of the lesson. Between each turn, the other student pair was completing their turn of the game, which is not documented in the DLA shown here. The grey arrows and letters represent the active moves made by the student, which were documented through the lesson observation notes. The quotations reflect relevant excerpts of the audio recording transcripts that aligned to the respective turn being observed.

Figure 1

Student 1: Consecutive Turns Using Multiple and Flexible Strategies



The DLA presented in Figure 1 shows the various thinking strategies Student 1 demonstrated over multiple turns when solving single-digit addition sums in relatively quick succession. To accurately interpret Student 1's thinking strategies for each turn, it is necessary to integrate both the visual tracking on The Keyboard (grey arrows) with the associated student explanations. Attending solely to the arrows makes it difficult to directly link the moves on The

Keyboard with the associated sums being solved. For instance, Turn 2 could be misinterpreted as 10 + 2 instead of 5 + 7, as the arrows show Student 1 skip counts in 5s to 10 and then adds on 2 more. However, following the arrows in conjunction with the explanations enables clear and informative interpretations of student strategies to be identified. For example, in Turn 1, by focusing solely on the arrows, it is unclear why Student 1 starts on 5, moves 2, counts 3, and moves 5 when the sum they needed to solve was 7 + 8. When reading this turn with the accompanying explanations, it becomes clearer that this student had decomposed 7 and 8 (into 5, 2 and 3, 5 respectively) and used this decomposition strategy to flexibly work with The Keyboard benchmarks (i.e., the black keys). Deconstructing each of Student 1's turns to this extent shows that over the three turns, a range of processes are used such as skip counting, benchmarking, and decomposition. Furthermore, it provides clear evidence to support the assertion that Student 1 has established a level of conceptual understanding whereby they can flexibly and efficiently manipulate single-digit numbers with proficiency. The ability to not only notice, but also understand and follow Student 1's mental computation knowledge to this extent is a strength of the DLA design.

The second DLA, presented in Figure 2, is focused on Student 2 and was constructed to extract and visually replicate just a single turn lasting less than 2 minutes. In this instance, repeated Keyboard images have been used (i.e., part A and B) to highlight the two separate thinking processes this student demonstrated to solve 7 + 6 in one turn.

Figure 2

Student 2: Single Turn Requiring Two Different and Disconnected Strategies

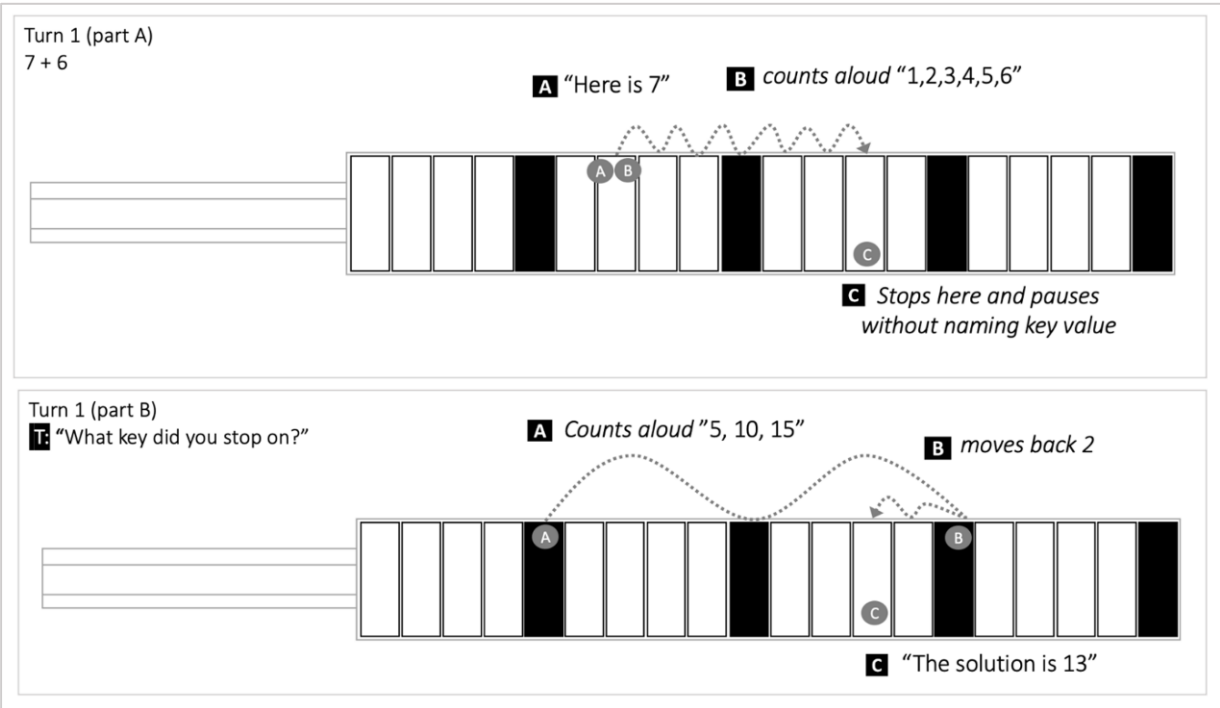


Figure 2 presents the deconstruction of a single game turn which offers critical insights into how Student 2 had to use two different and disconnected strategies to first solve the single digit addition sum and second to show the key to representing the sum solution. Had this same game turn been observed using a conventional method, it is unlikely that these subtle yet important aspects of Student 2's learning would have been identified. For example, even though this student does demonstrate a correct process for adding 7 + 6 in Turn 1, Part A (i.e., starting on 7 and counting up 6 more), the annotated comments provide evidence that this has been done procedurally rather than conceptually, as there are no connections made between how the parts of 7 and 6 comprise the whole of 13. Further evidence of procedural application can be identified in Turn 1, Part B, when Student 2 demonstrates a process disconnected from 7 + 6 by applying skip counting by 5s to work out the value of the 13 key. Again, this reinforces the interpretation that Student 2 is not identifying important connections between the problem strategy and the answer. Being able to discern this level of detail about students' understanding and proficiency offers potential to more accurately plan the necessary targeted instruction for future learning.

