Affordances of a Novel Mathematical Manipulative: Teacher Perspectives on the Keyboard

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This study explores two Year 2 teachers' perceptions of the affordances of a novel mathematical manipulative, the Keyboard, based on their observations of its use in the classroom. Analysis of the semi-structured interviews indicated that the Keyboard's linear structure, design features, and hands-on nature promoted flexible number strategies, relational reasoning, and student engagement. It was viewed as a valuable complement to other tools, with potential applications extending beyond its initial design. These findings contribute to research on the role of specific manipulatives in primary mathematics education and highlight implications for instructional practice.

Mathematical manipulatives can be defined as "concrete objects that can be used to help students understand and solve mathematics problems" (Bouck & Park, 2018, p. 66). Hattie et al. (2017) argued that using physical manipulatives alongside visual, symbolic, verbal, and contextual representations can facilitate deep learning by enabling students to "make concepts concrete and visible, look for patterns, make connections, and form generalizations" (p. 170). Indeed, there is evidence to suggest that incorporating mathematical manipulatives into the teaching and learning of primary mathematics can positively impact student achievement (Byrne et al., 2023), although the magnitude of such effects is contingent on the specific ways in which manipulatives are implemented, including:

- *when* (length of instructional time, with shorter and medium durations yielding greater effects than extended periods),
- *how* (level of instructional guidance, with high guidance proving more effective),
- *who* (students' developmental stage, with greater effects observed for those at the concrete operational stage),
- *what* (mathematical topic, with stronger effects for fractions compared to arithmetic or algebra), and
- *which* (type of manipulative, with neutral designs—e.g., plain wooden blocks—being more effective than those resembling real-world objects, such as plastic money or pizzas) (Carbonneau et al., 2013).

In terms of research into teacher perspectives on mathematical manipulatives, teachers commonly believe that manipulatives improve student understanding, provide multisensory learning opportunities, and make abstract ideas more concrete (Marshall & Swan, 2008; Quigley, 2021). They are also widely regarded as a means to enhance engagement and enjoyment in mathematics (Tan, 2020; Quigley, 2021). However, manipulatives alone are not necessarily sufficient to improve students' mathematical understanding, as their effectiveness depends on how they are used (Marshall & Swan, 2008). Additionally, studies have identified several constraints on their use, including limited resource availability, the time required for setup and organisation, and teachers' confidence and pedagogical knowledge in integrating them effectively (Tan, 2020; Marshall & Swan, 2008).

We note that in previous studies into teacher beliefs about mathematical manipulatives, it is rare that specific manipulatives are named. Consequently, the reader is left wondering which

(2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 397–404). Canberra: MERGA.

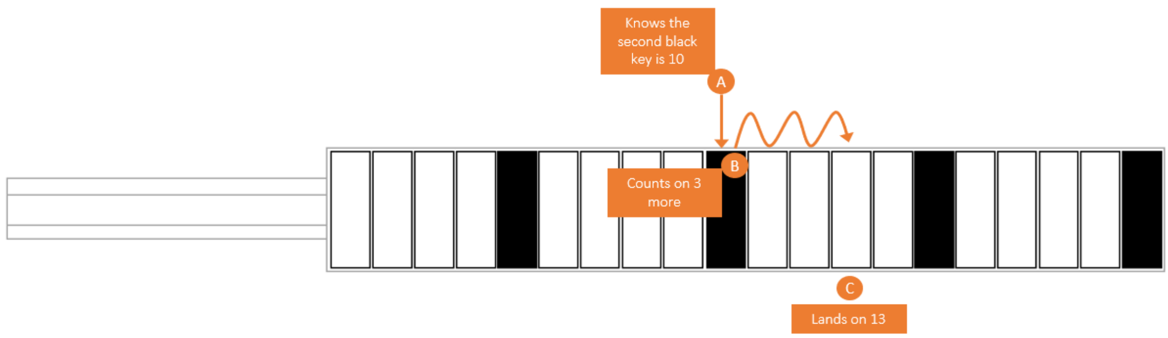
manipulatives, in which grade level, and for which content, did the teachers have in mind as they expressed their views. To address this gap, we conducted a small-scale teaching experiment in two Year 2 classrooms, introducing a novel manipulative (the Keyboard) and gathering observational insights from classroom teachers. The research examined: *What are the perceived affordances and constraints of a novel mathematical manipulative (the Keyboard) as identified by Year 2 classroom teachers?*

