

# How to Teach the Meaning of Limit Intuitively in Calculus: A Didactical Design Research

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Many students struggle to grasp the concept of limits, leading to persistent learning obstacles. This study employed a didactical design research (DDR) approach involving 56 mathematics education students at Universitas Khairun, divided into two groups. Data from tests, interviews, and teaching interventions revealed three key obstacles: epistemological (e.g., confusion between indeterminate and undefined forms like  $0/0$ ), ontogenic (e.g., difficulties with factoring and graphing), and didactical (e.g., misaligned instruction). A didactical design based on the Theory of Didactical Situations (TDS) was developed to address these challenges.

## Introduction

Introducing the concept of limits to students involves a variety of pedagogical approaches, each presenting distinct challenges. Visual and intuitive examples drawn from disciplines such as art, physics, and engineering have been shown to improve student engagement and conceptual understanding (Raviv, 2022). Several studies advocate for teaching limits as a notion of proximity, integrating both static and dynamic perspectives (Shigeno, 2020; Nagle et al., 2017). However, students frequently struggle to synthesise different forms of limit concepts—such as sequences, limits at a point, and limits at infinity—when these are taught in isolation (Fernández-Plaza & Simpson, 2016).

Research on the limit concept in calculus reveals substantial differences between high school and university approaches. At the high school level, limits are often taught as a process, with students relying on dynamic conceptions and procedural understanding (Nagle, 2013). In contrast, university instruction tends to emphasise formal definitions and structural understanding, with relatively consistent praxeology across different countries (Viirman et al., 2022). This shift often poses difficulties for students, who may struggle with abstract mathematical language and the coordination of multiple conceptual processes (Bansilal & Mkhwanazi, 2021). Common misconceptions include formal epsilon-delta definitions (Nagle, 2013). Researchers argue that university curricula frequently overlook students' prior learning experiences, resulting in discontinuities in their mathematical understanding (Bloch et al., 2000). Addressing these misconceptions is crucial for improving calculus instruction at both levels (Winarso & Toheri, 2017).

Students' understanding of limits in calculus reveals several recurring themes. Many struggle with formal definitions, often relying instead on intuitive or dynamic interpretations (Nagle et al., 2017; Tall & Vinner, 1981). Visual and allegorical approaches have been shown to support intuitive understanding (Shaldehi et al., 2022; Raviv, 2022), while algorithmic contexts may serve as stepping stones to deeper conceptual knowledge (Pettersson & Scheja, 2008). Language plays a crucial role, with terms like "tends to" potentially reinforcing misconceptions (Jaffar & Dindyal, 2011). The others are students' mathematical beliefs, particularly their sources of conviction, which can influence their limit comprehension (Szydlik, 2000). Common misconceptions include viewing limits as fixed boundaries or unattainable targets (Szydlik, 2000). To address these challenges, researchers suggest the use (2025). In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 437–444). Canberra: MERGA.

of mental imagery and real-world examples to build intuition before introducing formal definitions (Shaldehi et al., 2022; Raviv, 2022). This study investigates students' misconceptions and learning obstacles in developing an intuitive understanding of limits.

Learning obstacles arise when students understand concepts superficially and are unable to relate them to other mathematical concepts. These obstacles are generally categorised into three types: didactical, ontogenic, and epistemological (Brousseau, 2002; Suyadi, 2019). Ontogenic obstacles relate to students' mental maturity and readiness to acquire new knowledge. Didactical obstacles arise from the instructional sequence—such as the order in which material is presented by the lecturer or tasks are completed by students. In contrast, epistemological obstacles stem from the nature of the learning tasks and students' limited ability to extend their thinking beyond procedural knowledge.

