Experiences of Pre-Multiplicative Thinking Junior Secondary School Students

Susan Mabb
University of the Sunshine Coast
Susan.Mabb@research.usc.edu.au

The Australian Curriculum suggests that students have transitioned from additive to multiplicative thinking upon entry into secondary school. However, research shows that many students are thinking additively, placing them at a disadvantage to their multiplicative thinking peers. In this paper, early findings from a broader research project describe the mathematical experiences of pre-multiplicative thinking students in a secondary school context. Critical realism was applied to initial interview data, and a proposed model emerged to represent the pre-multiplicative thinking research space.

Multiplicative thinking sits within the study of mathematics. It is an essential cognition that is defined as the use of multiplication and division strategies to solve a range of numerical-based problems (Young-Loveridge et al., 2012). First appearing in the *Year 2 Australian Curriculum* (ACARA, 2024), multiplicative thinking is progressively developed across the primary school mathematics curriculum. Students are expected to think multiplicatively upon entry into secondary school (ACARA, 2024) because multiplicative thinking lays the foundation for accessing mathematical concepts prescribed by the secondary school curriculum (Beckmann et al., 2008). Unfortunately, research indicates that between 30% and 55% of Australian Year 8 students are not thinking multiplicatively (Siemon et al., 2019). This statistic points to a problem in mathematics education, implying that a significant number of middle school students may be struggling to negotiate the age-appropriate curriculum. The need arises for secondary mathematics teachers to contemplate how they work with pre-multiplicative thinking students to ensure these students can access learning at the age-appropriate level.

In this paper, preliminary findings from a secondary school project are shared to explore the research question: What do pre-multiplicative thinking students in the junior secondary school experience when learning mathematics? Participants chosen for the project were positioned at the cognitive threshold between additive and multiplicative thinking. These participants were representative of middle school students having some understanding of multiplication and division but not yet able to apply robust multiplicative thinking to answer numerical questions. Critical realism (CR) advocates for researchers to listen to those at the centre of the research (Sayer, 2010). The findings arising from CR data analysis on participants' initial interview responses are presented. As many middle school students are operating pre-multiplicatively, it will be meaningful to consider participants' thoughts and responses in relation to learning mathematics that are shared in this paper.

Critical Realism and Its Application in Research

CR originated through the writings of Bhaskar (2008) who challenged social scientists to consider whether people-centred research should be conducted using the same methods that scientists apply to the natural world. Sayer (2010) elaborated that when studying human beings and their actions, it is impossible to isolate variables as scientists do when studying natural phenomena. CR offers a useful way to study entities within a social context (Sayer, 2010). While CR has not yet been used in mathematics education research, its application has been suggested (Nunez, 2015). In this paper, students, teachers and learning, within a junior secondary school mathematics setting are viewed through the theoretical lens of CR.

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 277–284). Canberra: MERGA.

According to CR, an entity is of a human, physical, or conceptual nature (Venkatesh et al., 2013). Every entity owns a set of properties and, possesses a causal power or liability which may, or may not, be exercised (Sayer, 2010). For example, a teacher (entity), with a toolkit (property) holding many and varied pedagogical techniques, would be considered to possess a power. If the teacher effectively accesses techniques from their toolkit to assist students with learning, then the teacher is exercising their power. Because the teacher is a complex human entity, there is the possibility that the teacher does not exercise their power to effectively use the available pedagogical techniques in their toolkit. Alternatively, a teacher with a limited toolkit may possess a liability because the teacher may not be equipped to ably assist students with learning. Initial interview data identified examples of teachers who have exercised their power or liability in the classroom. Some of these examples are presented in this paper.

CR specifies the analysis of entities at different levels (Easton, 2010) so that researchers can identify the connectedness of an entity's data. Emergence is the name given to this analytical process because data at a higher level of an entity may emerge because of data at a lower level (Stylianou, 2019). During data analysis of initial interviews, the following levels were identified: Level 1 - student (L1); Level 2 - learning (L2); and Level 3 - policy (L3). This type of layered analysis elicits the following example: It emerges that a student may be reluctant to answer questions in front of the class (L2), because the student is feeling confused about the learning (L1). In CR, emphasis is also placed on relationships that exist between entities. Relationships help to explain events that occur in the CR research space (Easton, 2010).