Design and Intended Affordances of the Keyboard

The Keyboard is a newly developed manipulative designed to support students in engaging with number relationships through visual-spatial reasoning. It consists of a handle and a linear board with 20 evenly spaced keys, each representing a number from 1 to 20, but without numerical labels. To aid navigation, every fifth key is black, providing subitising cues that may help students structure their thinking around landmark numbers (e.g., 5, 10, 15, 20). The absence of written numerals ensures that locating numbers requires students to use relational strategies rather than passive number recognition. The design further supports iterative structuring, as multiple Keyboards can be linked together by sliding the handle into the end of another Keyboard thereby increasing the number of keys. For example, joining four keyboards creates a set of 80 keys and maintains a scaled subitising framework where students can see four sets of 20, four black keys per Keyboard, and four white keys between each black key. This layered repetition has the potential to encourage flexible numerical reasoning and deepen students' understanding of number magnitude and structure (Hopkins et al., 2023).

Figure 1

A Student Locates 13 on the Keyboard



Preliminary findings from Hopkins et al. (2025) provide the first empirical evidence that the Keyboard may support students in transitioning from counting-based strategies to retrieval-based strategies when solving single-digit addition problems. That study involved a one-on-one intervention with a Year 4 student who predominantly relied on counting strategies, using microgenetic methods to track changes in strategy use across 20 sessions. Structured activities and games using the Keyboard were introduced across seven consecutive sessions, with minimal explicit instruction to allow for intuitive exploration. Analysis of trial-by-trial strategy data revealed a shift toward retrieval-based strategies, including both derived-fact strategies (e.g., knowing that 8 + 5 is the same as 8 + 2 + 3, and that 8 + 2 = 10, with 3 more giving 13) and direct retrieval (e.g., just knowing that 8 + 5 = 13). Notably, once the student no longer had access to the Keyboard, they did not revert to counting strategies, suggesting that the Keyboard may support the internalisation of more efficient mental computation strategies.

Beyond its potential to support retrieval-based strategies (Hopkins et al., 2025), the Keyboard may also foster broader aspects of number sense and relational reasoning. Because it emphasises positional relationships over explicit quantity-based grouping, students must engage in spatially driven problem-solving when determining numerical values. For instance, to locate 13 (see Figure 1), students might first find the second black key (10) and count on three—or simply move forward three by subitising—or they might locate the third black key (15) and count back two. These actions can strengthen their understanding of numbers in relation to each other. This positional thinking could also support reasoning about numerical proximity, where students can directly compare magnitudes by placing small cubes beneath the Keyboard to visualise which number is closer to a reference point (e.g., "Which is closer to 16? 14 or 19?"). Additionally, by linking multiple Keyboards together, students may notice recurring positional patterns, such as how 7, 27, 47, and 67 always appear two keys after the first black key on each Keyboard. These features align with Siegler and Ramani's (2009) findings that linear number board games significantly enhance young children's understanding of numerical magnitude. Their research suggests that structured spatial representations, such as linear board games, help children develop a clearer sense of numerical order and relative magnitude. Similarly, the Keyboard's linear format may provide a useful spatial framework for reinforcing numerical relationships, as students must actively locate, compare, and navigate numbers through spatial reasoning rather than relying on symbolic recognition alone.

Compared to other mathematical tools, the Keyboard shares similarities with number lines and structured counting frameworks but offers unique advantages. Like bead strings, it provides a linear representation of number relationships, but unlike bead strings, it emphasises positional relationships rather than counting in groups. Similarly, while tens frames support subitising, they structure numbers in base-ten arrangements, whereas the Keyboard encourages iterative subitising across a larger scale, especially when multiple Keyboards are linked. Moreover, unlike bead frames and tens frames, the Keyboard allows multiple numbers to be represented simultaneously, enabling students to reason about relative distance and numerical difference. Perhaps most significantly, its lack of numerical labels sets it apart from number lines and number charts, as students must construct numerical meaning through spatial reasoning and relational thinking, rather than simply identifying pre-written values.

While Hopkins et al. (2025) provide promising early evidence of the Keyboard's instructional potential through a one-on-one intervention study, the current paper represents the first classroom-based investigation of the Keyboard's use, focusing specifically on teachers' perceptions of its affordances and constraints.

Method

Study Design and Participants

This study is best characterised as a teaching experiment (Cobb, 2000), in which the researchers introduced a novel mathematical manipulative (the Keyboard) into two Year 2 classrooms through a structured, two-week (eight-lesson) teaching sequence. The aim was to investigate how classroom teachers perceived the affordances and constraints of the Keyboard, based on their observations of the lessons. Giovanni (Class A) had eight years of teaching experience, exclusively in Years 2 and 3, and was more familiar with using manipulatives for developing early number sense, addition, and subtraction. Michael (Class B) had taught for seven years, primarily in Years 3 and 6, and was teaching Year 2 for the first time, with less experience using manipulatives designed for early number learning.

