Exploring the Complexity of Students' Inconsistent Relational Understanding of the Equal Sign

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Recently, the phenomenon of students holding simultaneous operational and relational understanding (SOR) of the equal sign has garnered increasing attention from researchers. Investigating this is crucial, as it helps in supporting students' transition toward a full relational understanding. This study utilised Rasch analysis and student interviews to delve deeper into the SOR stage, involving 1008 Chinese Grade 1 students. The findings validated the SOR stage and uncovered nuanced difficulties, barriers, and contributing factors underlying students' inconsistent reasoning about the equal sign.

While students' knowledge of the equal sign is a well-researched topic, new insights continue to emerge. One of these insights is the investigation of students' unstable understanding between a relational perspective (viewing the equal sign as a symbol indicating equivalence between two sides) and an operational perspective (interpreting the equal sign as a symbol denoting the calculated result). Researchers focus on the students' coexistence of relational and operational conceptions of the equal sign (e.g., Blanton et al., 2018; Lee & Pang, 2023), as well as the regression from relational understanding back to operational understanding (Molina & Ambrose, 2008; Sun et al., 2023). These findings revealed the areas that require attention during the transition as students develop their operational conception into a relational understanding. To better support students in grasping a full relational understanding of the equal sign, their conceptual difficulties, barriers, and shifts that emerged during the transition period need to be clearly examined and addressed (Lee & Pang, 2023; Sofuoğlu, 2024). However, the literature in this field, particularly regarding students' simultaneous operational and relational understanding (SOR) of the equal sign, still remains under-researched (Lee & Pang, 2023). This study utilises a sample of Chinese students in Grade 1 to further investigate the phenomenon of the coexistence of two understandings, aiming identify the specific difficulties and barriers students encounter in developing a robust relational understanding of equal sign.

Literature Review

In a seminal work, Matthews et al. (2012) proposed four developmental stages of knowledge of the equal sign: 1) *rigid operational*, viewing it as indicating results calculated from left to right (e.g., accepting only $a + b = c$); (2) *flexible operational*, recognising results calculated in either direction (e.g., accepting $c = a + b$ and $a = a$); (3) *basic relational*, understanding the equal sign as indicating equivalence (e.g., accepting $a + b = c + d$); and (4) *comparative relational*, building on the third stage, students compare relationships between numbers when solving or evaluating number sentences like $a + b = c + d$ instead of performing full calculations. Similarly, Stephens et al. (2013) recognised three stages of understanding: *operational*, *relational-computational*, and *relational-structural*. These two constructs are widely recognised as foundational frameworks for assessing students' developmental stages in understanding the equal sign. Recently, building on two constructs, more nuanced stages of students' development of the conception of the equal sign have emerged. Lee and Pang (2023) observed that while many students were able to perceive the equal sign relationally in certain scenarios (e.g. solving and evaluating $a + b = c + d$), in other cases (e.g. evaluating $a + b = c + d = e$) they demonstrated an operational understanding (e.g. considering $2 + 3 = 5 + 7 = 12$ is true). Therefore, they proposed an additional stage between *flexible operational* and *basic relational*, which is *simultaneous operational and relational* (SOR). Likewise, Blanton et al.