Three levels of reality are defined in CR. The *empirical* level defines events that are visible through experience or observation (Easton, 2010), such as how a student feels when learning mathematics. The *actual* level represents events that are harder to measure as they may be concealed or observed but not clearly understood (Easton, 2010). An example of an actual event is a student's observation of their teacher's disposition to teaching mathematics. The observation is made from a second-hand perspective so is not fully understood by the student. The third level of reality, named the *real*, represents hidden factors, known as causal mechanisms (Easton, 2010). These mechanisms, like an entity's power or liability, may, or may not be exercised. The aim of CR is to discover mechanisms that, when exercised, may cause the research event of interest. While not easy to identify and examine, critical realists work to study actual and real events that occur as undercurrents in their research. Thus, a future research goal will be to identify mechanisms that support middle school students' transition into multiplicative thinking.

The Multiplicative Thinking Continuum

An eight-zone multiplicative thinking continuum, describing students' shift from additive to multiplicative thinking, was developed during the *Scaffolding Numeracy in the Middle Years* (SNMY) project with students from Years 4 to 8 (Siemon et al., 2006). The continuum, named the *Learning and Assessment Framework for Multiplicative Thinking* (LAF), included key ideas and teaching advice for each LAF zone, from Zone 1-*primitive modelling* through to Zone 8-*reflective knowing*. During the SNMY project, assessment tasks were created to determine a student's zone position on the LAF, and targeted teaching resources were developed to assist with students' growth along the multiplicative thinking continuum. SNMY researchers identified that many middle school students find it difficult to transition from additive thinking (Zones 1-4) to robust multiplicative thinking and beyond (Zones 5-8).

Recent research (Siemon et al., 2019) reveals that there has been no significant change in the multiplicative thinking landscape compared with the earlier study (Siemon et al., 2006). It remains that many middle schoolers are operating pre-multiplicatively (Siemon et al., 2019). The critical juncture between additive and multiplicative thinking lies at LAF Zones 3 and 4. Participants described in this paper are Year 7 students positioned at this critical multiplicative thinking juncture. Zone 3 participants: use helpful numbers, such as 2 and 5; use doubling and halving strategies; and are beginning to work with large numbers by applying additive thinking. Zone 4 participants: use common two-digit multiplication; use additive thinking for larger numbers and decimals; can begin proportion problems; and are only able to provide minimal mathematical explanations (Siemon et al., 2006).

Research Design

The research design describes the CR data analysis that was conducted on the interviews of ten pre-multiplicative thinking participants from a broader research project. At the start of the project, participants from a Queensland Year 7 mixed-ability mathematics class were assessed against the LAF. From this original participant group, six participants at LAF Zone 3 and four participants at LAF Zone 4 were chosen to represent students at the critical juncture between additive and multiplicative thinking. These individuals formed the project's focus group, and their mathematical experiences expressed in the initial interviews, are presented in this paper.

Each participant undertook a 30-minute initial interview which involved answering 16 questions. Verbal and written responses were video recorded and transcribed. The initial interview protocol was informed by a literature review as recommended in CR research (Hoddy, 2019). Additionally, a grounded theory interview protocol (Kennedy, 2011) was referenced during question development. Kennedy's (2011) protocol was designed to capture interactions between disengaged middle school students and their teachers. Initial interview questions aimed to capture participant voice because CR places importance on social scientists listening to the understandings of their participants (Sayer, 2010). Therefore, questions encouraged participants to express their feelings and perspectives in relation to learning mathematics. The first interview question asked participants to draw a picture or use words to describe how it feels when learning mathematics. Two further questions illustrate how participant voice was captured: What challenges do you face in learning maths? If you could give your maths teacher one piece of advice to assist students with their learning, what would it be?

