Analysis of Common Errors in Simplifying Algebraic Problems among Grade 10 Learners

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This paper analyses learners' common errors in simplifying algebraic problems. 102 Grade 10 learners from three rural schools in South Africa participated in the study. Following a quantitative approach, content analysis of learners' responses to algebraic tests revealed that while learners commit several errors in algebraic problems, encoding and transformation errors were the most prominent. These errors are mainly due to the inability to utilise correct methods, misapplication and overlooking algebraic rules, and failure to provide final answers to problems. The study highlights the need for remedial intervention, which has implications for improving mathematics teaching and learning.

Algebra is a discipline characterised by its abstract nature and governed by a set of rules, principles, and theories. However, errors and misconceptions arise when these rules, principles, and theories are disregarded or improperly implemented. In the South Africa's Further Education and Training phase (Grade 10 - 12), solving algebraic expressions and equations typically requires the understanding and integration of various mathematical concepts, including equations, factorisation, division, and exponents (Baidoo, 2019). As such, it is important to recognise that understanding algebra is beneficial for simplifying and solving mathematical problems in various subjects, including functions, calculus, trigonometry, financial mathematics, and others (Luneta & Makonye, 2010). This is because proficiency in algebra is essential for achieving success in higher-level mathematics.

Notwithstanding the South Africa National Diagnostic Report (2022) stating that algebra and equations make up 30% of the curricula for Grade 10, 25% for Grade 11, and 17% for Grade 12, learners exhibit deficiency in solving algebra. In fact, Grade 10-12 learners sometimes struggle to demonstrate fundamental and elementary algebraic skills that should have been gained in the early years of education, resulting in difficulties responding to algebraic problems (National Diagnostic Report, 2022). Similarly, researchers across several countries have reported that learners have challenges in comprehending the simplification of algebraic problems (Adeniji & Baker, 2023; Luneta & Makonye, 2010). This could be due to the requirement of performing multiple mathematical algorithms concurrently and utilising additional concepts such as exponents, factorisation, and division. The inability to remember algebraic rules or the inappropriate application of the rules forms the basis for encountering difficulties in solving algebraic problems, leading to several errors or misconceptions (Luneta & Makonye, 2010; Mamba, 2012; Pournara et al., 2016). The present study examines the causes of learners' errors in algebra and the possible solutions to avoid them. Specifically, this paper answers the following research question: *Which errors are the most prevalent when solving algebraic expressions among Grade 10 learners?*

Literature Review

The role mathematics plays in our everyday lives and in understanding other subjects of the curriculum cannot be overemphasised (Ndemo & Ndemo, 2018). Algebra is the bedrock of advanced mathematics, and it is related to all branches of mathematics (Department of Education, 2011; Moru & Mathunya, 2011). Agustyaningrum et al. (2018) stated that algebra

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 333–340). Canberra: MERGA.

consists of axiomatic theories that provide opportunities to consider various mathematical systems, whether special cases or not. In addition, Adeniji and Baker (2023) stated that solving algebraic equations plays a crucial role in developing learners' critical thinking and supporting learners' journey to higher education mathematics. Despite its significance, the performance of learners and the errors they make have raised concerns across the world (Ndemo & Ndemo, 2018). Several studies, such as Akhtar et al. (2020), Mncube (2016), and Ndemo and Ndemo (2018), revealed that learners' limited understanding of algebra often results in a negative attitude and poor performance. The National Senior Certificate (2022) diagnostic report also highlighted that South African learners still lack algebraic skills because many learners lack fundamental skills that should have been acquired in lower grades. Based on Newman's Error Analysis (NEA), this study is limited to the prominent types of errors identified by Makonye and Hantibi (2014), specifically, systematic errors, transformation errors, and encoding errors. First, systematic errors are those common errors that are being made by learners continuously for a long period, usually as a result of failing to understand specific concepts, rules, or operations (Mncube, 2016). Learners sometimes construct incorrect theories, assumptions, and misconceptions that they become attached to, and those theories and misconceptions interfere with learning (Mbewa 2013). For instance, Mulungye et al. (2016) argued that when most learners were given a task such as $(a + b)^2$ majority of the learner's answer was $a^2 + b^2$ instead of $a^2 + 2ab + b^2$. In this instance, learners fail to use the distributive property and the rules of exponents correctly.