Some studies propose strategies and approaches to tackle the misconceptions and learning obstacles. Liang (2015) used conceptual conflict strategies with graphing to introduce limits. Others suggest reformulating the definition as a local approximation (Bokhari & Yushau, 2006) or focusing on covariational reasoning (Nagle et al., 2017). Research indicates that students often rely on dynamic conceptions and struggle with formal definitions (Nagle, 2013). Common metaphors used by students include collapsing dimensions and physical limitations (Oehrtman, 2009). While dynamic interpretations can be intuitive, care must be taken to align them with formal limit concepts (Nagle et al., 2017). Combining mental imagery with puppet allegories can make understanding limits easier and more enjoyable (Shaldehi et al., 2022).

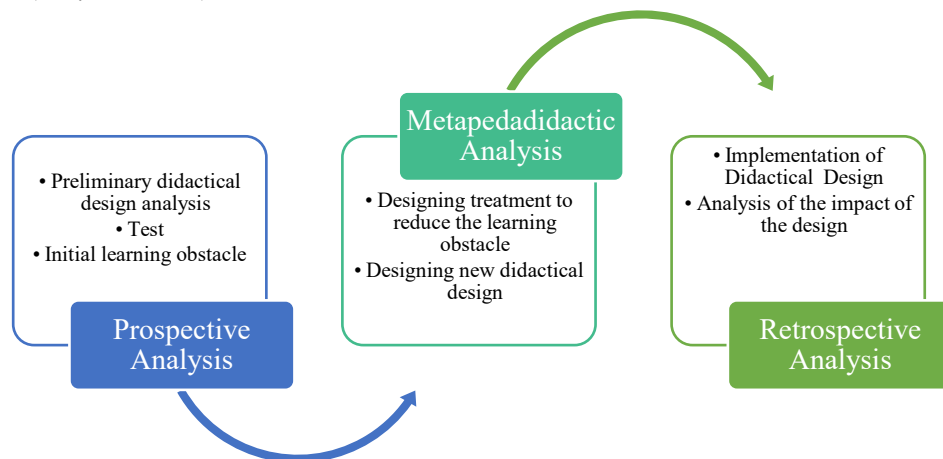
Despite the variety of instructional strategies discussed earlier, learning obstacles in students' intuitive understanding of limit concepts persist. This suggests that current teaching designs still require further refinement to better support students' thinking processes. Thus, it is necessary to develop a new didactical design to overcome problems. The fact that students face learning obstacles suggests that the didactical design that was previously used has flaws (Astriani et al., 2022). The didactical design refers to the stages of Theory Didactical Situation (TDS) by Brousseau (2002) which consist of 4 situations: Action, formulation, validation, and institutionalisation. It's important to identify the student's learning obstacles when working on introducing limits in calculus at university in order for instruction to modify challenges so that they either do not occur again or are mitigated through the didactical design in the teaching phase. These approaches aim to make limit concepts more accessible and intuitive for students beginning their study of limits. The approaches in this study also aim to bridge the gap between intuitive understanding and formal mathematical definitions of limits.

This paper addresses two research questions: 1) What types of initial learning obstacles do students encounter when developing an intuitive understanding of the limit concept? and 2) How can a didactical design be developed to support students in overcoming these obstacles and enhancing their intuitive understanding of limits? The first question identifies the specific learning challenges, while the second focuses on the application of the Theory of Didactical Situations (TDS) to guide the design and its intended outcomes.

## **Method**

### **Research Design**

This study employs a Didactical Design Research (DDR) approach, grounded in interpretive and critical paradigms (Suryadi, 2019). The interpretive paradigm guides the analysis of how learning obstacles emerge, while the critical paradigm informs the design and evaluation of instructional interventions. Based on these paradigms, the study follows three DDR stages, outlined in Figure 1.

**Figure 1***DDR Stages (Suryadi, 2019)*

## Participants

This research was conducted at Universitas Khairun from May to December 2024. Participants consisted of two groups: (1) 23 third-semester mathematics education students who had completed a differential calculus course and were given test questions to identify initial obstacles; and (2) 33 first-semester students currently enrolled in the same course.

## Data Collection and Analysis

There were 3 instruments used for data collection in this research. Two experts in mathematics education, DDR, and the mathematics analysis field validated the instruments.