Due to a lack of methodological guidance in the literature on how to apply CR, the suggestion to use grounded theory methods was taken (Hoddy, 2019). Audio data from initial interview video recordings were transcribed. Open coding from grounded theory was applied which broke apart the qualitative data and allowed for the creation of eleven initial codes to represent its discrete parts (Ho et al., 2025). After 10 open coding iterations, 34 codes were identified, each holding a bank of descriptive data from the 10 participants. Next, axial coding from grounded theory was iteratively applied to rebuild the data in a meaningful way by identifying how the codes were connected through general themes (Ho et al., 2025). Finally, axial coding, applied through a CR lens, assisted with synthesising the data into general themes to produce a CR model representative of the research space. In a future research stage, selective coding will be applied, through the lens of CR, to identify causal mechanisms which will point to reasons for students' multiplicative thinking delay.

Participants' Accounts of Learning Mathematics

Open coding and primary axial coding exposed four general themes in the data: participants' affective responses; participants' experiences; participants' insights about teachers; and participants' advice to teachers.

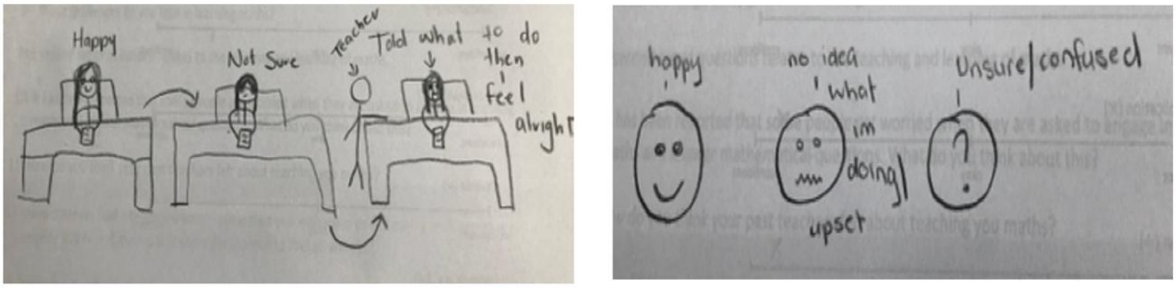
Participants' Affective Responses

Participants reported two types of affective responses in relation to learning mathematics: *emotional responses* and *responses to the cognitive challenge*.

Nine participants reported *emotional responses* when learning mathematics. Most reported feeling discomfort (8), including responses of "stressed" and "anxious" when learning mathematics. Other discomfort responses included feeling overwhelmed and feeling worried about passing mathematics. Three of the eight participants who had reported feeling discomfort, also reported experiencing a series of emotions. In response to the first question of the initial interview, Karen and Indy (pseudonyms) drew pictures to show the series of emotions they have experienced during a mathematics learning episode (see Figure 1). In addition, many of the participants believed that some people get worried when learning mathematics. Three other *emotional responses* were recorded. Participants said they felt "happy" when understanding the learning (2), and "annoyed" (1) and "bored" (1) when learning mathematics.

Figure 1

The Series of Feelings That Karen and Indy Experienced when Learning Mathematics



(a) Karen

(b) Indy

Responses to the cognitive challenge of learning were also articulated. Seven participants used the word "confused" to express how they have felt when learning mathematics. Five of these participants described times when they had not understood the learning being delivered.

Participants' Experiences

Participant voice revealed the following codes: *learning mathematics is difficult*; *time affects uptake of learning*; *reluctance to answer questions in front of the class*; *reluctance to ask for help when not understanding*; *factors that support learning*; and *working with others helps with learning*.

Learning mathematics is difficult was further coded into *difficulty with using large numbers and generalising*; *significant cognitive load*; *mathematics is harder than other subjects;* and *trying to understand the question and getting stuck*. Seven participants articulated how *learning mathematics is difficult*, with five reporting *difficulty with using large numbers and generalising* in the SNMY assessment. Uma retells her difficulty with moving from a count-all approach to one that requires generalising a number pattern.