Semi-structured interviews were conducted at two time points and transcribed for analysis. Giovanni's interviews captured his evolving perspectives across all eight lessons observing the Keyboard in use, while Michael's interviews contrasted his observations of lessons involving a bead frame in the first four sessions with those involving the Keyboard in the subsequent four sessions, allowing for a direct comparison of the perceived affordances of each manipulative. Data were analysed using thematic analysis, with themes generated inductively from participant responses. The analysis prioritised capturing teachers' perspectives in their own terms, ensuring that key insights emerged naturally from the data.

Throughout the lessons, both teachers were positioned in an observational role rather than leading instruction. This allowed them to focus on noticing how students interacted with the Keyboard and how instruction was structured to support learning. Stepping out of an active teaching role can help teachers attend more closely to student thinking and pedagogical decisions that might otherwise be overshadowed by classroom management demands (Clarke et al., 2013). By observing rather than directing instruction, teachers were able to form reflections based on classroom observations and interactions, which informed their insights during interviews. Finally, we note that roles were deliberately delineated in this study: the lesson sequence was collaboratively planned by the first and fourth authors, but all teaching was led solely by the first author; the Keyboard was designed by the second author; and the interviews with the observing teachers were conducted by the third author. This research was approved by the Monash University Human Research Ethics Committee (Project ID: 36277). All participants provided informed consent prior to their involvement in the study.

Lesson Structure

The broader teaching experiment consisted of eight 45-minute mathematics lessons, with the same lesson content delivered to both classes. Each lesson followed a three-part sequence:

1. Familiarisation with the manipulative (5–10 minutes): Students engaged in whole-class activities designed to help them develop familiarity with the structure of the manipulative. For example, they might be asked, "Show me 17 on the Keyboard. How did you find it? Is there another way you know it's 17?" These tasks encouraged students to develop a sense of its spatial and numerical organisation, recognising key patterns and relationships that underpin its design.
2. Mathematical game (20–30 minutes): Students played a structured game where the manipulative was used as either a game object or a thinking tool. Class A used the Keyboard in all lessons, while Class B used a bead frame for the first four lessons before transitioning to the Keyboard for the final four lessons. An example game was a modified version of Nearest to the Gnarly Number (Russo, 2017), where students aimed to get as close as possible to a target number using the available cards.
3. Mathematical investigation (10–15 minutes): The game was extended into a problem-solving task requiring deeper reasoning. For instance, students might be asked, "If the Gnarly Number was 19, and my opponent's total was 16, what might my total have been if I had won? Show me on the Keyboard (bead frame) and explain your reasoning". These investigations encouraged students to engage in strategic thinking and numerical comparisons while also deepening their understanding of the manipulative's structure.

Findings

Our thematic analysis of the interview transcripts resulted in the identification of four interrelated themes that described specific affordances of the Keyboard. These are unpacked below, with quotations from the two teacher-participants used to illustrate each theme. While the interview prompts were open to both affordances and constraints, the two teachers did not explicitly raise any limitations of the Keyboard in their reflections.

Linear Representation of Number

The linear representation of number was perceived as a particular strength of the Keyboard, especially in contrast with the bead frame. Michael noted the similarities with a number line, as students could track increasing magnitude along one dimension.

> One problem with the bead frame is – well… One strength the keyboard has, it's just a number line… You don't have to worry about it going down to the next line and then losing count.

Michael also noted that, when using the Keyboard, students were less inclined to adopt inefficient counting-based strategies, and more inclined to work from known benchmark numbers and 'trusting the count', again compared with the bead frame:

> I think there was a bit less of kids counting. When we used the bead frames… there is a bit of kids still counting the beads on the row. Even if it was ten, they would go one, two, three… They weren't trusting the count. Whereas because this was broken down more, it was easier for them to use. I played with the same student with the keyboard and she was able to go five… six, seven, eight...

This finding is consistent with research suggesting that linear number tracks are superior for developing early number sense, in particular, an appreciation of number magnitude, because they better map onto the desired mental representation (Siegler & Ramani, 2009). However, the Keyboard is distinct from a number line in that it did not contain number labels. Giovanni noted that this design feature served to increase student cognitive engagement in the associated activities, as students had to bring to mind the relevant number relationships:

> I would be using a number line… But the numbers would be visible for the kids to see to help them understand where on that line that they are. With this, they had to use their memory, what they know, their knowledge and really visualise what position everything is, based on the pattern.