Second, transformation errors occur due to the inability of learners to correctly choose the right mathematical solution methods during the process of solving, simplifying, or rearranging variables in algebraic problems (Abdullah et al., 2015). An example of transformation error is illustrated in a study by Moru and Mathunya (2022), where learners misapplied the rules of exponents. For instance, the learners were given $y^4 + y^4$, and the answer that was given by learners was y^8. The learners argued that they added the exponents because "when the bases are the same, we add the powers" (Moru & Mathunya, 2022, p.136), which means learners misunderstood the rule of adding algebra to that of multiplying exponents. Lastly, Abdullah et al. (2015) defined encoding errors as mismatch between the actual relationships in the problems, misusing mathematical symbols, confusing dependent and independent variables, mixing units and scales, and inability to express the final answer. Furthermore, Fitriani et al. (2018) added that encoding errors usually occur when learners are not used to writing conclusions in math problems. For instance, in a study conducted by Abdullah et al. (2015), learners were required to solve this problem: *studies show that* $\frac{5}{6}$ *of the students play sepak takraw.* $\frac{1}{2}$ *of the students who play sepak takraw also play badminton. If there are 132 pupils, what is the number of students who play sepak takraw and badminton?* The findings of the study showed that many learners got the answer 55 right, but they failed to provide a concluding statement that relates the answer to the problem context as 55 students played both sepak takraw and badminton.

Research Methodology

This study employed the positivist paradigm and quantitative research approach to analyse learners' errors in solving algebraic problems. According to Kaboub (2008, p. 343), positivist paradigm asserts that real events can be empirically observed and explained with logical analysis. For this study, data were collected from learners in three schools from the Amathole West district in the Eastern Cape province of South Africa. The schools were purposefully selected from the rural district, and Grade 10 mathematics learners served as the participants. Given that learners at the Further Education and Training (FET) phase in South African schools are at liberty to choose between mathematics and mathematical literacy, the researcher purposefully selected schools with higher mathematics enrolment. As such, the sample of mathematics learners in schools X, Y, and Z was 23, 23, and 56, respectively, making a total of 102 participants.

Data were gathered through a content-based test and an examination of the document released by the Department of Education. The content-based test enabled participants to make errors related to algebraic expressions. The structured questionnaire was employed to reveal the fundamental causes of the errors made in the written test. Based on the researchers' understanding and experience, a protocol detailing descriptions of the three types of errors committed in algebraic problems was created. All the instruments for data collection were validated by two Professors of mathematics education.

Data Analysis

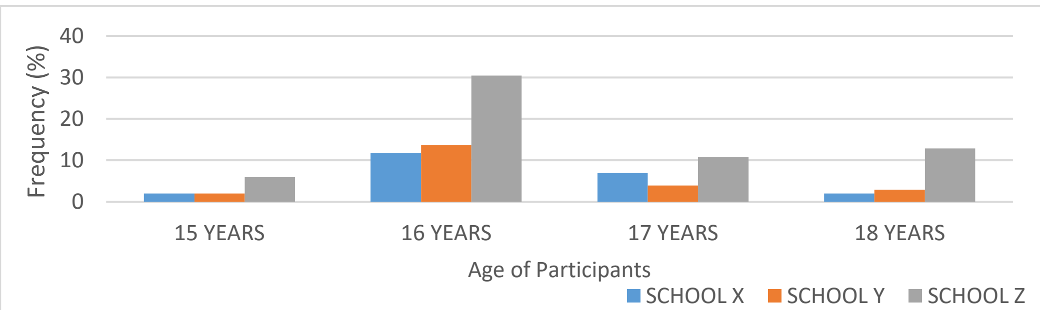
A quantitative approach to content analysis was employed to answer the research question. The use of content analysis in this study provides opportunities to systematically analyse and interpret learners' responses to algebraic problems (Rourke & Anderson, 2004). It can be utilised to discern similarities and contrasts, patterns, and relationships. Content analysis was employed for the present study because it allows the researchers to handle large samples and draw more robust conclusions, enhancing the external validity of analysing learners' errors in solving algebraic problems. Additionally, descriptive statistics provide information about the distribution, relationship between variables, and frequency variability (Cohen, Manion & Morrison, 2017). The statistical analysis involves calculating the frequencies and percentages of correctly answered questions and identifying common errors among the samples.

Results and Discussion of Findings

First, participants' characteristics in terms of gender and age from the three schools are presented. In this context, the majority of the learners are female, with 63 (62%) participants being girls and 39 (38%) being boys. Specifically, the breakdown of female participants in each of the three schools was as follows: School X 56.5%; School Y 60.9%; and School Z 64.3%, which aligns with the general population breakdown in our societies. Additionally, Figure 1 illustrates the age distribution of the participants from each participating school. According to the South African Schools Act, 84 of 1996, the statistical age norm per grade is the grade number plus 6. As such, the grade 10 learners' average age is expected to be 10+6 = 16 years. Accordingly, in this study, a total of 9.9% of the participants are aged 15 years in all three schools, while the majority of the participants fall within the appropriate age of 16 years old (55.9%). About 34.4% of the participants were over the required age, with 21.6% being aged 17 years and 12.8% being 18 years, which is a stipulated schooling age (South African Schools Act 84 of 1996, Amended 1998).

