



Mathematics Education Research Group of Australasia

Beyond Boundaries: Strengthening Futures in Mathematics Education



**Proceedings of the 48th Annual Conference
of the Mathematics Education Research Group of Australasia
6–10 July 2025, Parramatta, NSW**

Nathan Berger, Matt Thompson, John Ley, & Cristi Wilsmore (Editors)

Beyond Boundaries: Strengthening Futures in Mathematics Education

Proceedings of the 48th Annual Conference of the Mathematics Education Research Group of Australasia

Edited by Nathan Berger, Matt Thompson, John Ley, & Cristi Wilsmore

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Preface

This is a record of the Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia (MERGA), held under the theme of *Beyond Boundaries: Strengthening Futures in Mathematics Education*. The conference was hosted in Parramatta by Western Sydney University and the Proceedings were published online at the MERGA website.

Western Sydney University's Parramatta South campus is situated on the land of the Burramattagal people of the Dharug Nation. Deriving from the Dharug word *Burramatta*, meaning 'the place where eels lie down', the locality sits at the head of the Parramatta River, which winds eastward for some 24 kilometres before meeting Sydney Harbour. As enduring Traditional Custodians, the Burramattagal people have lived on and cared for this Country for thousands of years, maintaining a deep and sustaining relationship with the river, the bushland, and all living things within them. We acknowledge their Elders past, present, and emerging, and recognise their enduring connection to this special place.

Parramatta is one of Australia's oldest European settlements, established in 1788 as the colony's second settlement after Sydney Cove, when Governor Arthur Phillip recognised the area's fertile land as essential to the survival of the fledgling colony. Parramatta quickly became a centre of colonial government, agriculture, and industry. Old Government House, the oldest surviving public building in Australia, served as the country residence of the first ten Governors of the colony and is today a World Heritage-listed museum, while Elizabeth Farm, built in 1793, is Australia's oldest surviving European dwelling.

The Parramatta South campus itself occupies grounds and buildings with a long and complex history. The site was home to the Parramatta Female Factory (1821-1848) where convict women and their children lived and worked. The site later became the Parramatta Lunatic Asylum, one of Australia's oldest psychiatric institutions, which operated in various forms from 1848 into the twentieth century. The campus also borders the former Parramatta Girls Home, a place of profound trauma for many Aboriginal and non-Aboriginal girls who were institutionalised there under government care. We invite delegates to explore the contemporary and historic buildings and grounds of the campus, and to move through these spaces with an awareness of the many layers of history they hold.

Western Sydney University remains committed to honouring these histories while building a future grounded in equity, inclusion, and community. Today the campus has been transformed into a thriving centre of learning and research. Western Sydney University was established in 1989 and has grown rapidly to become a dynamic, multi-campus institution serving over 47,000 students across Greater Western Sydney. The School of Education carries on a long tradition of teacher education in the region, tracing its roots through the Nepean College of Advanced Education and its predecessor, the Westmead Teachers' College, established in 1969. Today, the School prepares both undergraduate and postgraduate students for careers in teaching across early childhood, primary, and secondary settings, with a particular focus on serving the diverse communities of our region and beyond.

It is a particular pleasure to welcome the MERGA community to engage with the conference theme in a place that has itself been defined by the crossing of geographic, cultural, institutional, and historical boundaries. This year we welcome delegates from Australia, Germany, Ghana, Indonesia, Japan, Korea, New Zealand, the Philippines, Singapore, and the United States of America who through scholarly and collegial discussions here in Parramatta collaboratively strengthened the future of mathematics education.

Keynotes

The keynote by Professor Jinfa Cai, Texas A&M University, *Fostering Students' Mathematical Understanding through Problem-Posing-Based Learning (P-PBL)*, presented a theoretical framework conceptualising mathematical understanding across five interconnected aspects: factual, procedural, conceptual, metacognitive, and affective. The address explored how Problem-Posing-Based Learning, in which students generate their own mathematical problems as an instructional approach, can push beyond the traditional boundaries of mathematics teaching by positioning students as active creators of

knowledge rather than passive recipients. Professor Cai proposed five complementary lenses through which researchers and practitioners can examine and support students' mathematical understanding, and pointed to future directions including the use of AI to synthesise existing research and strengthen the field's collective capacity to foster deep mathematical understanding.

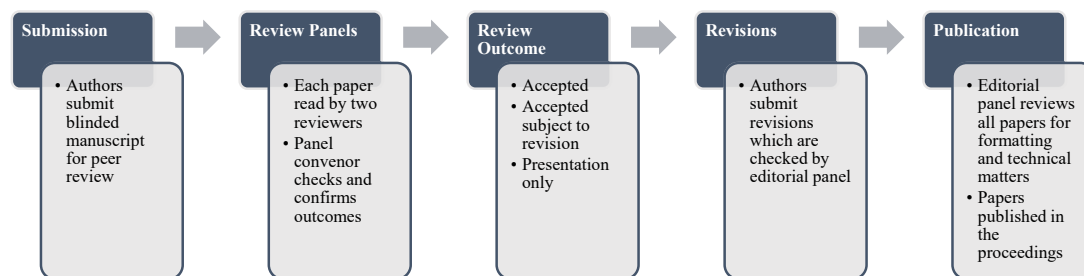
The keynote by Professor Tamsin Meaney, Western Norway University of Applied Sciences, *Advocacy and Subversion: Using the Past to Think About Mathematics in the Future*, examined how mathematics educators can resist or redirect policy pressures that narrow the purposes of mathematics education, and in doing so, move beyond the boundaries of what is currently considered possible. Drawing on Bernstein's pedagogic device and examples from her own research across Norway, New Zealand, and Australia, the address explored how 'unthinkable' possibilities for practice can emerge through advocacy (working with participants to create new spaces) and subversion (repurposing existing structures in unexpected ways) at the curriculum, institutional, and classroom levels. Professor Meaney argued that identifying and acting on these possibilities is essential if mathematics education is to contribute meaningfully to the complex societal challenges that will shape the futures of the students we teach.

The Clements-Foyster lecture was delivered by Professor Kim Beswick. In the presentation, *Affect in Mathematics Education: Blurring the Boundaries*, Professor Beswick examined the status of affect (attitudes, beliefs, confidence, and emotion) as an educational outcome in mathematics, arguing for a more meaningful integration of affective and cognitive dimensions of learning and research, one that blurs rather than reinforces the boundary conventionally drawn between them. Drawing on examples from her own program of research, spanning teacher knowledge, pedagogical content knowledge, STEM leadership, inclusive mathematics teaching, and critical mathematical thinking, Professor Beswick illustrated approaches to integrating cognition and affect and presented systems thinking as a productive lens for understanding the two as interconnected rather than separable. She argued that strengthening future mathematics education depends on taking affective outcomes as seriously as cognitive ones, recognising that knowing mathematics well also means feeling positively and confidently about it.

Proceedings

The published Proceedings include the plenary papers, research papers, and abstracts of short communications and round tables. All submitted research papers were double-blind reviewed in the Main round of submissions by panels of mathematics educators with expertise in the field. Figure 1 captures phases of the review process.

The Proceedings illustrate the breadth of mathematics education research undertaken in the region and beyond by the MERGA research community. The Editorial Team would like to thank the authors for sharing their research. Gratitude also goes to the Review Panel Convenors and all the reviewers for their professionalism and effort in reviewing papers and award nominations and providing constructive feedback. The review process ensured that the high academic standards of the MERGA community were upheld.

Figure 1*MERGA Review and Revision Process for Research Papers***Thank you**

We would like to thank all the many colleagues who have helped bring MERGA48 together, the Local Organising Committee, the MERGA Executive, the School of Education at Western Sydney University, AAMT CEO Allan Dougan, and our sponsors.

- Platinum Level: School of Education, Western Sydney University
- Gold Level: The Australian Academy of Science
- Bronze Level: AAMT, Springer Nature

A special word of thanks goes to Dr Michelle Locke, Dharug woman and Senior Lecturer in the School of Education for conducting the Welcome to Country at the Opening Ceremony.

Professor Catherine Attard (Conference Convenor)

Associate Professor Nathan Berger (Chief Proceedings Editor)

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Clements-Foyster Lecture

The Clements-Foyster Lecture acknowledges an eminent mathematics education researcher from Australia, New Zealand, or a South East Asian rim country, who is invited to present a keynote address at the annual MERGA conference. This annual keynote address is named in honour of Ken Clements and John Foyster who initiated and organised the first Mathematics Education Research Group of Australia Conference at Monash University in 1977. This led to the establishment of the organisation now known as MERGA.

Affect in Mathematics Education: Blurring the Boundaries

Kim Beswick

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In this paper I describe the status of affect as an educational outcome and argue for more meaningful integration of affective and cognitive aspects of mathematics learning and mathematics education research. I provide examples from projects with which I have been involved of directions that could be pursued towards this goal and present systems thinking as a perspective that offers considerable explanatory value and can foreground the whole of cognition and affect as more than its constituent parts allowing a more realistic approach to and view of the likely impacts of interventions.

The study of affect in mathematics education has been of interest to researchers for many decades with foci following developments in funding and access to research participants as well as conceptual progress. There is consensus that students should, in addition to learning mathematical content, develop positive attitudes, self-efficacy, and confidence in relation to the subject, and the ability to flexibly apply the mathematics that they know. To assist students in acquiring these capabilities, teachers need also to have them. There has, consequently been considerable attention to the affect of teachers, particularly primary school teachers, often preservice, many of whom have negative views of mathematics and of their own ability to learn and teach it (Itter & Meyers, 2017). Mathematics anxiety has been of particular interest in relation to these teachers as well as students.

The field has been characterised by a proliferation of terms to describe the aspects of affect that are being considered, along with pleas, only partly successful, for researchers to define precisely what they mean by whichever terminology they use. Attempts to systematise the study of affect have been built on recognition that affect and cognition are not neatly separable. They have included continua ranging from constructs considered most affective (i.e., emotions) to those considered to have a considerable cognitive component (i.e., beliefs) (McLeod, 1992), and models that take account of other dimensions of affective constructs. Hannula (2012), for example, proposed categorising affective constructs according to three dimensions: cognition, motivation, and emotion, each of which can be considered as states or traits and can be examined from physiological, psychological or social perspectives.

Much attention has been devoted to examining the beliefs of teachers of mathematics and how they might be influenced in ways considered desirable by researchers and, although the literature is replete with accounts of changed practices resulting from specific projects some of which have included a focus on affect, the teaching of mathematics at scale has changed little (Beswick et al., 2023). In keeping with the theme of this conference, *Beyond Boundaries: Strengthening Futures in Mathematics Education*, there have also been calls to integrate affect into content focussed mathematics education research which have resulted in aspects of affect being considered in addition to mathematical attainment. These attempts have, however, often fallen short of genuine integration. It is worth bearing in mind that, in research, analytic approaches whereby broad concepts are broken into separately examinable components to allow detailed investigation are useful, but the work of re-constituting the pieces is arguably as important yet receives less attention.

In my research I have considered what teachers believe about the nature of mathematics and the logical implications of these beliefs for their beliefs about how mathematics is best taught and learned (e.g., Beswick, 2012). There is explanatory value in considering beliefs as constituting systems. In these systems teachers' beliefs about themselves in relation to

mathematics and mathematics teaching and about their students' capacities to learn the subject are especially consequential for their practice (Beswick, 2018). Although teachers' knowledge and their beliefs can and usually are considered separately (even within single studies) I have argued that distinguishing these constructs, while empirically useful, disguises the practical influence of beliefs relative to knowledge. Beswick (2011) provided a detailed argument for this stance.

Affect as an educational outcome

Although from a theoretical standpoint the boundaries between affect and cognition are not clear cut there is considerable and consequential divergence in the ways in which they are treated and valued in practice. Affective constructs feature, for example, in high-level statements about the aims of education such as the Alice Springs (Mparntwe) Education Declaration (Council of Australian Governments Education Council, 2019) which includes in the second of its two goals for education that all young Australians become "confident and creative learners" (p. 4). This is unpacked in terms of a range of affective constructs including self-worth, self-awareness, self-identity, emotional wellbeing, personal values such as honesty, loyalty, responsibility and respect, resilience, adaptability, optimism, initiative, creativity, and imagination. Other aspects of the goals refer to motivation, confidence, moral and ethical integrity, empathy, and civic values.

Curriculum documents that have been created since the Declaration have mentioned many of these. The Australian curriculum (Australian Curriculum and Reporting Authority, [ACARA], 2023) for example, makes explicit reference to the declaration. The most recent version, version 9, comprises three dimensions, namely learning areas or subjects, general capabilities, and cross-curriculum priorities. General capabilities include critical and creative thinking which is purported to support, among other things, "Dispositions such as inquisitiveness, reasonableness, intellectual flexibility, open- and fair-mindedness, and a readiness to try new ways of doing things", ethical understanding that includes an understanding of the ways in which their values impact others, and personal and social capability, elements of which are self-awareness encompassing personal awareness including of their feelings, and emotional awareness. Social awareness includes empathy and both relational and community awareness (ACARA, 2023).

Numeracy is another of the general capabilities included in the Australian Curriculum. Its definition includes knowledge and skills along with the ability and confidence to apply mathematics in everyday life (ACARA, 2023). This formulation resonates with the description of numeracy of Geiger et al. (2015) who additionally emphasised the importance of a critical orientation that supports questioning, and critique as an overarching aspect of numeracy. Numeracy thus defined includes both cognitive and affective aspects.

Despite the prominence of affective outcomes in key documents intended to guide the teaching of mathematics, such outcomes appear to receive little attention in practice compared with knowledge and skills. At least part of the reason for this is likely the relative difficulty of assessing affective outcomes in ways analogous to the regular and familiar approaches to assessing cognitive mathematical outcomes. Although there is a plethora of instruments used to measure various aspects of affect in psychological studies, assessing students' affect in everyday classrooms is necessarily subjective and unreliable compared to assessments of cognitive outcomes (although this is not as clear cut as might be assumed). As a result, it is difficult to imagine how affective outcomes might be taken as seriously as cognitive outcomes (i.e., achievement). Nevertheless, high achievement with negative affect does not suggest a student likely to exhibit numerate behaviour or to continue studying mathematics beyond when it is compulsory or of other utilitarian value, yet we know that this describes a great many students at the end of secondary schooling. One wonders whether such an outcome is (or should

be) more or less acceptable than its converse: relatively low achievement with positive affect. Of course we want both. One possible direction in which an answer to this conundrum might be found is in the integration of affective and cognitive outcomes such that achievement is construed and assessed as entailing relevant affect. In addition, there are pedagogical approaches and organisational choices in schools that can be part of a commitment to taking student affect seriously.

Approaches to integrating cognition and affect

In this section I describe in chronological order five examples from projects with which I have been involved to illustrate attempts at incorporating affect and cognition to demonstrate ways in which the integration of cognition and affect might be achieved. Some examples provide evidence of the utility of basing research on frameworks that include both cognitive and affective constructs. In many, if not all, one can reasonably infer that beliefs about the self (i.e., aspects of identity) underpin observable phenomena. Each offers an invitation for further research using or adapting the tools described or further developing or challenging the theoretical ideas and frameworks.

Empirical evidence for the integration of affect and cognition

As part of the Providing the Mathematical Foundation for an Innovative Australia within Reform-Based Learning Environments (MARBLE) project, Beswick et al. (2012) conceptualised teacher knowledge as including the seven types of knowledge identified by Shulman (1987), the knowledge categories identified by Ball (Ball et al., 2008) along with confidence to teach various aspects of the mathematics curriculum (e.g., fractions, mental computation, chance and data), and beliefs about such things as numeracy in the classroom (e.g., Teachers of mathematics should be fascinated with how children think and be intrigued by alternative ideas), and numeracy in everyday life (e.g., I often use mathematics to make decisions and choices in everyday life). The profile instrument comprised open response and Likert type items and was completed by 62 Year 5-8 teachers at the start of their involvement in a professional learning program. The use of Rasch measurement techniques showed that the profile was a measuring a single underlying construct thereby demonstrating that teacher knowledge can be considered a rich uni-dimensional construct that incorporates many elements.

Considering the many facets of knowledge on a single scale allowed us to see that confidence, which is often regarded as a dimension of attitude (Beswick et al., 2006), was evident at levels of knowledge that demanded little by way of content or pedagogical content knowledge suggesting that pursuing this aspect of positive affect for pre-service teachers independent of attention to cognitive aspects of knowledge may not be wise and certainly not sufficient (Beswick et al., 2012). The authors noted that there is, “value in pursuing holistic conceptions and measures of mathematics teachers’ knowledge as well as ongoing detailed analyses of categories”. There is scope for the use of this of similar instruments in investigations of teacher knowledge, broadly defined including to test whether the findings apply to teachers in other contexts and with differing mathematical backgrounds.

Extending the conceptualisation of mathematical pedagogical content knowledge

Beswick and Chick (2020) presented a framework for teachers’ pedagogical content knowledge that built on prior work by Chick and colleagues (e.g., Chick et al., 2006; Chick and Beswick, 2017; and Dymont et al., 2018) and that incorporated aspects of teachers’ beliefs. Beliefs related to each of the three categories of PCK: first, aspects of PCK in which content and pedagogy are indistinguishable (Clearly PCK), second, aspects of PCK in which content knowledge appears to be prominent but deployed in a pedagogical context, and third, the converse in which pedagogical knowledge is used in a content context. Table 1 provides excerpts from the framework that concern beliefs.

Table 1*Aspects of PCK concerning teachers' beliefs in Beswick and Chick (2020)*

<i>PCK category</i>	<i>PCK for School Mathematic Teachers (SMTPCK)</i>	
	<i>Evident when the teacher ...</i>	<i>Example</i>
<i>Clearly PCK</i>		
Beliefs about The Nature of Teaching and Learning	Discusses or uses personal or established philosophies and approaches to teaching the discipline	Discusses the use of pedagogies consistent with a constructivist view of learning as a key facet of teaching mathematics; requires students to memorise times tables because of a belief that students should have fast recall of key number facts
Student Affect (in relation to content)	Discusses or addresses students' affective responses to particular mathematics content	Recognises that adolescent students may have negative emotional reactions to the prospect of learning algebra
<i>Content Knowledge in a Pedagogical Context</i>		
(Beliefs about) The Nature of Content	Expresses an appreciation of the nature of mathematics that goes beyond the school curriculum and aligns with mathematicians' views of the discipline	Compares the aesthetic qualities of two solution methods
<i>Pedagogical Knowledge in a Content Context</i>		
Student affect (general)	Describes how student affect influences pedagogical approach	Knows a particular student will respond negatively to being asked for an answer in a large group session

The framework was used as a lens to consider the PCK of experienced mathematics teachers comprising two focus groups in which they discussed, among other things a student's non-standard but mathematically correct response to a fraction division task as part of the Powerful Knowledge project led by Rosemary Callingham (Beswick & Chick, 2016). The inclusion of beliefs in the framework allowed their interaction with knowledge (traditionally defined) to be seen more clearly than had been the case with earlier versions. In addition to the belief categories incorporated in the framework, beliefs about themselves and their efficacy as teachers of mathematics and beliefs about the capabilities of their students were prominent in the teachers' conversations. Beliefs about themselves and particularly how difficult they found the task, appeared to influence the teachers' judgements of the cognitive demand of the task. Beliefs about student affect were evident in such things as their attribution of motives (e.g., laziness, enjoyment) to students' behaviours. Beliefs in these two areas are arguably among the most important in shaping teachers' practice (Beswick, 2018) and Beswick and Chick (2020) conceded there is a case for their inclusion in the framework. Further research testing the value and comprehensiveness of the framework and modifications thereof is warranted along with its use to examine PCK for mathematics teaching in highly granular and nuanced ways that can get more precisely than other measures to what it is that teachers of mathematics know and use in their teaching.

Conceptualising Science, Technology, Engineering and Mathematics capability

Geiger et al. (2023) developed a framework for principals' Science, Technology, Engineering and Mathematics (STEM) capability that drew on earlier work on numeracy (e.g., Goos et al., 2014). The framework, which underpinned the Principals as STEM Leaders (PASL) project, comprises four aspects: STEM discipline and interdisciplinary knowledge and practices, Task, Context, and Dispositions, all embedded in a Critical Orientation. Each of these aspects were unpacked as a series of elements that recognised the role and responsibilities of the principal. For example, the knowledge aspect was framed in terms of awareness rather than as an expectation that a principal should have knowledge of STEM disciplines in the same way that a teacher of a STEM discipline would need. The elements related to dispositions are shown in Table 2.

Table 2

Elements of STEM capability for principals related to Dispositions

Dispositions
Belief that all students and teachers can develop/enhance their STEM capability
Openness to the implementation of innovative STEM teaching practices
Confidence that innovative STEM programmes can be initiated, managed and brought to fruition
Flexible and adaptable change leadership practices so that all feel supported and encouraged in their engagement with the STEM project, regardless of differences in levels of knowledge, skill and confidence
Willingness to be personally and actively involved in establishing and sustaining the school STEM teaching and learning programmes
Flexible and adaptive thinking as their institution continues to innovate (e.g., openness to changes in school curriculum delivery structures and practices)

As can be seen from Table 2, Dispositions include beliefs and confidence along with qualities such as openness, flexibility, willingness and adaptability. These latter qualities are likely underpinned by principals' beliefs about themselves, their role and their efficacy in that role especially as it relates to fostering a positive STEM culture in their school.

In relation to dispositions Fraser et al., (2021) reported that principals in the PASL project were committed to equity through developing the STEM capability of all students, not only high achievers in mathematics and science, and that teachers agreed that principal dispositions were key to progressing STEM education in their schools. Among the recommendations in the final report of the project was that:

Greater attention should be paid to the development of positive dispositions towards STEM among principals, teachers, students, and school communities; to build parental awareness of the importance of STEM learning as relevant for all; and to acknowledge its various demands in principals' professional learning programs.

Inclusive mathematics teaching as site for difficult questions about affect

Inclusive education is founded on a series of rights declarations (UNESCO 1994, 2020; United Nations 1989, 2006) and subsequent legal requirements (e.g., in Australia the Disability Discrimination Act (1992), Disability Standards for Education (2005)). The Disability Discrimination Act includes three requirements that constitute legal obligations for Australian schools and teachers. These are:

1. The right to education on the same basis as other students.

2. Reasonable adjustments are made to ensure that students with disability have opportunities to participate in education and training.
3. Consultation between the school, parents/carers and student takes place prior to adjustments being made.

In addition, teachers feel a moral imperative to meet the learning needs of all their students (Beswick et al., 2025).

Inclusion appears to be a context in which the obligations and demands upon, and the aspirations of teachers to do the best for their students meet in ways that are emotionally charged as a result of the perceived difficulty of the task and inadequate resourcing, and conflicting beliefs about how teachers perceive their role: is it to teach those able to learn from standard approaches or to endlessly differentiate for individuals? Nilsson (2026) used affect to describe emotions or feelings which she considered dynamically constructed through interactions within classrooms, schools and the broader society to analyse what she described as a negative mood surrounding inclusive education in Denmark. Central to her analysis was the “inclusion-child”, a construct to which teachers’ feelings of frustration, guilt and fear adhere construing inclusion as an impossibly burdensome task. Conversely, Nilsson (2026) also noted that some teachers expressed pride and satisfaction when they believed they had achieved something positive in relation to inclusion suggesting that a positive reframing is at least possible. Nilsson’s framing of emotions as attaching to objects and carrying meaning is consistent with the view that emotions, like beliefs, constitute systems (DeBellis & Goldin, 2006). Such systems can be considered to exist separately from or as part of belief systems.

The Learning Year Level Appropriate Curriculum (LYLAC) project, led by Rhonda Faragher, and its antecedent projects have been sites in which we have confronted and considered sometimes uncomfortable issues about inclusion. In terms of affect we have wondered whether affective outcomes take on greater importance for students with intellectual or development difficulties compared with their higher attaining peers because positive affect is implicitly seen as a consolation prize rather than the main objective of schooling. This highlights our earlier questioning of the status of affective relative to cognitive outcomes of education. Faragher et al. (2019) reported findings from a smaller study involving students with Down syndrome that focussed on the positive affect these students experienced in inclusive settings. In each case the students were also able to engage with the same lesson as their year level peers with minimal adjustments or none at all. We have wondered also whether low attainers are uniquely scrutinised as to the extent of their ‘understanding’ of mathematical concepts: We know that many students successfully complete mathematical tasks with minimal conceptual understanding, yet the veracity of their attainment is rarely questioned.

Conceptualising critical mathematical thinking

Geiger et al. (2023) provided a conceptualisation of critical mathematical thinking (CMT) as the basis for critical citizenship. That is, CMT is a quality that empowers individuals to deliberate upon and make decisions about mathematical solutions to real world problems taking account not only of the mathematics but also of the social and environmental implications of various courses of action. They have argued that responses to social and ecological disruptions that increasingly characterise human experience require more than purely cognitive analyses (Geiger et al., 2025). Their framework comprises five dimensions: Mathematical capability, Critical capability, Evaluating, Reasoning, and Dispositions with the latter in its initial form defined as “Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, and displaying perseverance”. This aspect is key to students’ engagement with socio-ecological issues in ethical and responsible ways (Geiger et al., 2024).

The framework was the basis for the development of the Critical Mathematical Thinking Noticing Instrument (CMTNI) which has been used with teachers involved in the project

(Geiger et al., 2025). The instrument comprises six 2–3-minute video vignettes of teacher-student and/or student-student interactions. The teachers can see each vignette just once and then respond to a series of questions related to their noticing of aspects of CMT with reference to the CMT framework. Initial use of the CMTNI has demonstrated differences in the breadth of aspects of CMT to which teachers refer in their responses.

The framework has also been used by teachers as they plan to implement the tasks used in the project and to define the components of CMT for which the teacher is looking as their students work on the tasks. Geiger et al. (2025) reported on the use of one of the project tasks with a Year 10 class. The task is shown in Figure 1. In each class in which the task was used there were some students who responded from a purely financial perspective, asserting the right of owners to do as they wish with their property. In the example described by Geiger et al. (2025) the teacher's prompts for students to consider ethical issues and the plight of those disadvantaged by the current housing market were insufficient to move all students beyond consideration of the financial impacts on AirBnB owners. Subsequent discussions with teachers resulted in one coining the term *empathy primer* to describe the use of an introductory discussion focussing on the importance of empathy and compassion in CMT before a specific problem context is presented. This approach has been broadly adopted across the project and appears to be successful in broadening students' thinking to include the implications of possible solutions. The disposition dimension of the CMT framework has consequently been amended to read, "Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, displaying perseverance, openness to alternative solutions, *preparedness to empathise and show compassion*" (Geiger et al., 2025).

Figure 1

AirBnB and the rental crisis

Are short-stay properties like Airbnb the reason for the rental crisis?

The following video and media report discuss the apparent shortage of affordable rental accommodation because owners are making their properties available for short-term arrangements. This raises the concern that such properties are taken out of the pool of available housing for permanent residents.

Please discuss whether the Brisbane City Council (media report) made the right decision to impose a registration fee on properties that offer short-stay opportunities through online agents (e.g., Airbnb and Stayz) or are they overreacting? Justify your response using available information about the availability of affordable accommodation.

Investigate the rental prices and short stay accommodation in Gympie. Based on your findings suggest whether the Gympie Regional Council should implement a short stay registration fee as imposed in Brisbane. Who would be advantaged or disadvantaged? Suggest other possible avenues to alleviate the rental problems in Gympie. Justify all responses using available information.

It will be interesting as the project draws to a close to analyse our data which include considerable video footage for evidence of the range of aspects of affect that teachers and students display, and the ways in which the affective aspects of CMT interconnect with other dimensions of the construct.