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(2018) also noted that some students exhibited an *emergent relational understanding*, which refers to a 'hybrid' of operational and relational understandings. Echoing them, Sun et al. (2023) found that about 55% of tested Chinese Grade 1 students (n = 497) can correctly solve the number sentences in form of $a + b = c +$ __, whereas only about 36% of tested students successfully solve the form $a + b =$ __ $+ c$. This appeared to indicate that many students could demonstrate the relational understanding in the former case, but their conception of the equal sign seems to revert to an operational level when dealing with the latter case. Such a regression is also evidenced in other literature (Molina & Ambrose, 2008). Introducing a SOR stage potentially explains that, in two studies mentioned above, students' conceptions of the equal sign did not 'regress' from the relational to the operational level; rather, they had not yet reached the pure relational level and could be considered to be at the SOR stage. It could be argued that the misconceptions students typically experienced in SOR stage could prevent these students from developing a robust relational understanding of the equal sign. Therefore, to support students in developing a full relational understanding, it is insufficient to examine their thinking solely within the dichotomy of relational and operational levels. More attention needs to be given to the SOR stage, which represents a conflation of two levels. However, as Lee and Pang (2023) showed, very few literature focussed on SOR or distinguished it from pure relational understanding. Therefore, students' difficulties, challenges encountered at SOR stage, along with related supportive pedagogies, warrant further attention and exploration (Lee & Pang, 2023; Sofuoğlu, 2024). It is worth mention that some studies note that experienced mathematics learners or mathematicians can interpret the equal sign flexibly—operationally or relationally—depending on context (e.g., McNeil & Alibali, 2005). However, the SOR stage that will be investigated in this study does not reflect such a flexibility. Instead, it refers to specific misconceptions that can result in a problematic understanding of mathematical structures, such as interpreting $5 + 2 = 7 + 6 = 13$ as true.

Furthermore, Lee and Pang (2023) pointed out that the existing instruments commonly used to assess students' conceptions of the equal sign, such as evaluating and solving non-conventional forms of number sentences such as $a = b + c$ and $a + b = c + d$, are difficult to identify the SOR stage. Therefore, they introduce the form $a + b = c + d = e$. Moreover, based on Sun et al. (2023), it appears that for the form $a + b = c + d$, many students demonstrated different understanding of equal sign when they were solving $a + b =$ __ $+ d$ and $a + b = c +$ __. In this sense, adding to Lee and Pang (2023), this study considers that the number sentences solving task involving these two forms potentially have capacity to probe students' SOR stage.

Accordingly, the purpose of this study is to explore further insights about SOR stage. First, the construct of the SOR stage as hypothesised by Lee and Pang (2023), has yet to be scientifically validated as a developmental stage in the understanding of the equal sign. Therefore, one goal of this study is to use Rasch modelling to validate the SOR stage. This leads to the first research question (RQ1): How can the SOR stage be validated through Rasch modelling analysis, and what insights does it provide into the complexities of students' reasoning at this stage? Furthermore, the second research question (RQ2) explores what conceptions held by students identified at SOR stage and the potential sources of their confusion. By doing so, this study contributes to the knowledge of how to support students transitioning from emerging to solid levels of relational understanding of the equal sign, benefiting both research and classroom practice.

Methodology

To validate the SOR stage (addressing RQ1), this study employed the Rasch modelling (elaborated below), and used students interviews to probe their thinking (addressing RQ2). The researchers (the research grant and ethical approval see the acknowledgement) collected the data at four primary schools in Changchun City, Jilin Province, China, with the participation of

1,008 Grade 1 students, in September 2023. Sun et al. (2023) have showed that many tested Grade 1 students exhibited a mixture of operational and relational understandings. Therefore, Grade 1 can be considered as a transition period for many Chinese students in progressing towards the full relational conception of the equal sign. This underpins the choice of Grade 1 students for this study. The following sections will outline how Rasch modelling was applied and how the interviews were administered.

Rasch Modelling Analysis

Given students' responses to assessment items or tasks, Rasch modelling will match students' abilities to the assessment item difficulties, and group them simultaneously (Matthews et al., 2012). A strength of Rasch modelling is its probabilistic nature: students' abilities are determined by calculating the probabilities of their correct responses to different items, while item difficulties are similarly assessed based on the probabilities of being answered correctly by students. This can reduce the impact from unexpected distortions, for example, high ability students' incorrect response due to careless mistakes, and a low ability students' correct response due to guessing (Linacre & Wright, 2000). Therefore, Rasch modelling is less influenced by random factors and is more reliable and flexible (Matthews et al., 2012).