These two exemplar DLAs show how multiple sources of qualitative data can be compiled to create simple yet cohesive and comprehensive accounts of individual student thinking. In addition, there are affordances in using these data collectively to evaluate the diversity across cohorts and consider how this will influence teachers' future instruction. For example, being able to compare and evaluate the different strategies utilised by Student 1 and Student 2 highlights that, even though both students were able to correctly answer the addition sum, there was a considerable distinction in how each outcome had been reached. Student 1 demonstrated productive and agile computation strategies, whereas Student 2 had to rely on more superficial and procedural application of knowledge. As the class teacher, having the means to recognise such discrepancies beyond the correct solution is critical in planning for instruction that caters to the diverse needs of the class. Moreover, developing an acute awareness of how some students may appear to be thinking, but are actually replicating procedures without understanding, will better prepare teachers to anticipate and respond to students authentically in the actual lesson. Such levels of evaluation have previously been associated with experienced teachers who have been supported to develop their pedagogical knowledge for in-lesson noticing. Therefore, an affordance of using the DLA methodology in the ways described here is that it creates greater accessibility for teachers of all levels of ability to engage with student thinking in meaningful and productive ways.

Concluding Thoughts

Presented here was a novel and alternative method for evaluating student thinking in mathematical games through the generation of Dynamic Learning Artefacts (DLAs). Capitalising on the strengths of existing data collection processes—such as visual perception enabled through video footage (Thanheiser & Melhuish, 2023) and reasoning from student interviews (Jacobs et al., 2010)—this approach allowed for the salient aspects of the learning experience to be leveraged in a 'fit for purpose' unit of analysis (Santagata, 2010). Constructed from a pictorial base and overlayed with observational annotations, DLAs reflected students' active thinking trajectories across a static image, offering accurate insights into a moment of learning through combining what was said with what was done (Sfard, 2001).The careful selection and alignment of lesson observations, photographs and student descriptions offered vivid accounts of the mental computation strategies students accessed and applied when solving single addition sums, enabling distinctions between procedural and conceptual understandings to be identified (Schoenfeld, 2016). Previously, the ability to make such specific evaluations about student thinking processes has been challenging, as classroom observations are generally less definitive, requiring inferential conclusions to be made beyond output production.

This work extends on the research reported by Hubbard (2023; 2025) in narrowing the focal point of the DLA to a granular level (i.e., 3 to 5 minutes), as opposed analysing student learning over the duration of a lesson. Modifying the unit of analysis in this way enabled the focal point to be conveyed more clearly than would be possible in attempting to replicate all of the game turns throughout the entire lesson. Pre-determining the focal point and constructing the DLA around the specific moves made on The Keyboard provided a consistent interpretation of student thinking, not only across each student's turns, but also between different students.

Attending to these moves, rather than the sum being solved, addresses concerns raised by Mason et al. (2010) that too often when evaluations of student thinking are made, priority is given to the mathematical content instead of the strategies students use that demonstrates their conceptual understanding. The simplicity and precision with which such thinking can be communicated makes DLAs an ideal mode to introduce less experienced teachers to the importance of developing their practices of noticing to underpin their mathematical instruction (Hill 2019; Jacobs & Empson, 2016).

Several limitations exist regarding the generation of DLAs and their broader applicability that are important to consider. For instance, the initial development process may be perceived as resource intensive, requiring time commitment to conduct the observations, synthetise the data then generate the DLAs. In addition, it is critical to recognise that without a specific pre-determined learning focus, making sense of the copious information sources readily available within classrooms may be overwhelming for an untrained observer. This reiterates Santagata's (2010) concerns— that without accounting for a specific focus, the analysis of the DLA may be compromised.

In conclusion, the use of DLAs has also been shown to be highly effective in conveying the various thinking strategies students demonstrated during mathematical games, and how these strategies extend beyond fluency recall to emulate the proficiencies of reasoning, understanding, and problem solving (Russo et al., 2021). Given the frequency with which teachers readily use games as part of their mathematics programs, there is merit in considering how DLAs can further support the identification and analysis of students' mental computational strategies within such contexts. Just as developing teachers' noticing skills has improved their pedagogical content knowledge for problem solving (Franke et al., 2001), closely attending to the thinking of students within a mathematical game context may enable teachers to readily recognise the potential for meaningful interactions that are supportive in building conceptual understanding. As the DLAs reported here were based on a game that used a specific manipulative (i.e., The Keyboard), there is scope for further exploration and research into the ways in which this approach can be adapted within broader game contexts that draw upon different visual representations and models when supporting student thinking processes in mathematics.

Acknowledgements

Although this was a non-empirical paper, it is important to note that data collected for the purpose of illustrating the DLAs took place within a broader study, for which ethics approval was granted by Monash University [Ref: 36277], and participants gave informed consent.

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