Systems thinking

The examples presented in this paper relied with varying degrees of explicitness on an underlying assumption that beliefs and other aspects of affect exist not in isolation but as parts of systems. Systems thinking has been relatively underutilised in mathematics education research but has the potential to offer much to efforts to integrate cognition and affect. Indeed, it has much to offer the study of these domains separately as well. To avail ourselves of the opportunities system thinking affords will require ongoing innovation in our theorising and methodological approaches. Following is an overview of systems thinking and an outline of

two forms of such thinking that have been useful and that show promise for progressing our understanding of affect in mathematics education research.

Systems thinking arose in the 20th century, principally in the biological sciences. It offers a way of holistically considering relationships between the interacting parts that comprise a system and the system as a whole. A system can exhibit properties that are not present in the parts, that is, the system has properties that are emergent from the interaction of its constituent parts. As a result, analytic approaches that break systems into parts can fail to discern the crucial features of the system as a whole.

Belief systems (Green, 1971)

Beliefs (and other aspects of affect) have been acknowledged to constitute systems. It is unreasonable and likely misleading, for example, to expect that a particular belief will correspond with a particular behaviour even when that belief appears to be the logical antecedent of the behaviour. Instead, behaviours are based on dynamically interacting groups of beliefs whose composition and relative importance to the behaviour of interest vary with context. Failure fully to appreciate this has been a reason for which observations such as that a teacher professes to value group work but is not observed using it have been characterised as contradictions between espoused and enacted beliefs. Green's (1971) description of belief systems has been a powerful and widely used for many years (e.g., Grootenboer & Marshman, 2016; Haser & Doğan, 2012; Mewborn, 2002) as a means of understanding such phenomena in terms that honour the integrity of teachers.

According to Green, beliefs vary in terms of the strength with which they are held with more strongly held beliefs considered more central. Centrality is also a function of the extent to which a belief is connected to others in the system, a fact that makes beliefs acquired earlier in life more likely to be central and that contributes to central beliefs being resistant to change. It has been argued that the beliefs an individual has about themselves are among their most central (e.g., Beswick, 2006; 2018). Belief systems are structured such that some beliefs are the consequences of others, that is they are derivative of other beliefs, chains of which can be traced by questioning the basis of successive beliefs offered as reasons for another until the chain ends with a belief that is primary in that it is not derived from another (Green, 1971). Green also suggested that beliefs can be held in clusters, that is groups of beliefs that are held in isolation from others. This is most likely to occur when beliefs arise in different contexts and means that an individual can unknowingly hold conflicting beliefs. There is also evidence that clusters might be created, probably unconsciously, to avoid the need to reconcile conflicting beliefs. Schuck (1999), for example, suggested that pre-service primary teachers who believed that learning mathematics should be enjoyable, but who had negative beliefs about their ability to teach the subject constructed the belief that knowing mathematics is not necessary for teaching it effectively. This allowed them to preserve a belief in themselves as potentially effective teachers. Beswick (2006) observed, based on Schuck's work, that clusters of beliefs, for example about mathematics and mathematics teaching, may form, or emerge, from the system, in order to avoid confronting conflicts. Finally, Green (1971) pointed to beliefs being either evidentially or non-evidentially held, with the latter impervious to contradictory evidence that since they are held on the basis of the authority of the source of the belief or because they support other, likely central, beliefs.

In contrast to beliefs which are largely cognitive, emotions are “feelings or states of consciousness, distinguished from cognition” (Philipp, 2007, p. 259) and include a physiological response (Hannula, 2002). They have been described as the energy that drives cognition (Evans, 1999) and as influencing cognition by biasing attention and memory (Hannula, 2002). Beswick (2011) argued that beliefs and emotions, although possibly constituting separate systems (DeBellis & Goldin, 2006) are intimately connected. Beliefs may

be associated with emotions and emotions with beliefs but both beliefs and emotions may also encode information independently, that is without such associations. Beliefs may also evoke emotions and emotions may evoke beliefs with the order often difficult to discern. For example, when confronted with an apparently difficult mathematics problem a student may be reminded that they have been unsuccessful with similar problems in the past, evoking both the belief that they are unlikely to be successful and the emotion of anxiety. This reaction may surface a further belief that despite the discomfort they have sometimes been successful with difficult problems when they have remained calm and made an attempt, leading to a moderation of their emotional state and, should success ensue, emotions of joy and pride and strengthened beliefs in their capacity to solve difficult problems.

An individual's most central beliefs such as those about themselves are likely to be strongly connected to emotions with the linkages with emotions adding to the sum of their connections and hence their centrality (Beswick, 2011). Challenge of centrally held beliefs is likely to give rise to negative emotions manifesting as resistance and adding to the upheaval that is both cognitive and affective (Beswick, 2011). Emotions seem also to be related to memory formation with strong emotional responses at the time of an event becoming part of the memory (McClay et al., 2023).

Complex systems

Complexity science offers an enriched understanding of Green's description of beliefs systems that adds to its explanatory power. Complex systems typically involve collections of living, but not necessarily conscious, agents that are at some level autonomous. Interactions among agents give rise to characteristics that are not attributable to any particular individuals in the system but rather are emergent properties of the system. Complex systems are commonly nested within other complex systems forming hierarchical structures. For example, cells, organs, individuals, social groups and societies can all be regarded as complex systems with each emerging from the interactions of agents comprising the previous level (Davis & Simmt, 2003). In addition, complex systems are self-organising in that they form patterns in the absence of central coordinator (e.g., flocks of birds) assisted by multiple feedback loops and non-linear interactions among agents. These features contribute to the ability of complex systems to embody their histories and adapt or learn such that the reaction of the system to an influence upon it may differ on different occasions (Miller et al., 2001).

Davis and colleagues (Davis, 2004; Davis & Simmt, 2003; Davis & Sumara, 2005) have proposed five necessary but insufficient conditions for the emergence of complexity from the interactions of a collective. These are *diversity* which refers to differences among agents in the system that allow the possibility of new responses. It is balanced by *redundancy*, the common ground among agents that allows meaningful interactions. *Enabling constraints* limit the activity of the collective in ways that provide focus while still allowing diversity to be expressed. *Decentralised control* emphasises the autonomy of agents from whose actions complexity emerges in ways that cannot be planned or predicted. *Neighbour interactions* occur between agents and are facilitated by redundancy. In educational contexts they most importantly refer to interactions between ideas.

Beswick (2006; 2018) described in detail how belief systems, understood according to Green's (1971) description can be considered complex. She argued that they are characterised by self-organisation, emergence, nestedness, and include networks of connections facilitating feedback loops and non-linear interactions among the agents which are in this case individual beliefs. If we accept that belief systems are complex, it makes sense that the five necessary but insufficient conditions for complex emergence identified by Davis and colleagues may also apply. Belief systems are characterised by diversity both in terms of the subjects of beliefs and the range of beliefs that can be held about specific objects. Redundancy arises from the

multiplicity of connections among beliefs in a system. Beswick (2018) provided the example of teacher who believes that students should do regular homework perhaps because they also believe that establishing regular study habits is important and that homework is an important way to consolidate new learning. Each of these beliefs may also be derivative in that they follow from the teacher's own experiences as a learner, though not necessarily the same ones. Derived beliefs may not, therefore, necessarily be undermined if one or maybe many of the related beliefs was no longer held and in that sense some beliefs are redundant. Redundancy thus seems connected to the centrality of beliefs because more centrally held beliefs have more connections to other beliefs in the systems and hence are in a sense protected by the redundancy in the system that the connections afford.

Much of the activity of belief systems appears to happen unconsciously rather than being controlled by a central authority and therefore are characterised by decentralised control. Nevertheless, the conscious mind is able to direct an individual's attention casting it in the role of providing enabling constraints analogously to the way that Davis (2005) described the teacher in the complex system of a classroom as the consciousness of the collective, making decisions about the focus and direction of collective activity. Neighbour interactions exist in the connections between beliefs.

Complexity also reminds us that despite our efforts to direct learning in both classrooms and professional learning contexts, the learning (new knowledge and affect) that emerges may be surprising. Our best efforts can set conditions that make the emergence of intended outcomes more likely but cannot guarantee their achievement.

Conclusion

Although positive affective outcomes are universally endorsed as valuable their value in practice remains limited by difficulties in assessing them. This difficulty is particularly evident in mathematics where traditional pen and paper tests that focus on the accurate and timely performance of algorithmic solutions continue to be the norm. I have argued that removing or blurring the boundaries between cognitive and affective outcomes offers possibilities for taking affective outcomes more seriously. If knowing mathematics is understood as not only being able to solve mathematical problems but also to have productive beliefs about the subject, positive emotions in relation to it, well-founded confidence in one's ability to use mathematical tools in unfamiliar as well as routine tasks, and a critical orientation to the use of mathematics, then supporting the development of students' positive affect becomes not just a nice additional outcome but central to what it means to teach mathematics.

The explicit inclusion of affective constructs in frameworks such as those described in the examples provided in this paper offer opportunities to explore interconnections among aspects of teachers' and students' affect, for example between their beliefs and emotions (compassion and empathy) related to CMT, or between aspects of affect and cognition, such as between knowledge, beliefs and confidence in the MARBLE project. A systems perspective makes this an entirely natural and comprehensible thing to do.

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References

- Australian Curriculum, Assessment and Reporting Authority. (2023). *Australian Curriculum: Version 9.0*. <https://www.australiancurriculum.edu.au/>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education*, 59(5), 389-407.
- Beswick, K. (2011). Knowledge/beliefs and their relationship to emotion. In K. Kislenko (Ed.), *Current state of research on mathematical beliefs XVI: Proceedings of the MAVI-16 conference June 26-29, 2010* (pp. 43-59). Institute of Mathematics and Natural Sciences, Tallinn University.
- Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics: An International Journal*, 79, 127-147.
- Beswick, K. (2018). Systems perspectives on mathematics teachers' beliefs: Illustrations from beliefs about students. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3-18). PME.
- Beswick, K., Callingham, R., & Watson, J. M. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education*, 15(2), 131-157. <https://doi.org/10.1007/s10857-011-9177-9>
- Beswick, K., & Chick, H. (2020). Beliefs and content knowledge for teachers of mathematics. In D. Potari & O. Chapman (Eds.), *International handbook of mathematics teacher education: Knowledge, Beliefs and identity in mathematics teaching and teaching development* (Vol. 1, pp. 185-209). Brill Sense.
- Beswick, K. & Chick, H. (2016). Responses to "The Scary Question": How teaching challenges impact the use of knowledge and its development. In Csíkos, C., Rausch, A., & Szitányi, J. (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2 (pp. 91-98). Szeged, Hungary: PME.
- Beswick, K., Faragher, R. & Cuskelly, M. (2025). Reasonable adjustments of inclusive teaching? The potential of Universal Design for Learning. In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia*, MERGA: Canberra.
- Beswick, K., Rawlings-Sanaei, F., & Tuohilampi, L. (2023). Interactive mathematics teacher activities. In *The evolution of research on teaching mathematics: International perspectives in the digital era*. (Eds. A. Manizade, N. Buchholtz, & K. Beswick) (pp. 135-155). Cham: Springer.
- Beswick, K., Watson, J. M., & Brown, N. (2006). Teachers' confidence and beliefs and their students' attitudes to mathematics. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces: Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 68-75). MERGA.
- Chick, H. L., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Křátek, & N. Stehlíková (Eds.), *Proceedings of the 30th annual conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 297-304). Prague: PME.
- Chick, H., & Beswick, K. (2017). Teaching teachers to teach Boris: a framework for mathematics teacher educator pedagogical content knowledge [journal article]. *Journal of Mathematics Teacher Education*, 1-25. <https://doi.org/10.1007/s10857-016-9362-y>
- Council of Australian Governments Education Council. (2019). *Alice Springs (Mparntwe) Education Declaration*. Education Services Australia Retrieved from <https://www.education.gov.au/alice-springs-mparntwe-education-declaration>
- Davis, B. (2004). *Inventions of teaching: A genealogy*. Lawrence Erlbaum.
- Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education*, 34(2), 137-167. (137-167)
- Davis, B., & Sumara, D. (2005). Complexity science and educational action research: towards a pragmatics of transformation. *Educational Action Research*, 13(3), 453-464.
- DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational studies in mathematics*, 63(2), 131-147.
- Disability Discrimination Act of 1992. <https://www.legislation.gov.au/C2004A04426/latest/text>.
- Disability Standards for Education of 2005. <https://www.legislation.gov.au/F2005L00767/latest/text>
- Dymont, J. E., Chick, H. L., Walker, C. T., & Thomas, P. N. (2018). Pedagogical content knowledge and the teaching of outdoor education. *Journal of Adventure Learning and Outdoor Learning*. <https://doi.org/10.1080/14729679.2018.1451756>

- Evans, J. (1999). Building bridges: Reflections in the problem of transfer of learning in mathematics. *Educational studies in mathematics*, 39, 23-44.
- Faragher, R., Beswick, K., Cuskelly, M., & Nankervis, K. (2019). The affective impact of inclusive secondary mathematics for learners with Down syndrome: "I just love it!". In G. Hine, S. Blackley, & A. Cooke (Eds.), *Mathematics Education Research: Impacting Practice (Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia)* (pp. 260-267). MERGA.
- Geiger, V., Beswick, K., Fielding, J., Goos, M., Kaiser, G., Scheiner, T., & Schmid, M. (2025). The role of compassion and empathy when engaging with tasks concerned with social and environmental justice. In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia*, MERGA: Canberra.
- Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., Goos, M., & Fernandez, K. (2023). Investigating critical mathematical thinking when applying mathematics to real-world problems. In P. Drijvers, C. Csapodi, H. Palmer, K. Gosztonyi, & E. Kónya (Eds.), *Proceedings of the thirteenth congress of the European Society of Research in Mathematics Education* (pp. 1177-1184). European Society of Research in Mathematics Education. http://erme.site/wp-content/uploads/2024/01/CERME13_proceedings_frontmatter-1.pdf
- Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., Goos, M., & Fernandez, K. (2024). *Capturing snapshots of teachers' critical mathematical thinking noticing capabilities in the context of mathematical modelling tasks* The 15th International Congress on Mathematical Education, Sydney, Australia.
- Geiger, V., Beswick, K., Fraser, S., & Holland-Twining, B. (2023). A model for principals' STEM leadership capability. *British Educational Research Journal*, 49(5), 900-924. <https://doi.org/https://doi.org/10.1002/berj.3873>
- Geiger, V., Scheiner, T., Beswick, K., Fielding, J., Goos, M., Kaiser, G., Fernandez, K., & Awan, S. (2025). Critical mathematical thinking: A framework and illustrative examples. In *Proceedings of the 27th ICMI Study Conference: Mathematics Education and the Socio-Ecological*. International Commission on Mathematical Instruction.
- Goos, M., Geiger, V., & Dole, S. (2014). Transforming professional practice in numeracy teaching. In Y. Li, E. A. Silver, & S. Li (Eds.), *Transforming Mathematics Instruction: Multiple Approaches and Practices* (pp. 81-102). Springer International Publishing. https://doi.org/10.1007/978-3-319-04993-9_6
- Green, T. F. (1971). *The activities of teaching*. McGraw-Hill.
- Grootenboer, P., & Marshman, M. (2016). *Mathematics, affect and learning: Middle school students' beliefs and attitudes about mathematics education*. Springer.
- Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: embodied and social theories. *Research in Mathematics Education*, 14(2), 137-161.
- Haser, Ç., & Doğan, O. (2012). Pre-service mathematics teachers' belief systems. *Journal of Education for Teaching*, 38(3), 261-274.
- Itter, D., & Meyers, N. (2017). Fear, loathing and ambivalence towards learning and teaching mathematics: Preservice teachers' perspectives. *Mathematics Teacher Education and Development*, 19(2), 123-141.
- McClay, M., Sachs, M. E., & Clewett, D. (2023). Dynamic emotional states shape the episodic structure of memory. *Nature Communications*, 14(1), 6533. <https://doi.org/10.1038/s41467-023-42241-2>
- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). Macmillan Publishing Company.
- Mewborn, D. S. (2002). *Examining mathematics teachers' beliefs through multiple lenses* Annual Meeting of the American Educational Research Association, <https://files.eric.ed.gov/fulltext/ED465526.pdf>
- Miller, W. L., McDaniel, R. R., Crabtree, B. F., & Stange, K. C. (2001). Practice jazz: Understanding variation in family practices using complexity science. *The journal of family practice*, 50(10).
- Nilsson, I. A. (2026). Caught in a bad mood: The affective economy of inclusive education. *International Journal of Inclusive Education*, 1-14.
- Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (Vol. 1, pp. 257-315). Information Age Publishing.
- Schuck, S. (1999). Teaching mathematics: A brightly wrapped but empty gift box. *Mathematics education research journal*, 11(2), 109-123.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.
- UNESCO. (1994). The Salamanca Statement and Framework for Action on Special Needs Education. <https://unesdoc.unesco.org/ark:/48223/pf0000098427>. UNESCO. 2020. Final Report of the International

- Forum on Inclusion and Equity in Education – Every Learner Matters.
<https://unesdoc.unesco.org/ark:/48223/pf0000372651>.
- United Nations Committee on the Rights of Persons with Disabilities. (2016). General comment No. 4.
United Nations. <http://www.ohchr.org/Documents/HRBodies/CRPD/GC/RighttoEducation/CRPD-C-GC-4.doc>
- United Nations. (1989). Convention on the Rights of the Child. <https://www.ohchr.org/en/instruments-mechanisms/instruments/convention-rights-child>.
- United Nations. (2006). Convention on the Rights of Persons with Disabilities. <https://www.ohchr.org/en/instruments-mechanisms/instruments/convention-rights-persons-disabilities>.

Keynote Presentations

Fostering Students' Mathematical Understanding through Problem-Posing-Based Learning (P-PBL)

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This paper examines the nature of mathematical understanding and explores how Problem Posing-Based Learning (P-PBL) can foster it using a framework proposed by Cai et al. (2026) that conceptualizes mathematical understanding through five interconnected aspects: factual, procedural, conceptual, metacognitive, and affective. It also presents five lenses for examining and fostering understanding: mathematical, cognitive, social, diagnostic, and instructional. Future directions for research are proposed, including the use of the proposed framework to synthesize existing studies using AI and further investigate how P-PBL can support mathematical understanding.

What is mathematical understanding? How should we foster students' mathematical understanding? These are the two most debated questions in mathematics education (Cai et al., 2026), and they are important because fostering students' mathematical understanding is a widely accepted goal in the teaching and learning of mathematics (Hiebert & Carpenter, 1992). In this paper, I will first discuss a framework to understand mathematical understanding and then discuss how Problem Posing-Based Learning (P-PBL) can foster students' understanding.

What Is Mathematical Understanding?

Processes versus Products

Mathematical understanding is a critical topic that has been extensively studied at least since Brownell (1935). There are at least two major debates surrounding mathematical understanding. The first is the product-process debate. Greeno (1978) stated that understanding is a process leading to "a structure that represents the relations among components of the idea that is understood" (p. 267). Similarly, Pirie and Kieren (1992, 1994) viewed mathematical understanding as a dynamic and continual process of organizing one's knowledge structure. Other researchers have used a product view to examine understanding. For example, Davis (1992) perceived understanding as "when a new idea can be fitted into a larger framework of previously assembled ideas" (p. 228). Hiebert and Carpenter (1992) defined mathematical understanding as an internal network of representations of mathematical ideas, procedures, and facts in which there are two kinds of connections: one based on the similarities and differences between different external representations or within the same form of representation and the other referring to hierarchical relationships, such as the ones between specific and general cases. The more and stronger the connections one constructs, the deeper the mathematical understanding they achieve. The word "understanding" has dual meanings, being both the present participle of the verb "to understand" (process) and a gerund acting as a noun (product).

Procedural versus Conceptual

The second major debate surrounding mathematical understanding is the procedural-conceptual debate. Hiebert and Lefevre (1986) defined conceptual understanding in terms of the richness of relationships and procedural understanding as one's familiarity with individual symbols or procedures/rules for solving mathematical problems. In the National Assessment of Educational Progress (NAEP), procedural and conceptual understandings refer to two fundamental but distinct types of mathematical knowledge tested to foster a student's mathematical understanding. Procedural understanding refers to the ability to select, apply, and

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 16–24). MERGA.

execute mathematical rules, algorithms, and procedures correctly and efficiently. It includes executing computations, following step-by-step formulas, and using standard notations. In contrast, conceptual understanding refers to deep comprehension of underlying mathematical principles, operations, and relationships. Rather than just memorizing steps, it focuses on understanding why a formula works and how to apply those ideas to new or real-world problem settings (National Assessment Governing Board, 2002). These definitions were challenged by Star (2005), who argued that the existing literature had emphasized and valued conceptual understanding while overlooking procedural understanding. His view is consistent with the 2001 National Research Council report *Adding It Up*, which uses procedural fluency, defined as skill in carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately. In this view, procedural understanding is not rote memorization of algorithms but rather a dynamic understanding of *when* and *how* to use mathematical algorithms and rules.

Features of Mathematical Understanding

Based on a literature review, Cai and Ding (2017) pointed toward the field's agreement on the following: (1) Understanding is conceptualized as both a process ("understanding" as a verb) and a result of that process ("understanding" as a noun); (2) understanding is both the making of connections and a result of connection making in a social environment; (3) understanding is a dynamic and continual process; (4) understanding may have different levels and kinds; and (5) the goal is to reach a deep level of mathematical understanding.

How to Foster Students' Mathematical Understanding?

Two Perspectives of Learning

There are two perspectives of learning which can explain ways of fostering students' mathematical understanding (Cobb, 1994). For example, in solving mathematical problems, students can use any approach they can think of, draw on any piece of knowledge they have learned, and justify their ideas in ways they feel are convincing. Learning takes place during the process of students constructing their own problem-solving solutions. In the meantime, students can share their own solutions in the classroom with their peers. The learning environment provides a natural setting for students to present various solutions to their group or class and learn mathematics through social interactions, meaning negotiation, and reaching shared understanding. Such activities help students clarify their ideas and acquire different perspectives of the concept or idea they are learning (Cai, 2003).

The same can be exhibited in P-PBL (Cai, 2022). When students pose their own mathematical problems based on a situation, they must make sense of the constraints and conditions from the given information to build connections between their existing understanding and a new understanding of related mathematical ideas (Cai & Rott, 2024). Although problem-posing activities are cognitively demanding tasks, they are adaptable to students' abilities and thus can increase students' access such that students with different levels of understanding can still participate and pose potentially productive problems based on their own sensemaking. Once students share their posed problems in the class, they not only clarify the problems they posed but also discuss possible ways of solving those posed problems.

Teachers' Learning Perspectives

In the past, mathematical understanding has mainly represented researchers' perspectives (Davis, 1992; Hiebert & Carpenter, 1992; Pirie & Kieren 1992, 1994; Richland et al. 2012; Simon, 2006). Given the important role of teachers in supporting learning, Richland et al. (2012) proposed building pedagogical frameworks that address how teachers process knowledge. They suggested the development of classroom routines that would facilitate students' mathematical understanding, including worked examples and connection making.

Cai and Ding (2017) reported that Chinese mathematics teachers emphasize the view that understanding is a web of connections, which results from continuous connection making. In contrast to the popular view which separates understanding into conceptual and procedural, Chinese teachers perceive understanding in terms of concepts and procedures. To achieve mathematical understanding, Chinese teachers take a more practical approach by making connections. Such perspectives of classroom practitioners not only inform the long-debated issue of the meaning of mathematical understanding but also provide ways to achieve it.

A Framework on Mathematical Understanding

It is quite useful to examine teachers’ perspectives about how to foster students’ mathematical understanding because of the important role teachers play in fostering that understanding in the classroom. A more comprehensive view of mathematical understanding is needed to provide a tool for organizing the kinds of mathematical understanding past studies have investigated. To that end, Cai et al. (2026) proposed a theoretical framework (see Table 1) outlining five aspects of mathematical understanding—factual, procedural, conceptual, metacognitive, and affective—and discussed five lenses employed by mathematics educators—mathematical, cognitive, social, diagnostic, and instructional—when considering how to foster students’ mathematical understanding. The five aspects of mathematical understanding are related to the kinds of understanding reached. Researchers approach the issue of how to foster students’ mathematical understanding in multiple ways, employing mathematical, cognitive, social, diagnostic, and instructional lenses that each focus on a different set of concerns.

Table 1
Fostering Students’ Mathematical Understanding: A Framework

Lenses	Aspects of Mathematical Understanding				
	Factual	Procedural	Conceptual	Metacognitive	Affective
Mathematical	<i>Fostering Students’ Mathematical Understanding</i>				
Cognitive					
Social					
Diagnostic					
Instructional					

Five Aspects of Mathematical Understanding

The five aspects of mathematical understanding, including factual, procedural, conceptual, metacognitive, and affective (Cai et al., 2026, pp. 3-4), are described as follows.

The *factual* aspect of mathematical understanding is knowledge that is basic to mathematics. It includes essential facts, terminology, details, or elements students must know or be familiar with to understand mathematics or solve a mathematical problem.

The *procedural* aspect of mathematical understanding refers to very specific skills, algorithms, techniques, and particular methods in mathematics. The U.S. NAEP mathematics assessment refers to procedural knowledge as students’ ability to follow algorithms, perform calculations, and execute steps in a specific order to reach a solution.

The *conceptual* aspect of mathematical understanding refers to “interrelationships among the basic elements within a larger structure,” which includes knowledge of classifications, principles, generalizations, theories, models, or structures pertinent to mathematics. The NAEP refers to conceptual understanding as students’ understanding of the underlying mathematical

concepts and relationships rather than just the ability to perform calculations. Students draw on conceptual understanding to explain the “why” behind mathematical operations and procedures.

The *metacognitive* aspect of mathematical understanding is awareness of one’s own thinking and particular cognitive processes. It is strategic or reflective knowledge related to monitoring whether solution strategies are appropriate and evaluating if answers are reasonable.

The *affective* aspect of mathematical understanding falls into a noncognitive dimension including but not limited to students’ attitudes, beliefs, motivation, identity, and emotion.

Five Lenses for Fostering Mathematical Understanding

Because the process of developing students’ mathematical understanding is inherently complex and multidimensional, when exploring questions of how to foster that understanding, researchers usually adopt a lens suitable for that exploration. In fact, any research in mathematics education is both situated and oriented by a choice of theoretical stance and structure as well as ways of interpreting phenomena that match the purpose and nature of the research questions (Hiebert et al., 2023). The five lenses Cai et al. (2026) discussed are interrelated and often complementary for examining how to foster mathematical understanding.

The mathematical lens refers to the analysis of the mathematics itself and its practices through unpacking specific mathematical ideas and teasing out what it means to understand that idea mathematically. For example, mathematically, understanding the arithmetic average involves (1) a procedural understanding of the idea as an algorithm, (2) a conceptual understanding of the idea as an algorithm, and (3) a conceptual understanding of the concept as a statistic (the mean) to describe and make sense of and compare data sets (Cai, 2000; Mokros & Russell, 1995; Watson & Moritz, 2000).

The cognitive lens refers to applying cognitive theories to students’ learning of specific mathematical topics. For example, Wearne and Hiebert (1988) examined middle school students’ learning of decimal numbers, focusing on the semantic phase of the cognitive process, specifically connecting and developing. With this perspective, researchers may investigate students’ difficulties understanding mathematical ideas.

The social lens acknowledges that mathematics is a social activity, one in which students can learn together through interacting with each other and teachers (Cobb, 1994). The use of the social lens in mathematics education has gained increasing attention in recent decades, reflecting the “social turn” identified by Lerman (2000) and later echoed by Inglis and Foster (2018). Research on fostering mathematical understanding that employs a social lens frequently involves language, such as mathematical discourse (e.g., Herbel-Eisenmann & Wagner, 2010) and classroom discussions (Pirie & Schwarzenberger, 1988).