Research has evidenced that Rasch modelling is an effective tool for identifying and validating the developmental stages of mathematical concepts (Bao & Stephens, 2024; Matthews et al., 2012). In this study, the assessment was designed to probe students' understanding of the equal sign, so students' abilities refer to their various conceptions on the equal sign. Students' (n=1008) responses to 19 assessment items were entered into an Excel spreadsheet and then loaded into *Winsteps* software for Rasch modelling analysis.

Assessment

This study employed an adapted version of Mathematics Equivalence Assessment [MEA] (Simsek et al., 2021). Students were required to evaluate True/False for a range of number sentences, such as forms of $a + b = c$, $a = a$, $a = b + c$, $a + b = c + d$, $a + b = c + d = e$, and $a + b + c = d + e = f$, and they were required to solve number sentences such with the forms of $a + b = \_\_ + c$ and $a + b = c + \_\_$, involving both addition and subtraction. These items are commonly used in examining students' understanding of the equal sign. Notably, the forms of $a + b = c + d = e$, $a + b + c = d + e = f$, $a + b = \_\_ + c$ and $a + b = c + \_\_$ are included as the literature suggests they potentially probe students' SOR (Lee & Pang, 2023; Sun et al., 2023). In this study, the arithmetic operations involved number sentences with numbers less than 10, adapting to Chinese Grade 1 students' general mathematics ability. The specific items are shown in Figure 1 below. The reliability and validity of MEA and its adapted version are well-documented in the literature (e.g., Simsek et al., 2021; Sun et al., 2023), and it meets the assumptions of Rasch modelling (e.g., Matthews et al., 2012). In addition, as will be seen the result of the Rasch analysis of this research demonstrate this assessment instrument is suitable. Students' responses to each item were coded dichotomously (0 for incorrect or 1 for correct). With this coding scheme, a possible concern is that a student with a relational understanding might provide an incorrect answer due to calculation error, resulting in a score of 0, which may not accurately reflect their actual understanding. However, the probabilistic nature of Rasch modelling can accommodate such a circumstance, ensuring the results remain robust. The reliability of this coding convention is also evidenced in other studies (e.g., Matthews et al., 2012).

Interview

Of students who were potentially identified as being at SOR stage, 20 were interviewed by the researchers. They were asked to explain their thinking in detail when solving and evaluating specific items, as well as to reflect on the past learning experiences that contributed to their reasoning. The interview was semi-structured, with opening questions designed to prompt responses, such as, "*Could you please explain why you filled this number here?*", "*Could you explain why you think this one is true*?" and "*Where and how did you learn this before*?"

Results and Discussion

The *Winsteps* program evaluated the data's fit to the Rasch model, with a key indicator being the Infit MNSQ values (IMNSQ) for test items and testers. The acceptable range for this value is 0.5 to 1.5, with values closer to 1.0 indicating a better fit to the Rasch analysis (Linacre & Wright, 2000). The IMNSQ of the assessment data of this study for students and items are 0.98 and 0.97, respectively, evidencing a good fit of this study's data to Rasch analysis. Furthermore, the analysis showed that the SEPARATION values for students and items are 2.25 and 14.55, respectively, demonstrating the assessment performs well in distinguishing both the abilities of students and item difficulties (Linacre & Wright, 2000). Therefore, the assessment shows a good reliability and discriminative power, making it a high-quality measurement. These results are important as it provides strong statistical evidence for this study's claim about SOR stages that will be presented later.