### *Test for Limit at a Point*

The test instrument comprised two sub-tasks focusing on finding the limit and comparing two functions. Student answers were analysed to reveal learning obstacles in their understanding of the meaning of limits

**Table 1***The Question of Test Limit at a Point*

Indicators	The question
Determine limit of a function	Find the limit of $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$
Compare two functions and determine whether both are similar function	<p>Given the function <math>f(x) = \frac{x^2 - 1}{x - 1}</math> and <math>g(x) = x + 1</math></p> <p>Answer the following questions and give reasons!</p> <p>Are the functions <math>f(x)</math> and <math>g(x)</math> similar? Draw the graph (via application)</p> <p>Find <math>\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}</math> dan <math>\lim_{x \rightarrow 1} (x + 1)</math></p> <p>Give a conclusion for the two functions</p> <p>Explain the meaning of the limit for <math>f(x)</math>.</p>

## Interview

In-depth interview guidelines were used to verify students' answers (Turner, 2010). Participants were selected based on the diversity of their responses to the limit questions. Three students were chosen for in-depth interviews to explore their thought processes, understand the reasons behind their differing answers, and gain insight into their conceptual understanding.

### *A Didactical Design of Limit at a Point*

This instrument was developed based on several key sources: 1) the results of the initial analysis of students' learning obstacles when answering the questions; 2) a comparison of calculus textbooks commonly used in Indonesia, including those by Varberg et al. (2013), Martono (1999), and Stewart (1999); 3) relevant articles on the limit at a point or intuitive understanding of limits; and 4) validation of the didactical design through a focus group discussion (FGD) with Calculus experts.

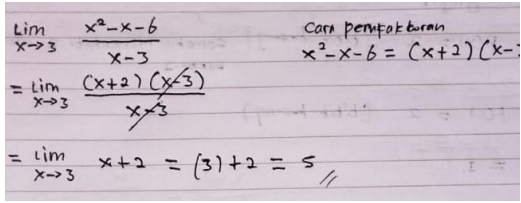
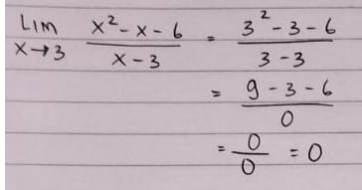
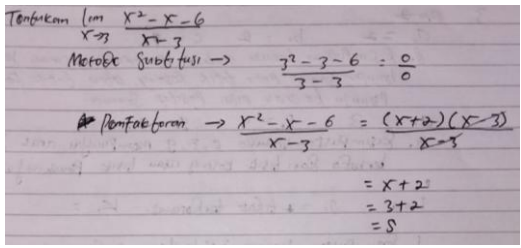
## Result

### The Initial Learning Obstacles

The obstacles identified at this stage offer insight into students' intuitive understanding of the limit concept. Table 2 presents responses to the first limit-at-a-point test question, while Table 3 summarises responses to the second. Based on the data presented in Tables 2 and 3, the initial learning obstacles are summarised below.

**Table 2**

*The Answer of the Test Limit at a Point Number 1*

The answer	Interview conclusion
<p>M1 answer:</p> 	<p>M1 was able to correctly find the limit value, but did not fully understand that the expression <math>x - 3</math> in the numerator does not yield an absolute value of zero, but rather approaches zero as <math>x</math> approaches 3. The same misunderstanding occurred with the expression <math>x - 3</math> in the denominator, leading M1 to assume that both could simply be crossed out. M1 also mentioned that 0 divided by 0 is undefined.</p>
<p>M7 answer:</p> 	<p>Based on the answer, it was determined that M7 used direct substitution in solving the problem. M7 forgot how to factor so he solved it by substitution. M7 misunderstood that 0 divided by 0 results in 0.</p>
<p>M12 answer:</p> 	<p>Based on the answer, information was obtained that M12 did direct substitution first, but because the value was 0, they used factoring. M12 did not understand that 0/0 was indeterminate. M12 only crossed out directly if the denominator and numerator were the same, and did not understand the concept of division. M12 also could not differentiate how to do ordinary algebra with solving function limits.</p>