The diagnostic lens recognizes the important role that assessment plays in supporting mathematics teaching and student learning. In this lens, teachers’ teaching with understanding is based on the assumption that the more teachers know about students’ thinking, the better they can foster students’ understanding and learning. In the technology era, an example of active assessment is computerized formative assessment, which monitors students’ progress and provides automatic feedback (Chang, 2025).

The instructional lens often considers learners’ characteristics when exploring instructional approaches that may better facilitate students’ understanding of a mathematical topic. The instructional lens is built on mathematical, cognitive, social, or diagnostic lenses. For example, Hiebert (1992) coordinated mathematical, cognitive, and instructional lenses in his analysis of the understanding of decimal fractions and how it may be fostered.

Fostering Students' Understanding through P-PBL

Problem posing in mathematics education refers to several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context (referred to as the *problem context* or *problem situation*; (Cai, 2022; Silver, 1994). Stanic and Kilpatrick (1989) proposed three points of view in problem-solving research: problem solving as a cognitive activity, problem solving as a learning goal unto itself, and problem solving as an instructional approach. Researchers have also conceptualized problem posing as a cognitive activity, as an instructional goal, and as an instructional approach (Cai, 2022). Fostering students' mathematical understanding through P-PBL involves viewing problem posing as an instructional approach to teach mathematics. That is, students will understand and learn mathematics through their engagement in problem posing.

Both theoretically and empirically, problem posing supports students' understanding of mathematics. In a recent meta-analysis, Ran et al. (2025) found a very positive effect of engaging students in problem-posing interventions, indicating a statistically significant positive relationship between problem-posing interventions and learners' cognitive mathematical learning outcomes (Hedges' $\bar{g} = 0.53$) and affective learning outcomes (Hedges' $\bar{g} = 0.67$).

What is Problem Posing?

In the classroom, students engage in problem-posing tasks to develop a deep understanding of mathematical concepts. Problem-posing tasks, then, are those instructional tasks that position students as generators or shapers of new problems based on real-life or mathematical situations. A problem-posing task usually includes two parts: a situation and a prompt (Cai & Hwang, 2023). The problem situation is what provides the context and data that the students may draw from (in addition to their own life experiences and knowledge) to generate problems. The prompt lets students know what they are expected to do. For example, the following is a problem-posing task used in Silver and Cai (1996): *Jerome, Elliott, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliott. Elliott drove twice as many miles as Jerome. Jerome drove 50 miles. Pose three different mathematical problems that can be solved based on this information.* In this problem, “Jerome, Elliott, and Arturo took turns driving home from a trip. Arturo drove 80 miles more than Elliott. Elliott drove twice as many miles as Jerome. Jerome drove 50 miles” is the situation; “Pose three different mathematical problems that can be solved based on this information” is the prompt. For the same situation, a different prompt could be used, such as “Pose one easy problem, one moderately difficult problem, and one difficult problem that can be solved based on this information.”

A Case of the Gym Membership Task

One case is a lesson on linear functions described in Hwang et al. (2024). The middle school teacher in this case found a problem-solving-based lesson in her curriculum unsatisfactory and subsequently redeveloped it to better target her students' learning. Specifically, the teacher modified the lesson to have students pose problems based on the two equations $y = 1200 + 3x$ and $y = 1200 - 3x$. This activity was designed to foster students' understanding of the concept of slope, including both positive and negative slope. She also replaced the original problem-solving task with a problem-posing gym membership task: *I want to go to a gym to exercise, and I am told that there are two ways to pay for using the facility: (a) Apply for an annual card and add \$1,000 to it. \$10 will be deducted from the card every time I use the gym; (b) No annual card, but a charge of \$30 every time I use the gym. What kind of mathematical problems do you think you can pose based on this situation to test your peers? The more problems you pose, the better. Write them on this worksheet and solve them.*

The teacher gave the students time to think about and pose problems for the gym situation. The students posed a variety of problems, such as “After n visits, how much money would she

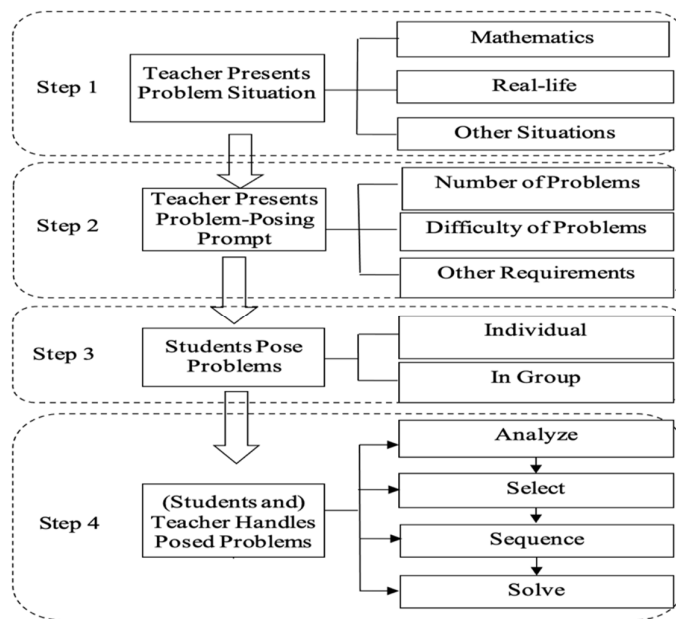
pay under each plan?” Through this activity, the teacher was able to elicit ideas of the y -intercept, slope, and so forth, connecting them to the symbolic representation.

The five lenses for exploring how to foster mathematical understanding can be applied to analyze what is being attended to or centered in this case. First, the mathematical lens is exhibited in students’ understanding of linear functions. Their cognitive understanding can be seen in their process of posing various problems. Socially, the students engaged in this task in the classroom, with their activity necessitated by interactions with each other. Moreover, the task could be used as a diagnostic tool for the teacher to gauge students’ understanding of linear functions. Finally, the task is an instructional tool used to further the learning goals of the lesson, such as increasing students’ engagement with linear functions.

The five aspects of mathematical understanding can also be fostered through the P-PBL approach, which has increasingly been the target of research exploring students’ mathematical understanding. P-PBL clearly provides a promising avenue through which to foster students’ mathematical understanding along all five aspects of mathematical understanding, and research on P-PBL has made use of all five lenses.

P-PBL Instructional Model

Having shown the value of P-PBL to foster students’ understanding through the above case, I now turn to presenting the P-PBL instructional model, which outlines the four steps involved in teaching through problem posing: (a) the teacher introduces a problem-posing situation; (b) the teacher provides a problem-posing prompt alongside the situation; (c) students pose problems individually or in groups; and (d) teachers address the posed problems in alignment with learning objectives, guiding students in solving selected problems (Cai, 2022).



The first two steps, the teacher introducing the problem-posing situation and the teacher providing a problem-posing prompt with the situation, are typically presented together; however, they are intentionally separated here to highlight the significance of considering both the situation and the prompt when designing problem-posing instruction. Step 3 involves students actively engaging in problem posing, either independently or collaboratively, and Step 4 focuses on how teachers handle posed problems in the classroom. This model shows an instructional flow when

implementing problem posing in the classroom. It also serves as a guide for empirical studies. In what followings, I use this model to show the progress and potential of research related to the teaching and learning of mathematics.

One of the biggest challenges in P-PBL is handling students’ posed problems. Although there has been wide recognition of the promise of P-PBL, additional attention needs to be paid to the specifics of handling students’ posed problems (Cai et al., 2024). The four practices (analyse, select, sequence, and solve) seem to work well. In this section, I’d like to summarize the previous discussion into three main points.

The first is the importance of analyzing students' posed problems. It is very clear that students can pose very different problems. Some students even pose unexpected responses such as statements or irrelevant problems. Thus, the analysis step is critical. Analysis can be based on classification as well as clarification of the problems. Such practices can not only help students structure their problems based on different classifications but also examine the appropriateness of each of their posed problems. One recent study showed that even unexpected responses like nonmathematical problems or unclear responses can be very useful in the classroom (Ran et al., 2025). Thus, the analysis stage can help students see how such unexpected responses could be modified to become more desirable mathematical problems.

There is no way that teachers can discuss or solve all the problems students pose. Current research suggests that, based on the learning goals, teachers need to select the kinds of problems to be solved. For example, even though a problem might be very interesting and mathematically sound, if the problem is irrelevant to the lesson learning goals, teachers might decide to assign that problem as homework instead of discussing it in the lesson. Although it sounds natural to sequence the problems from simple to more difficult, it may occasionally make more sense to sequence them from difficult to simpler. In any case, it is very important to select problems to solve. Solving problems not only helps students learn further with respect to related mathematical ideas but also reinforces the problem-posing aspects of the lesson.

Looking into the Future

In this paper, I have presented a framework about mathematical understanding and fostering mathematical understanding. I have briefly discussed the potential ability of P-PBL to foster students' mathematical understanding. To close, I discuss two areas of possible future research.

The first is for the mathematics education field to use this framework to examine existing literature, including cross-cultural studies, and identify the successes and challenges we have documented regarding promoting students' mathematical understanding and ways to better support them. The framework can serve as a guide not only for individual research (as well as practice) but also for the field as a whole as we work collectively to study and deepen students' mathematical understanding. Mathematics educators will benefit from carefully evaluating how extant approaches to fostering mathematical understanding have (or have not yet) addressed each of the aspects of mathematical understanding. Moreover, mathematics education as a field has benefitted from the multiplicity of lenses that researchers and practitioners have employed, as each lens provides a complementary perspective on mathematical understanding. Especially given the rapidly evolving technological landscape, including the rise of generative artificial intelligence (AI), such synthesis will be possible and feasible to identify specific and effective practices for fostering students' understanding of specific mathematical ideas. In addition, it will be critical to use these lenses to focus future research on how such technologies can support the development of each aspect of mathematical understanding in practice.

The second is continued effort on the use of P-PBL to foster students' mathematical understanding. Current research on mathematical understanding tends to focus on factual, procedural, and conceptual aspects of mathematical understanding with less research on the affective aspect even though students' deep engagement with problem posing is well-documented. Future research should explore how problem posing can be effectively integrated into mathematics instruction to foster not just students' cognitive but also affective and metacognitive development, including investigating how teachers can explicitly address students' enjoyment, curiosity, and motivation when engaging in problem-posing activities. By integrating cognitive and affective dimensions in both instructional practice and research methodologies, mathematics education can develop more effective strategies for nurturing students' problem-posing skills and intrinsic motivation.

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References

- Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In W. D. Reeve (Ed.), *The teaching of arithmetic (Tenth Yearbook of the National Council of Teachers of Mathematics)*; pp. 1–31). Columbia University.
- Cai, J. (2000). Understanding and representing the arithmetic averaging algorithm: An analysis and comparison of U.S. and Chinese students' responses. *International Journal of Mathematical Education in Science and Technology*, 31(6), 839–855.
- Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. Lester (Ed.), *Research and issues in teaching mathematics through problem solving* (pp. 241–254). National Council of Teachers of Mathematics.
- Cai, J. (2022). What research says about teaching mathematics through problem posing. *Éducation & Didactique*, 16-3, 31–50. <https://doi.org/10.4000/educationdidactique.10642>
- Cai, J., & Ding, M. (2017). On mathematical understanding: Perspectives of experienced Chinese teachers. *Journal of Mathematics Teacher Education*, 20(1), 5–29. <https://doi.org/10.1007/s10857-015-9325-8>
- Cai, J., Ding, M., & Hwang, S. (2026). Fostering students' mathematical understanding: Toward a theoretical framework. *Asian Journal for Mathematics Education*. Online first. <https://doi.org/10.1177/27527263261442721>
- Cai, J., & Hwang, S. (2023). Making mathematics challenging through problem posing in the classroom. In R. Leikin (Ed.), *Mathematical challenges for all* (pp. 115–145). Springer.
- Cai, J., Koichu, B., Rott, B., & Jiang, C. (2024). Advances in research on mathematical problem posing: Focus on task variables. *The Journal of Mathematical Behavior*, 76, 101186.
- Cai, J., & Rott, B. (2024). On understanding mathematical problem-posing processes. *ZDM-Mathematics Education*, 56, 61–71. <https://doi.org/10.1007/s11858-023-01536-w>
- Chang, H.-H. (2015). Psychometrics behind computerized adaptive testing. *Psychometrika*, 80(1), 1–20.
- Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13–20. <https://doi.org/10.3102/0013189X023007013>
- Davis, R. B. (1992). Understanding “understanding.” *The Journal of Mathematical Behavior*, 11(3), 225–241.
- Greeno, J. G. (1978). Understanding and procedural knowledge in mathematics instruction. *Educational Psychologist*, 12(3), 262–283. <https://doi.org/10.1080/00461527809529180>
- Herbel-Eisenmann, B., & Wagner, D. (2010). Appraising lexical bundles in mathematics classroom discourse: Obligation and choice. *Educational Studies in Mathematics*, 75, 43–63. <https://doi.org/10.1007/s10649-010-9240-y>
- Hiebert, J. (1992). Mathematical, cognitive, and instructional analyses of decimal fractions. In G. Leinhardt, R. T. Putnam, & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 283–322). Erlbaum.
- Hiebert, J., Cai, J., Hwang, S., Morris, A. K., & Hohensee, C. (2023). *Doing research: A new researcher's guide*. Springer.
- Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65–97). Macmillan.
- Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1–27). Lawrence Erlbaum Associates.
- Hwang, S., Xu, R., Yao, Y., & Cai, J. (2024). Learning to teach through problem posing: A teacher's journey in a networked teacher–researcher partnership. *Journal of Mathematical Behavior*, 73, 101120.
- Inglis, M., & Foster, C. (2018). Five decades of mathematics education research. *Journal for Research in Mathematics Education*, 49(4), 462–500. <https://doi.org/10.5951/jresmetheduc.49.4.0462>

- Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.). *Multiple perspectives on mathematics teaching and learning* (pp.19–44). Ablex.
- Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, 26(1), 20–39. <https://doi.org/10.2307/749226>
- National Assessment Governing Board. (2002). *Mathematics framework for the 2003 National Assessment of Educational Progress*. U.S. Department of Education, The College Board.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.
- Pirie, S., & Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics*, 23, 505–528. <https://doi.org/10.1007/BF00571470>
- Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, 26, 165–190. <https://doi.org/10.1007/BF01273662>
- Pirie, S. E., & Schwarzenberger, R. L. E. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in Mathematics*, 19, 459–470. <https://doi.org/10.1007/BF00578694>
- Ran, H., Cai, J., Muirhead, F., & Hwang, S. (2025). An analysis of unexpected responses in middle school students' mathematical problem posing from the perspective of problem-posing processes. *Educational Studies in Mathematics*, 119, 467–486. <https://doi.org/10.1007/s10649-025-10399-9>
- Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational Psychologist*, 47(3), 189–203. <https://doi.org/10.1080/00461520.2012.667065>
- Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, 14(1), 19–28.
- Silver, E. A., & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education*, 27(5), 521–539.
- Simon, M. A. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking and Learning*, 8(4), 359–371. https://doi.org/10.1207/s15327833mtl0804_1
- Stanic, G., & J. Kilpatrick. (1989). Historical perspectives on problem solving in the mathematics curriculum. In R. Charles & E. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 1–22). National Council of Teachers of Mathematics.
- Star, J. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education*, 36, 404–411. <https://doi.org/10.2307/30034943>
- Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning*, 2(1-2), 11–50. https://doi.org/10.1207/S15327833MTL0202_2
- Wearne, D., & Hiebert, J. (1988). A cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers. *Journal for Research in Mathematics Education*, 19(5), 371–384. <https://doi.org/10.2307/749172>

Advocacy and Subversion: Using the Past to Think About Mathematics in the Future

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As we go into the second quarter of the twenty-first century, we enter a troubling time for mathematics education in educational institutions at all levels. This troubling time comes with a multitude of challenges that seem to restrict how mathematics educators can incorporate research-based understandings into their practices. Using Bernstein's pedagogic device, I discuss where possibilities for identifying the "unthinkable" lie at the curriculum level, the institutional level and in the classroom, using examples from my own research. These possibilities can be linked to both advocacy with participants to create new spaces as well as to subversion in which existing spaces are used in unexpected ways, to proactively find alternative ways to act upon the unthinkable, so we can respond to the challenges that affect our practices.

In a recent editorial for the *Journal of Research in Mathematics Education*, entitled "Mathematics education at the end of the world", Horn (2026) stated "education in general and mathematics education in particular currently exist in precarious environments across the globe" (p. 1). It seems that rather than seeing mathematics education as a contributor to solutions for the more complex climate and geo-political crises engulfing our world, politicians seem intent on redirecting attention away from these difficult-to-tackle crises to instead present the current incarnation of education as being the problem. By doing so, they are then able to offer a solution that will save the economy and to a lesser extent the society. For example, Hattam et al. (2009) showed how an Australian Minister for Education used the media, through a specific contact, to present school education at that time as being in crisis and to blame teachers and their innovative pedagogies for this crisis. The consequence of this displacement of attention onto these so-called educational crises resulted in discussions being shifted from the purposes of education and what it could contribute to the wider society. This can be seen in the findings from research that Troels Lange and I did on public discourse to do with the introduction of the MySchool comparison website for the results of the Australian national tests (NAPLAN). A focus on these test results resulted in teachers and parents blaming each other for children's lack of attainment in mathematics (Lange & Meaney, 2014). In the media, the Federal Minister for Education had highlighted the importance of adding value to children, situating them as commodities which could be evaluated based on their achievement levels. Thus, rather than potentially discussing how education could contribute to the aims of society, it became possible for the Minister to situate the discussion of mathematics education as being about who was to blame for individual children not doing well on a national test.

In Norway, also with Troels Lange (Lange & Meaney, 2018), we investigated how the Norwegian Minister for Education used media discussions to make a case for the need for major changes to Early Childhood Education and Care (ECEC). These included more focus on "learning/teaching" and away from working with children's own interests, which had been the foundation for ECEC in Scandinavia (Bennett, 2005). This was a major shift from the philosophical foundations of ECEC and how mathematics education should be considered within it (Schaanning, 2015). The Minister's argument for this change was tenuous at best, in that in a debate piece in a newspaper, he tried to situate the economic future of the country as dependent on the society gaining more PhDs in mathematics and science. To gain these PhDs, the drop-out rate from upper secondary school needed to be reduced. For this to happen, more

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 25–33). MERGA.

students needed to pass the exams at the end of junior secondary school. This would happen if interest in mathematics and science was awakened earlier in ECEC. In structuring this argument, the Minister does not explicitly state that his intention is to shift the focus to more direct teaching of those subjects but by ignoring that the ECEC curriculum already required children to engage with mathematics and science, he presented the situation as a crisis he knew how to overcome. This is an example of what Smyth (2006) described as the use of the media to promote policy change, “when the ideological agenda of government needs to be concealed, for example, in the desire of government to more closely and tightly control the work of teachers and schools, it is convenient to disguise the real intent” (p. 310). As a result of lobbying by those who worked in ECEC, the teachers, the proposed policy change was rejected (Fosse et al., 2018), but there remain pressures to undermine the current philosophical foundations of ECEC.

Thus, to determine how mathematics education could contribute to resolving the actual crises facing society, such as climate change and geopolitical tensions, there is a need to identify ways to resist proposed policy changes through advocacy or to subvert their focus. To discuss the opportunities for doing this, I first briefly describe Bernstein’s (2000) pedagogical device, which he developed to show how knowledge was transmitted from the level of policy to the level of classroom practices. In this device, Bernstein (2000) discussed how the contestation of what constituted valuable knowledge allowed for spaces to think the “unthinkable”. Three examples from previous research illustrate how recognition and implementation of the “unthinkable” at each level of the device could be achieved through subversion and advocacy.

Bernstein’s Pedagogic Device

Bernstein (2000) argued that the pedagogic device controls what knowledge comes to be valued at different levels of the education system, which he labelled the fields of production, recontextualization and reproduction. For example, the selection of “official knowledge” by policy makers in the field of production, results in teachers having few independent choices in the field of reproduction. However, the transmission of policy initiatives, as “official knowledge” down through the recontextualising and reproductions fields, is never straight forward. This is because there is always competing valued knowledge within and between each field, which results in adaptations being made. Bernstein was most interested in the processes, which led to the adaptations. These processes he labelled the rules of the pedagogic device:

1. *Distributive rules*: These rules distributed forms of knowledge to different social groups. In this way, distributed rules distributed different forms of consciousness to different groups. Distributive rules distributed access to the ‘unthinkable’, that is, the possibility of new knowledge, and access to the ‘thinkable’, that is, to official knowledge.
2. *Recontextualising rules*: These rules constructed the ‘thinkable’, official knowledge. They constructed pedagogic discourse: The ‘what’ and the ‘how’ of that discourse.
3. *Evaluative rules*: These rules constructed pedagogic practice by providing the criteria to be transmitted and acquired. (Bernstein, 2000, p. 114)

Although Bernstein focused on the processes on how valued knowledge comes to be selected in each field, previous research has shown that the rules can support knowledge in the lower fields contributing to changes in what is value at the policy level (Meaney, et al., 2013). The example of the rejection of the proposed changes to ECEC policy in Norway, could be consider an example of the highly valued knowledge of ECEC teachers reasserting its importance at the policy level.

In the next sections, I discuss examples of advocacy and subversion in regard to the “unthinkable” possibilities which can become available when the selection of valued knowledge from different groups in the education system are brought together.

Opportunities and compromises in the production field

Decisions about educational policy currently seem to be made with limited input from educators, including teachers, teacher educators and researchers. Nonetheless, there are likely to be competing discourses about what is valuable knowledge from specialists that should be selected for inclusion in curricula. For example, Bernstein (2000) highlighted the role of subject specialists, such as mathematicians. Mathematicians “distributed” their knowledge, alongside that of other specialists who were deemed to have relevant knowledge or requirements, such as industry. Curricula writers select from what is made available to determine the official knowledge of the curricula, based on interpretations of what is needed by the society. Three decades ago, Ellerton and Clements (1988) suggested that the role of mathematicians had diminished in curricular planning in Australia, with others such as teachers and mathematics educators having a more prominent role. This may no longer be the case with fulfilling industry demands for specialised workers, seemingly a more legitimate purpose for education, than solving climate and geo-political crises.

The impact of economic demands on the delivery of education, such as the instigation of New Public Management in the 1980s and 1990s, has been examined in regard to general education policy (see for example, van der 7et al., 2015), but less so in regard to mathematics education. New Public Management was instigated “to make educational policy more effective and more efficient. Often, budget reductions were also at stake” (van der Sluis et al., 2017, p. 304). As a result of these economic policies, in New Zealand:

The Tomorrow’s School legislation (Lange 1988) devolved responsibility for providing education to local communities under a charter system. As well, some of the functions previously carried out by a large Department of Education, including curriculum development, were contracted to external providers. However, this legislation mandated state control of curriculum development and implementation as a means of ensuring schools were accountable (Codd 1995). National curriculum development was to be one of the new Ministry of Education’s goals for the 1990s. (McMurchy-Pilkington et al., 2013, p. 344)

For Māori, these changes provided possibilities to gain funding for the schools that they had set up, outside the government education systems, to ensure their children gained fluency in *te reo Māori*, their Indigenous language. *Te reo Māori* had been recognised as under dire threat of being lost at the end of the 1970s (Meaney, et al., 2012). The compromise from accepting state funding was that the school communities had to be accountable to the Ministry of Education by, for example, showing how they followed curricular requirements.

For these Māori educators, who Bernstein (2000) would have labelled “individuals *outside the field of production*” (p. 115), having their valuable knowledge incorporated into curricula was not straight-forward, but occurred as a result of identifying the “unthinkable”. The decision to revise curricula in the 1990s gave Māori mathematics educators the opportunity to insist on a version written in *te reo Māori*. Although the learning objectives could not be changed, the translated version, contributed to the standardisation of mathematical terms and expressions in *te reo Māori*, allowing students to move between Māori schools without having to learn new vocabulary (McMurchy-Pilkington et al., 2013). It also began the process of Māori mathematics educators gaining skills in policy and curriculum development that had not previously been available. In subsequent curricular revisions, recognition of their knowledge and skills led to being able to adapt the mathematics curriculum to better suit the aspirations of Māori communities. These revisions have also provided inspiration for other Indigenous communities to agitate for their own mathematics curriculum (see for example, Sámedigge, 2025). Even though political forces are currently working to deny the value of this expertise, the skills and expectations about what can or should be achieved by Māori for the benefit of Māori students, going beyond training them to be workers for the economic benefit of a society, does not disappear.

In summary, the unthinkable appears when valued knowledge and skills juggle with expectations about the purpose of education at the policy level. Subversion of policy makers' expectations can occur when the unthinkable is recognised within the contestation.

Opportunities and compromises in the recontextualisation field

In the pedagogic device, official knowledge becomes recontextualised as specialists in the education field, such as teacher educators, textbook writers and national examination writers, make sense of it within existing organisational structures (Bernstein, 2000). The independence of these specialists from each other can lead to contestation about what is important knowledge. How much space there is for recontextualising these different interpretations of the official knowledge in curricula will also depend on how independent the recontextualisation field is from the production field. For example, when the policy level has pre-eminence in an education system, teacher educators may be limited in what they can discuss with preservice teachers. When the first national curriculum was introduced into Australia, Briant and Doherty (2012) found that some teacher educators, including one mathematics teacher educator, were determined to maintain their commitment to social justice within their work with preservice teachers, while acknowledging the new pressures on doing so. In contrast, other teacher educators, including another mathematics teacher educator, felt pressured to align their teacher education with the foci in the curriculum, reducing the importance that they gave to raising social justice issues. In recent times, the independence of teacher education in places such as Australia to decide what should be provided to preservice teachers has become restricted, perhaps without a clear research basis for these changes (Knipe & Fitzgerald, 2017).

Yet, in some countries, like Norway, teacher education remains fairly independent of the schooling systems, with an understanding that preservice teachers' work can continue well beyond the immediate situations found in schools. This independence allows for teacher educators to identify and develop their teacher education on issues that teachers need both now and in the future. Previous research at our institution had suggested that preservice teachers tended to situate multilingual students as problematic if they were not yet fluent in the language of instruction or when they were fluent, the same as Norwegian students, thus requiring no further adaptations to their teaching (Rangnes & Eikset, 2019). To tackle this issue, in an externally-funded project, we recontextualised valued knowledge from different sources to develop understandings about how to support preservice teachers.

The core values of the curriculum included developing students' identity and cultural diversity in all subject areas (Utdanningsdirektoratet, 2019). This provided a legitimate basis for raising with preservice teachers how to work in multilingual mathematics classrooms to support students' cultural identities. However, we recognised that our interpretation or recontextualising was perhaps not quite what policy makers had envisioned. Initially we identified the kinds of teaching practices and the knowledge about their students, teachers would need. We also included the role of being advocates (Lange & Meaney, 2019), as understanding what students need for their learning is of limited value if there is no clear path to ensure that it is provided. In Norway, educators can also make submissions to policy proposals and have opportunities to debate policy issues in the media. Thus, knowing more about both in-class and outside the classroom advocacy was deemed important (Meaney & Rangnes, 2024).

To develop our understandings about the role of advocacy and how to raise it with preservice teachers, knowledge about what our preservice teachers might already know about teaching in multilingual classrooms and learning about their students from previous research was brought together. Recontextualising these different kinds of knowledge led us to utilise the contradictions in the preservice teachers' conflicting understandings about multilingual students learning mathematics as starting points for discussing the role of being an advocate.

As supporting preservice teachers to engage in these kinds of reflections was new for us as teacher educators, we are still learning how to develop our teacher education practices. As a result, we continue to bring in new understandings as part of our recontextualisation, such as the core values, of the curriculum, so that preservice teachers have a wider knowledge base from which to develop their classroom practices (e.g., Kasari & Steffensen, 2026).