The Validation of SOR

The results of Rasch analysis are presented in Figure 1 below. The left column displays the

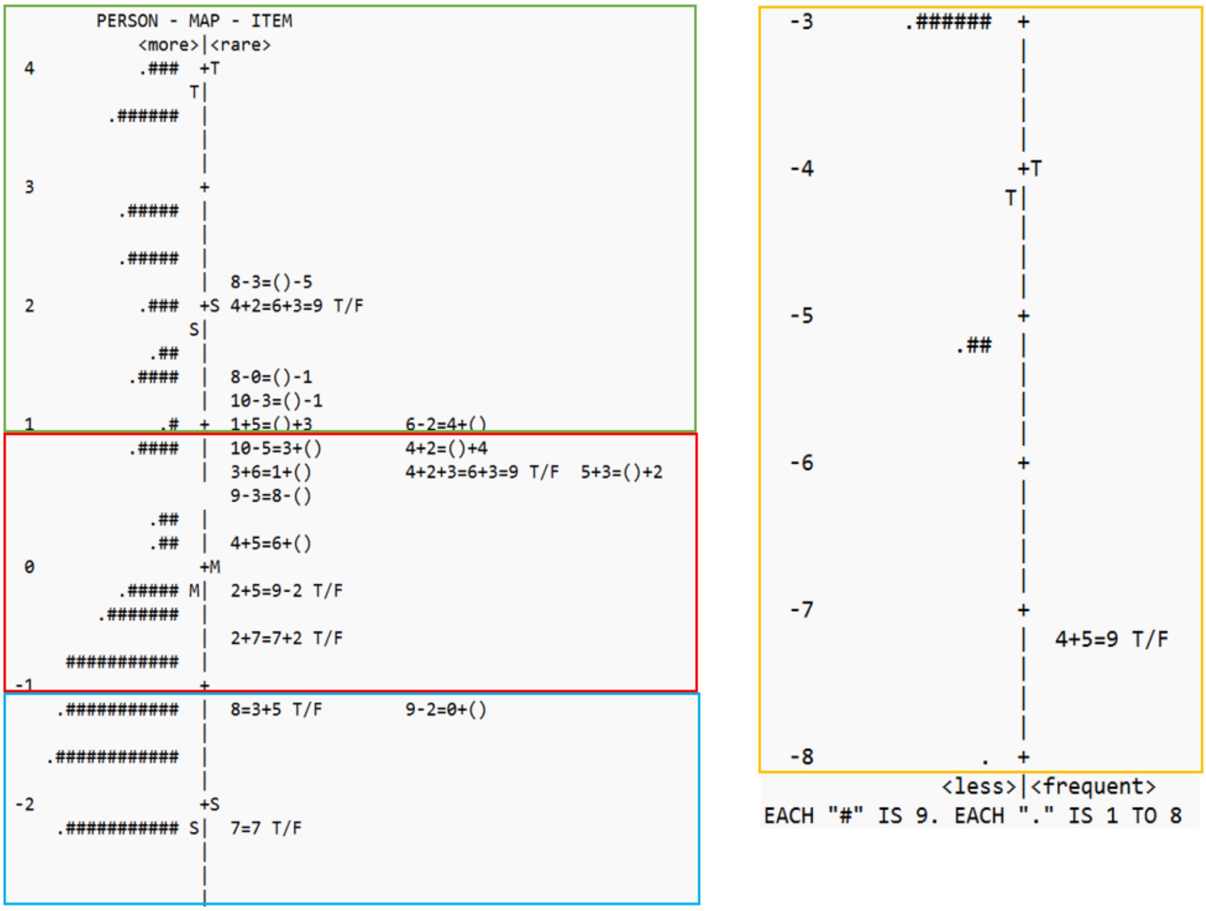

Figure 1. The Variable Map of Rasch Analysis (Analysis Results)