Epistemological obstacle: (1) Do not know the reason for crossing out or eliminating the same terms or factors between the numerator and denominator; (2) Cannot distinguish between indefinite and definite forms; For example, by stating that  $0/0$  is 0, or  $0/0$  is undefined; (3) Cannot distinguish between limit and algebraic forms due to the loss of limit notation in the process of finding the limit value; (4) Cannot distinguish between undetermined and undefined; (5) Limitations in understanding the similarities in 2 different functions  $f(x)$  and  $g(x)$ ; (6) Limitations in understanding the differences between  $f(x)$  and  $g(x)$  are how to solve them using factoring and substitution; and (7) Cannot explain the meaning of limit intuitively.

Ontogenic obstacle: (1) Cannot undertake factoring; (2) Errors in solving factoring by not writing limit notation or can be said to be unable to distinguish between ordinary algebraic forms and limit forms; and (3) Unable to draw a function graph.

**Table 3**

*The Answer of Test Limit at a Point Number 2*

## The answers

## Interview conclusion

M1 answer:

2. b.  $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x \rightarrow 1} x+1 = 1+1 = 2$

•  $\lim_{x \rightarrow 1} (x+1) = 1+1 = 2$

c. kesimpulan terhadap Fungsi  $f(x)$  dan  $g(x)$

- Fungsi  $f(x)$  harus dicari dengan metode pemfaktoran karena  $f(1)$  tidak ada atau jika di grafik membentuk titik bintik kosong
- Fungsi  $g(x)$  dapat langsung dicari nilainya dengan metode substitusi karena  $g(1)$  ada

Jadi,  $f(x)$  dan  $g(x)$  berbeda.

d. makna limit pada  $f(x)$

Limit pada  $f(x)$  adalah suatu nilai yang semakin mendekati nilai fungsi  $f(x)$ , pada  $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$  ketika nilai  $x$  mendekati 1 membentuk titik kosong pada grafik nilai limit fungsi  $f(x)$  hanya semakin mendekati angka 2.

M1 demonstrated understanding that the way to find the limit value was by substituting the value of  $x$  approaching  $c$  but because it produces a divisor of 0 which means it is undefined, then M1 used the factoring method. M1 understood that  $f(x)$  and  $g(x)$  are different from the solution process.  $f(x)$  is sought by factoring because if the value of  $f(1)$  is substituted there is no value, while  $g(x)$  can be directly substituted for its value.

*Translation part c) and d):*

c) conclusion on  $f(x)$  and  $g(x)$ :  $f(x)$  must be found using the factoring method because  $f(1)$  does not exist or if it is on the graph it forms an empty dot.  $G(x)$  can be directly found using the substitution method because  $g(1)$  exists. So  $f(x)$  and  $g(1)$  are different.

d) meaning of limit on  $f(x)$ : limit on  $f(x)$  is a value that is getting closer to the value of the function  $f(x)$  on  $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$  when it approaches 1 it will form an empty dot on the graph so that the limit value of the function  $f(x)$  only approaches 1.

M12 answer:

2.  $f(x)$  dan  $g(x) = \frac{x^2-1}{x-1}$  dan  $g(x) = x+1$

Misal:  $x = 0$ , maka  $f(x) = \frac{0^2-1}{0-1} = 1$   
  $g(x) = 1$

$x = 1$ ,  $f(x) = \text{tak tentu } (\frac{0}{0})$   
  $g(x) = 2$

$x = 2$ ,  $f(x) = \frac{2^2-1}{2-1} = \frac{3}{1} = 3$   
  $g(x) = 2+1 = 3$

a. sama, apabila dimasukkan nilai 0 dan 2,

$x$	0	1	2
$f(x) = \frac{x^2-1}{x-1}$	1	$\frac{0}{0}$	3
$g(x) = x+1$	1	2	3

b.  $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1} = \frac{1^2-1}{1-1} = \text{tak tentu}$

$\lim_{x \rightarrow 1} (x+1) = (1+1) = 2$

c. pada  $f(x)$  apabila persamaannya dimasukkan nilai 1) maka didapat nilai tak tentu, selanjutnya fungsi  $f(x)$  dan  $g(x)$  sama.