Recontextualising policy will always happen as it comes into contact with established understandings at different levels of the education system. Yet, making use of what had previously been unthinkable during the contestation of knowledge may not always lead to benefitting society. Horn (2026) described how in the USA, “socially advantaged parents” made use of the “democratic visions of mathematics education encapsulated in ‘mathematics for all’”, described in the National Council for Teachers of Mathematics (1989) Standards, as “they hoarded educational opportunities for their own children” (p. 2). Thus, rather than benefitting the society, education became a way of adding value to their children, while reducing this possibility for others.

Opportunities and compromises in the reproduction field

For Bernstein (2000), the evaluative rules determine what becomes appropriate pedagogical practices, in the reproduction field. These practices are affected by the content which needed to be taught and how it should be made available to which student groups, depending on aspects such as age. To determine appropriate pedagogical practices for specific classrooms is affected by curricula, as well as established traditions within the schooling system. Nevertheless, there remains possibilities for identifying new previously unthinkable practices, although recognition of these possibilities becomes more difficult, the more regulated the schooling system becomes. For example, we know that high-status tests, even if low-stakes for students, can have an impact of what teachers consider is possible to do in their classrooms (Lange & Meaney, 2012)

In Norway, another core value in the curriculum is the requirement to develop students’ democratic competencies, which could be interpreted in a multitude of ways, including those which took a broad view of what democratic competencies could be. In work with Trude Fosse, we have considered how this core value could be developed into pedagogical practices that could provide students with skills to responding to the crises facing the planet. Our focus was on supporting young students to reflect on societal issues, such as fairness, using mathematical understandings. We did this by using an established practice of having children pose and solve a written mathematical problem, which could include pictures and symbolic representations, known as a *regnefortelling* (Fosse & Meaney, 2024). We chose to do this after an investigation of textbooks for Grade 2 provided few possibilities for connecting the mathematics children were expected to learn to their everyday experiences (Fosse & Meaney, 2021), let alone to some of the important issues facing society and the world. Consequently, we asked teachers, we knew, to have their Grade 2 students produce a *regnefortelling*, about a fair and an unfair situation, using an example of *regnefortelling* from earlier research as a prompt.

In Figure 1, there are two examples of *regnefortelling*, about unfair, *urettferdig*, and fair, *rettferdig*, situations. There are no written problems, with the drawings providing clues to what the problems are. In the first drawing, Mia is paid 5 kroner for cleaning while Mie is paid only 2 kroner. Mie is very unhappy, reflected by the mouth turned downward. In the fair situation, both children get 5 kroner. This *regnefortelling* illustrates how mathematics provides a way to compare the pay rates for cleaning and could be used to discuss issues of equal pay for equal work. In the second drawing, the unfair situation is when Peder gets only 2 pieces of pizza, with Peder crying to show his unhappiness. Mia on the other hand gets six pieces of pizza. The fair situation is when both people get four pieces of pizza, providing an opportunity to discuss fractional parts. The distribution of resources, often food items, provided insights into these children’s understandings of fair and unfair.

Figure 1

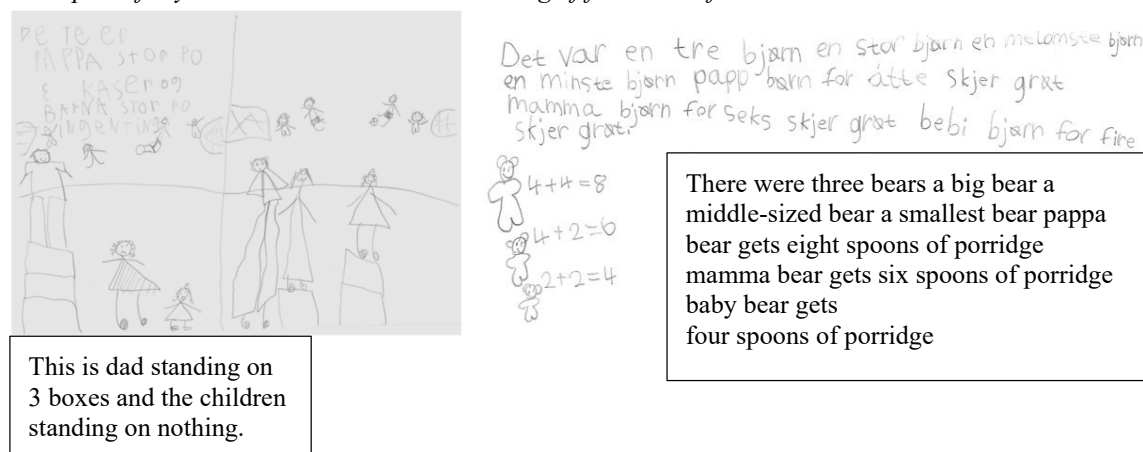
Examples of 7-year old children's understanding of fair and unfair situations



In a later project, Grade 1 students were asked to produce *regnefortellinger* in which the fair outcomes involved an uneven distribution of resources. Examples from two six-year olds can be seen in Figure 2. In the first set of drawings, the child sets out how when dad gets all the boxes to stand on, then the children cannot see. A redistribution of the boxes allowed everyone to see over a fence. In the second picture, the child used the story of Goldilocks and the three bears to also show an uneven but fair distribution of resources. Although it is clear that the children understand that some circumstances require uneven distribution of resources to be fair, writing problems and solutions about these situations seems difficult. For this approach to be developed into a pedagogical practice that contributes to children's understandings about how mathematics could be used to respond to societal issues, more knowledge is needed about the kinds of prompts that would support children to produce their problems and solutions.

Figure 2

Examples of 6-year old children's understanding of fair and unfair situations



This is dad standing on 3 boxes and the children standing on nothing.

Det var en tre bjørn en stor bjørn en mellomstor bjørn
en minste bjørn pappa bjørn for åtte skjeer grøt
mamma bjørn for seks skjeer grøt baby bjørn for fire
skjeer grøt

$4+4=8$
 $4+2=6$
 $2+2=4$

There were three bears a big bear a middle-sized bear a smallest bear pappa bear gets eight spoons of porridge mamma bear gets six spoons of porridge baby bear gets four spoons of porridge

In this example, the previously unthinkable in the reproduction field was the possibility to take an established practice, that of writing *regnefortellinger*, to develop children's democratic practices through working with more complex issues than typically found in textbooks. If mathematics is to be used in solving some of the issues now facing our society and the world we live on, there is a need to previously unthinkable possibilities to support students to do this.

Conclusion

Bernstein's (2000) pedagogic device illustrates how contestation between the valued knowledge of different groups at different levels of the education system can provide opportunities to identify the unthinkable. Given the pressure that education, including mathematics education, is facing, it is important to find ways to bring different knowledges

together so that new possibilities for educational practices are identified, developed and implemented, which could contribute to the good for society.

However, bringing together different knowledges so that previously unthinkable possibilities can be identified is not easy, nor is it easy to implement these new possibilities into existing structures. Subverting the intention of curricula, intentionally or unintentionally, can result in making unsatisfactory compromises, unless the aim for implementing the unthinkable possibilities remains clear. Recontextualising traditional practices, such as having children produce *regnefortellinger*, or interpreting core elements of curricular, such as identity and culture, to support preservice teachers to learn to be advocates, are forms of subversion. Although still requiring further trialling and refining, the identification and implementation of such new practices is becoming even more necessary. For example, supporting preservice teachers to gain understandings into advocacy has become critically important as funding for students who are still learning Norwegian as an additional language, is now being severely limited with the councils who fund the schools, looking for ways to save money. Redefining how the laws pertaining to teaching in the home language are to be interpreted has resulted in some students no longer being eligible. This has resulted in these students working in general classrooms, without even the small amount of support that these dedicated support teachers had provided. However, to ensure that new and alternative practices can be incorporated into different levels of the pedagogic device may require mathematics education researchers and teacher educators to become advocates in order to promote these new and different practices.

The three examples give insights into how the interaction between the valued knowledge of different groups provides spaces for new understandings about situations to arise and be acted upon. Although the crises that we face today are different to those of the past, looking back at how mathematics educators have previously tackled challenges, gives “critical hope” (Horn, 2026) about how to move forward in the future. Understanding how official knowledge, such as the curriculum, is adapted as it moved down through the pedagogic device provides insights into how this “critical hope” can be realised, through subversion of existing practices. However, for knowledge about these new practices to move up through the device, as shown in previous research (Meaney, et al., 2013), so that it becomes valued in discussions about curriculum, then mathematics educators need to develop their own advocacy skills.

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References

- Bennett, J. (2005). Curriculum issues in national policy-making. *European Early Childhood Education Research Journal*, 13(2), 5–23. <https://doi.org/10.1080/13502930585209641>
- Briant, E., & Doherty, C. (2012). Teacher educators mediating curricular reform: anticipating the Australian curriculum. *Teaching Education*, 23(1), 51–69. <https://doi.org/10.1080/10476210.2011.620605>
- Ellerton, N. F., & Clements, M. A. (1988). Reshaping school mathematics in Australia 1788–1988. *Australian Journal of Education*, 32(3), 387–405. <https://doi.org/10.1177/00049441880320>
- Fosse, T., Lange, T., Lossius, M. H. & Meaney, T. (2018). Mathematics as the Trojan horse in Norwegian early childhood policy? *Research in Mathematics Education*, 20(2), 166–182. <https://doi.org/10.1080/14794802.2018.1473162>

- Fosse, T. & Meaney, T. (2021). Milieus of learning in a Norwegian textbook. In G. A. Nortvedt, N. F. Buchholtz, J. Fauskanger, F. Hreinsdóttir, M. Häikiöniemi, B. E. Jessen, J. Kurvits, Y. Liljekvist, M. Misfeldt, M. Naalsund, H. K. Nilsen, G. Pálsdóttir, P. Portaankorva-Koivisto, J. Radišić, & A. Wernberg, (Eds.), *Bringing Nordic mathematics education into the future: Preceedings of The Ninth Nordic Conference on Mathematics Education, NORMA 20, Oslo, 2021* (pp. 81-88). NCM & SMDF. https://www.ucviden.dk/ws/files/144566789/NORMA_20_preceedings.pdf
- Fosse, T. & Meaney, T. (2024). Young students' representations of mathematics in posing problems about fairness. *Quaderni di Ricerca in Didattica, Numero speciale n. 13*, 19-28. https://sites.unipa.it/grim/CIEAEM74_Pproceedings_QRDM_Issue_2024_Suppl_13.pdf
- Hattam, R., Prosser, B., & Brady, K. (2009). Revolution or backlash? The mediatisation of education policy in Australia. *Critical Studies in Education*, 50(2), 159–172. doi: 10.1080/17508480902859433.
- Horn, I. S. (2026). Mathematics education at the end of the world. *Journal for Research in Mathematics Education*, 57(1), 1–8. <https://doi.org/10.5951/jresmetheduc-20250229>
- Kasari, G., & Steffensen, L. (2026). Exploring the potential of mathematical modelling for integrating issues about multilingual mathematics classrooms in teacher education. *Mathematics Education Research Journal*, 1–27.
- Knipe, S., & Fitzgerald, T. (2017). Caught between competing worlds: Teacher education in Australia. In J. Nuttall, A. Kostogriz, M. Jones, & J. Martin (Eds.), *Teacher education policy and practice: Evidence of impact, impact of evidence* (pp. 129–142). Springer. https://doi.org/10.1007/978-981-10-4133-4_9
- Lange, T. & Meaney, T. (2012). The tail wagging the dog? The effect of national testing on teachers' agency. In C. Bergsten, E. Jablonka & M. R. Sundström (Eds.), *Evaluation and comparison of mathematical achievement: dimensions and perspectives. Proceedings from Madif8* (p. 131-140). Linköping: Svensk Förening för Matematikdidaktisk Forskning.
- Lange, T. & Meaney, T. (2014). It's just as well kids don't vote: The positioning of children through public discourse around national testing. *Mathematics Education Research Journal*, 26(2), 377–397. <https://doi.org/10.1007/s13394-013-0094-3>
- Lange, T. & Meaney, T. (2018). Policy production in the media: The case of more mathematics in early childhood education. In M. Jurdak & R. Vithal (Eds.) *Social and political dimensions of mathematics education* (pp. 191–207). Springer. https://doi.org/10.1007/978-3-319-72610-6_11
- Lange, T. & Meaney, T. (2019). Discussing mathematics teacher education for language diversity. In J. Subramanian (Ed.) *Proceedings of the tenth international mathematics education and society conference* (Vol. 2). Hyderabad, India: University of Hyderabad.
- McMurphy-Pilkington, C., Trinick, T., & Meaney, T. (2013). Mathematics curriculum development and Indigenous language revitalisation: Contested spaces. *Mathematics Education Research Journal*, 25(3), 341–360. <https://doi.org/10.1007/s13394-013-0074-7>
- Meaney, T. & Rangnes, T. E. (2024). Introducing the role of being an advocate in mathematics teacher education. *Journal of Mathematics Teacher Education*. 27, 809–830. <https://doi.org/10.1007/s10857-024-09629-3>
- Meaney, T., Trinick, T. & Fairhall, U. (2012). *Collaborating to meet language challenges in Indigenous mathematics classrooms*. Springer. <https://doi.org/10.1007/978-94-007-1994-1>
- Meaney, T., Trinick, T. & Fairhall, U. (2013). One size does not fit all: Achieving equity in Māori mathematics classrooms. *Journal of Research in Mathematics Education*, 44(1), 235-263. <https://doi.org/10.5951/jresmetheduc.44.1.0235>
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Author.
- Rangnes, T. E. & Eikset, A. S. B. (2019). Preservice teachers' reflections on language diversity. In U. T. Jankvist, et al. (Red.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (s. 1754–1661). Freudenthal Group & Freudenthal Institute, Utrecht University and ERME. https://hal.science/hal-02435368/file/TWG09_24.pdf
- Sámedigge (2025). *Læreplan i matematikk 1.-10.trinn – samisk plan*. {Curriculum in mathematics for Grades 1-10} – Sámi plan}. Sametinget. https://sametinget.no/_f/p1/i65d2a7d5-a22c-40b4-94e8-ce442ca4d0fc/lareplan-i-matematikk-1-10trinn-samisk-plan.pdf
- Schaanning, E. (2015). Hvis skolematematikken ikke fantes [If school mathematics would not exist]. *Arr – idéhistorisk tidsskrift*, (4).
- Smyth, J. (2006). The politics of reform of teachers' work and the consequences for schools: Some implications for teacher education. *Asia-Pacific Journal of Teacher Education*, 34(3), 301–319. doi: 10.1080/13598660600927208.

- Utdanningsdirektoratet (The Norwegian Directorate for Education and Training). (2019). *Læreplan i matematikk 1.–10. trinn* (Curriculum for mathematics, grades 1-10). Author <https://www.udir.no/lk20/mat01-05?lang=eng>
- van der Sluis, M. E., Reezigt, G. J., & Borghans, L. (2017). Implementing new public management in educational policy. *Educational Policy*, 31(3), 303–329. <https://doi.org/10.1177/0895904815598393>

Research Papers

Defining Mathematical Thinking for Classroom Practice

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The term mathematical thinking is frequently used in research and policy writing as a key construct for mathematics education. Despite a clear shift from content-focused skills toward broader, holistic processes, a review of the use of the term revealed persistent ambiguity and tensions in how mathematical thinking was defined and conceptualised. This paper presents an overview of the evolution of mathematical thinking and its components, in order to propose a clear and contemporary working definition of mathematical thinking to support teachers' reflections on their pedagogical decision-making.

As a secondary mathematics teacher, the first author noticed students who could explain their thinking were often more engaged, persistent, and generally achieved higher academic outcomes. Whereas students who struggled to think of ways to approach problems or lacked confidence to discuss strategies disengaged quickly, avoided difficult questions, and had poorer long-term outcomes. These observations provided the impetus to better understand how to support students' in developing their mathematical thinking for future success.

Understanding mathematics is essential for secondary students, as it underpins many trades and professions and influences future employment opportunities. Limited mathematical proficiency has been found to restrict social privileges, employment, and economic security, while strong mathematics ability is associated with up to 28% higher wages (OECD, 2024). Mathematical thinking is widely recognised as an important goal of mathematics education (Stacey, 2006) to prepare students to participate in an increasingly technological and globalised society (Geiger et al., 2023). Although mathematical thinking has been identified as essential for future Science, Technology, Engineering and Mathematics (STEM) workers (Geiger et al., 2023), a powerful way of learning mathematics (Goos & Kaya, 2019; Isoda & Katagiri, 2012; Stacey, 2006), and involves the development of interpretative and sense-making dispositions (Goos, 1997), a review of contemporary research offered limited clarity regarding a contemporary definition of mathematical thinking.

In accepting the importance of helping students to develop their mathematical thinking, we sought a clear and contemporary definition. In this paper, we present an overview of the evolution of mathematical thinking and its components to support future research on teachers' pedagogical reasoning/decision-making.

Evolution of Mathematical Thinking

Historically, mathematical content was positioned separately from mathematical thinking in problem-solving. Burton (1984) conceptualised mathematical thinking as “operations, processes and dynamics” (p. 35), proposing “entry, attack and review” (p.41) affective phases of thinking. Students would experience curiosity and productive tension to initiate persistence during entry, confidence sustained problem-solving during attack, and review focused on checking and refining solutions. Building on Polya's work, this model was later extended to include heuristic strategies (e.g., drawing diagrams, simplification, working backwards) positioned as skills to be practiced and routinised for fluency (Schoenfeld, 2016).

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Mathematical thinking was also explicitly framed as a set of skills supporting procedural fluency (Schoenfeld, 1985; Stacey, 2006). Grounded in Schoenfeld's (1985) problem-solving framework, early research examined productive behaviours and explicitly taught heuristics to support efficient solution development (Kilpatrick et al., 2001). Instruction emphasised guided support, self-belief, and the application of practised strategies to support knowledge comprehension, and application (Stacey, 2006), which are lower-order cognitive processes in Bloom's taxonomy (Bloom et al., 1956; Schoenfeld, 2016). From this perspective, mathematical thinking was conceptualised as identifiable skills, including numerical fluency and conceptual understanding across content areas, such as measurement, algebra, geometry and operations (Burton, 1984).

Leaning more into processes rather than static skills, Goos (2014) described mathematical thinking as "the act of sense-making" (p. 196), grounded in the processes of specialising, generalising, conjecturing, and convincing. This emphasised that processes could be strengthened through classroom activity and social interaction. Although Stacey et al. (1985) initially positioned generalising and specialising as skills of problem-solving, mathematical thinking was later reframed as a dynamic process encompassing "deep mathematical knowledge, general reasoning abilities and knowledge of heuristic strategies" (Stacey, 2006, p. 41).

Problem-solving became a clear focus in the 1980's, which saw a pedagogical shift from rote learning of discrete skills to viewing mathematical thinking as integrating content knowledge and techniques within broader problem-solving processes. Although closely linked to problem-solving, mathematical thinking was described using varied terminology, including specialising, generalising, conjecturing and convincing (Stacey et al., 1985); formulating, representing, and solving mathematical problems (Goos & Kaya, 2019; Schoenfeld, 1982); and abstraction, generalisation, symbolisation, and logical reasoning (Stacey, 2006). These processes were conceptualised as actions through which students explored complex ideas and extended mathematical understanding (Schoenfeld, 2016; Stacey et al., 1985). However, substantial variation persisted in defining these processes. For example, Stacey (2006) emphasised specialising, generalising, conjecturing, and convincing, whereas Tall (2008) highlighted abstraction, synthesis, modelling, problem-solving and proof. More holistically, mathematical thinking was framed as both a mathematical perspective and use of mathematical tools such as abstraction and symbolic representation (Schoenfeld, 2016). More recent literature aligned mathematical thinking with higher-order thinking skills (HOTS), emphasising abstraction, reasoning, generalising, hypothesising, and proving (Brown, 2017; Stacey et al., 2015).

This conceptual shift carried important pedagogical implications. When mathematical thinking was framed as a set of discrete skills, instruction emphasised procedural fluency, strategy instruction, and guided practice (Kilpatrick et al., 2001; Stacey, 2006). By contrast, process-oriented perspectives positioned teachers as facilitators who design learning environments that promote exploration, reasoning, and sense-making (Goos, 2014). Students are required to generalise, conjecture, and justify through collaborative discourse (Stacey, 2006), requiring practices beyond repetitive exercises to foster flexible thinking (Isoda & Katagiri, 2012; Tall, 2008). Supporting mathematical thinking requires higher-order pedagogical strategies, including critical questioning and exploratory practices that structure knowledge, deepen conceptual connections (Van Zoest et al., 2017), and enable real-world application (Schoenfeld, 2016; Stacey, 2006). This pedagogical shift reflect movement from

developing skills towards processes cultivating inquiry, dialogue, and the construction of mathematical meaning (Schoenfeld, 2016).

Despite this apparent shift to processes, some contemporary authors continued to define these actions as skills. For example, Delima et al. (2018) framed specialising, generalising, conjecturing, and convincing as skills of procedural competency. Similarly, Tong et al. (2020) emphasised the skills of “predicting results, assessing knowledge, analysing and generalising” (p. 484). These examples illustrate the continued lack of consensus regarding whether mathematical thinking is best conceptualised as a set of skills or as a set of processes.

Not all definitions emphasised problem-solving in mathematical thinking. Tall (2008) conceptualised mathematical thinking as a developmental progression through conceptual-embodied (perception and action), proceptual-symbolic (calculation and manipulation), and axiomatic-formal (definitions and proof) worlds, where mathematical processes are compressed into procepts (thinkable concepts) enabling the construction of formal axiomatic theories that can link back to more sophisticated forms of embodiment and symbolism (Tall, 2008). Conversely, mathematical thinking was positioned as a multidimensional construct of mathematical attitudes (mindset), methods (abstraction, generalising, specialising and symbolising), and content, where students are nurtured to be independent learners (Isoda & Katagiri, 2012). Devlin (2012) argued mathematical thinking reflected how mathematicians *think* when problem-solving, developing ideas, and constructing arguments for real-world contexts. In addition to logical and analytical thinking, and quantitative reasoning, this framework suggests *doing* mathematics through collaboration will support engagement and understanding (Devlin, 2012).

Mathematical Thinking as a Multifaceted Construct

Positioning problem solving, reasoning, and communication as central to conceptual understanding (National Council of Teachers of Mathematics, 1989) broadened the focus of mathematics beyond procedural skills to include; the articulation of reasoning, recognition of the affective domain encompassing students’ beliefs, feelings, and dispositions toward mathematics (Grootenboer & Marshman, 2016). Productive disposition was introduced as the fifth strand of mathematical proficiency, alongside curriculum aspects of conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning (Kilpatrick et al., 2001). Productive disposition has a learner focus, and the “tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (Kilpatrick et al., 2001). From this perspective, teacher beliefs, modelling of mathematical value, and pedagogical choices were thought to cultivate students’ productive dispositions through sustained engagement and opportunity for productive struggle (Marshman & Goos, 2018). Consequently, the development of productive disposition appears closely intertwined with mathematical thinking, as students who perceived mathematics as meaningful were more likely to engage in the reasoning, problem-solving and reflection that underpin deep understanding (Kilpatrick et al., 2001).

Throughout this evolution, we see an unresolved tension between skill-based and process-oriented perspectives. Simultaneously, we have a broadening of conceptualisations, such as those that include reasoning, metacognition, discourse (Schoenfeld, 2016), the affective domain (Grootenboer & Marshman, 2016), productive disposition (Kilpatrick et al., 2001), a deepening of understandings including axiomatic theories, embodiments, and symbols, with a persistent enticement for simpler notions of skills and processes. This unresolved tension between perspectives and underpinning components explains why mathematical thinking remains

inconsistently defined in literature, difficult for teachers to operationalise, and only vaguely referenced in policy documents. The lack of definitional clarity has pedagogical implications, as skill-oriented approaches prioritise procedural fluency and accuracy, while process-orientated perspectives emphasise inquiry, reasoning, and communication (Goos, 2014; Schoenfeld, 2016). Therefore, a clear and contemporary definition of mathematical thinking that can support teachers to develop shared understanding and classroom applications is needed. In this paper we propose a clear, contemporary definition.

Components of Mathematical Thinking

Despite the ambiguity in definitions, several common components are identified across mathematical thinking literature. Across the literature, communication frequently emerged as crucial in enabling students to express and justify mathematical ideas, alongside problem-solving, reasoning, and metacognition as closely related components of mathematical thinking (Goos & Kaya, 2019; Schoenfeld, 2016). Within this framing, processes including specialising and generalising, conjecturing and convincing have been commonly discussed as key components of mathematical thinking (Burton, 1984; Stacey, 2006). It is noted that while these components have been commonly discussed, agreement on definitions has not always been achieved.

Communication

Communication was widely recognised as a fundamental component of mathematical thinking, functioning both as a skill (Bloom et al., 1956) and a process that underpins understanding and proficiency (Goos, 1997). It encompasses verbal, non-verbal, written, and symbolic forms through which students express, interpret, and refine mathematical ideas, translating between representations, employing precise language, and connecting ideas to support sense-making (Goos, 1997; Kilpatrick et al., 2001). Building on the work of both Polya and Vygotsky, communication involves dialogue, gestures, visual representations, diagrams, and concrete materials (Schoenfeld, 2016; Stacey, 2006). From a socio-cultural perspective, collaborative discourse between teacher and student, or among peers may mediate learning, make thinking visible, and support reasoning and mathematical understanding (Brown, 2017). Teacher-led discussions, modelling, and inquiry-based strategies further foster engagement, reasoning, and the development of mathematical thinking, highlighting the importance of effective pedagogy (Van Zoest et al., 2017).

Problem-Solving, Reasoning, and Metacognition

Problem-solving appears to be intertwined with mathematical thinking (Isoda & Katagiri, 2012; Schoenfeld, 1985), although the relationship is contested. Some research has suggested mathematical thinking is a fundamental mental activity that is activated when solving problems (Danoebroto et al., 2024), whereas other studies argued that problem-solving is a context for developing mathematical thinking (Burton, 1984; Isoda & Katagiri, 2012). Although Polya's (1975) four-phase model (understanding, devising, carrying out, and reflecting) remains influential for problem solving, contemporary frameworks extended this perspective to include affective domain elements of beliefs, attitudes, and metacognition (Schoenfeld, 2016). Schoenfeld (2016) argued student's affective domain beliefs shape their problem-solving behaviours. Problem-solving was also framed as iterative and socially mediated, with collaborative discourse supporting perseverance, reasoning, and strategic competence (Grootenboer & Marshman, 2016; Kilpatrick et al., 2001). In each of the Queensland senior mathematics syllabi, problem-solving and mathematical thinking are intertwined, stating "Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples

without understanding” (Queensland Curriculum and Assessment Authority, 2024, p. 8). However, no definition of mathematical thinking is provided. This creates conceptual ambiguity about what constitutes mathematical thinking in classroom practice.

Mathematical reasoning is central to proficiency, enabling logical thought, explanation, justification, reflection, and application in real-world contexts (Kilpatrick et al., 2001; Stacey, 2006). The Australian Curriculum defines mathematical reasoning as:

central to thinking and working mathematically . . . analysing, proving, experimenting, modelling, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, and prove that something is true or false. (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2025, Reasoning, para. 1)

According to this definition, students are required to communicate their mathematical thinking to show how they reached their conclusion. The curriculum further defines mathematical processes as “the thinking, reasoning, communicating, problem-solving and investigation *process skills* (emphasis added) involved in working mathematically” (ACARA, 2025, Mathematical Processes, para. 1). It is interesting to note the use of the term ‘process skill’ here furthering the lack of clarity of these terms. Mathematical reasoning is simultaneously mandated and underspecified, leading to uncertainty for teachers.

Metacognition is defined as a students’ knowledge of their own cognitive processes, and monitoring and regulation of thoughts during mathematics tasks (Schoenfeld, 2016). These processes support cognitive regulation during complex tasks, with expert problem solvers demonstrating planning, checking, and strategy use to become autonomous learners (Goos & Kaya, 2019; Schoenfeld, 1982; Schoenfeld, 2016). Teachers who support the development of metacognition through pedagogical choices may be inherently fostering effective mathematical thinking.