grouping of students based on their abilities (i.e. their conceptions on the equal sign), ranging from logit -8 to 4, while the right column displays the grouping of items according to their difficulties, ranging from logit -7 to 3. The logit values can be used to calculate the probability of a student correctly responding to an item ($\Pr(correct) = \frac{1}{1+e^{-(student\ logit - item\ logit)}}$). For instance, a student at level -3 has approximately a 11.2% chance of correctly evaluating a level -1 item, such as 2 + 7 = 7 + 2, as true, and only an 4.74% chance of correctly finding the missing number for a level 0 item 4 + 5 = 6 + __. Both items are forms of the number sentence *a* + *b* = *c* + *d*, which is commonly used to assess whether a student's understanding of the equal sign has achieved relational level. This result suggested that even when a student with an operational understanding occasionally provided correct response to the items requiring relational thinking by guessing (more likely guessing correctly on evaluation items than on solving items), he/she can still be categorised in the operational understanding group (the yellow-highlighted group, will be elaborated later). This adds confidence that the identified stages accurately reflect students' genuine understanding of the equal sign. In Rasch analysis, the logit values also identify significant shifts in item difficulties, revealing the 'natural breaks' that indicate changes in cognitive skills (Griffin, 2020). In this sense, since the items in this research are designed to assess students' knowledge of the equal sign, the logit values can serve as thresholds between different stages of understanding.

The stages of students' understanding of the equal sign identified in this study aligns with developmental stages proposed by Matthews et al. (2012). The students in the yellow-highlighted group accepted the conventional form: *a* + *b* = *c* (4 + 5 = 9) but failed to grasp the forms *a* = *a* and *a* = *b* + *c* (7 = 7 and 8 = 3 + 5). Therefore, these students are placed in the yellow group indicating a *rigid operational* stage. Students in the blue-highlighted group understood *a* = *b* + *c* and *a* = *a* but struggled to make sense to the form *a* + *b* = *c* + *d*, whether in evaluating or solving tasks (e.g., True/False 2 + 7 = 7 + 2, or solve 4 + 5 = 6 + __). These students can be categorised into *flexible operational* stage. It is noted that some students at this stage can solve 9 – 2 = 0 + ___. It could be explained this number sentence is special because of the presence of '0', so students are likely to perceive this one the same as 9 – 2 = __. Later interview results supported this claim. Students in red-highlighted and green-highlighted groups can appreciate the forms *a* + *b* = *c* + *d*. In line with Matthew et al. (2012), they can be considered at the *basic relational* stage. For the *comparative relational* stage, this study did not include assessment items, as it was beyond the scope of the research.

However, when closely examining the *basic relational* stage, two gourps can be further distinguished, with logit 1 as a threshold (i.e. green and red highlighted groups). First, it is clear that within logit 0, the number sentences are almost with the form *a* + *b* = *c* + __ (red-higlighted group), and majority of items of form *a* + *b* = __ + *c* are within logit 1 and beyond (green-higlighted group). This suggests the students in the red-highlighted group could confidently deal with form *a* + *b* = *c* + __ but struggled with form *a* + *b* = __ + *c*. For instance, many students in this group filled 5 for 8 – 3 = __ – 5, demonstrating operational understanding. Subsequent interviews corroborated their thinking (see below). This finding echoed Sun et al. (2023) who discovered a significant lower proportion of Chinese Grade 1 students correctly responding to items in the form *a* + *b* = __ + *c* than *a* + *b* = *c* + __. Sun et al. (2023) showed this result with a response distribution chart. This study takes it a step further by using Rasch analysis to confirm the existence of SOR students exhibiting such behaviours (approximately 30% of all tested students). It is noted that this red-higlighted group was likely to correctly respond to two *a* + *b* = __ + *c* form number sentences 4 + 2 = __ + 4 and 5 + 3 = __ + 2. This study conjectured that since 4 + 2 = __ + 4 can be considered as a special instance of *a* + *b* = *b* + *a*, and so may activate students' relational view towards this item. The interview response supported this explanation (see below). For 5 + 3 = __ + 2, this study speculates that students' familiarity with addition may make a number sentence exclusively involving addition more likely to activate relational thinking for those at SOR stage. This requires a further exploration. Meanwhile, the results confirm that the True/False item 4 + 2 = 6 + 3 = 9 is beyond the abilities of red-highlighted group, indicating that many students who were comfortable with $a + b = c + d$, mistakenly accepted this item as true. This finding echoed Lee and Pang (2023), who proposed the existence of SOR stage by observing this misconception. Notably, a similar True/False item $4 + 2 + 3 = 6 + 3 = 9$ is located at red-highlighted group, suggesting that students in this group are very likely to provide the correct evaluation. This finding appears to show that these students hold contradictory views about the role of the equal sign in these two number sentences, demonstrating a relational understanding for $4 + 2 + 3 = 6 + 3 = 9$ but an operational view for $4 + 2 = 6 + 3 = 9$.