Figures 1

Age Distribution of Participants per School



The analysis of the content-based test was presented below based on the types of errors committed.

Systematic Errors in Algebra

The content-based test consisted of three questions, each testing a type of error. For example, Question 1, which has five sub-questions (1.1-1.5), tests systematic errors. Participants were allowed to provide solutions to the algebraic problems on paper provided for them. The participants illustrated various approaches below based on a variety of questions.

Question 1: Simplify 2(x – 2y) (x2 + xy + y2)

For this question, learners are expected to solve algebraic expressions by first understanding the basic components of algebra, such as variables, constants, and coefficients. Then, simplify the expression by eliminating the parentheses using the distributive property, applying exponent rules, combining like terms, applying the order of operations, and factorisation. All these align with the South Africa Department of Education's (2011. p.59) expectations of Grade 10 learners.

Table 1 revealed that learners in schools X, Y, and Z performed well (i.e., 100%) on question 1.1. However, on question 1.2. Only learners in schools X and Z have responses that are error-free at 100%, while school Y performed at 60.86%, depicting that 39.14% of the responses have errors. Table 1 further revealed that 41.17% errors were committed across all three schools with question 1.5 posing a greater challenge to the majority of the learners. From the research findings, the majority of the learners employed an inappropriate use of the distributive property. Similarly, some of the learners misapply properties, neglect negative signs, apply signs in one direction, and fail to combine like terms properly, which often leads to incorrect solutions. Hence, the primary reasons learners struggle with algebra are inadequate conceptual understanding and sometimes inattention. This finding is similar to those of Mulungye, Miheso, and Ndethiu (2016), who iterate that some systematic errors that are made by learners in algebra include the inappropriate use of the distributive property, which states that a (b + c) = ab + ac.

Table 1

Comparison of Learners' Responses and Systematic Errors Committed in Answering Question 1

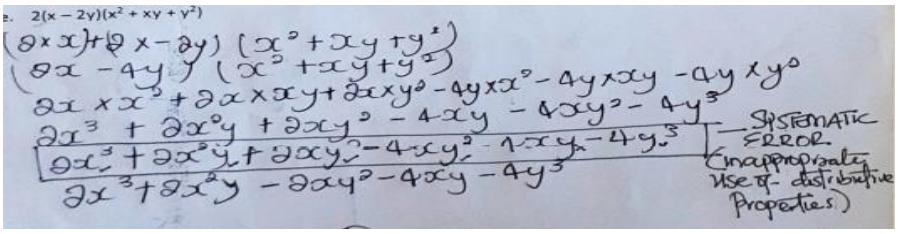
Question No.	School X		School Y		School Z		Total (committed errors)	
	Freq	%	Freq	%	Freq	%	Freq	%
1.1	23	100.00	23	100.00	56	100.00	00	0.00
1.2	23	100.00	14	60.86	56	100.00	09	8.82
1.3	21	91.30	19	82.60	51	91.07	11	10.78
1.4	19	82.60	10	43.47	38	67.85	35	34.31
1.5	22	95.65	11	47.82	27	48.21	42	41.17
Ave. %		93.91		66.95		81.42		

Figure 2 illustrates a sample of learners' responses showing the error committed in question 1. This response shows a systematic error in line 5 where the learner forgets to appropriately distribute -4y across the terms. Even though the steps were clearly shown, many learners commit this error, neglecting some terms when applying the distributive property, leading to systematic error.

Analysis revealed that when some learners were given a positive smaller number to be subtracted from a negative bigger number or vice versa, they consequently ignored or failed to apply the rule that states that the answer takes the sign of the bigger number. Furthermore, errors committed were based on a faulty line of thinking, which is likely to recur since they were built on false ideas. This resonates with the argument of Mncube (2016), who argued that systematic errors are the result of learners' failing to understand concepts or rules. As such, Mbewa (2013) stated that learners construct the rules and misconceptions that they become attached to, and those rules and misconceptions interfere with learning. Systematic errors in algebraic expression are common among many mathematics learners because, in many schools, algebra is often taught as a series of procedural steps rather than a coherent system built on logical principles. As such, learners frequently memorise algorithms without grasping the underlying concepts, leading to systematic errors when they encounter problems that require flexible thinking. For many learners, algebraic symbols are abstract and lack meaning, making it challenging to view equations as representations of real-world situations (Mbewa, 2013; Mulungye et al., 2016; Biney et al., 2023). Similarly, learners often face difficulties with the transition from arithmetic to algebra, a shift that necessitates a more abstract way of thinking. The ability to manipulate variables and understand their relationships is complex and can be overwhelming for learners who have not fully internalised basic mathematical principles. This lack of understanding often manifests as errors, particularly when learners are asked to simplify expressions or solve equations that deviate from familiar patterns.