Despite variations in emphasis, problem-solving, reasoning, and metacognition were consistently recognised as interdependent key components of mathematical thinking.

Specialising, Generalising, Conjecturing and Convincing

Although definitions of mathematical thinking varied, the interrelated processes of specialising, generalising, conjecturing, and convincing repeatedly appeared in the literature related to mathematical thinking despite shifts in conceptual emphasis. Specialising uses specific examples to explore and build understanding through inductive engagement with concrete representations, enabling students to recognise emerging patterns (Burton, 1984; Danoebroto et al., 2024). Building on this, generalising requires identifying regularities across examples and abstracting common elements that could be expressed symbolically or verbally, allowing learners generalise from specific examples (Stacey, 2006), through inductive and analogical reasoning (Isoda & Katagiri, 2012; Tall, 2008). Conjecturing occurs as students formulate and test hypotheses based on observed patterns, engaging in the cyclical process of hypothesising and revising (Goos & Kaya, 2019). Convincing involves validating conjectures through explanation, argumentation, or proof, reflecting the social and discursive nature of mathematics and supporting the development of adaptive reasoning (Kilpatrick et al., 2001).

Collectively, these processes were not enacted automatically by students but were shaped by teachers’ beliefs about mathematical thinking and their pedagogical choices, including task selection, questioning, and the facilitation of discourse. Teachers who valued exploration, justification, and productive struggle were more likely to design learning environments that made these processes explicit, thereby supporting students’ engagement in mathematical thinking as an enacted classroom practice rather than a fixed set of skills (Goos & Kaya, 2019; Schoenfeld, 2016).

Teachers' understanding of mathematical thinking

Teachers' beliefs about mathematics are generally regarded as internalised understandings about the world that an individual considers to be true, shaping their dispositions toward pedagogical practice (Marshman, 2021). Teachers' beliefs shape how mathematical thinking is interpreted and supported in their classroom. These beliefs influence instructional decisions, including task selection, questioning strategies, and tolerance for productive struggle, with students' views often mirroring those of their teachers (Marshman, 2021). When teachers value reasoning, problem-solving, and metacognition they are more likely to design learning environments that promote conceptual understanding and productive disposition, where mathematics is perceived as meaningful and attainable (Kilpatrick et al., 2001; Schoenfeld, 2016). For teachers to be able to support students to develop the essential construct of mathematical thinking, they need to know what constitutes mathematical thinking in their contemporary classrooms. This requires a clear and contemporary definition of the multifaceted nature of mathematical thinking.

Final Thoughts

This lack of clarity in the definition of mathematical thinking creates challenges for teachers seeking to intentionally foster this essential construct. A clear and contemporary definition is required to enable teachers to foster it in their classroom and enabling further research into mathematical thinking. The following definition is therefore proposed for mathematical thinking:

Mathematical thinking refers to the multidimensional processes through which students actively engage with, express, and develop mathematical ideas through reasoning, problem-solving, and reflective regulation of their thinking. It involves the coordinated processes of specialising, generalising, conjecturing, and convincing, and is enacted through language, representations, and often collaborative discourse to achieve idea refinement or consensus. Mathematical thinking integrates cognitive and affective dimensions, including metacognition, beliefs, and dispositions, enabling students to make connections, to move from concrete exploration toward abstraction and formal reasoning, justify conclusions, and apply mathematics beyond procedural execution into real-world contexts.

With a contemporary working definition of mathematical thinking for teachers, further research is needed to better understand teachers' perceptions about mathematical thinking and their reflections on their mathematical classroom pedagogies.

References

- Australian Curriculum, Assessment and Reporting Authority (ACARA). (2025). *Australian curriculum: Mathematics* v9.0. <https://www.australiancurriculum.edu.au/curriculum-information/understand-this-learningarea/mathematics>
- Bloom, B. S., Englehart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives, handbook 1: The cognitive domain*. David McKay Co. Inc.
- Brown, R. (2017). Using collective argumentation to engage students in a primary mathematics classroom. *Mathematics Education Research Journal*, 29(2), 183-199.
- Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal for Research in Mathematics Education*, 15(1), 35-49.
- Danoebroto, S. W., Suyata, & Jailani. (2024). Teachers' Efforts to Promote Students' Mathematical Thinking Using Ethnomathematics Approach. *Mathematics Teaching Research Journal*, 16(2), 207-231.
- Delima, N., Rahmah, M. A., & Akbar, A. (2018). The Analysis of Students' Mathematical Thinking based on Their Mathematics Self-Concept. *Journal of Physics: Conference Series*, 1108(1). <https://doi.org/https://doi.org/10.1088/1742-6596/1108/1/012104>
- Devlin, K. (2012). *Introduction to Mathematical Thinking*. Keith Devlin.

- Geiger, V., Beswick, K., Fielding, J., Scheiner, T., Kaiser, G., & et.al. (2023). *Investigating critical mathematical thinking when applying to real-world problems*. In Eds, P. Drijvers, C Csapodi, H. Palmer, K. Gosztonyi & E. Konya, *Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*, Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Budapest, Hungary.
- Goos, M. (1997). *Making sense of mathematics: The teacher's role in establishing a classroom community of practice*. The Annual Postgraduate Research Conference of the Graduate School of Education, Queensland, Australia. <https://search.informit.org/doi/10.3316/aeipt.110749>
- Goos, M. (2014). Researcher-Teacher relationships and models for teaching development in mathematics education. *ZDM*, 46, 189-200. <https://doi.org/10.1007/s11858-013-0556-9>
- Goos, M., & Kaya, S. (2019). Understanding and promoting students' mathematical thinking: a review of research published in ESM [Review]. *Educational Studies in Mathematics*, 103(1), 7-25. <https://doi.org/10.1007/s10649-019-09921-7>
- Grootenboer, P., & Marshman, M. (2016). *Mathematics, Affect and Learning: Middle School Students' Beliefs and Attitudes About Mathematics Education*. Springer. <https://doi.org/10.1007/978-981-287-679-9>
- Isoda, M., & Katagiri, S. (2012). *Mathematical Thinking: How to develop it in the classroom*. In (Vol. 1). World Scientific Publishing Company.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Research Council, National Academy Press. <https://nap.nationalacademies.org/read/9822/chapter/6#116>
- Marshman, M. (2021). Learning to teach mathematics: How secondary prospective teachers describe the different beliefs and practices of their mathematics teacher educators. In *The learning and development of mathematics teacher educators: International perspectives and challenges* (pp. 123-144). Springer.
- Marshman, M., & Goos, M. (2018). The beliefs about mathematics, its teaching and learning of those involved in secondary mathematics pre-service teacher education. *Mathematics Education Research Group of Australasia (MERGA)*, 519-526.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. NCTM.
- OECD. (2024). *Mathematics for life and work: A comparative perspective on mathematics to inform upper secondary reform in England*. OECD Publishing. <https://doi.org/10.1787/26f18d39-en>
- Polya, G. (1957) *How to solve it. A new aspect of mathematical method. 2nd Edition*, Princeton University Press.
- Queensland Curriculum and Assessment Authority (QCAA). (2024). *Mathematical Methods general senior syllabus 2025 v1.3*: General senior syllabus. 1.3. Queensland Curriculum and Assessment Authority. <https://www.qcaa.qld.edu.au/senior/senior-subjects/syllabuses/mathematics/mathematics-methods>
- Schoenfeld, A. H. (1982). *Expert and Novice Mathematical Problem Solving. Final Project Report and Appendices B-H*.
- Schoenfeld, A. H. (1985). *Mathematical Problem Solving*. Academic Press.
- Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of education*, 196(2), 1-38.
- Stacey, K. (2006). *What is mathematical thinking and why is it important?* Proceedings of the APEC-Tsukuba International Conference 2007: Innovative Teaching mathematics through lesson study. University of Tsukuba.
- Stacey, K., Burton, L., & Mason, J. (1985). *Thinking Mathematically*. Addison-Wesley Publishers.
- Stacey, K., Vincent, J., Stephens, M., & Holton, D. (2015). *Desktop review of mathematics school education: pedagogical approaches and learning resources* [Report]. <https://search.informit.org/doi/10.3316/aeipt.215847>
- Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, 20(2), 5-24.
- Tong, D. H., Loc, N. P., Uyen, B. P., & Son, T. H. (2020). Enhancing creative and critical thinking skills of students in mathematics classrooms: An experimental study of teaching the inequality in high schools [Article]. *Universal Journal of Educational Research*, 8(2), 477-489. <https://doi.org/10.13189/ujer.2020.080219>
- Van Zoest, L. R., Stockero, S. L., Leatham, K. R., Peterson, B. E., Atanga, N. A., & Ochieng, M. A. (2017). Attributes of Instances of Student Mathematical Thinking That Are Worth Building on in Whole-Class Discussion. *Mathematical Thinking and Learning: An International Journal*, 19(1), 33-54. <https://doi.org/10.1080/10986065.2017.1259786>

The Planning and Implementation of AI-Generated Rich Tasks in Primary Mathematics Classrooms

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This paper examines how three Australian primary teachers used generative AI to design and implement rich mathematical tasks. Drawing on multiple data sources analysed using three theoretical lenses, findings indicate that while generative AI expanded planning possibilities, the richness of tasks depended on teacher enactment rather than AI output. Teachers with deeper mathematical knowledge for teaching crafted more intentional prompts, critically evaluated outputs, and adapted tasks responsively, positioning professional knowledge as decisive in whether AI-supported task design enhances or constrains meaningful mathematical learning.

The emergence of publicly available generative AI (GenAI) in late 2022 has significant implications for mathematics education. Evidence suggests that AI holds considerable potential to support and enhance teaching practice across educational contexts (Wayne et al., 2023). Digital resources have long been recognised for their capacity to disrupt traditional pedagogical approaches (Attard & Holmes, 2020), and, in the case of GenAI, may also contribute to alleviating high teacher workloads (Granziera et al., 2025). However, research consistently demonstrates a lag between the emergence of new technologies and scholarly examination of their implications for teaching and learning (Granić, 2022).

Although research on the use of GenAI tools in education continues to emerge, there are clear gaps. For example, literature that specifically refers to mathematics education tends to focus on high school or tertiary levels (Egara & Mosimege, 2024). There is a gap in empirically informed literature that addresses the impacts of primary teachers' use of GenAI tools to develop quality tasks and resources for mathematics education.

This paper presents findings from a study conducted in Australian primary mathematics classrooms examining the potential value and impact of GenAI as a tool to create rich mathematical tasks for primary classrooms. The paper presents findings pertaining to three teachers' experiences planning and implementing AI-generated tasks.

Background

Teacher adoption of GenAI for instructional purposes is accelerating, though uptake remains uneven. US data indicates that teachers of mathematics report lower use for planning or instruction, and teachers in low-socioeconomic schools report lower adoption rates (Kaufman et al., 2025). Australian large-scale data are limited, though early evidence suggests weekly AI use for planning, assessment, and feedback (Blundell et al., 2025). Teachers under high stress and time pressure are more likely to value AI tools (Collie & Martin, 2024), yet efficiency-driven uptake may foster uncritical use of materials designed for hypothetical rather than specific classrooms.

Research on AI-generated mathematics content supports this concern. Teachers can perceive AI-generated lesson plans as generic and vague, lacking the specificity needed for immediate classroom use (Walkington, 2025). Similarly, Rutherford et al. (2025) found that teachers valued ChatGPT for generating ideas but were frustrated by limitations in producing cohesive resources; however, lessons developed with ChatGPT resulted in more motivationally supportive teacher language and reduced student boredom. Mathematics teachers describe AI

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lesson plan outputs as a useful starting point that requires iterative modification through prompting to meet classroom needs (Aga et al., 2024). However, the quality of this iteration can vary; Dilling and Herrmann (2024) found pre-service mathematics teachers' interactions with large language models (LLMs) were primarily superficial, with teachers using a limited range of prompts. Effective prompt crafting is itself an emerging professional skill grounded in teachers' pedagogical and content knowledge (Biton et al., 2025).

Despite these affordances, GenAI can produce mathematically inaccurate or developmentally inappropriate content (Sawyer & Aga, 2024). Pre-service teachers (PSTs) may struggle to detect such errors; Kim et al. (2026) reported 33.8% of errors in ChatGPT-generated fraction problems went undetected. Critically, PSTs with stronger Mathematical Knowledge for Teaching (MKT) were more critical of AI-generated plans (Aga et al., 2024), and more accurate in identifying pedagogical weaknesses (Gurl et al., 2025). These findings suggest the capacity to effectively negotiate AI outputs for mathematics lesson planning may depend on the depth of teachers' own mathematical and pedagogical knowledge.

The findings also have particular implications for the design and use of rich mathematical tasks. Rich mathematical tasks are broadly characterised by being authentic to real world applications, accessible but challenging for students at multiple levels, having multiple solution pathways, and provide opportunities for reasoning, mathematical connections, and discussion (Moulds, 2002). Critically, research consistently demonstrates that a task's richness is not an inherent property of the task itself but is realised through its enactment in the classroom (Piggott, 2011), and tasks frequently decline in cognitive demand during classroom implementation (Tekkumru-Kisa et al., 2020). This enactment-dependent nature is particularly significant in the context of AI-generated lesson plans, where the tool may produce something that structurally resembles a rich task but cannot ensure its mathematical potential is realised in practice.

The way that GenAI is perceived by teachers impacts their uses, and student learning outcomes of this use (Alwaqdani, 2025). Teachers play a central role in interpreting and integrating AI into mathematics learning, and this is particularly the case in the planning and implementation of rich mathematical tasks. This paper presents an investigation of how three primary teachers used GenAI to design rich mathematical tasks, and how their MKT shaped both the planning process and classroom enactment of those tasks.

Methods and Methodology

This qualitative multiple case study aimed to develop a nuanced understanding of the implications of using GenAI to support the development of rich tasks for teachers of primary mathematics. The study was guided by the following research questions:

- In what ways do teachers use generative AI to produce primary mathematics rich tasks?
- What are the implications for the use of GenAI to design rich tasks for primary teachers?
- What are the implications of the use of AI-generated rich tasks for primary students?

Data informing this paper are drawn from a larger study comprising 34 case studies across nine schools (in NSW, WA, and the ACT), from a range of socioeconomic areas. Each case consisted of a teacher and their students. Teachers were recruited via professional networks, and data was collected from during 2025. Ethics approval was granted by the Western Sydney University Human Research Ethics Committee. Schools and participants were anonymised, and pseudonyms used throughout. Three cases were selected to represent the variation in the implementation of AI across different school contexts. Table 1 summarises the cases analysed and demographic information for the participants included in this paper.

Table 1

	Case A	Case B	Case C
Teacher Name (Pseudonym)	Olivia	Audrey	Tania
School	Non-government, large	Government, medium	Government, small
Demographic	school in high SES area	school in mid SES area	school in mid SES area
Teacher Career Stage	Early	Mid	Late
Grade	5	3/4	1
Lesson Title	‘Can You Beat the Spinner?’	‘Decimal Dilemma at the Market’	‘Fraction Lines’
Task Topic	Chance and probability	Decimal place values	Linear fractions
Task Activity	Students designed and tested a spinner game in pairs, predicting fairness by recording outcomes across 20 spins.	Students created a shopping list, using place value and decimals to calculate totals and stay within a set budget	Students divided lolly snakes and Play-Doh into halves and quarters to explore linear fractions.

Data collection consisted of pre- and post-interviews with teachers, one AI planning session, three classroom observations per teacher, student focus groups, and AI chat planning artefacts. Pre-interviews explored teachers’ existing attitudes toward and experience with GenAI, while post-interviews captured reflections on the planning and implementation process. In the AI planning session facilitated by the research team, teachers interacted with a GenAI tool to design three rich mathematical tasks and accompanying lesson plans. Teachers in NSW government schools used NSWeduChat, their approved platform, while the other teachers selected their preferred tool (ChatGPT or Claude). During the sessions the research team provided guidance, but teachers were given autonomy to interact with the GenAI directly. To support them, teachers were given a structured set of elements to address: a negotiated definition of a rich task; task duration; curriculum outcomes; a warm-up task; learning intentions and success criteria; opportunities for explicit teaching; a resource list; enabling and extending prompts; differentiation ideas; reflection prompts; an assessment rubric; and a word list. These served as a scaffold to guide interactions with the AI while providing flexibility.

Teachers were observed across three lessons corresponding to the three tasks. Observations were conducted by both authors, recording detailed notes using an observation schedule. Student focus groups of up to six students (teacher selected) were conducted following implementation, with each class participating in at least one focus group session per case.

Following data collection, all sources were collated and prepared for analysis. Interviews and focus groups were transcribed verbatim, and GenAI planning logs and artefacts were formatted with attention to teacher prompts. The full dataset was imported into NVivo 15, where a coding framework grounded in the three theoretical lenses was applied. Researchers first coded the broader dataset independently then collaboratively, reviewing codes to ensure consistency. For the three cases reported here, data were coded independently and then analysed comparatively to identify patterns and variation in AI implementation and task quality.

To address the research aims and capture the complexities and nuances of AI uptake, teacher planning, and classroom practice, three theoretical frameworks were applied to the data. First, the Technology Acceptance Model (TAM) (Davis, 1989) was used as the ‘adoption’ lens to help us understand teachers’ perceptions of the value of generative AI and their attitudes and intentions for future use. The TPACK Developmental Model developed by Niess et al., (Niess et al., 2009) was used as a ‘practice’ lens alongside TAM to identify how the teachers used the generative AI along with the level of assimilation of the tool into their teaching practices.

Finally, the Mathematical Knowledge for Teaching (MKT) model (Ball et al., 2008) assisted us as the ‘mathematics and pedagogy’ lens in investigating the depth of mathematics knowledge required to ensure teachers’ interactions with the generative AI produced tasks that were appropriate and of high quality.

Findings and Discussion

To address the research questions, we first examine findings related to the design and classroom enactment of the three rich tasks. We then analyse the broader impact of using GenAI as a planning tool for the three teachers and their students.

Task Design using GenAI

As detailed, the participating teachers were at different career stages and had varying prior experience with GenAI. Analysed through the Technology Acceptance Model (TAM) (Davis, 1989), all three demonstrated sustained positive attitudes towards AI use. Teachers reported strong perceived usefulness for rich task development through AI, and for Tania and Audrey, perceived ease of use translated into ongoing behavioural intention, with continued use across mathematics and other areas. These findings suggest that technological resistance was not a primary barrier; rather, the critical work occurred at the pedagogical level.

Viewed through the TPACK Development model (Niess et al., 2009), participation appeared to shift teachers’ integration of AI within their pedagogical practice. Tania and Olivia began at the Accepting phase, consistent with their voluntary participation. Tania was already experimenting and was in the Adapting phase as described by Niess et al. On conclusion of the study, Audrey and Olivia had moved toward the Exploring phase, recognising how GenAI had extended their pedagogical repertoire, as evidenced in Olivia’s reflection: “There’s so many things that I think I needed to contextually base the lesson more, and that can be done using AI...”, in contrast to the findings of Walkington (2025). Similarly, Audrey described increasingly strategic prompting to contextualise tasks and further differentiate learning: “...if I’m not happy with one or two questions, then I would put it through again. ‘Can you differentiate this question and include students with mixed abilities?’ It’s easy.”

While TAM and TPACK assist in explaining adoption and developmental progression, the MKT framework (Ball et al., 2008) illuminates how teachers mediated AI-generated suggestions through professional judgements during task design. Knowledge of Content and Curriculum (KCC) played a particularly important role during planning. Although the teachers entered specific curriculum content descriptions for each lesson, they were at times required to prompt the AI to modify the content when it did not align with the curriculum requirements.

Knowledge of Content and Students (KCS) was evidenced in a planning discussion between the research team and Olivia regarding her lesson on chance and probability. When ChatGPT suggested students construct their own spinners, Olivia initially considered using a virtual spinner via PolyPad for reasons of efficiency. However, she ultimately decided that having students physically create their own spinners would enhance their conceptual understanding of randomness and probability, while PolyPad could be used to explore variations in likelihood or to conduct extended experiments. This episode illustrates how KCS informed pedagogical decision-making in ways that mediated the AI-generated suggestions.

Further evidence of KCS and Knowledge of Content and Teaching (KCT) emerged in the adaptations Audrey made between planning and implementation. She initially modified her ‘Decimal Dilemma at the Market’ task by personalising it as ‘Audrey’s Ice-Cream Parlour’, replacing the suggested fictional market price list from the NSEduChat lesson output: (“Provide students with a fictional market price list that includes items priced in whole numbers and decimals”) with a context she believed would increase student engagement. This provided a context for explicit modelling prior to independent problem solving. She then extended the

lesson beyond the AI-generated structure by incorporating real Woolworths supermarket catalogues. This additional task required students to apply their understanding of decimals to select healthy snacks within a given budget.

Similarly, Tania negotiated with NSWeduChat during task design, prompting: “I said I would prefer Play-Doh. ‘Can you give me a lesson with Play-Doh?’ And that’s what it came out with.” Her request reflected a judgement about the practical suitability and appropriateness of materials for her class, shaping the AI-generated task to better align with her students’ needs and the realities of her classroom context. Together, these modifications illustrate KCS in anticipating how particular materials and contexts would function for specific learners, and KCT in the deliberate selection and structuring of tasks to support conceptual development.

While the planning phase illustrated how the teachers mediated GenAI through their MKT, its role became more complex during implementation. Task design decisions were further shaped by real-time pedagogical judgement and student responses. The following section examines how AI-supported tasks were implemented in practice and how teachers’ professional knowledge continued to shape their enactment.

Classroom Enactment of AI-Generated Rich Tasks

Classroom observations of the rich task implementation made visible the central role of MKT in mediating how AI-supported tasks were enacted in practice. Across all three lessons, the teachers were observed making in-the-moment decisions in response to student thinking, an aspect they later acknowledged in their final interviews.

In Audrey’s lesson, many adaptations were observed. These changes reflected her depth of MKT and her recognition of the teaching and learning opportunities embedded within the rich task. Audrey made this comment in relation to her enactment of the task:

You have the time component, you still have explicit teaching, but it’s up to the teacher how you’re going to do your delivery... It’s not really part of the activity, it’s written there. But when I did my lessons, I always focused on vocab because from experience, that’s a big thing in my class.

Such real-time pedagogical adjustments cannot be anticipated by AI. Pedagogical decisions such as if and how students are grouped, potential responses to student misconceptions, or specific questioning techniques are dependent on the teacher’s knowledge. This was evident in Tania’s lesson where she adapted the task in the moment, explaining, “Well, it was just off the cuff. I saw that one of the kids wanted to know...that comes with just being a teacher for so long”. This comment highlights the role of accumulated experiential knowledge within MKT.

In contrast, fewer pedagogical adjustments were observed in Olivia’s lesson. During the lesson, the construction of physical spinners appeared to shift students’ attention towards the craft elements of the activity rather than the intended mathematical focus. No pedagogical shifts were made to address this during the lesson. However, her final interview, Olivia reflected:

I felt like for that group of children, it just kind of was about the spinners...Upon reflection, I would’ve done a virtual spinner just because I feel like it was very overwhelming for them to be cutting out or and colouring...It kind of took away from the maths...I should have done it digitally and had the confidence to maybe alter that with AI.

Olivia’s reflection demonstrates emerging professional judgement, suggesting that while the pedagogical implications were not fully recognised in the moment, they became visible through post-lesson reflection. In this way, the enactment phase further illustrates how MKT shapes not only immediate instructional decisions, but also teachers’ ongoing professional development.

In Olivia’s lesson, some of these affordances were not taken up during implementation. For example, students constructed pre-printed spinners divided into equal sections, which limited opportunities to explore unequal partitions and their probabilistic implications. Similarly, potential connections to angle measurement and to addressing emerging misconceptions during

the reflection phase were not pursued in the moment. Importantly, these observations are not presented as a critique of individual practice. Rather, they underscore that recognising and capitalising on the mathematical richness of tasks, particularly those generated or supported by AI, depends on teachers' ongoing development of MKT. As such, the findings highlight the importance of sustained professional learning to support teachers in identifying and enacting the full range of mathematical possibilities embedded within AI-generated rich tasks.

A significant observation arising from the comparison of classroom enactments with the original GenAI outputs was the variation in how teachers recognised and leveraged the mathematical 'richness' embedded within the tasks. Audrey and Tania frequently paused to provide explicit instruction, extend discussion, and adapt the tasks to deepen student thinking across multiple mathematical concepts. Their enactment reflected well-developed Specialised Content Knowledge (SCK), KCS, and KCT, particularly in identifying moments where mathematical ideas could be made more explicit.

Implications for Teachers

The use of GenAI to design rich tasks presents both opportunities and responsibilities for teachers of primary mathematics. Participants reported that AI generated fresh ideas, particularly for experienced teachers such as Tania and Audrey. A key affordance was the capacity to tailor tasks through deliberate prompting. As Audrey noted, she could adapt tasks to suit her class rather than rely on existing published resources: "I could make it something that they're familiar with rather than having the general terms as used in the department units."

However, contextualising a task to a specific classroom extends beyond embedding it in a familiar scenario. It requires deep KCS, including awareness of prior understanding, likely misconceptions, and classroom dynamics. While teachers can input contextual details, AI cannot comprehend the nuanced realities of a live classroom. As Olivia reflected, teachers must pause and ask if a task truly fits their context rather than accepting the first output generated.

Prompt crafting therefore emerges as a contemporary enactment of MKT within digitally mediated planning environments, a finding that complements those of Biton et al., (2025). Knowing what to ask of the AI is itself a pedagogical act, requiring clarity about mathematical intentions and anticipated student responses. The quality of AI-generated tasks is shaped less by the tool and more by the professional knowledge within the prompts that guide it.

Teachers also identified practical considerations, including unrealistic time allocations within AI-generated lessons. This reinforced the need for teachers to critically interpret outputs, adjust expectations, and revisit plans prior to implementation. Although all three teachers agreed that refined prompting could significantly reduce planning workload, they cautioned that efficiency must not replace professional judgement. As Audrey noted, AI is only as useful as a teacher's capacity to 'unpack' the activity, and Tania similarly warned that without strong pedagogical knowledge, AI-generated tasks risk lacking depth.

While GenAI expands planning possibilities and may reduce workload, its pedagogical value remains contingent upon teachers' MKT and their willingness to actively negotiate, refine, and take ownership of AI outputs. These teacher-level implications have important consequences for students' engagement and experiences of differentiated learning.

Implications for Students

The teacher-level implications of GenAI use were reflected in students' classroom experiences. Across the three cases, in alignment with the findings from Rutherford et al., (2025), AI-supported tasks were associated with heightened engagement, particularly when teachers actively adapted and contextualised tasks in response to student needs. During lesson observations and focus groups, students expressed enthusiasm for tasks connected to familiar contexts, with one student commenting, "It was fun, and we could understand it. It was creative

and we're still learning" (Focus Group, Case A). The hands-on and creative elements embedded within the tasks were also noted as distinct from students' typical mathematics lessons, particularly in Olivia's class.

At the same time, students in all three classes expressed a desire for greater differentiation. In Audrey's class, a student reflected, "I would've changed the difficulty level because I think most people found it pretty easy and the people who found it hard only struggled with a few bits". Similarly, students in Tania's and Olivia's classes suggested that additional challenge or extension opportunities would have strengthened the tasks. These comments indicate that while engagement was high, not all learners experienced an optimal level of cognitive demand.

Students also recognised that the rich tasks integrated multiple mathematical ideas. As one student in Audrey's class observed, "It's different because we usually just do one topic for math, but today we did a bit more...". This suggests that AI-mediated task design has the potential to support more connected and conceptually integrated mathematical experiences. However, the extent to which these affordances translate into deep learning remains dependent upon how effectively teachers refine and differentiate the tasks during planning and enactment.