> I was getting used to drawing the tables and the people sitting at the tables. When it came to the larger numbers, I tried to add up the tables and times the tables, but I got them incorrect cos I didn't do something right.

Five participants reported a *significant cognitive load*. Casper said, "when I'm thinking too much, I put too much effort in, and I get a headache". Narelle reported her challenge.

> Sometimes you [the teacher] go through all three [problems] at once and it gets too much. It's like my brain is going a hundred k's an hour. You just can't organise all three things. It comes in one big clump.

Abby reported feeling overwhelmed when trying to copy the work and listen to her teacher's explanation at the same time. Four participants believed that *mathematics is harder than other subjects*, while three participants reported *trying to understand the problem and getting stuck*.

Seven participants articulated that *time affects uptake of learning* with reports of not having enough time to grasp the learning, and responses of "too fast", "too much", "can't get the work done in time" and "can't catch up". Bonny described not being ready for the harder questions as she had not yet grasped the concept. Uma explained she needs time to read, reread and interpret worded problems. Indy described how it can take several visits, over time, to grasp a concept. When Indy ultimately grasped the concept, she reported it was "whoa, mind-blowing". Abby shared a positive learning experience: "I like Maths Pathway because you can spend more time thinking on the questions and have more time to answer them."

Three participants reported a *reluctance to answer questions* in front of the whole class. Bonny described how her nervousness to answer questions overshadows her learning: "Sometimes I'm scared to say the answer in case I get it wrong, then everyone laughs."

Three participants articulated a *reluctance to ask for help* when not understanding the learning. Narelle reported worry that she will anger her teacher if she asks for help.

> "Oh, we just learned that. You didn't listen clearly." (quoting a teacher) When the test comes, I don't know how to do it. I didn't understand that question and I didn't ask for help. I would pretend it's all okay. When it comes to the exam, I don't know anything.

Participants identified *factors that support learning* (6). Karen discussed how doing and visualising assisted with conceptualisation of the Cartesian plane.

> We were doing Cartesian plane, and we did battleship. We had to play the game and then you had to try and figure out the coordinates to sink the other person's ship. We had to do that on a piece of paper so we could actually see the Cartesian plane. Playing battleship probably made it a lot more fun to learn, and it probably made it easier, cos you actually visualise what it was.

Participants reported that *working with others helps with learning*, specifically that talking with others helps (5). Working in small groups was reported as valuable (4). Karen detailed working with her peers: "I like one-on-one. See how they do theirs. Ties into what I am doing."

Participants' Insights About Teachers

Participant voice revealed that students notice *teachers' disposition to teaching mathematics,* and students are appreciative of *teachers who articulate their desire to help, followed by enacted help*.

Four participants recalled *teachers' dispositions to teaching mathematics*, with three describing teachers who liked teaching mathematics. Nikki shared her observations: "My Year 6 teacher was, I don't know whether you'd call it a mathematician, but she studied maths. So, we did lots of maths." Two participants reported variation in teachers' dispositions. Bonny articulated the contrast in disposition between teachers: "We'd pretty much only do maths [Year 6 teacher]. But then other teachers, they tried to avoid maths as much as possible. So that's probably where they're stronger at, where they like to teach."

Two participants, Indy and Casper, reported they were appreciative of *teachers who articulate their desire to help, followed by enacted help*. Indy spoke about two teachers who had revealed this quality, a primary teacher and her current Year 7 mathematics teacher.

> She [primary teacher] taught me a lot about math and helped me understand. I think she felt like it was a good thing that she could help me learn better. He's [Year 7 maths teacher] always trying to help us understand. He always shows us diagrams and stuff to help us understand it [the learning].

Participants' Advice to Teachers

Participants provided advice to teachers around the following themes: *provide clarity around learning*; *use models to represent a concept*; *differentiate learning*; *check in with students*; *use different strategies to teach a concept*; *make time for learning*; *get students to reflect on mistakes*; and *do not use worksheets to teach*.