Taken together, the Rasch analysis shows the students in the red-highlighted group are more likely to demonstrate inconsistencies in their conceptions of the equal sign. By contrast, students in the green-highlighted group appear more robust in solving and evaluating varied forms of number sentences, exhibiting a more solid relational understanding. In this sense, this study argues the red-highlighted and green-highlighted groups should be considered as two developmental stages of the understanding of the equal sign, which align with stages of SOR and PBR (pure basic relational) proposed by Lee and Pang (2023). In addition, the Rasch analysis appears to reveal that within the SOR stage, students can be further differentiated as many students located between logit -1 and 0 are likely to exhibit relational understanding when dealing with True/False items but reverting back to operational thinking when asked to solve missing number sentence. These students' relational understanding of the equal sign is less robust than those who can solve $a + b = c +$ __ (i.e. students located between logit 0 and 1). This finding has implications for item design cautioning on over reliance on True/False items.

Students' Reasoning and Possible Source of Inconsistency

To address RQ2, this study interviewed 20 students in the red-highlighted group to explore their reasoning behind inconsistencies in solving and/or evaluating specific number sentences, as well as the factors contribute to such reasoning. The data from two students, Ming and Yang, will be presented and discussed. Ming was a typical student who could correctly responded to the form $a + b = c +$ ___, but had confusions with the form $a + b =$ __ $+ c$ and $a + b = c + d = e$. Yang was a student located at bottom area of red-highlighted group (logit -1-0), who was correctly evaluate the form $a + b = c + d$ but was less confident to solve the missing number in number sentences of the same form. The interview responses from both students are representative of the patterns observed among all interviewed students.

Ming

Ming explained when he was solving 8 – 3 = __ – 5, he filled the 5 as he saw the blank straight after '8 – 3 ='. When the researcher asked Ming whether he saw another '– 5' follow the blank, he responded, "I don't need to care about another minus 5, I only need to calculate the answer for 8 – 3". Then the interviewer asked him about the approach to 10 – 5 = 3 + __ and 4 + 2 = __ + 4, Ming's response is shown below:

Interviewer: So how did you do this one, 10 – 5 = 3 + __, you did this correctly.

Ming: 10 – 5 is 5 so I need to find out 3 plus another number equals 5, so it's 2.

Interviewer: So now the missing number here is not the answer 10 – 5 not like the one in 8 – 3 = __– 5?

Ming: No, 3 has been already after the equal sign, 10 – 5 obviously is not 3, so I need to think about all numbers on the right-hand side equals to the left-hand side.

Interviewer: Ok, could you please tell me what does the equal sign mean in this one [10 – 5 = 3 + __]?

Ming: Two sides are the same.

Interviewer: Ok, what about this one 4 + 2 = __ + 4, why the missing number you filled is 2, not 4 + 2 = 6?

Ming: Because 4 + 2 obviously is the same as 2 + 4, I know two sides are the same.

Interviewer: Good, if you know for 4 + 2 = __+ 4, two sides are the same. Could you please reconsider 8 – 3 = __ – 5?

Ming: Oh, I see, the 8 – 3 should equal the whole blank minus 5, so it should equal 10.