Conclusion

This paper reports on three cases only and does not claim broad generalisability. However, the combined use of TAM, the TPACK Development Model, and MKT enabled a layered analysis of AI adoption, pedagogical development, and classroom enactment. Across cases, findings show that while GenAI expands possibilities for task design and planning efficiency, its impact is neither automatic nor uniform.

The effectiveness of AI-supported rich tasks was mediated by teachers' MKT. Teachers who crafted intentional prompts, critically evaluated outputs, and adapted tasks responsively enhanced enactment and student engagement. Although students valued contextualised and conceptually connected tasks, their call for greater differentiation reinforces that meaningful learning depends on sustained teacher expertise rather than technological capability alone.

While GenAI may reduce workload and broaden pedagogical options, its value remains contingent upon teachers' professional knowledge and confidence to negotiate, rather than accept, AI outputs. Further research is needed to identify effective professional learning approaches and clarify the conditions under which AI may enhance student outcomes. Ultimately, GenAI does not diminish teacher expertise; rather, it repositions MKT as the decisive factor in determining whether AI-supported task design enhances or constrains meaningful mathematical learning.

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References

- Aga, Z. G., Sawyer, A. G., & Wolfe, M. (2024). More confidence more critical: Investigating secondary mathematics preservice teachers' use of an Artificial Intelligence chatbot as a curriculum development tool. *American Journal of Creative Education*, 7(2), 63–78. <https://doi.org/10.55284/ajce.v7i2.1278>
- Alwaqdani, M. (2025). Investigating teachers' perceptions of artificial intelligence tools in education: P2otential and difficulties. *Education and Information Technologies*, 30(3), 2737–2755. <https://doi.org/10.1007/s10639-024-12903-9>
- Attard, C., & Holmes, K. (2020). *Technology-enabled Mathematics Education: Optimising Student Engagement*. Routledge.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59, 389–407.

- Biton, Y., Segal, R., & Alush, K. (2025). How utilizing generative AI when addressing pedagogical and mathematical events contributes to mathematics teacher educators' TPACK (Technological Pedagogical Content Knowledge). *International Journal of Education in Mathematics, Science and Technology*, 13(4), 895–913.
- Blundell, C. N., Mukherjee, M., & Nykvist, S. (2025). Adopting generative AI in K-12 teaching and learning: Australian teachers' actions through the lens of innovation theory. *Education and Information Technologies*, 30(16), 24009–24028. <https://doi.org/10.1007/s10639-025-13699-y>
- Collie, R. J., & Martin, A. J. (2024). Teachers' motivation and engagement to harness generative AI for teaching and learning: The role of contextual, occupational, and background factors. *Computers and Education: Artificial Intelligence*, 6, 100224. <https://doi.org/10.1016/j.caeai.2024.100224>
- Davis, F. D. (1989). Perceived usefulness, perceived ease of use, and user acceptance of information technology. *MIS Quarterly*, 13(3), 319–340. <https://doi.org/10.2307/249008>
- Dilling, F., & Herrmann, M. (2024). Using large language models to support pre-service teachers mathematical reasoning—An exploratory study on ChatGPT as an instrument for creating mathematical proofs in geometry. *Frontiers in Artificial Intelligence*, 7. <https://doi.org/10.3389/frai.2024.1460337>
- Egara, F. O., & Mosimege, M. (2024). Exploring the integration of artificial intelligence-based ChatGPT into mathematics instruction: Perceptions, challenges, and implications for educators. *Education Sciences*, 14(7), 742. <https://doi.org/10.3390/educsci14070742>
- Granić, A. (2022). Educational technology adoption: A systematic review. *Education and Information Technologies*, 27(7), 9725–9744. <https://doi.org/10.1007/s10639-022-10951-7>
- Granziera, H., Collie, R. J., Roberts, A., Corkish, B., Tickell, A., Deady, M., O'Dea, B., Tye, M., & Werner-Seidler, A. (2025). Teachers' workload, turnover intentions, and mental health: Perspectives of Australian teachers. *Social Psychology of Education*, 28(1), 149. <https://doi.org/10.1007/s11218-025-10113-w>
- Gurl, T. J., Markinson, M. P., & Artzt, A. F. (2025). Using ChatGPT as a lesson planning assistant with preservice secondary mathematics teachers. *Digital Experiences in Mathematics Education*, 11(1), 114–139. <https://doi.org/10.1007/s40751-024-00162-9>
- Kaufman, J. H., Woo, A., Eagan, J., Lee, S., & Kassan, E. B. (2025). *Uneven Adoption of Artificial Intelligence Tools Among U.S. Teachers and Principals in the 2023–2024 School Year*. https://www.rand.org/pubs/research_reports/RRA134-25.html
- Kim, Y. R., Park, M. S., & Joung, E. (2026). Exploring the integration of artificial intelligence in math education: Preservice Teachers' experiences and reflections on problem-posing activities with ChatGPT. *School Science and Mathematics*, 126(1), 9–23. <https://doi.org/10.1111/ssm.18336>
- Moulds, P. (2002). Rich tasks: developing student learning around important tasks. *Australian Science Teachers' Journal*, 48(4).
- Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C., Browning, C., Ozgun-Koca, S. A., & Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, 9, 4–24.
- Piggott, J. (2011). *Rich Tasks and Contexts*. <http://nrch.maths.org/5662>
- Rutherford, T., Rodrigues, A., Duque-Baird, S., Veng, S., Mykyta-Chomsky, R., Cao, Y., Chisholm, K., & Bergwall, E. (2025). “I just think it is the way of the future”: Teachers' use of ChatGPT to develop motivationally-supportive math lessons. *Computers and Education: Artificial Intelligence*, 8, 100367. <https://doi.org/10.1016/j.caeai.2025.100367>
- Sawyer, A. G., & Aga, Z. G. (2024). Counterexamples to demonstrate artificial intelligence chatbot's lack of knowledge in the mathematics education classroom. *Connections*.
- Tekumru-Kisa, M., Stein, M. K., & Doyle, W. (2020). Theory and research on tasks revisited: Task as a context for students' thinking in the era of ambitious reforms in mathematics and science. *Educational Researcher*, 49(8), 606–617. <https://doi.org/10.3102/0013189X20932480>
- Walkington, C. (2025). The implications of generative artificial intelligence for mathematics education. *School Science and Mathematics*, n/a(n/a). <https://doi.org/10.1111/ssm.18356>
- Wayne, H., Fengchun, M., & UNESCO. (2023). *Guidance for generative AI in education and research*. UNESCO Publishing.

Using Generative AI To Develop Maths Tasks With Focus On Social Justice Perspectives

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This paper reports on the use of ChatGPT to develop Social Justice Mathematics (SJM) tasks for year 5 and 6 students. Two educators engaged in an iterative prompting process, drawing on a task design framework. Findings indicate that while ChatGPT can produce tasks situated in authentic, equity-oriented contexts, outputs frequently described mathematical activity rather than operationalising it. This requires teachers to apply their pedagogical content knowledge to evaluate and re-direct AI generated content. The SJM task framework proved useful both as a prompt structure and as an evaluative lens for teachers to use for designing authentic social justice-based tasks.

Introduction

Generative Artificial Intelligence (AI) models, such as ChatGPT, can be powerful tools for enhancing teachers' professional learning and pedagogical practices (Cameron & Mesiti, 2024). These technologies present both affordances and challenges for primary school teachers in mathematics education. AI tools have the potential to support teachers in creating curricular materials and designing meaningful learning activities that encourage critical thinking and effective collaboration among students (Baidoo-Anu & Ansah, 2023). However, a key challenge for teachers is knowing how to craft questions or prompts that align with the broader pedagogical intentions of a lesson. At present, the use of AI to generate prompts for developing rich tasks that emphasise the importance of critical and informed thinking when addressing social issues (Skilling & Park, 2025) is limited in educational research and practice.

A large body of literature in mathematics education calls for broadening the school mathematics curriculum to include instructional approaches that engage students with contemporary social issues (e.g. Frankenstein, 2009; Gutstein, 2016; Skovsmose, 2023). More specifically, it is argued that students should be offered opportunities to critically examine complex real-world problems (Steflitsch & Kolloosche, 2025) and develop their identities as reflective and responsible citizens (English, 2025).

This paper explores how generative AI tools can be used to adapt maths tasks to engage with personal and societal issues using a Social Justice Mathematics (SJM) framework informed by the literature (e.g., Gutstein, 2016; Skovsmose, 2023). We report on an iterative prompting process used to refine ChatGPT's output for the same mathematics task across two trials. The following research questions are addressed in this paper:

What prompts are useful for developing SJM tasks using ChatGPT?

To what extent did the tasks generated by ChatGPT align with SJM task principles?

Background

This paper will consider two core themes including Generative AI and task design, and social justice mathematics tasks. To help frame the understanding, each theme will be explained in the following sections.

AI and Task Design

Generative AI tools, particularly large language models such as ChatGPT, are increasingly being explored as supports for teachers in the design of instructional materials and learning tasks. These models are trained on large corpora of publicly available and licensed texts and generate responses by identifying patterns and associations across this training data, producing structured text in response to user prompts (Cameron & Mesiti, 2024). In educational contexts, AI can function as a support for primary school teachers by acting as a ‘guide on the side’ or as a co-designer of curriculum materials, including lesson plans and learning sequences. However, while AI can generate coherent and structured outputs, the decision-making processes underpinning these outputs remain largely opaque to users (Daugherty & Carter, 2025), and the pedagogical expertise required to evaluate and refine AI-generated teaching and learning materials continues to reside with teachers. Effective use therefore requires new forms of professional knowledge, including the capacity to craft prompts, interpret outputs critically, and iteratively refine generated content so that it aligns with pedagogical intent (Cameron & Mesiti, 2024).

Within this emerging area, attention has increasingly turned to the role of prompts in shaping the nature and quality of AI-generated outputs. Prompt engineering is commonly described as the practice of designing inputs that guide a language model to generate precise, relevant, and task-appropriate responses (Spasić & Janković, 2023). Practitioner-oriented literature further suggests that prompts are often structured around a defined task, supported by additional elements such as instructions, roles, or key terms that help focus the generated response (Ibrahim, 2023). These techniques highlight how relatively small changes in prompt structure can influence the specificity and orientation of AI generated content.

More systematic investigation of prompt design has begun to appear in research literature. Spasić and Janković (2023), for example, examined the use of standard prompting techniques in the preparation of lesson plans and found that the inclusion of role specification, explicit instructions, and seed words led to more detailed and targeted outputs. Their work highlights that the structure of a prompt can directly shape the form and focus of the materials generated by AI, suggesting that careful prompt construction is an important consideration when using these tools for educational task design.

Features of Social Justice Mathematics (SJM) Tasks

Students’ ability to critically evaluate complex information, make inferences, and consider multiple solutions to real-world problems, requires engaging them with hands-on activities that address authentic challenges (Daugherty & Carter, 2025). Within SJM pedagogy, students learn to use mathematics to justify decisions and examine societal issues (Gutstein, 2006). This approach encourages critical engagement with complex social problems while supporting students to develop agency and capacity for action through mathematical problem-solving (Ernest, 2007). In SJM classrooms, mathematics tasks often draw on cultural, socio-political, and economic contexts (D’Ambrosio & D’Ambrosio, 2013), enabling students to explore issues of justice, such as inequitable access to resources (Ödmo et al., 2023).

One way of implementing SJM pedagogy is by designing mathematics tasks that promote collaborative problem-solving. Through this approach, students can work together to negotiate

alternative strategies to addressing complex problems, justify their solutions, and learn from their classmates' perspectives (Stefflitsch & Kollosche, 2025). Maintaining the cognitive demand of tasks is essential, to ensure that all students engage with rich mathematical concepts and connect solutions with their prior mathematical knowledge (Matthews et al., 2022). Creating contextualised tasks derived from students' life-worlds can further strengthen engagement, motivating students to use mathematics to interpret and question the world around them (Gutstein, 2006). At the same time, tasks framed around broader real-world problems can deepen students' understanding of both mathematical concepts and issues of justice and equity by exposing them to social realities beyond their immediate experience (Frankenstein, 2009).

Despite these benefits, designing such tasks can be challenging. Teachers often face constraints related to time, curriculum demands and the complexity of developing appropriate instructional activities (Daugherty & Carter, 2025). Guidelines or frameworks that can be used as evaluative tools for teachers to develop or select appropriate SJM tasks can help with addressing these barriers. Table 1 provides a framework for designing tasks with a SJM focus, based on elements commonly identified in the literature as being characteristic of SJM tasks (e.g., Frankenstein, 2009; Gutiérrez, 2012; Gutstein, 2006; Skovsmose, 2012).

Table 1

A Framework for Designing Maths Tasks with a Social Justice Focus

Design principles	Associated literature
1. Embedded in social or equity-based issues	(Gutiérrez, 2012; Gutstein, 2006; Skovsmose, 2012)
2. Mathematically rich & cognitively demanding	(Matthews et al., 2022)
3. Promotes critical reflection and action	(Gutstein, 2006; Stinson & Wager, 2012)
4. Encourage multiple perspectives and solutions	(Canogullari & Radmehr, 2025; Sullivan et al., 2012)
5. Student agency and decision making	(Gutstein, 2007; Wright, 2021)
6. Contextualised tasks	(Frankenstein, 2009; Gutstein, 2016)
7. Uses real-world data	(Bragg, 2021; Frankenstein, 2009)

The framework informed the analysis of mathematics tasks generated by ChatGPT in this study and is used in the Results section to evaluate the alignment of the generated tasks with principles of SJM pedagogy.

Methodology

Research Design

This study employed a comparative qualitative case study design (Stake, 1995) to examine how iterative prompting influenced the generation of SJM tasks using ChatGPT. Two educators independently entered an identical initial comprehensive prompt (see Figure 1) requesting a mathematically rich, socially critical task addressing inequitable access to healthcare in Australia. When devising the prompt, we were informed by Spasić and Janković's (2023) findings which advised specifying a role for ChatGPT, along with instructions and seed words. Each participant then engaged in iterative prompting, refining the generated output until the task was judged to be coherent, implementable, and aligned with principles of socially critical mathematics pedagogy. The participants worked independently during this phase.

All prompts and corresponding ChatGPT outputs were retained and treated as textual artefacts. Consistent with principles of document analysis (Bowen, 2009), each prompt–response pair constituted a unit of analysis. Within-case analysis traced shifts across iterations, focusing on linguistic accessibility, mathematical specificity, structural clarity, and classroom readiness. Iterative changes were summarised in tabular form to document refinements and limitations, which are outlined in the Results section of this paper. The final task produced in

each case was then evaluated against established characteristics of SJM tasks. Cross-case comparison is presented in the Discussion section.

Figure 1

Prompts used in ChatGPT

Can you apply a mathematical critical thinking and social justice lens on a critical societal issue like accessing healthcare in different regions of Australia. Generate a maths task by making social justice lens explicit in the task. Ensure that the task is mathematically rich and cognitively demanding, introduces a context from students' familiar settings and engage students in socio-political enquiries that support students' understanding of the relationship between mathematics and social justice issues of individual and global relevance. You could include teaching prompts such as: do you know the number of people living in metropolitan cities and regional/remote areas of Australia? Also, when was the last time you visited a doctor and how long did it take to reach the clinic or hospital? Then add a prompt – do you think that everyone in Australia should have equal access to primary healthcare? The task should be able to be completed within 60-90 minutes.

Results

This section presents the results from each case sequentially. For each case, we provide an overview of the iterative prompting process, including the key prompt shifts, the outputs generated by ChatGPT, and identified limitations. Due to the lengthy nature of the prompts and ChatGPT's responses, this information is summarised in tabular form to highlight changes across iterations and to distinguish refinements made, such as linguistic, structural, and mathematical modifications. A brief analytic commentary follows each table before presenting the final task artefact produced in that case. The final output for each case is then examined in relation to principles of SJM tasks referred to earlier in this paper. A cross-case comparison is presented in the discussion section.

Case 1: Single comprehensive prompt with iterative language refinement

Table 2 provides an overview of the prompts used, the outputs produced, the nature of improvement, and perceived limitations for each iteration in Case 1.

Table 2

Summary of Prompts and Outcomes for Case One

Iteration	Prompt	Output	Nature of Improvement	Limitations
1	Initial comprehensive prompt requesting socially critical, mathematically rich task	Generated full task description with authentic Australian context; explicit equity framing; suggested use of ratios, rates, modelling and representations	N/A	Abstract, teacher-facing language; no concrete dataset supplied; mathematics described rather than operationalised
2	Request to rewrite as student-facing and engaging	Reframed with relatable scenario ("Imagine you wake up feeling unwell..."); simplified language; retained ratio/representation work and fairness prompts	Increased accessibility; clearer audience alignment; maintained mathematical intent	Still lacked specific numerical data; inquiry questions potentially removed focus from modelling
3	Request for teacher rationale linking to mathematical critical thinking	Added explicit pedagogical justification articulating mathematical reasoning and equity focus	Clarified instructional intent; supported teacher implementation	Did not strengthen mathematical structure of task itself; dataset still unspecified

As can be seen from Table 2, the refinements to the task were primarily linguistic and pedagogical rather than structural in nature. The first output demonstrated strong conceptual alignment with socially critical mathematics, but lacked concrete numerical data and was too abstract for students to easily access. The second iteration improved accessibility and audience alignment while still maintaining mathematical relevance. The third iteration added pedagogical explanation but did not substantively modify the task design. While the final artefact integrated mathematics and social justice coherently, movement across iterations were based on clarification and justification, rather than increased mathematical specificity or advice on implementation.

Generated SJM Task One

The initial output indicated that the model successfully incorporated several core design features aligned with socially critical mathematics tasks. Specifically, the task situated learning in an authentic and nationally relevant context (healthcare access in Australia), foregrounded equity and fairness as central investigative questions, positioned mathematics as an analytical tool for interrogating social conditions, embedded opportunities for proportional reasoning, ratios, rates, comparison of distributions, and data representation, and structured the activity as an inquiry rather than a procedural exercise. The second iteration resulted in the task described in Figure 2, which was considered the ‘final’ version of the task, in that the third iteration involved adding a rationale, rather than further task refinement. As Figure 2 shows, while the task required students to analyse population distributions, compare service access, and reflect on fairness, no datasets were provided in the AI response, or guidance around what mathematical modelling might be required. The response did allude to social justice elements of equity-based issues along with the potential to consider multiple perspectives and solutions, but these were implied and not inherent requirements of the task.

Figure 2

The Final Version of SJM Task One

Imagine you wake up feeling unwell. You need to see a doctor. How far would you have to travel? How long would it take you to get there?

In this task, you will use mathematics to investigate whether people in different parts of Australia have fair access to primary healthcare, such as local doctors and clinics.

You will start by thinking about your own experiences.

When was the last time you went to a doctor or clinic?

How long did it take to get there?

Was it close to your home?

Next, you will explore data about where people live in Australia — in cities, regional towns, and remote areas — and where healthcare services are located. You will use maths to compare these regions and look for patterns, differences, and gaps. This might include: comparing numbers of people and clinics; working with distances, travel times, or ratios; using tables, graphs, or maps to show your thinking.

As you work, you will be asked to think about fairness, not just numbers. Should everyone in Australia have the same access to healthcare? What does fair access actually mean?

Can maths help us show when something is unfair?

By the end of the task, you will use mathematical evidence to explain what you think about healthcare access in Australia. You will be using maths not just to solve problems, but to understand a real issue that affects people's lives and to ask important questions about fairness and justice.

Case 2: Iterative development of task

Table 3 provides an overview of the prompts used, the outputs produced, the nature of improvement, and perceived limitations for each iteration in Case 2. Table 3 shows that the refinements to the task in the second case involved improvements in organisational clarity and classroom usability, with mathematical steps made more explicit. The first output generated a structured investigation with explicit data sets that potentially was already mathematically rich. Prompted by ChatGPT's offer of providing a worksheet, the task instructions were made more precise, but were still lengthy, potentially leading to cognitive overload, and hence considered not ‘classroom ready’. Improvements tended to centre on presentation and usability, rather than

conceptual strengthening, as the initial task was already quite strong in this regard. There was no need to request data sets as they were supplied in the response. Movement across the iterations reflected streamlining, rather than structural enhancement to the task. The third iteration retained the dataset and key questions and kept the focus on the mathematics. The final iteration resulted in the task shown in Figure 3.

Table 3

Summary of Prompts and Outcomes for Case Two

Iteration	Prompt	Output	Nature of Improvement	Limitations
1	Initial comprehensive prompt requesting socially critical, mathematically rich task	Request to summarise into half-page task card (and A5 layout)	N/A	Too much information; included conversational tone; formatting dense; some redundancy
2	Request to convert to printable worksheet	Reorganised into sequenced sections (calculations, modelling, representation, reflection); included tables for working	Improved organisational clarity and classroom usability; strengthened visibility of mathematical steps	Lengthy; potential cognitive overload; not classroom 'ready'
3	Request to summarise into half-page task card (and A5 layout)	Condensed to core investigative steps (measure access, model solutions, justify decision); retained dataset and key question	Increased concision and focus; preserved rigour while improving efficiency	Assumes a lot of prior knowledge; would require teacher scaffolding to support students

Generated SJM Task Two

As Figure 3 shows, ChatGPT generated a classroom ready task card that could be used directly with students. The task integrated inequity into the desired context and provided explicit numerical data that students could use to allocate resources. It required students to make decisions and justify their reasoning. The final task incorporated many, if not all, of the SJM design principles outlined in Table 1.

Figure 3

The Final Version of SJM Task Two

Maths Investigation Card

Is Healthcare Access Fair?

In Australia, where you live can affect how quickly you can see a doctor. Some people live minutes from a clinic, while others travel hours or rely on flying services for care.

Your Challenge
You are advising the government on where to build **30 new health clinics**. Use mathematics to decide what is fair.

Key Question
Does equal always mean fair? Who benefits from your decision, and who might still miss out?

Healthcare Access Data

Region	Population	Clinics	Avg Travel Time (min)
Metro	16,000,000	800	12
Inner regional	5,000,000	160	28
Outer regional	2,200,000	60	55
Remote	350,000	10	140

What to Do

- Calculate people per clinic and clinics per 100,000 people.
- Compare regions using tables, graphs, or ratios.
- Test different ways to allocate the 30 new clinics.
- Decide which plan is fairest and justify using maths.

Discussion and Conclusion

The first research question asked what is the iterative process of designing and refining ChatGPT prompts for SJM tasks, and what limitations does it present? Across both cases, the

prompting process was effective in moving outputs from abstract, teacher-facing descriptions towards student-facing tasks with accessible language and inquiry, consistent with Spasić and Janković's (2023) finding that structured prompts with explicit contextual cues generate more pedagogically coherent outputs. However, the two cases illustrate different prompting trajectories. In Case 1, revisions were primarily about the language, not the structure, and explicit prompting was required so that the task provided students with actual data to work with. In Case 2, the initial output was mathematically sound compared with Case 1. This suggests that the quality of the prompting process depends on the teacher's capacity to recognise what is missing, both pedagogically and mathematically (content).

The second research question asked to what extent tasks generated by ChatGPT aligned with the characteristics of the SJM framework. Both tasks from each case demonstrated meaningful alignment with several core design principles of the framework. However, Task 1 implied the social justice elements, rather than embedding them structurally, leaving the mathematical modelling open, whereas Task 2 came closer to meeting the dual criteria of being both socially meaningful and mathematically rich (Matthews et al., 2022). In task 2, the mathematics was explicit and intentional, with the 'real data' (Frankenstein, 2009) included.

The iterative prompting process itself offers a useful model for teacher engagement with generative AI tools. Rather than treating a single prompt as a finished product, teachers need to act as an active evaluator, requesting successive revisions to ensure their pedagogical content knowledge is driving what the AI is producing (Cameron & Mesiti, 2024). Primary school teachers could use the characteristics of the SJM task framework both as a generative prompt structure, and as an evaluative lens, when working with AI tools. By understanding what distinguishes a mathematically rich, equity-focused task from one that is merely contextualised in social language, teachers are better positioned to direct AI tools toward outputs of genuine pedagogical value. In doing so, they could use the characteristics of the SJM task framework to generate mathematical tasks that improve their students' ability to analyse complex societal problems and evaluate a range of potential solutions, thereby promoting informed citizenship.

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References

- Baidoo-Anu, D., & Ansah, L. O. (2023). Education in the era of generative artificial intelligence (AI): Understanding the potential benefits of ChatGPT in promoting teaching and learning. *Journal of AI*, 7(1), 52-62. <https://doi.org/10.61969/jai.1337500>
- Bowen, G. A. (2009). Document analysis as a qualitative research method. *Qualitative Research Journal*, 9(2), 27-40. <https://doi.org/10.3316/QRJ0902027>
- Bragg, L. A. (2021). A social justice approach to teaching mathematics. *Australian Primary Mathematics Classroom*, 26(1), 25-29. https://drive.google.com/file/d/1JP00gD6TBVCW_0w6QwKCKiYqzAfpCFUM/view
- Cameron, S., & Mesiti, C. (2024). What Kind of Mathematics Teacher Is ChatGPT? Identifying the Pedagogical Practices Preferred by Generative AI Tools When Preparing Lesson Plans. In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 135-142). Gold Coast: MERGA.
- Canogullari, A., & Radmehr, F. (2025). Task design principles in mathematics education: a literature review. *International Journal of Mathematical Education in Science and Technology*, 1-33. <https://doi.org/10.1080/0020739X.2025.2457365>
- D'Ambrosio, U., & D'Ambrosio, B. S. (2013). The role of ethnomathematics in curricular leadership in mathematics education. *Journal of Mathematics Education at Teachers College*, 4(1). <https://doi.org/10.7916/jmetc.v4i1.767>

- Daugherty, M. K., & Carter, V. (2025). Ways of thinking and STEM-based problem solving: Toward the future. In L. D. English & T. Lehmann (Eds.), *Ways of Thinking in STEM-based Problem Solving* (pp. 263-272). Routledge. <https://doi.org/10.4324/9781003404989-19>
- English, L. (2025). STEM-based problem solving in a new era. In L. D. English & T. Lehmann (Eds.), *Ways of Thinking in STEM-based Problem Solving* (pp. 1-14). Routledge. <https://doi.org/10.4324/9781003404989-1>
- Ernest, P. (2007). Mathematics education ideologies and globalization. *Philosophy of Mathematics Education*, 21.
- Frankenstein, M. (2009). Developing a critical mathematical numeracy through real real-life word problems. In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modelling Verbal Descriptions of Situations* (pp. 111-130). Brill. https://doi.org/10.1163/9789087909383_008
- Gutiérrez, R. (2012). Context Matters: How Should We Conceptualize Equity in Mathematics Education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in Discourse for Mathematics Education: Theories, Practices, and Policies* (pp. 17-33). Springer Netherlands. https://doi.org/10.1007/978-94-007-2813-4_2
- Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. Taylor & Francis. <https://doi.org/10.4324/9780203112946>
- Gutstein, E. (2007). So one question leads to another”: Using mathematics to develop a pedagogy of questioning. In N. S. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 51-68). <https://www.tcpres.com/improving-access-to-mathematics-9780807747285>
- Gutstein, E. (2016). “Our issues, our people—Math as our weapon”: Critical mathematics in a Chicago neighborhood high school. *Journal for research in mathematics education*, 47(5), 454-504. <https://doi.org/10.5951/jresmetheduc.47.5.0454>
- Ibrahim, J. H. (2023). *The art of asking ChatGPT for high-quality answers: A complete guide to prompt engineering techniques*. Independently published.
- Matthews, L. E., Jones, S. M., & Parker, Y. A. (2022). *Engaging in Culturally Relevant Math Tasks, K-5: Fostering Hope in the Elementary Classroom*. Corwin Press. <https://doi.org/10.4135/9781071841655>
- Ödmo, M., Chronaki, A., & Bjorklund Boistrup, L. (2023). Bringing Critical Mathematics Education and Actor–Network Theory to a Statistics Course in Mathematics Teacher Education: Actants for Articulating Complexity in Student Teachers’ Foregrounds. *Education Sciences*, 13(12). https://mdpi-res.com/d_attachment/education/education-13-01201/article_deploy/education-13-01201-v3.pdf?version=1702298673
- Skilling, K., & Park, W. (2025). STEM in a changing world: ‘Insider’ and ‘outsider’ perspectives. In L. D. English & T. Lehmann (Eds.), *Ways of Thinking in STEM-based Problem Solving* (pp. 15-28). Routledge. <https://doi.org/10.4324/9781003404989-2>
- Skovsmose, O. (2012). *An invitation to critical mathematics education*. Springer Science & Business Media. <https://doi.org/10.1007/978-94-6091-442-3>
- Skovsmose, O. (2023). *Critical mathematics education*. Springer Nature. <https://doi.org/10.1007/978-3-031-26242-5>
- Spasić, A. J., & Janković, D. S. (2023). *Using ChatGPT standard prompt engineering techniques in lesson preparation: role, instructions and seed-word prompts* In 58th international scientific conference on information, communication and energy systems and technologies. IEEE. <https://doi.org/10.1109/ICEST58410.2023.10187269>
- Stake, R. E. (1995). *The art of case study research*. Sage.
- Stefflitsch, D., & Kollosche, D. (2025). Students’ perceptions of Critical Mathematics Education: an exploration. *Educational Studies in Mathematics*, 120(2), 225-247. <https://doi.org/10.1007/s10649-025-10423-y>
- Stinson, D. W., & Wager, A. (2012). A sojourn into the empowering uncertainties of teaching and learning mathematics for social change. *Teaching mathematics for social justice: Conversations with educators*, 1(1), 3-18. National Council of Teachers of Mathematics (NCTM).
- Sullivan, P., Clarke, D., & Clarke, B. (2012). *Teaching with tasks for effective mathematics learning* (Vol. 9). Springer Science & Business Media. <https://doi.org/10.1007/978-1-4614-4681-1>
- Wright, P. (2021). Transforming mathematics classroom practice through participatory action research. *Journal of Mathematics Teacher Education*, 24(2), 155-177. <https://doi.org/10.1007/s10857-019-09452-1>

Research Participation as a Space for Teachers' Reflection and Re-evaluation of Student *Success*

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This paper explores how participation in research activities can function as a space for teachers of mathematics to share, negotiate, and re-evaluate their conceptions of student *success*. Drawing on a case example from a broader mixed-methods study, the paper illustrates how dialogic, activity-based research prompted moments of hesitation, tension, and reflection that extended beyond data collection. The findings suggest that research participation itself can support professional sense-making and reveal the conditions that shape how *success* is conceived and enacted in mathematics classrooms.