Five participants recommended that teachers *provide clarity* around learning. All five used the term "explain" during the interviews. Uri said, "explain it more, in more detail". Keith suggested, "do it with me or do it on the board; walk us through it".

Five participants requested that teachers *use models*. Two participants noted the use of place value blocks. Two participants suggested how the array representation was useful. Bonny described how the use of concrete props to represent variables and constants when developing an expression was helpful. She also described how visual cues were helpful.

> He [teacher] drew pictures on the board like the crocodile and a boat and it would show you where they were swimming. It was just a number line but with more pictures, so you'd understand it. That helped. I think for some of the class, like me, [who are] much more visual, like watching that [picture] so then they can focus on it better.

Participants requested that teachers *differentiate learning* (3). Karen gave detailed advice.

> Work with them [students] in groups. It helps. You [teacher] get a whole group of students who are trying to figure out the same thing and then do some more different ways of trying to do it. And it helps in ways cos there's less people. They [teacher] have more focus on you [the student] particularly, and they kind of see what you need help with, so then they can actually help you with whatever you're learning.

Three participants recommended teachers *check in with students* rather than assuming students will let teachers know when they need help. Indy focussed on non-verbal cues: "Look back at them [students] to see if they're doing it. Watch their facial expressions, and if they haven't written anything down [then they don't understand]."

Three participants suggested that the teacher *use different strategies* to teach a concept. Karen described a teacher whom she believed could not help students who were struggling because he was unable to explain the concept in a way that the students could understand.

Two participants recommended that teachers *make time* for learning. Abby noted, "give students more time to copy the work; if you don't get all the information down, you'll have to copy off someone else's book or you just don't get the information at all".

Participants suggested that teachers *get students to reflect on mistakes* they have made (2). Narelle remembered how reflecting on her mistakes had been useful.

> I got a question wrong, and it was like four times eight and I remember I got 16. Then he [her teacher] knew I was trying to do double double, but I had only done double. He told me to reflect on that. I think it made me learn times four better because I know it's double double now, not just double.

Two participants advised: *do not use worksheets to teach*. Bonny reported that worksheets put students in a "bad mindset". Keith described his experience of being given a worksheet and told by the teacher to "learn this". Keith advised, "that doesn't help; go through it on the board".

Analysis and Discussion

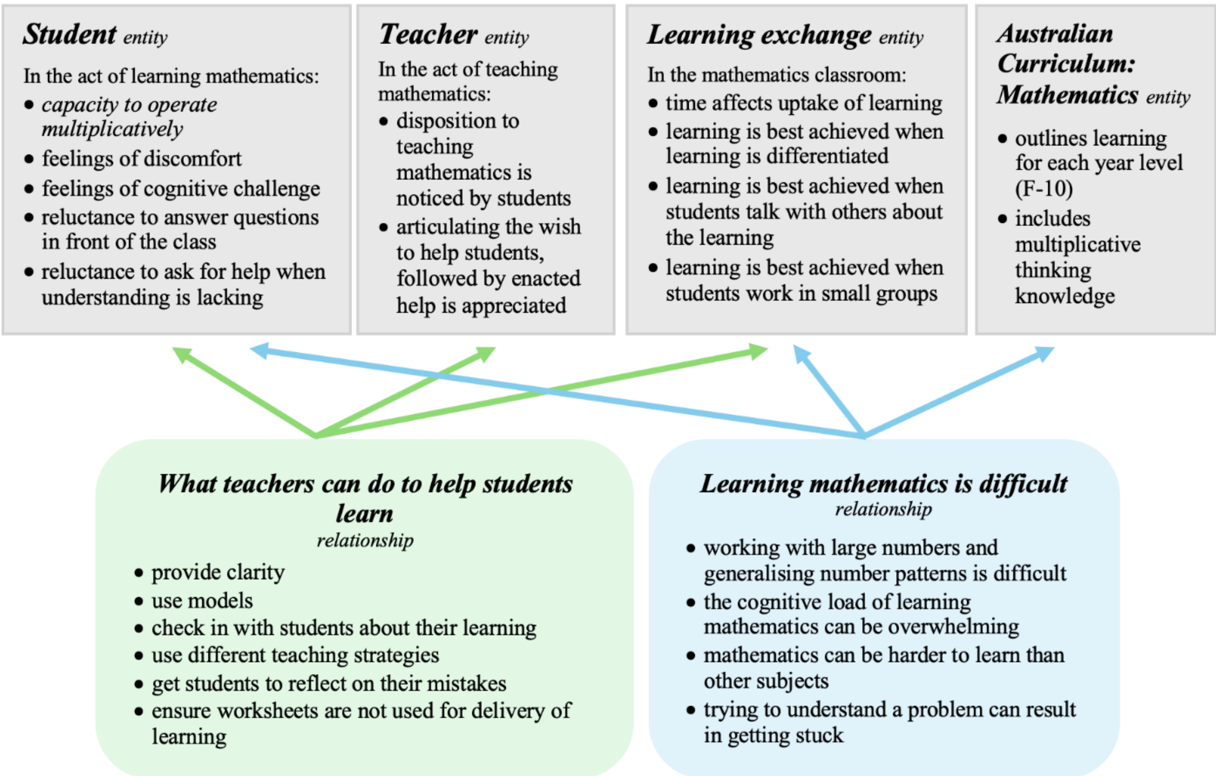
After open coding had initially been applied to the interview data, identifying 34 codes, it was noted that instances of repeating participant descriptions occurred across some codes. For example, *time assigned to learning* appeared under the codes of *participant challenges* and *advice for teachers*. Examples of these repeating participant descriptions are shared in the qualitative data presented above and emphasise the importance of the associated messaging in participant descriptions.