Ming's responses tended to reveal that in the form of $a + b = c +$ __, the number after the equal sign suspended his operational view and lead him to consider the equal sign relationally. However, when having $a + b =$ __ $+ c$, his operational view is spontaneously activated. For the special $a + b =$ __ $+ a$ form, the commutative law of addition made him readily switch to the relational view again. Subsequently, based on his conception on 4 + 2 = __ + 4, Ming realised the same approach should be applied to 8 – 3 = __ – 5 and he made a correction. Finally, Ming was asked about his evaluation of 4 + 2 = 6 + 3 = 9. He explained that during arithmetic drill practice at home, his parents demonstrated that multi-step operations questions could be written sequentially in this manner. As a result, this approach became deeply ingrained in his thinking. This response was widely provided by many other interviewed students. For Chinese Grade 1 students, while this multi-step operations are presented in the classroom, they also practice arithmetic drill extensively at home. It is possible that on one hand, teachers in the classroom show students the correct approach, on the other hand they are influenced by parents' misleading. Consequently, students may consider both approaches (relational and operational) are legitimate.

Yang

Yang explained his thinking to evaluate two True/False items with the form $a + b = c + d$: "The calculated results of two sides are the same, so they are true". When he was asked about the strategy to solve the number sentences such as 4 + 5 = 6 + __ and 5 + 3 = __ + 2, he responded, "This one (5 + 3 = __ + 2) is easy, 5 + 3 = 8, so it's 8. But for this one (4 + 5 = 6 + __), I am not sure, how come 4 + 5 = 6? I am not sure" Yang's responses suggest that his relational understanding is activated when the calculated results are already provided. However, whenever he needs to perform the calculations himself, his operational thinking appears to take over, suggesting that True/False items are more likely activate students' relational understanding, as the Rasch analysis showed. When Yang was asked about the evaluation of form: $a + b = c + d = e$, his response was similar to Ming's.

Combining Ming's comments above, it appears that for students at SOR stage, number sentences where a number immediately follows the equal sign (i.e. True/False items and $a + b = c +$ __) are likely to encourage relational thinking. In contrast, items requiring students to fill a number immediately after the equal sign tend to lead to operational thinking. Students' responses appeared to indicate the similarity between the form $a + b =$ __ $+ c$ and their everyday conventional arithmetic practice ($a + b =$ __) contributes to such thinking. Especially for students at SOR stage, greater efforts may still be needed to prevent their relational understanding from being disrupted by the influence of conventional arithmetic drill.

Concluding Remarks: Implication for Research and Practice

This study validates the SOR stage proposed by Lee and Pang (2023), uncovering fine-grained complexities about students' conception of the equal sign during the transition period towards the full relational understanding. First, this study differentiated students' thinking by identifying their nuanced responses to various equivalence assessment items. For instance, previous literature has provided limited exploration of the discriminative power of these items. By contrast, this study confirmed that True/False items (i.e., $a + b = c + d$ and $a + b = c + d = e$ forms), together with number sentences requiring solution in the forms $a + b = c +$ __ and $a + b =$ __ $+ c$, are effective in differentiating levels of emerging relational understanding. As Lee and Pang (2023) argued, such a differentiation is important as it helps to provide targeted support for students with diverse needs. Therefore, this study could inform any future research agendas on investigating applying these nuanced items to assess and/or enhance students' conceptions on equal sign. Furthermore, this study has an implication for everyday classroom practice as these items can help teachers pinpoint the critical misconceptions in students' understanding of the equal sign and provide tailored scaffolding.

Second, this study finds that students' emerging relational understanding of the equal sign can still be strongly influenced by their experiences with conventional arithmetic practice, which may reactivate an operational view when similar structures are encountered. This experience occurs not only in the classroom but also potentially at home, where students' parents may inadvertently use inappropriate approaches to number sentences. This highlights the need for classroom teachers to place greater emphasis on reinforcing the relational meaning of the equal sign in students' everyday arithmetic experiences. In particular, the critical misconceptions, such as the sequential approach $a + b = c + d = e$, must be addressed carefully.

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