Introduction

Educational research aims to support and improve the experiences of students and teachers in classrooms; however, a persistent research-practice gap remains, with this disconnect particularly pronounced in mathematics education (Goos, 2014). Teachers' work unfolds within "a complex ecology of people, policies, social practices, and social norms," where "teachers have to grapple with expectations and demands that are made *of* them and *on* them by various stakeholders" (Tan et al., 2015, p. 1584). Within this complexity, tensions and dilemmas frequently arise but are more often managed than resolved (e.g., Berry, 2007). Among these tensions are questions of what counts as *success*. Although frequently invoked in educational documentation, research, and discourse, *success* is rarely, if ever, explicitly defined. This lack of clarity compounds existing tensions, as teachers navigate competing expectations across the spaces and roles they simultaneously occupy. This paper draws on data from a larger study of how teachers of mathematics conceive and determine student *success*. What follows is a single illustrative case that traces one participant's journey across multiple research phases, marked by reflective moments, productive uncertainty, and tensions. This analysis reveals not only how *success* was conceived, but also how research participation itself can create opportunities for teachers to think aloud, wrestle with tensions, and engage in professional sense-making that they may rarely be afforded.

Literature Review

Australian educational policies and national curriculum documents frequently cite the notion of "successful learners" as a central goal of schooling (e.g., Department of Education, Skills and Employment, 2019). Yet, these commitments are rarely accompanied by explicit definitions of what such *success* entails. Similarly, educational research – including mathematics education research – often references *success* when framing research aims, outcomes, and claims of impact, yet without specifying what this *success* entails. In the absence of clear descriptions despite diverse applications, meaning is left to be inferred by readers or assumed on contextual cues, leading to ambiguity and inconsistent interpretations.

Literature reflects markedly diverse – and at times conflicting – conceptions on what it means to be *successful* in mathematics education. In some accounts, *success* is associated with academic achievement and cognitive development (e.g., York et al., 2015), while others characterise *success* as students' ability to reproduce lower-order procedures in predetermined ways (e.g., Shepard, 2000). In contrast, other conceptions foreground – individually and in combination – problem-solving, creativity, collaboration, and engagement with mathematics in meaningful and socially situated contexts (e.g., Clarke, 1997; Ernest, 2015; Watt, 2005).

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 58–65). MERGA.

Success also appears to be determined in different contexts, by different methods, and at different points in time by different stakeholders. For instance, *success* is associated with marks, grades, ranks, and scientific measurement, as well as through summative tasks or standardised assessment (e.g., Reid, 2019; Shepard, 2000; Watt, 2005). Other accounts link *success* with self-efficacy, affective domains, and productive dispositions that are self-determined and less readily measurable (e.g., Ernest, 2015; Kilpatrick et al., 2001). Collectively, these varying and often conflicting perspectives situate *success* as a complex and contested construct within mathematics education. In this study, investigating teachers' conceptions of *success* afforded unexpected opportunities to reflect on the tensions they experienced as they navigated these conflicts. This paper, therefore, asks: How can participation in research activities function as a space for teachers of mathematics to share, negotiate and re-evaluate their conceptions of student *success*?

Methodology

The aim of the larger study was to examine how Australian primary and secondary teachers conceived and determined student *success* in mathematics classrooms. It was conducted in three phases: an online survey (n =139), activity-oriented focus groups (n=40 across 18 groups) and semi-structured individual teacher interviews (n=10). The analysis presented in this paper draws primarily on the focus groups and interviews, as these phases generated rich instances of shared reflection, negotiation, and re-evaluation. To avoid resembling "individual interviews done in group settings" (Colucci, 2007, p. 1423), the focus groups incorporated four interactive activity-based tasks designed to create participant-led dialogic spaces in which participants articulated their reasoning, explored conflicting conceptions, and negotiated interpretations collectively. The interviews further explored teachers' conceptions, experiences, and assessment of *success*, as well as inviting participants to reflect on preliminary findings.

Focus groups, conducted either in-person or virtually, involved teachers from early childhood to senior secondary years across government, Catholic, and independent sectors in six of the eight Australian states and territories. Ten of these teachers, representing similar demographic diversity, participated in individual interviews. Recruitment for both focus groups and individual interviews occurred through voluntary expressions of interest linked at the conclusion of the initial online survey. All sessions were video recorded and transcribed. Initial manual coding informed an inductive thematic analysis following Braun and Clarke's (2006) six phases, with illustrative excerpts and emerging themes organised iteratively across broader analytic stages. From this process, several particularly insightful case examples emerged. This paper focuses on one such case involving "Isaac", a secondary mathematics teacher and Head of Mathematics at a Queensland independent school, with 12-15 years of experience. Isaac participated in all three phases of the study and, during focus groups, was randomly paired with "Frank," a secondary mathematics teacher from the government sector in New South Wales whom he had not previously met.

Results

Like other focus groups, the discussion began with how their schools currently measure student *success*. Isaac, like Frank, described NAPLAN, PAT-M, grades, and a strong emphasis on in-class exams that were "*pretty prescriptive*" with "*pretty closed, procedural questions*", as well as a strong focus on Problem-Solving and Modelling Tasks aimed to "*try and maximise the Year 12 results*". Isaac added that Year 9 focused more on "*how well students are engaging with real-world maths*," though this is not included in their formal reporting.

Focus Group Activity 1:

The pair began the first activity – an open-response Y-Chart task – in which participants were asked what *success* looks like, sounds like and feels like in an ideal mathematics

classroom. For the most part, they worked quietly and individually, recording complementary ideas such as “*kids debating/disagreeing with each other*,” “*excitement, cheering and congratulation each other on solving a difficult problem*” (sounds like), “*teacher not the hardest worker in the room*,” “*calm, confident, anxiety free*” (feels like), “*kids working ‘messily’ and quickly – just trying to get their thinking down in writing*,” and “*collaboration*” (looks like). Notably, despite earlier discussion of their schools’ measures of *success*, there was no mention of standardised assessment, grades or exams. Before moving on, I asked whether there was anything they wished to add. At this point, Isaac offered:

Yeah, I just...just an observation that this is not...doesn’t look maths specific to me. Like I would probably write this about many subjects.

Clarifying whether this was simply an observation or whether he felt more “*mathematics-specific*” ideas should be included, Isaac replied:

Well, I was trying to think of more mathematical because I’m a maths teacher, so I’m like...[pause] feel like there should be more maths stuff in there, but they’re just the things that came to me.

Although nothing further was added, the exchange appeared to prompt a moment of reflection for Isaac – one that seemed to unsettle him and which he would later return to.

Focus Group Activity 2

The group proceeded to the next activity, which involved 26 statements representing different ways *success* might be indicated through engagement with learning mathematics. Participants were asked to collectively evaluate the statements aloud and select their ten most valuable representations of/for *success*, before prioritising nine of those from Very High Priority to Very Low Priority (Diamond 9). As Isaac and Frank discussed the indicators, attention turned to students justifying their thinking and communicating solution pathways. At this point, Isaac returned to his earlier observation:

I think that’s probably what I was talking about before, Rebecca, about it [Y-Chart responses] not being necessarily mathematical. But this one that does...I do think it speaks to mathematics.

This comment stood out because Isaac’s tone suggested he perceived the Y-Chart responses as constrained in some way. Yet the task was deliberately open-ended, with participants retaining full control over what they recorded. Isaac’s return to the idea of “*mathematical*” appeared driven less by the task design itself and more by an underlying tension between how *success* was being conceived and what he may have expected from the activity, a reading supported by subsequent reflections. Nevertheless, as the group continued to evaluate and select their ten statements, Isaac began to make broader connections, reflecting aloud on what they had selected and discarded:

It’s an observation here...the ones that we’ve ditched...they’re indicators of some *success*. But I feel like if you get the green ones right first [their selections], then the other stuff kind of follows.

The “*green ones*” referred to students’ willingness to give things a go, adopting a positive relationship with mathematics, ‘seeing’ the relevance of mathematics ideas to their world, justifying their mathematical workings, and solving problems without solution methods provided. By contrast, the discarded statements included good scores on summative and standardised assessments, report grades, and mastery of curriculum standards. As he worked, Isaac appeared to be evaluating their selections, using the process to actively reflect on his own conceptions of *success* rather than simply completing the task solely for research purposes.

Despite never having met, Isaac and Frank collaborated thoughtfully and productively, advocating, negotiating and challenging one another. At times, Isaac referred to statements as “*another mathematical one*” or “*a maths-specific one*” – a subtle echo of his earlier comments. Yet during the prioritisation stage, he was observed relegating these more discipline-specific indicators in favour of dispositional and process-oriented indicators (i.e. the “*green ones*”).

However, when reviewing the final selections, Isaac added a rushed comment, spoken almost under his breath:

It's very, they're all...we've picked very, like, general. [Quick, low mumbling] Maybe that's part of the trap. But, um, I'm happy enough!

While Frank readily agreed with the final choices, Isaac's fleeting remark – “*maybe that's part of the trap*” – hinted at a growing unease with the indicators of student *success* they had chosen, despite their close alignment with the earlier Y-Chart responses. Quickly buried, the comment nonetheless raises questions about why Isaac felt caught or tricked, exposing another small crack in his certainty – one that would become increasingly visible as the tasks progressed.

Focus Group Activity 3

The third task presented 18 statements representing different approaches to determining *success* in mathematics classrooms, again asking participants to select their ten most valuable. Isaac and Frank quickly selected indicators such as a shift in student confidence, teacher questioning, teacher observation, and a shift in student engagement. Similarly, they discarded A-E grades and NAPLAN results. However, a subtle distinction was observed whereby Frank appeared to approach the task through the lens of what he valued – consistent with the task's intent – whereas Isaac reasoned more in terms of what he typically did or did not implement.

Focus Group Activity 4

Following a similar format, the final task presented 16 statements related to assessment design and implementation. Participants were asked to identify the ten factors they most considered when implementing assessment to determine student *success*. While Frank immediately engaged, Isaac's readiness to participate – unlike in earlier tasks – noticeably slowed. His directness appeared to waver, alongside a shift in body language. Isaac's tone became more cautious, even tentative, as he explained:

I usually try and plan my teaching and planning first, and, like, I usually think about the assessment as the final product as opposed to...picking an assessment method that is valuable to my [planning].

For Isaac, ‘assessment’ appeared to be almost automatically associated with summative and reporting purposes, and far less as a means to support learning or inform teaching and planning. These reflections appeared to unsettle Isaac; his posture shifted as he slumped back in his chair, fist resting against his face, becoming more inwardly focused.

Appearing more challenged by this activity, the pair hesitated over their selection; their reasoning became less clear and, at times, contradicted earlier convictions. For example, their previous emphasis on students' broader mathematical understanding and the role of feedback was set aside when asked to consider assessment design specifically. Sensing their struggle, I attempted to clarify how they were approaching the task in relation to *success*, to which Isaac – still leaning away, hands clasped behind his head – carefully, almost haltingly explain that:

In reality, I never have an assessment task where I'm **not** trying to determine student *success*. I never really implement...deliberately implement assessment tasks where I'm not interested to determine student *success*. Although I definitely recognise there are shortcomings in the way I do it.

Stumbling through his words and accompanied by strained non-verbal cues, Isaac eventually admitted, “*The reality is I just write assessment tasks.*” This statement – along with his self-correction around “*deliberately*” – revealed both resignation and struggle: resignation in the sense that ‘assessment’ is treated as something habitual and procedural, and struggle in recognising that such proceduralism may conflict with deeper aspirations and conceptions of *success* in mathematics classrooms. He again reasserted that he would “***never set out to implement assessment tasks where [he's] not interested in determining student success.***” Frank, too, appeared to grapple with his own reflections.

At this stage, their visible struggle prompted a slight pivot from the script. To keep the task constructive, I gently drew on earlier activities to prompt reflection. But before I could continue, Isaac interjected firmly, stressing that the task asked which factors they *currently* consider, not which factors “*should be*” considered. His tone carried a defensive edge, and sensing this shift, I immediately stepped back and affirmed his interpretation, although this distinction was not explicitly specified. Isaac then softened slightly, noting that his responses would differ under a “should consider” framing. As Frank reflected aloud on “typically structured assessment,” it was then – almost out of nowhere – that Isaac articulated the crux of his struggle:

I think probably what I’m finding – just again, it’s just a reflection here – is that...what I consider to be successful in a maths classroom, when I look at that and I compare that to the techniques that I employ when I’m implementing assessment, they kind of don’t match up. That’s what I’m...that’s what I’m seeing here. Things that I do for assessment aren’t really geared at the things I said were most important. So that’s kinda why I think I’m having trouble with this one, just because [pause] I recognise that what I’m doing is not ideal. Hmmm... what else?

Isaac did not linger on this reflection. Yet, after waves of resistance, deflection, and frustration, he had openly acknowledged a misalignment between his professed values of *success* and the assessment practices he routinely employed. His swift turn away from this admission may reflect both the weight of that admission and the challenge of sustaining such recognition. But, in that brief moment, Isaac gave voice to a central concern echoed across the study: teachers’ stated conceptions of student *success* often “*don’t match up*” with the practices they feel compelled to enact within institutional spaces. Despite initial hesitation, the pair soon re-engaged with the task, actively discussing statements and sharing examples. Isaac remained physically leaned into the activity, and the confident pace of his contributions returned. Earlier emotions and side-comments settled, and nothing further of note emerged. Before concluding, Isaac appeared – even amid the discomfort – motivated to pursue these ideas further.

Semi-Structured Individual Interviews:

I was unsure whether Isaac would volunteer for the semi-structured individual interviews conducted several months later. Yet his willingness to do so – and the depth of engagement that followed – proved invaluable. Isaac showed little hesitation in sharing his thoughts and experiences, even when grappling with challenging or uncomfortable ideas.

From the outset, Isaac was forthcoming and reflective, frequently thinking aloud, pausing to reframe ideas, clarify his thinking in real time, and sharing detailed personal anecdotes. Like many participants, he differentiated *success* from achievement and performance, emphasising how *success* in mathematics is accessible to *all* students but becomes unrealistic when narrowly (mis)interpreted as academic performance alone. He explained that “*kids are on different time frames*” and argued that reducing fear and encouraging a willingness to have a go better equip them to draw on mathematics when and however they may need it in their lives – echoing *success* as an individual endeavour. As Isaac attempted to describe a ‘*successful student*’ and an ‘*unsuccessful student*’, his thinking shifted in real-time. Initially, he drew on what he called “*the traditional view*,” describing ‘*successful students*’ as those who “*think abstractly*,” “*pick up things*” quickly, and can “*perform mathematical tasks*.” Yet, he reflected aloud that as “*articulate[d] it*” – particularly when attempting to describe an ‘*unsuccessful student*’ – other ideas emerged. Instead, he concluded that “*the successful student is the one that gets in and gives it a go*.” While acknowledging teachers’ role in developing students’ mathematical capacities, Isaac returned to the centrality of dispositions, noting the difficulty of fostering confidence, attitudes, and willingness to participate when students, parents, and broader school communities often associate *success* with grades, ranks, and comparisons. He further described how, in practice, the exhausting and unforgiving pace of his courses often relegates students’ mathematical learning to a secondary concern. At times, his words became unsteady as he grappled with his own complicity in this shared professional reality:

The reality is...I've got 25 kids. I have pressures. Teachers, I think, don't make it their number one priority to get kids into learning. I have to move on. I have to proceed. I can't wait for you...so teachers are, like, I think, we, myself, think I aim for it, but it's not a prerequisite for me to keep going with my course like I'm going to do. I'm going to do what I can in the time and energy that I have to try and get you "in". But that's pretty limited. Then, ready or not, here we go!

This reflection captures the relentlessness of teaching within tightly timed structures, where the demand to 'keep going' eclipses deep learning, and systemic pressures to meet coverage goals undermine intentions to foster authentic mathematical engagement and student *success*. Despite his earlier attention to discipline-specific indicators, Isaac became increasingly critical of how mathematics itself is framed and enacted in classrooms. He described a prevailing emphasis on "*right and wrong answers*" – a framing that acts both as a comfort and a constraint:

[Sigh] We often gear the learning towards, like, there are right or wrong answers because that's something we really love about maths. For a lot of people, they love that. And so...if you make questions simple enough, then everyone can experience success of, like, getting something right. So, I think we've institutionally trained students to be like, well, I'm successful/not successful based on if I'm getting right or wrong answers.

Isaac acknowledged how this culture limits what students come to experience as *success*:

But there's a whole other...world of maths out there that is not about right or wrong answers, and we kind of don't spend much time on that, traditionally. So, kids never get to experience the *success* of like...learning something new or seeing an application of something or seeing something in a way that no one else in the class saw. Like, I think they're all measures of *success*, but kids don't really get to access that too much.

He also reflected – with a mix of frustration and empathy – on how teachers of mathematics, himself included, may unintentionally perpetuate this perception:

I think...we do it out of a good place but we're like, oh, you got that right? Go to the next one!...We think you've spent years failing in maths. You're going to come into my classroom and succeed and get things right. But then we kind of perpetuate that cycle of maths is right or wrong.

These reflections illustrated Isaac's growing awareness of entrenched pedagogical tensions in which well-intentioned efforts to build students' confidence by simplifying mathematics can inadvertently reinforce the very right-or-wrong dichotomy teachers hope to disrupt. His critique extended beyond content or method – areas he initially foregrounded – to question the socio-cultural logics of mathematics teaching itself, and how norms operating across institutions and classroom spaces constrain conceptions of *success*.

Later in the interview, when presented with preliminary findings from earlier phases of the study, including the Y-Chart themes, Isaac described mathematics as "*a vehicle*" rather than the *focus* of *success*. He emphasised broader dispositions – "*being active, being hands-on, being curious, working with other people... being able to deal with a struggle*" – as the foundations for lifelong growth. These reflections are particularly striking given his earlier frustrations during the focus groups – including during this very activity. Not someone inclined to acquiescence, through the process of participation and reflection, Isaac appeared to work through these tensions and actively re-examine his own conceptions of student *success*.

Reflecting on other preliminary findings, Isaac spoke candidly about how systemic pressures shape the day-to-day classroom realities. He observed that the performance metrics most dominant in schools are precisely those least valued by teachers. He again described the tyranny of pace in the pursuit of grades and scores, noting how accountability measures often eclipsed deeper mathematical engagement, dispositional and process-oriented qualities and teachers' preferences for formative, real-time interactions with students. Yet he admitted that he rarely plans for these higher-valued ideals because they are not what institutions prioritise. Isaac described feeling "*almost divided*" between what he valued professionally and what institutional spaces prioritised. He articulated a core dilemma echoed across the study: what teachers value in their professional spaces is often precisely what institutional spaces fail to

recognise, reward or make room for. Amid these admissions, however, he imagined an alternative grounded in professional trust – one in which he felt permitted to foreground a mathematics classroom where his conceptions of *success* were seen, felt and heard, trusting that summative measures would “*just automatically fall into place*”.

Still, Isaac remained acutely aware that teachers of mathematics “*exist within bigger systems*” – a dynamic that felt inescapable, regardless of intention or effort – and that these “*system expectations*” can themselves become “*barriers*” to enacting the mathematics classrooms they value. The emotional intensity of these tensions became most visible when Isaac spoke not only of professional expectations but also of livelihood, family and fear – words that, at times, seemed difficult for him to articulate:

Teaching is such a stable job. And yet, there’s this sense...there’s this...it’s not...we don’t live in fear, but we’re like...our job security and our family and like...these are factors.

His hesitant phrasing – “*it’s not... we don’t live in fear, but...*” – exposed an unease rooted less in a fear of professional inadequacy than in a fear of non-compliance; of straying too far from expectations and norms perceived to safeguard stability and security. He continued:

It’s like I don’t know if I’m prepared to go out on a limb. I don’t know if I’m prepared to do something different. I’ll just play it safe. And that comes back to like, well, I’m feeding a family, and this is not...I don’t spend all day, every day doing this. So, it’s gotta be manageable as well. And I think there’s a real element of playing it safe, too. I think – even if we’re not told to prioritise NAPLAN, ATAR and grades – inherently, we’re like, what’s the **safest** course of action here? I know if I do this, I won’t get in trouble...I don’t even know if we’re conscious of that.

The proximity of safety and fear highlights the emotional cost of navigating institutional spaces that equate conformity with security, where professional safety may come at the expense of the very professional freedoms needed to pursue broader, more enduring conceptions of student *success* in mathematics education.

Discussion

This paper examined how participation in research activities can function as a space for teachers of mathematics to share, negotiate, and re-evaluate their conceptions of *success*. The case of Isaac illustrates how engagement in dialogic, activity-based research can act as a catalytic professional space, prompting moments of hesitation, discomfort, reframing, and critical reflection as he candidly grappled with nuanced conceptions of *success* and the realities between intent and enactment. Although the study was not designed to explicitly seek out professional tensions, Isaac’s participation was marked by moments of destabilisation and re-evaluation as he confronted tensions between professional values, institutional expectations, and classroom practices. Many of Isaac’s tensions resonated with Berry’s (2007) binary illustrations, including ‘confidence and uncertainty’, ‘action and intent’, ‘safety and challenge’, and ‘valuing and reconstructing’. As Berry (2007) notes, “as interconnections between tensions became apparent, new knowledge of practice is brought to light” (p. 120), offering new frames for interpreting and organising experiences. In this case, research participation created conditions in which tensions were not only articulated but became generative sites for reflection. For example, Isaac observed that the performance metrics most dominant in schools were often those least valued by teachers, drawing attention to the constraining influence of measurement culture and limited professional trust.

Like other participants, Isaac’s engagement extended beyond responding for the purposes of the study, suggesting a growing readiness to grapple with his own professional thinking and, possibly, an increasing trust in the research space. Participation prompted reflection on practice itself, transforming the research process from a data-collection exercise into a professionally resonant encounter that may feed forward into future thinking, learning and teaching. Isaac’s moments of candid reflection and self-interrogation reinforce that research participation can enable forms of professional meaning-making that may be rarely afforded within tightly

structured and time-pressured education systems. This reflective engagement contributed to richer and more nuanced understandings of student *success* across the broader study, as teachers were afforded time, space, and permission to pause, interrogate, and play within their professional spaces.

Implications

Rather than positioning participants as data sources alone, as teachers of mathematics shared, negotiated, and re-evaluated their conceptions of student *success*, the research activities functioned as a reciprocal space through which they gained clarity and insight into their own beliefs and experiences while also generating deeper insight into teachers' conceptions for the purposes of the study. Importantly, this occurred without the study being framed as action research or professional development. Instead, the research design positioned participation as a productive site for professional inquiry, supporting teachers' sense-making while also informing the study. Attending to how research spaces are experienced may therefore offer one way to narrow the research-practice gap by allowing research to simultaneously serve analytic and practice-oriented purposes. For researchers – particularly early-career researchers – these findings underscore the value of designing methodologies that foreground participants' meaning-making rather than only research outcomes.

Acknowledgments

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References

- Berry, A. (2007). Reconceptualizing teacher educator knowledge as tensions: Exploring the tension between valuing and reconstructing experience. *Studying Teacher Education*, 3(2), 117-134.
- Braun, V., & Clarke, V. (2006). *Using thematic analysis in psychology*. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Colucci, E. (2007). "Focus groups can be fun": The use of activity-oriented questions in focus group discussions. *Qualitative health research*, 17(10), 1422-1433.
- Clarke, D. (1997). *Constructive assessment in mathematics: Practical steps for classroom teachers*. Key Curriculum Press.
- Department of Education, Skills and Employment. (2019). *Alice Springs (Mparntwe) Education Declaration*. <https://nla.gov.au/nla.obj-2391375250/view>
- Ernest, P. (2015). The social outcomes of learning mathematics: Standard, unintended or visionary? *International Journal of Education in Mathematics, Science and Technology*, 3(3), 187-192.
- Goos, M. (2014). Researcher–teacher relationships and models for teaching development in mathematics education. *ZDM*, 46(2), 189-200.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press. <https://doi.org/10.17226/9822>.
- Reid, A. (2019). *Changing Australian education*. (1st ed.). Allen & Urwin.
- Shepard, L. A. (2000). The role of assessment in a learning culture. *Educational Researcher*, 29(7), 4–14. <https://doi.org/10.3102/0013189X029007004>
- Tan, A.L., Chang, C.H. & Teng, P. (2015). Tensions and dilemmas in teacher professional development. *Procedia-Social and Behavioural Sciences*, 174, 1583-1591. <http://doi.org/10.1016/j.sbspro.2015.01.808>
- Watt, H. M. (2005). Attitudes to the use of alternative assessment methods in mathematics: A study with secondary mathematics teachers in Sydney, Australia. *Educational Studies in Mathematics*, 58(1), 21–44.
- York, T. T., Gibson, C., & Rankin, S. (2015). Defining and measuring academic success. *Practical Assessment, Research, and Evaluation*, 20(1), 1-20. <https://doi.org/10.7275/hz5x-tx03>

A Preliminary Framework for the Effective Use of Generative Artificial Intelligence (GenAI) by Mathematics Teachers

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This paper reports a framework for the effective use of generative artificial intelligence (GenAI) by mathematics teachers. Informed by the findings of a rapid literature review, the framework outlines the knowledge, skills, dispositions and conditions that support the effective use of GenAI. The preliminary framework provides immediate implications for the development of pre-service and in-service professional learning that aims to support mathematics teachers to work effectively with GenAI for teaching and learning.