Figure 2

Sample of Response to Question 1



Transformative Error in Algebra

Question 2 aims to identify and analyse transformative errors in algebraic products involving various operations. When faced with such questions, the algebraic product may involve terms that contain variables and constants and may require the application of the distributive property, factoring, and simplifying. Thus, question 2, which comprises of algebraic product, has five questions (2.1 – 2.5). Analysis indicated that participants illustrated various approaches to solve this kind of question; however, below is a sample of common errors, identified as transformative errors.

Question 2: Simplify $(-2y^2 - 4y + 11)(5y - 12)$

Figure 3

Sample of Response to Question 2

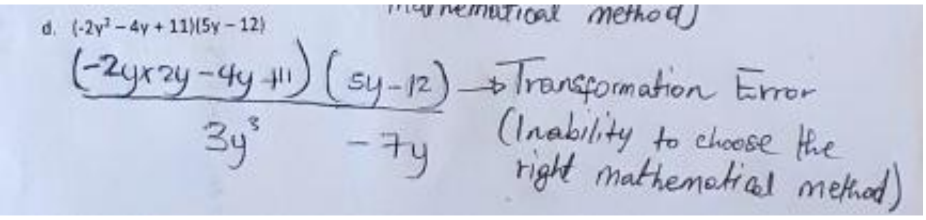


Table 2 revealed that only school Y performed well (i.e., 100%) in question 2.1. However, learners from all three schools (X, Y, and Z) struggled with questions 2.3 – 2.5 with varying causes of transformation error. Based on the average percentages presented in Table 2 below, all three schools managed to perform a little above 50%. For learners to be able to solve algebraic products, they are expected to develop a clear understanding of algebraic concepts, the arithmetic and algebraic rules, and how to apply these to simplify or manipulate the expression.

However, many of the learners across the three schools struggle with questions 2.1-2.5 in general as a result of their inability to choose the right solution methods. Table 2 further revealed a 70.58% committed errors across all three schools with question 2.4 posing a greater challenge to the majority of the learners. From the research findings, many learners seem not to fully grasp the properties of arithmetic and algebra, while some learners over-generalise specific strategies or rules, neglecting the nuances of different algebraic expressions.

Table 2

Comparison of Learners' Responses and Transformative Errors Committed in Answering Question 2

Question No.	School X		School Y		School Z		Total (committed errors)	
	Freq	%	Freq	%	Freq	%	Freq	%
2.1	16	69.56	23	100.00	44	78.57	19	18.62
2.2	16	69.56	13	56.52	39	69.64	34	33.33
2.3	13	56.52	09	39.13	28	50.00	52	50.98
2.4	12	52.17	06	26.08	12	21.42	72	70.58
2.5	09	39.13	08	34.78	32	57.14	53	51.96
Ave. %		57.38		51.30		55.35		

Furthermore, findings revealed that even when learners already understand the needs of the question, they still fail to choose appropriate mathematical operations. For instance, when a learner mishandles arithmetic rules, a multiplication sign is taken as an addition sign and vice versa or confused about whether to apply addition or multiplication when dealing with exponents which leads to transformative errors. In addition, most of the learners fail to observe the rules for their ordering, which normally goes by the acronym BODMAS, meaning “Bracket, Off, Division, Multiplication, Addition, and Subtraction”. This finding corresponds with those of Abidin and Ali (2015), who argued that learners make transformation errors when they already understand the needs of the question but still fail to choose the correct mathematical operations that are involved. For example, in a study conducted by Moru and Mathunya (2022), learners were given $y^4 + y^4$ and the answer that was given by learners was y^8. According to the authors, learners argued they added the exponents because “when the bases are the same, we add the powers,” which means learners mistook the rule of adding algebra to that of multiplying exponents. As such, Abdullah, Abidin, and Ali (2015) argued that when learners lack the ability to choose the right mathematical solution method when solving algebraic expressions, they often commit transformative errors.

Encoding Error in Algebra

Question 3 focuses on algebraic products involving several variables and coefficients. It consists of five questions (3.1-3.5). The participants illustrated various approaches, and below is a sample of common errors made based on a variety of questions given.