After primary axial coding had been conducted to develop general themes (Hoddy, 2019), secondary axial coding was undertaken to place the qualitative interview data into a proposed CR model (Easton, 2010), representing the experiences of pre-multiplicative thinking students in a junior secondary school mathematics setting. Four emerging entities and their associated properties, as prescribed by CR, were identified to represent the research space: *Student, Teacher*, *Learning exchange*, and *Australian Curriculum: Mathematics*. Easton (2010) used an exchange entity to represent the ongoing communication between buyer and seller in his CR study. Hence, the *Learning exchange* entity was used to represent ongoing classroom interactions between the student and teacher. The *Australian Curriculum: Mathematics,* while not explicitly discussed in the initial interviews, was deemed a necessary entity as it conceptually underpins actions taking place between other entities in the research space. Secondary axial coding assisted with the appropriate placement of repeating participant descriptions and emerging connections in the data. Two themes that surfaced could be viewed as further entities in the proposed model but were instead classified as key relationships between existing entities: *What teachers can do to help students learn,* and *Learning mathematics is difficult.* In CR, a relationship that has been found to have come into existence and interacts in the research space may operate as an entity with its own set of properties (Easton, 2010).

Figure 2 depicts the proposed CR model that represents the pre-multiplicative thinking secondary school research space from a student's perspective. The four entities of *Student, Teacher, Learning Exchange* and *Australian Curriculum*: *Mathematics* are shaded in grey. The key relationships are coloured green and blue. Matching arrows show each relationship's connectivity with other entities in the research space. Participants' experiences that were axially coded into general themes using grounded theory were appropriately positioned across the proposed CR model. The secondary axial coding process using properties of entities and relationships was presented in bulleted lists. The first property of the Student entity has been italicised because it is the focus of the broader research project, and depicts the Student entity's power or liability, according to CR. At the time of the initial interviews, all participants possessed the property, *capacity to operate multiplicatively*, however the capacity had not yet been realised, therefore presenting as a liability. If, over the course of the project, a participant shifts into multiplicative thinking, then their liability will transform into their power.

Figure 2

Proposed CR Model of Pre-Multiplicative Thinking Junior Secondary Students' Learning Experiences



Conclusion

The findings presented in this paper describe the experiences of pre-multiplicative thinking junior secondary school students when learning mathematics, viewed for the first time from a

CR perspective. Detailed qualitative interview data provided the following key findings of students' mathematical learning experiences: feelings of discomfort; difficulties with understanding; challenges with cognitive load; and struggles with time constraints. In addition, students identified that talking with others and working in small groups supports their learning, and they identified ways in which their teachers can help them learn.