Integrated Instruction of Important Number Concepts

Beyond its linear design, teachers emphasised that the Keyboard facilitated effective instruction by allowing multiple mathematical concepts to be introduced in an integrated manner. As Giovanni stated:

> I thought they were great, because in one lesson, just using the keyboards, he (first author) was able to cover a number of different strategies, not just one focus. Just by introducing the keyboard, he was able to show how to add on, how to take away, having two more and two less, seeing patterns.

Through continued exposure to the Keyboard, Giovanni noted that students' "concept of less than and more than has naturally just evolved", noting that the Keyboard supported students in reasoning about numerical proximity and comparing magnitudes:

> So they would be able to say, not just by looking at it, but go, "Oh. she's closer because the gnarly number is here, or she's two away and I am one away, which means I am closer."

Additionally, teachers noted that the Keyboard was not necessarily a replacement for other manipulatives but could be used in conjunction with them to enhance understanding. Michael observed that different manipulatives provide unique affordances, and the Keyboard could complement other tools, particularly in supporting addition and subtraction strategies:

> Being able to do jumping strategy for addition and subtraction, that sort of thing… it's not either/or - it's in conjunction with a tool like the bead frame. (Michael)

Although not the focus of the lessons demonstrated during the project, Michael also noted that the keyboard has the potential to support student exploration of multiplicative ideas:

> It also is fantastic for doing… multiplicative thinking or splitting things into groups of five. Dividing, doing that sort of thing. I think it has a lot of utility.

Given that initial evidence suggests the Keyboard may assist some students in shifting from counting-based to retrieval-based strategies when solving single-digit addition problems (Hopkins et al., 2025)—a focus distinct from the current classroom-based study—its potential for supporting learning across a range of number concepts is particularly noteworthy.

Flexibility and Thinking Relationally

The design features of the Keyboard, in particular the fact that every fifth key is black, provided opportunities for students to think flexibly and relationally about number. Indeed, students' ability to notice and internalise the Keyboard's structure in a relatively short period of time was surprising to Giovanni:

> So, for them to be able to see the pattern of every black key in increments of five, I thought was going to be a challenge and for them to understand "Okay, what's two more than a black key?", I thought the wording for that example would have been tricky for them but I think they were able to process black keys of five. They can count by fives. One more after five must be six or eleven or so on... They were developing strategies that they didn't even realise they could do… Because, they were making a connection to the piano keys.

Students were able to identify a given number as being embedded in a rich set of relationships through their internalisation of the structure of the Keyboard. This enabled them to rapidly identify numbers despite there being no number labels. As Giovanni noted:

> I feel like they could clearly see the numbers between one and twenty. It wasn't just black-and-white now, they were actually seeing clear numbers in spots where there were no numbers. So, if they're on like, "Okay, you've picked up ten and seven, that's seventeen. Where would that be?" They would, without really thinking too much, go straight to the further end of the keyboard and place the counter past the black, and I am like, "That's great."

On occasion, students went beyond the physical keyboard to estimate the position of numbers beyond 20:

> Some kids looked at their keyboard… and they said "If we were going to add these two numbers, it would be" and they pointed at the keyboard but past the twenty mark on the keyboard saying "It would be roughly here". (Giovanni)

Building on from this, both teachers noted that an additional strength of the Keyboard as a tool was that numbers beyond 20 could be explored by connecting multiple Keyboards:

> I love that it can be kept as just twenty and you can make them bigger as they need to. (Michael)
>
> I loved it because, again start off real basic, introduce it on the first day and then just expand on their knowledge on the second day. As soon as you connected it, they were mind-blown… And then you go in further and add a third, and then their heads exploded. (Giovanni)

Sustained Student Engagement

Finally, the Keyboard was viewed as sustaining student engagement through allowing for 'hands-on learning', but in a manner that minimised student distraction. Giovanni indicated that the students appreciated being able to hold the mathematics in their hands:

> It was very engaging the fact that it was very tactile. The kids could hold it and play with it and explore with it in different ways. It kept them engaged the whole time.

In addition to promoting general engagement, the Keyboard appeared to be particularly beneficial for certain students who struggled with other manipulatives. Michael noted that some students found the Keyboard significantly easier to use than the bead frame, which in turn improved their confidence:

> I think seeing one student in particular who is typically low… seeing how much easier he found using the keyboard… He found it so, so much easier compared to the bead frame. Just seeing him play the games, he was so much more confident in using them.