M12 was able to draw the conclusion that  $f(x)$  and  $g(x)$  are more or less the same but at  $x=1$  they are different, although there are still errors because M12 cannot clearly provide a final conclusion that  $f(x)$  and  $g(x)$  are different only when  $x=1$  and the rest are the same.

*Translation part c)*

In  $f(x)$  if the value is entered into the equation, an indeterminate value is obtained, otherwise  $f(x)$  and  $g(x)$  are the same.

Didactical obstacle: The sequence of materials developed by the lecturer does not match the students' thinking process. For example, in the first meeting, the lecturer introduced the limit

intuitively by presenting a function that when  $x$  is substituted produces  $0/0$ , then the lecturer did not explain what  $0/0$  is. Therefore, from the initial learning obstacle the following didactical design recommendations were implemented to overcome these obstacles: (1) Explain the rationale for cancelling identical terms or factors between the numerator and denominator; (2) Provide examples that require determining limit values for functions yielding defined, undefined, and undetermined.

### The Didactical Design of Limit at a Point

A review of published literature has found no prior studies on teaching the concept of function limits using the Theory of Didactical Situations (TDS) that explicitly incorporates all four phases: action, formulation, validation, and institutionalisation. This design also consists of 3 situations that are deliberately designed to improve students' understanding of limits (see Table 4).

**Table 4**

*Teaching Activities Based on TDS*

Teaching activities based on TDS	Pedagogical didactic anticipation																
<p>Action situation</p> <p>Situation 1:</p> <p>Students are presented with a function,</p> $f(x) = \frac{x^2-x-6}{x-3}$ <p>Students are asked to identify possible values of <math>f(3)</math></p>	<p>If the students answer it as follows:</p> $f(3)= 0/0 = 0$ $f(3)= 0/0 = \text{none}$ $f(3)= 0/0 = \text{undefined}$ $f(3)= 0/0= \text{indeterminate}$ <p>The example of anticipation: Introducing <math>0/0</math> is an indeterminate form. Suppose <math>0/0 =x</math>      <math>0=x \cdot 0</math></p> <p><math>x</math> can be replaced by other numbers, namely 1, 2, 3. So <math>0/0</math> is indeterminate</p>																
<p>Formulation situation</p> <p>Because the value of <math>f</math> for <math>x = 3</math> cannot be determined, we need to identify the values of <math>f</math> for <math>x</math> around 3.</p> <p>Therefore, students are asked to draw a graph of</p> $f(x) = \frac{x^2-x-6}{x-3}$ <p>for <math>x \neq 3</math></p> <p>Then students are asked to determine the value of <math>f(x)</math> for the values of <math>x</math> in the table below</p> <table><tr><td><math>x</math></td><td>2.8</td><td>2.9</td><td>2.99</td><td>....</td><td>3</td><td>3.001</td><td>3.01</td></tr><tr><td><math>F(x)</math></td><td></td><td></td><td></td><td>...</td><td>?</td><td></td><td></td></tr></table>	$x$	2.8	2.9	2.99	....	3	3.001	3.01	$F(x)$				...	?			
$x$	2.8	2.9	2.99	....	3	3.001	3.01										
$F(x)$				...	?												
<p>Based on the graph and table above, students are asked to draw conclusions about the value of <math>f</math> around <math>x = 3</math>.</p> <p>Situation 2:</p> <p>To make the method of determining the limit value more efficient, students are asked to solve the problem below</p> $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x - 2}$	<p>The value of <math>f(x)</math> approaches 5 for <math>x</math> approaching 3</p> <p>If the students answer it as follows:</p> <p>Students work using a table of values</p> <p>The example of anticipation: Using tables is inefficient. Students perform factoring and substitution</p> $\lim_{x \rightarrow 2} \frac{x^2-2x-8}{x-2} = \lim_{x \rightarrow 2} \frac{(x\cancel{-2})(x+4)}{x\cancel{-2}} = \lim_{x \rightarrow 2} (x + 4) = 2+ 4 = 6.$ <p>The example of anticipation:</p>																