A proposed CR model depicting pre-multiplicative thinking junior secondary school students' learning experiences was developed. Four entities (*Student*, *Teacher*, *Learning Exchange*, and *Australian Curriculum: Mathematics*) and two relationships (*What teachers can do to help students learn,* and *Learning mathematics is difficult*) emerged. In addition to capturing key findings from the initial interview data, the proposed CR model illustrates elements and connections in the secondary mathematics classroom, from the perspective of the pre-multiplicative thinking student. Secondary mathematics educators are encouraged to consider the findings presented in this paper because they provide a unique insight into the experiences of pre-multiplicative thinking students when learning mathematics.

Acknowledgements

Project approval: Ethics S211675 University of the Sunshine Coast; and Brisbane Catholic Education. Student/parent informed consent. Pseudonyms used. Margaret Marshman and Anne Bennison are thanked for ongoing support and feedback provided on early drafts of the paper.

References

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2024). *Australian Curriculum: Mathematics.* https://v9.australiancurriculum.edu.au/

Beckmann, S., & Fuson, K.C. (2008). Focal points – Grades 5 and 6. *Teaching Children Mathematics*, *14*(9), 508–517.

Bhaskar, R. (2008). *The possibility of naturalism: A philosophical critique of the contemporary human sciences* (4th ed.). Routledge.

Easton, G. (2010). Critical realism in case study research. *Industrial Marketing Management, 39*(1), 118-128. https://doi.org/10.1016/j.indmarman.2008.06.004

Ho, L. & Limpaecher, A., (2025). *Open, axial, and selective coding in qualitative research: A practical guide.* https://delvetool.com/blog/openaxialselective#:~:text=Axial%20coding%20in%20grounded%20theory,you%20developed%20in%20open%20coding.

Hoddy, E. T., (2019). Critical realism in empirical research: Employing techniques from grounded theory methodology. *International Journal of Social Research Methodology 22*(1), 111-124. https://doi.org/gjdpfk

Kennedy, B. L., (2011). The importance of student and teacher interactions for disaffected middle school students: A grounded theory study of community day schools. *Urban Education, 46*(1), 4-33. https://doi.org/10.1177/0042085910377305

Nunez, I. (2015). Philosophical under labouring for mathematics education. *Journal of Critical Realism, 14*(2), 181-204. https://doi.org/10.1179/1476743015Z.00000000060

Sayer, A. (2010). *Method in Social Science* (Revised 2nd ed.). Routledge.

Siemon, D. E., Breed, M., Dole, S., Izard, J., & Virgona, J. (2006). *Scaffolding Numeracy in The Middle Years – Project Findings, Materials, and Resources, Final Report* submitted to Victorian Department of Education and Training and the Tasmanian Department of Education.

Siemon, D. Banks, N., & Prasad, S. (2019). Multiplicative thinking: A STEM foundation. In T. Barkatsas, N. Carr, & G. Cooper (Eds.), *STEM education: An emerging field of inquiry*. Brill Sense Publications.

State Government of Victoria. (2018). *Scaffolding numeracy in the middle years.* https://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/Pages/scaffoldnum.aspx

Stylianou, A., & Zembylas, M. (2019). Head teachers' spirituality and inclusive education: A perspective from critical realism. *International Journal of Inclusive Education, 23*(4), 419-435. https://doi.org/pqkg

Venkatesh, V., Brown, S.A., & Bala, H. (2013). Bridging the qualitative-quantitative divide: Guidelines for conducting mixed methods research in information systems. *MIS Quarterly, 37*(1), 21-54.

Young-Loveridge, J., & Mills, J. (2012). Deepening students' understanding of multiplication and division by exploring divisibility by nine. *Australian Mathematics Teacher, 68*(3), 15-20.