Question 3: Simplify $(3a - 5b)(3a + 5b)(a^2 + ab - b^2)$

Figure 4

Sample of response to Question 3

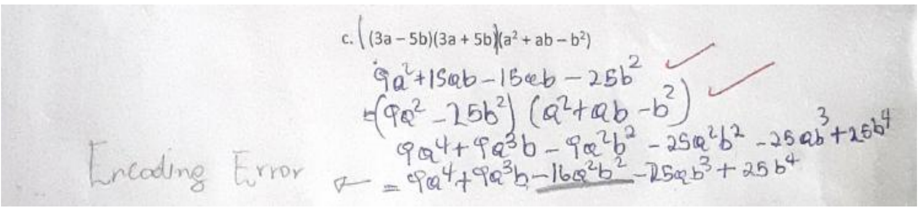


Table 3 revealed that none of the schools had 100% error-free responses in all five questions asked (i.e., from questions 3.1-3.5). Based on the average percentages presented in Table 3 , all three schools managed to perform a little above 50% (in all the five questions), which indicates that learners from all the sampled schools struggled with algebraic expressions involving various variables and coefficients. Table 3 further revealed a 66.66% committed errors across all three schools with question 3.3 posing a greater challenge to the majority of the learners. From the research findings, the errors majority of the learners encounter stem from a misunderstanding of symbols where learners confuse similar-looking symbols or use incorrect symbols, ambiguity in notations, and complexity of expressions in producing final answers.

In South African schools, learners are typically taught how to simplify algebraic expressions with multiple variables and coefficients through a structured and methodical approach. They were also taught how to factorise third-degree polynomials (Department of Education, 2011. P. 49).

Table 3

Comparison of Learners' Responses and Encoding Errors Committed in Answering Question 3

Question No.	School X		School Y		School Z		Total (committed errors)	
	Freq	%	Freq	%	Freq	%	Freq	%
3.1	12	52.17	22	95.65	38	67.85	30	29.41
3.2	16	69.56	13	56.52	51	91.07	22	21.56
3.3	13	56.52	07	30.43	14	25.00	68	66.66
3.4	09	39.13	12	52.17	27	48.21	54	52.94
3.5	11	47.82	09	39.13	19	33.92	63	61.76
Average %		53.04		54.78		53.21		

However, most of the learners struggled with these types of questions because of their inability to express the final answer. Even when learners can solve problems following the appropriate mathematical steps, they still fail to produce the correct answer. This finding is in line with Abdullah et al. (2015), who argued that encoding errors occur when learners can solve problems following the right mathematical steps but still fail to produce the right answer. Furthermore, the study findings also revealed that some learners see the final answer as incomplete because it has a plus or minus or more than one term. This finding corroborates those of Bayos (2020) and Fitriani et al. (2018), who assert that encoding errors usually occur because learners are not used to writing conclusions in mathematics problems. Hence, researchers (e.g., Baidoo, 2019; Khalo et al., 2022; Delastri & Lolang, 2023; Fumador & Agyei, 2018; and Iddrisu et al., 2017) urged that mathematics teachers should identify all the errors made by learners when solving algebraic questions and use this to enhance their teaching focus in algebra lessons.

Overall, this study reveals that learners often commit more transformation and encoding errors than systematic errors. Specifically, findings indicated that 19.01% of the learners made systematic mistakes, 45.09% made transformation errors, and 46.47% made encoding errors. This suggests that learners frequently make transformation and encoding mistakes due to the complexity of the cognitive processes involved in simplifying algebra. These results call for remedial intervention targeting the common errors identified.

Conclusion

The findings in this paper highlight that, despite algebra being a foundation for several other topics in mathematics, many Grade 10 learners commit various errors when solving algebraic problems. Two of the prominent errors identified are encoding and transformation errors, largely caused by the selection of inappropriate methods, confusion in applying algebraic rules and manipulating symbols, inaccurate representation of symbols, and unclear answers. The study highlights specific difficulties learners encounter when solving complex algebraic problems involving multiple steps. The findings of this study extend existing results from the literature and contribute to the need to improve learners' understanding of algebra to meet the expectations of the South African Department of Education. It is therefore recommended that mathematics teachers should pay close attention to the errors commonly committed in algebra and encourage constant practice, double checking operations when applying distributive law, factoring and isolating variables, appropriately breaking down complex problems by taking a step at a time, and reviewing and cross-checking steps leading to final answer if improved learning outcomes in algebra are desired.

Acknowledgements

This study was approved by the University of Fort Hare Ethics Committee with an approval number MNC021SOLA01.

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