<p>Situation 3:</p> <p>To distinguish the process of simplifying functions in the limit from functions in ordinary algebra. Then the following problem is presented</p> <p>Given the functions <math>f(x) = \frac{x^2-1}{x-1}</math> dan <math>g(x) = x + 1</math></p> <p>Answer the following questions and give reasons!</p> <p>Are <math>f(x)</math> and <math>g(x)</math> the same two functions?</p> <p>Are <math>\lim_{x \rightarrow 1} f(x)</math> and <math>\lim_{x \rightarrow 1} g(x)</math> the same value?</p>	<p>The lecturer asked why in step 2, the factor can be crossed out.</p> <p>If the students answer it as follows:</p> <p><math>f(x)=g(x)</math>  <math>f(x) \neq g(x)</math></p> <p>The example of anticipation: The lecturer asked, is <math>f(1) = g(1)</math>? Then present the graphs of <math>f(x)</math> and <math>g(x)</math></p>
<p>Validation &amp; institutionalisation situation</p> <p>To strengthen the understanding of determining limit values, students are presented with limit problems with various forms of function values and contexts. Determine the following limit values:</p> <p>1. <math>\lim_{x \rightarrow 1} \frac{2x^2-x-1}{x-1}</math>    2. <math>\lim_{x \rightarrow 1} \frac{x^2-2x+1}{x-3}</math>    3. <math>\lim_{x \rightarrow 4} \frac{\sqrt{x}-2}{x-4}</math>    4. <math>\lim_{x \rightarrow \frac{\pi}{4}} \frac{2x \cos(x-\pi)}{x-\frac{\pi}{4}}</math></p>	

## Discussion and Conclusion

Based on the analysis of initial learning obstacles, students were found to rely heavily on memorised procedures rather than developing an intuitive understanding of limits. This was evident in their tendency to cancel factors without grasping the mathematical justification for doing so. Furthermore, students were unable to explain whether two given functions were equivalent. Some simply stated that the functions were different because the methods used to find their values were different, such as using substitution versus factoring. Likewise, with the intuitive meaning of the limit, students said that the limit on  $f(x)$  is a value that is getting closer to the value of the function  $f(x)$  on  $\lim_{x \rightarrow 1} \frac{x^2-1}{x-1}$  when it approaches 1 it will form an empty dot on the graph so that the limit value of the function  $f(x)$  only approaches 1. Students can find the limit value but are still confused when explaining the meaning of limit. These findings align with Duru (2011) and Nagle (2017) who observed that although students may apply procedures correctly, they often fail to understand the underlying concept or apply it to related problems.

Based on the findings regarding initial learning obstacles, improvements are needed in structuring the learning process. To address this, a didactical design was developed, comprising four phases: action, formulation, validation and institutionalisation. The action and formulation phases include three instructional situations. Situation 1 aimed to determine the value of a function in the form  $0/0$ . Situation 2 described the process of determining the limit value. Situation 3 helped distinguish between simplifying functions in the context of limits and in ordinary algebra. While validation and institutionalisation phases aim to reinforce students' understanding of limit values across various functional forms and contexts. This design, rooted in the DDR framework, was intentionally constructed to support students' conceptual thinking and reduce learning obstacles (Hendriyanto et al., 2024; Sukarma, 2024). Further research is recommended to evaluate the effectiveness of this didactical design.

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