This suggests that for some students, the structure of the Keyboard provided an entry point into mathematical thinking that other tools may not have afforded as effectively. Interestingly, Michael also noted that there may be design advantages associated with the keyboard compared with other manipulatives that have more moving parts, such as bead frames, as its simplicity and static nature supported students to focus on the mathematics:

> They work a little bit better because there's less moving parts and there is less to fidget with and I think with the bead frames, there is a bit of doing a back and forth, shaking them, tipping them. It's not the end of the world, but just one less moving part, there is one less thing for kids to fidget with. It's just a good thing for attention, I think. Just having to be something that's static, but you can

> make it dynamic with counters, is a good thing because for kids who can't focus on too many things at once, it's good.

Collectively, these findings imply that the Keyboard may be particularly effective in sustaining student focus while facilitating structured, hands-on mathematical exploration.

Finally, beyond discussing its affordances, it is worth emphasising that both Giovanni and Michael were enthusiastic about the possibility of utilising the Keyboard to support future mathematics instruction in their own classrooms going forward, if given the opportunity:

> If we have those in the classroom, I would definitely use them. (Giovanni)

> I really love the idea of them. I would like to spend more time using them. I would be really interested to see where else it could be used. (Michael)

Discussion and Conclusion

These findings contribute to existing research on mathematical manipulatives in several ways. First, they reinforce the view that teachers perceive manipulatives as having distinct, tool-specific affordances, reinforcing the need for research that investigates teacher perceptions of particular tools rather than manipulatives in general. The results demonstrate that not all manipulatives are interchangeable; rather, their design features shape how students interact with mathematical concepts. Second, the Keyboard's linear structure aligns with prior research indicating that structured linear spatial representations enhance number sense (Siegler & Ramani, 2009). Siegler and Ramani's work specifically found that linear number tracks are more effective than circular ones for helping young children develop a clearer understanding of numerical magnitude. The Keyboard provides a similarly structured, linear representation of number, reinforcing magnitude relationships in a way that reduces reliance on inefficient counting strategies. Additionally, its lack of numerical labels required students to engage more deeply with number relationships rather than relying on rote recognition of pre-written numerals. Teachers observed that students internalised the Keyboard's repeating structure, recognising numbers based on their position relative to black keys and other reference points, rather than through simple recall. This suggests that both its structured, linear format and the absence of numerical labels contributed to students' ability to reason flexibly about number relationships. Third, several additional hypothesised benefits of the Keyboard, particularly its support for relational reasoning and flexible number strategies, were evident in teachers' reflections. This aligns with prior research on how manipulatives can enhance conceptual understanding of key number concepts, such as reasoning about numerical relationships (Marshall & Swan, 2008; Quigley, 2021). Additionally, the study provides further evidence that manipulatives with mathematically purposeful yet simple designs may be preferable, as suggested by Carbonneau et al. (2013). Teachers noted that the Keyboard's structured but uncluttered design helped students focus on number relationships rather than being distracted by moving parts. Finally, teachers perceived the Keyboard as enhancing engagement and enjoyment in mathematics learning (Tan, 2020; Quigley, 2021), particularly through the structured learning experiences that accompanied its use. The way it was introduced—through whole-class familiarisation, mathematical games, and problem-solving investigations—appeared to sustain student interest while fostering deeper thinking about numerical relationships. Findings also reinforce the idea that a manipulative's effectiveness depends not only on its design but also on how it is embedded within instruction (Marshall & Swan, 2008). The Keyboard's lack of numerical labels further contributed to this engagement, as students were required to construct meaning through active reasoning rather than passive identification of printed numbers. This reasoning-based interaction with the Keyboard may have also supported students' enjoyment, as teachers observed students sustaining interest and curiosity while identifying and reasoning about numbers in relation to the Keyboard's structure.

While this teaching experiment provided valuable insights into how teachers perceive the Keyboard's affordances, several limitations must be acknowledged. First, notably, neither teacher explicitly identified constraints associated with the Keyboard. While this may reflect their genuine enthusiasm, it may also reflect the structure of the lessons or the framing of the interviews. Future research could explore potential limitations more systematically. Second, the study was small in scale, involving only two teachers from a single school. Although their perspectives offer depth, broader generalisability requires further research across different school contexts and with a larger sample of educators. Third, our study focused on teacher perspectives rather than student learning experiences, meaning further research is needed to examine how students engage with the Keyboard and how it influences their mathematical reasoning over time.

Looking ahead, teachers also identified potential for the Keyboard to support multiplicative thinking, which extends beyond its original design focus on number sense and additive reasoning. This raises interesting possibilities for future research into how the Keyboard might support students' understanding of a broader range of mathematical concepts. Taken together, these findings highlight how the Keyboard, when embedded within a carefully structured lesson sequence, can support teachers to perceive new opportunities for fostering students' understanding of number relationships and mathematical reasoning.

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