Background and Significance

The use of generative artificial intelligence (GenAI) tools has exploded since the introduction of OpenAI's ChatGPT in 2022, with significant focus on how GenAI tools can be harnessed by teachers to support teaching and learning. Initially, the research focus was primarily on the potential time-savings afforded by these tools (e.g., Roy et al., 2024). More recently, the focus has shifted to exploring how teachers can use GenAI tools to support teaching and learning by, for example, using GenAI to brainstorm teaching examples and explanations, generate teaching materials and assessments, and plan units of work (e.g., Cameron et al., 2025; Pepin et al., 2025). The increased interest in GenAI has also seen an explosion in the number and range of professional learning opportunities focused on GenAI, primarily in response to the need for teachers to develop new knowledge and skills to work with the increasing range of tools. The need for effective professional development has been recognised at a systemic level, with the Australian Government calling for universities to work with education systems and both pre-service and in-service teachers to develop teacher capacity to work with GenAI to support effective teaching and learning (Commonwealth of Australia, 2024). In this study, effective use of GenAI is viewed as the ability to use GenAI in an efficient manner to complete a range of tasks to a high-standard whilst also positioning the teacher as the expert on mathematics teaching and learning.

The demand for professional learning on GenAI has far outpaced the current research base, thus creating an urgent need for evidence to inform professional learning initiatives that help teachers understand and use GenAI to enhance their teaching (e.g., Commonwealth of Australia, 2024). The study reported in this paper, involving a rapid review of literature, establishes an evidence base from which we were able to develop a preliminary framework for the effective use of GenAI by mathematics teachers. This framework will inform the design of GenAI-focused professional learning opportunities for pre-service and in-service teachers through by identifying the core knowledge, skills and dispositions mathematics teachers require to integrate GenAI effectively into their practice.

The overarching research question for this study was *What knowledge, skills and dispositions do mathematics teachers need to make effective use of GenAI for mathematics teaching and learning?*

Methodology

The work reported in this paper is part of a larger program of research exploring mathematics teachers' use of GenAI for mathematics teaching and learning. Inspired by previous frameworks which have provided structured guidance on integrating new and (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 66–73). MERGA.

powerful technologies into mathematics teaching and learning (e.g., Computer Algebra Systems; Pierce & Stacey, 2004), our framework offers a research-informed basis for considering teachers' effective use of GenAI and for the design of professional learning. The preliminary framework, presented in this paper, will serve as a foundation for future research and will be refined and validated in later stages of the study in collaboration with mathematics teachers who are familiar with using GenAI tools to support mathematics teaching and learning.

Rapid Literature Review

The rapid review protocol, when compared to systematic reviews, enables the accelerated synthesis of existing knowledge (Garritty et al., 2020). This accelerated process, which places less emphasis on critical appraisal than systematic reviews, serves to enable time-critical decision-making. This rapid approach aligned with our need to establish a preliminary framework capturing key concepts evident in the literature for use in later stages of our study. The review also contributes to the emerging evidence base that informs the development of professional learning for mathematics teachers working with GenAI. Furthermore, the ever-increasing evidence base and research interest in GenAI will require us to repeatedly sample the field and update the framework, a process that would be impractical given the time demands of a systematic review. Informed by the Cochrane rapid review guidelines (Garritty et al., 2024), this section details the process of identifying and screening relevant research articles.

Following the development of the research question, a research librarian was consulted to assist with developing the search strategy (see Figure 1 for sample search string). The research question was based upon existing research on technology use in mathematics which has shown the importance of skills, knowledge and affect for effective use (e.g., Pierce & Stacey, 2004). Databases were identified based on the research librarian's initial experimentation with different databases; this approach is consistent with the rapid review protocol which prioritises searching the databases that are most likely to contain relevant literature (Garritty et al., 2024). Databases searched were Discovery (including Complementary index, ERIC, Academic Search Complete, Directory of Open Access Journals, Science Direct), ProQuest Central, Dimensions and Scopus. Articles were restricted to English language publications published after 2020.

Figure 1

Sample Search String

(TI (math* teacher* OR numeracy* teacher*)OR AB (math* teacher* OR numeracy* teacher*)) AND (((AI OR "Artificial Intelligence" OR "artificial neural networks" OR Chatbot OR ChatGPT OR "CLAUDE" OR GenAI OR "Generative AI" OR "Google Gemini" OR "Large Language Models" OR LLM OR "MS Copilot" OR OpenAI))) AND (AB(Teacher* N2 (Aptitude OR Assumptions OR Attitude OR Capability OR Capacity OR Dislikes OR Efficacy OR Experiences OR fluency OR Grasp OR "Human-computer interaction" OR Insight OR Knowledge OR Likes OR Literacy OR Mindset OR Outlook OR Preferences OR Prerequisite OR proficiency OR Skills OR skill-level OR Talent OR Understanding OR Viewpoints)))

Articles were screened using Covidence software following the Cochrane rapid review protocol guidelines (Garritty et al., 2024). Extraction focused on demographic data (e.g., country of study, in-service cf. pre-service teachers, etc) and identifying findings that provide insight into the factors that support mathematics teachers to make effective use of GenAI tools. Screening and extraction were conducted by a single researcher. Peer debriefing (Nguyen & Spall, 2008) was used to review screening and extraction decisions, and to provide clarification and negotiate outcomes when required. A hybrid thematic approach (Fereday & Muir-Cochrane, 2006) was used to identify themes evident in the research findings that would then inform the development of the framework. Using a two-step approach, inductive coding of research findings established themes describing factors that are reported to support the effective use of GenAI before deductive coding was used to map these findings onto the three domains identified in the research question (i.e., knowledge, skills, dispositions).

Review Findings

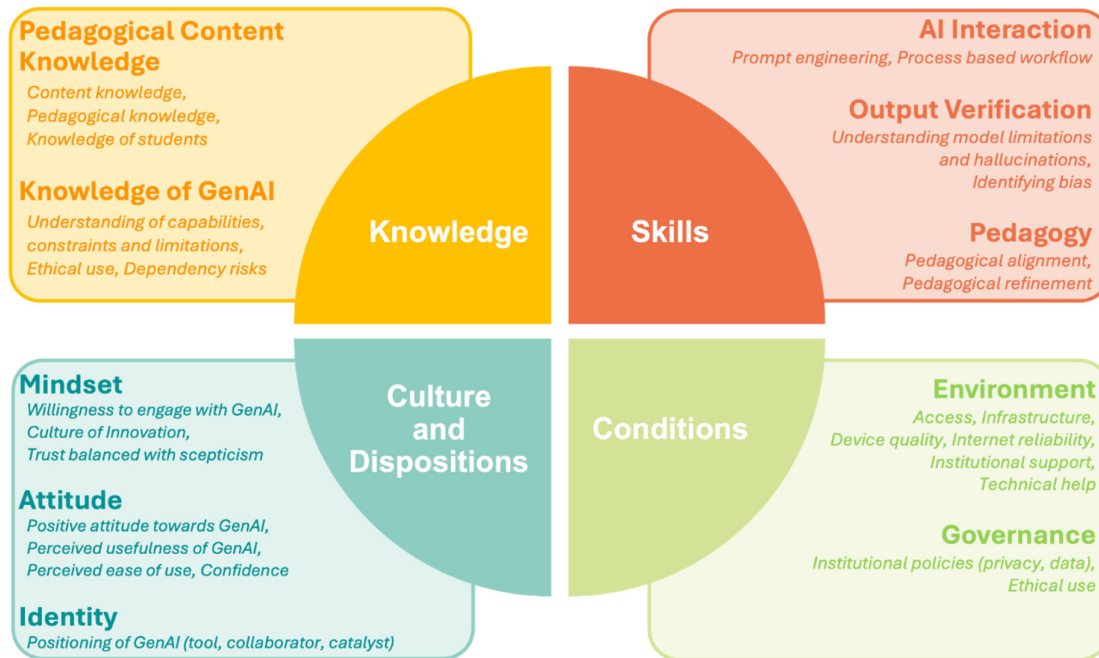
The rapid review identified 72 articles that provide insights into the skills, knowledge and dispositions that mathematics teachers need to make effective use of GenAI. Articles are roughly spread across in-service and pre-service teachers (43 cf. 39). Questionnaires are the primary data collection tool (36), although a significant number of studies (33) used questionnaires in conjunction with other data collection tools such as interviews, observations, review of chat logs, etcetera. Consequently, it is unsurprising that quantitative studies (33) were more common than qualitative (18) or mixed methods studies (11). A notable finding in the literature was the diverse range of countries included in the sample, with 20 countries featured in the 72 articles. This diversity indicates that the included articles provide a diverse data set in terms of participants, methods, and contexts, thus reducing bias in the research and improving the generalisability of findings across populations and contexts (Polanin et al., 2017).

The Effective Use of GenAI for Mathematics Teaching Framework

The preliminary framework consists of four domains (*Knowledge; Skills; Culture and Dispositions; Conditions*) that support mathematics teachers to make effective use of GenAI (see Figure 2). Each domain consists of several elements (e.g., *Pedagogical Content Knowledge* and *Knowledge of GenAI*) and sub-elements (e.g., *Content Knowledge*, *Pedagogical Knowledge*, *Knowledge of Students*) that describe the specific aspects of the domain that support the effective use of GenAI. Of note is the inclusion of the domain *Conditions*, which was not an initial focus of the research question and literature review. Upon conducting the inductive analysis of research findings, it was evident that this fourth domain was required to describe how external conditions support the effective use of GenAI. The following sections provide a description of elements and sub-elements within each domain.

Figure 2

The Preliminary Effective Use of GenAI for Mathematics Teaching Framework



Knowledge

The *Knowledge* domain describes the essential knowledge needed for mathematics teachers to work effectively with GenAI and highlights the importance of both *Pedagogical Content*

Knowledge (e.g., Shulman, 1986) and *Knowledge of GenAI*. Although Technological Pedagogical Content Knowledge (TPACK; Mishra & Koehler, 2006) has increasingly been used to investigate the knowledge required by teachers to work with GenAI given its affordance to combine PCK and technology specific skills (e.g., Li, 2025; Segal et al., 2025), in developing this framework we chose to separate *Pedagogical Content Knowledge* (PCK) and *Knowledge of GenAI* into two elements rather than applying the TPACK model. First, the PCK that supports effective GenAI use can be developed, and exists, without reference to technology and thus serves a broader purpose in supporting effective teaching. Second, studies in the review predominantly position AI as a tool to support mathematics teachers in their planning tasks rather than as a tool that teachers use in class to support mathematics learning. Consequently, the affordances of the TPACK model in being able to describe the knowledge needed to use technology to teach using specific pedagogies does not currently apply to this framework.

Pedagogical Content Knowledge (PCK) is essential when working with GenAI as it provides a lens through which users can critically engage with and evaluate GenAI outputs. Such evaluation is particularly important for recognising hallucinations within GenAI outputs and for planning adaptations that overcome these (Aga et al., 2024; Cameron et al., 2025; Copur-Gencturk et al., 2024). PCK also serves to inform how teachers adapt GenAI outputs for their own class contexts, with pre-service teachers with strong PCK reported as being more acute in their evaluation of GenAI outputs (Aga et al., 2024). PCK also enables teachers to adapt GenAI lesson plans to cater for diverse student needs, with pre-service teachers recognising that GenAI lesson plans are often teacher directed, repetitive and do not provide accessible learning opportunities for all students (Gurl et al., 2024).

Knowledge of GenAI incorporates understanding the capabilities, constraints and limitations of GenAI along with ethical considerations and knowledge of dependency risks is essential in supporting mathematics teachers to become critical users of GenAI. We opted not to refer to this element as AI literacy (e.g., Ng et al., 2021) given the specific focus on the implementation of GenAI in mathematics education compared to the broader focus on AI literacy.

The presence of hallucinations and mathematical errors in GenAI outputs are key limitations of Large Language Model technologies, and thus understanding the constraints and limitations of the technology help to prime teachers to be critical users of GenAI (e.g., Bernardi et al., 2025). Similarly, understanding the capabilities of GenAI to generate text and other outputs supports teachers to utilise GenAI whilst mitigating the aforementioned risks associated with their limitations (e.g., Bui et al., 2025). In mathematics, this includes understanding that GenAI tools are primarily text generators, not calculators, and that this presents unique challenges when using GenAI in mathematics contexts (e.g., Cameron et al., 2025).

Malik et al. (2025) detail the importance of understanding ethical use of GenAI, noting the potential risks to student privacy and teacher autonomy from unintended unethical use. They explain that the development of ethical understanding of GenAI, including understanding bias and cultural relevance, may act as safeguard to support the ethical use of GenAI. Further, knowledge of the risks of dependency of overuse of GenAI (e.g., Wijaya, Yu, et al., 2024) may enable teachers to develop and maintain ownership of their expertise and ensure they are using GenAI in a manner that supports, rather than replaces, their expertise.

Skills

The *Skills* domain, incorporating the key elements of *AI Interaction*, *Output Verification*, and *Pedagogy*, encapsulates the key skills that mathematics teachers require to make effective use of GenAI. The *AI Interaction* domain includes skills related to prompt engineering, which describes the skills needed to craft precise prompts to generate relevant, accurate, and pedagogically appropriate outputs. Grover et al. (2025) reported teachers' experiences of exploring prompt engineering, noting the steep learning curve associated with writing clear and

specific prompts. However, research also notes the need for iterative processes when working with GenAI (e.g., Yu et al., 2025), noting that outputs are unlikely to directly align with the teachers' needs. Key to this iterative process are skills related to *Pedagogy*, including the ability to review and identify when GenAI outputs do or do not align with the intended aims and preferred practices of the teachers (i.e., *pedagogical alignment*; e.g., Oh, 2025) and the ability to then refine outputs into a usable form that supports the aims of the teacher (i.e., *pedagogical refinement*; e.g., Oh, 2025). The ability to identify hallucinations and bias (i.e., *output verification*), also forms an important part of iterative processes for working with GenAI. Critical thinking skills, combined with a critical view of GenAI (see *Culture and Dispositions*) and a strong understanding of student learning needs, underpin a teachers' ability to identify mathematical and pedagogical errors (e.g., Şimşek, 2025).

Overall, the skills required for working effectively with GenAI are underpinned by the *Knowledge* domain. For example, the ability to assess a GenAI output for its pedagogical alignment, and to then refine the output prior to classroom use, needs to be supported by strong pedagogical content knowledge. This example highlights the interconnected nature of the elements and domains, and thus the need for holistic professional learning opportunities that aim to address multiple elements concurrently.

Culture and Dispositions

This domain describes the key *Cultures and Dispositions* (i.e., *Mindset, Attitude, Identity*) that support mathematics teachers to make effective use of GenAI. Overall, research indicates that a critical view of GenAI is necessary for supporting effective use as it positions teachers to be critical of outputs, thus supporting them to identify hallucinations and bias, and to engage in the processes of pedagogical alignment and refinement. Busuttil and Calleja (2025) reported that contemporary views about the teaching and learning of mathematics, underpinned by discovery and connectionist educational philosophies view ChatGPT positively for supporting both teaching and learning. The importance of belief structures is further emphasised by Lin and colleagues (2025) who note a tension between technology acceptance and teacher beliefs, observing that teachers' beliefs about mathematics teaching may sometimes conflict with the practices embedded within GenAI systems; a tension that has been identified through the analysis of GenAI created lesson plans (e.g., Cameron & Mesiti, 2024). This example again highlights the relationships between elements in the domain, with such conflicts between beliefs and GenAI potentially contributing to pedagogical misalignment.

Attitude has been shown to play a crucial role in supporting the effective use of GenAI (e.g., Li, 2025; Wijaya, Su, et al., 2024). Fostering positive attitudes towards GenAI can therefore enhance teachers' ability to use it effectively. Key factors in shaping these attitudes are perceived usefulness and ease of use (Börekci & Uyangör, 2025), suggesting that teachers need both the skills to operate GenAI tools easily and efficiently, and the ability to recognise and demonstrate how GenAI can enhance their mathematics teaching.

A recurring theme in the literature was the need for teachers to position themselves as critical users of GenAI. Several studies with pre-service teachers highlight the importance of being critical of GenAI due to its current limitations (e.g., Aga et al., 2024; Bernardi et al., 2025). Aga et al. expand upon this, advocating for GenAI to be seen as a "mere tool" (p. 74) that can support the expertise of teachers. Similar metaphors highlight the potential to view GenAI as an assistant, thought-partner or compass, where users can offload routine tasks so they can focus on higher-order tasks like scaffolding and differentiation through the process of pedagogical refinement (Oh, 2025).

The sub-elements reported here demonstrate the potential for teachers' attitudes, beliefs and identity to impact their use of GenAI. Notably, implementation of GenAI appears dependant on both affective engagement with GenAI and with mathematics more broadly, thus

highlighting the need for professional learning to consider how to support affective engagement with GenAI while encouraging reflection on what it means to teach and learn mathematics in the age of AI.

Conditions

Although not an initial focus of the research question, the review shows that external *Conditions*, including those related to the user's external *Environment* and relevant *Governance* frameworks, impact teachers' capacity to make effective use of GenAI. For example, Li (2025) reported a correlation between educational policies, technology availability, standardised assessments, curriculum, and parental and community engagement and the implementation of AI. Examples provided included the need for device and internet access, but also the support of stakeholders in creating an environment that supports AI use. A similar study using structural equation modelling also highlighted a link between educational systems and cultural norms, teachers' attitudes, and perceived behavioural control and their use of GenAI (Wijaya, Su, et al., 2024). The potential influence of the external domain is further highlighted by Albalawi (2025), who, based on data collected from teachers about their intentions to incorporate AI into their teaching, calls for policy makers to develop instructional frameworks that align AI tools with pedagogical practices and to provide "financial and moral incentives" (p. 896) to encourage AI integration. Various other models have also been proposed to support teachers in using GenAI, offering insights into the conditions that facilitate effective use of GenAI. Suggested strategies include pairing experienced GenAI teachers with novice teachers, implementing long-term professional learning programs, and establishing teacher support forums for teachers to discuss AI applications and troubleshoot challenges (Trung et al., 2025).

The *Conditions* domain demonstrates that the effective implementation of GenAI requires a broad range of support structures and is not only dependent on the knowledge, skills, culture and dispositions of individual teachers. This suggests that supporting effective use of GenAI may benefit from established support structures such as mentoring, communities of practices, and other whole school approaches to supporting innovation in teaching and learning.

Conclusions, Implications, and Further Research

This paper reports a framework for the effective use of GenAI that will be expanded and validated through future research with practising mathematics teachers. The framework, informed by a rapid literature review, describes the key domains (*Knowledge; Skills; Culture and Dispositions; Conditions*) that support mathematics teachers to make effective use of GenAI, with each domain illustrated through corresponding elements and sub-elements. While many sub-elements are specific to GenAI (e.g., *Knowledge of GenAI; AI Interaction*), the framework reinforces the importance of classroom teachers and the need for human oversight when working with GenAI (e.g., *Pedagogical Content Knowledge; Pedagogy; Identity*). This review highlights the continuing importance of the human element in mathematics teaching in the age of AI, with teachers acting as mediators between GenAI-generated outputs and the realities of daily classroom interactions.

The framework provides an organised evidence base for the development of pre-service and in-service professional learning about GenAI and provides several implications for the design and delivery of such programs. The complexity of the framework highlights that professional learning that only focusses on sub-elements are unlikely to be effective in supporting teachers to make effective use of GenAI. In many cases, GenAI professional learning will need to be grounded in discussions of pedagogy. For example, professional learning should extend beyond prompting techniques to include strategies for identifying pedagogical (mis)alignments between teachers' intentions and GenAI-generated outputs, as well as approaches for refining these outputs before classroom use (either online or offline).

As a preliminary framework derived from the literature, our future research will focus on validating and refining the framework across different local contexts. The framework will also benefit from the development of illustrations of practice for each sub-element. This additional detail, to be developed in collaboration with mathematics teachers who are experienced GenAI users, will provide an additional layer of evidence to inform the design of pre-service and in-service teacher education programs that aim to develop mathematics teachers' capacity to use GenAI effectively.

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References

- Aga, Z. G., Sawyer, A. G., & Wolfe, M. (2024). More confidence more critical: Investigating secondary mathematics preservice teachers' use of an Artificial Intelligence chatbot as a curriculum development tool. *American Journal of Creative Education*, 7(2), 63–78. <https://doi.org/10.55284/ajce.v7i2.1278>
- Albalawi, A. S. (2025). Integrating AI into education: Perspectives from mathematics teachers. *International Journal of Research in Education and Science*, 11(4), 896–921. <https://doi.org/10.46328/ijres.1316>
- Bernardi, M. L., Capone, R., Faggiano, E., & Rocha, H. (2025). Generative AI in mathematics education: Pre-service teachers' knowledge and implications for their professional development. *International Journal of Mathematical Education in Science and Technology*, 56(8), 1513–1530. <https://doi.org/10.1080/0020739x.2025.2490104>
- Börekci, C., & Uyangör, N. (2025). The role of academic self-efficacy in pre-service mathematics and science teachers' use of generative artificial intelligence tools. *Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, 27(2), 681–704. <https://doi.org/10.25092/baunfbed.1596547>
- Bui, P., Vinerean-Bernhoff, M., & Liljekvist, Y. (2025). *Comparing GenAI adoption in pre- and in-service mathematics teachers*. PsyArXiv. https://doi.org/10.31234/osf.io/w7nva_v1
- Busuttil, L., & Calleja, J. (2025). Teachers' beliefs and practices about the potential of ChatGPT in teaching mathematics in secondary schools. *Digital Experiences in Mathematics Education*, 11(1), 140–166. <https://doi.org/10.1007/s40751-024-00168-3>
- Cameron, S., & Mesiti, C. (2024). What kind of mathematics teacher is ChatGPT? Identifying the pedagogical practices preferred by generative AI tools when preparing lesson plans. In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 135–142). https://merga.net.au/wp-content/uploads/2024/07/MERGA46_2024_Mesiti_RP480_2024-06-29-01.pdf
- Cameron, S., Mesiti, C., & Fijn, P. (2025). How Knowledgeable is ChatGPT 4o? Assessing the pedagogical content knowledge of a generative artificial intelligence tool. In S. M. Patahuddin, L. Gaunt, D. Harris, & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 93–100). https://merga.net.au/wp-content/uploads/2025/08/MERGA47_2025_Cameron_RR.pdf
- Commonwealth of Australia. (2024). *Study Buddy or Influencer: Inquiry into the use of Generative Artificial Intelligence into the Australian Education System*. Commonwealth of Australia.
- Copur-Gencturk, Y., Li, J., & Atabas, S. (2024). Improving teaching at scale: Can AI be incorporated into professional development to create interactive, personalized learning for teachers? *American Educational Research Journal*, 61(4), 767–802. <https://doi.org/10.3102/00028312241248514>
- Fereday, J., & Muir-Cochrane, E. (2006). Demonstrating rigor using thematic analysis: A hybrid approach of inductive and deductive coding and theme development. *International Journal of Qualitative Methods*, 5(1), 80–92. <https://doi.org/10.1177/160940690600500107>
- Garrity, C., Gartlehner, G., Kamel, C., King, V. J., Nussbaumer-Streit, B., Stevens, A., Hamel, C., & Affengruber, L. (2020). *Cochrane rapid reviews. Interim guidance from the cochrane rapid reviews methods group*. https://methods.cochrane.org/sites/methods.cochrane.org/rapidreviews/files/uploads/cochrane_rr_-_guidance-23mar2020-final.pdf
- Garrity, C., Hamel, C., Trivella, M., Gartlehner, G., Nussbaumer-Streit, B., Devane, D., Kamel, C., Griebler, U., & King, V. J. (2024). Updated recommendations for the Cochrane rapid review methods guidance for rapid reviews of effectiveness. *BMJ*, 384, e076335. <https://doi.org/10.1136/bmj-2023-076335>

- Grover, S., Grunge, B., Lynch, R., & Clarkson, C. (2025). Use of generative AI for assessment creation by high school mathematics teachers. In S. Mishra, A. Kothiyal, S. Iyer, S. Sahasrabudhe, A. Lingnau, & R. Kuo (Eds.), *Proceedings of the International Conference on Technology 4 Education 2024, Volume 1* (pp. 360–373). https://doi.org/10.1007/978-981-96-5761-2_27
- Gurl, T. J., Markinson, M. P., & Artzt, A. F. (2024). Using ChatGPT as a lesson planning assistant with preservice secondary mathematics teachers. *Digital Experiences in Mathematics Education*, 11(1), 114–139. <https://doi.org/10.1007/s40751-024-00162-9>
- Li, M. (2025). Integrating artificial intelligence in primary mathematics education: Investigating internal and external influences on teacher adoption. *International Journal of Science and Mathematics Education*, 23(5), 1283–1308. <https://doi.org/10.1007/s10763-024-10515-w>
- Lin, T., Zhang, J., & Xiong, B. (2025). Effects of technology perceptions, teacher beliefs, and AI literacy on AI technology adoption in sustainable mathematics education. *Sustainability*, 17(8), 3698. <https://doi.org/10.3390/su17083698>
- Malik, R., Abdi, D., Wang, R., & Demszy, D. (2025). Scaffolding middle school mathematics curricula with large language models. *British Journal of Educational Technology*, 56(3), 999–1027. <https://doi.org/10.1111/bjet.13571>
- Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record*, 108(6), 1017–1054. <https://doi.org/10.1111/j.1467-9620.2006.00684.x>
- Ng, D. T. K., Leung, J. K. L., Chu, S. K. W., & Qiao, M. S. (2021). Conceptualizing AI literacy: An exploratory review. *Computers and Education: Artificial Intelligence*, 2, 100041.
- Oh, S. (2025). Integration of MATH41 and generative AI in pre-service mathematics teacher education: An empirical study on lesson design competency. *IEEE Access*, 13. <https://ieeexplore.ieee.org/abstract/document/11087693/>
- Pepin, B., Buchholtz, N., & Salinas-Hernández, U. (2025). A scoping survey of ChatGPT in mathematics education. *Digital Experiences in Mathematics Education*, 11, 9–41.
- Pierce, R., & Stacey, K. (2004). A Framework for Monitoring Progress and Planning Teaching Towards the Effective Use of Computer Algebra Systems. *International Journal of Computers for Mathematical Learning*, 9, 59–93. <https://doi.org/10.1023/B:IJCO.0000038246.98119.14>
- Polanin, J. R., Maynard, B. R., & Dell, N. A. (2017). Overviews in Education Research: A Systematic Review and Analysis. *Review of Educational Research*, 87(1), 172–203.
- Roy, P., Poet, H., Staunton, R., Aston, K., & Thomas, D. (2024). *ChatGPT in lesson preparation—A teacher choices trial: Evaluation Report*. Education Endowment Foundation.
- Segal, R., Biton, Y., & Alush, K. (2025). How utilizing generative AI when addressing pedagogical and mathematical events contributes to mathematics teacher educators’ TPACK (Technological Pedagogical Content Knowledge). *International Journal of Education in Mathematics, Science and Technology*, 13(4), 895–913. <https://doi.org/10.46328/ijemst.4928>
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4. <https://doi.org/10.2307/1175860>
- Şimşek, N. (2025). Integration of ChatGPT in mathematical story-focused 5E lesson planning: Teachers and pre-service teachers’ interactions with ChatGPT. *Education and Information Technologies*, 30(8), 11391–11462. <https://doi.org/10.1007/s10639-024-13258-x>
- Trung, L. T. B. T., Thang Trung, T., & Dung, T. M. (2025). ChatGPT in Vietnamese math classrooms: What are the influencing factors behind teachers’ adoption? *Journal of Pedagogical Research*, 9(2). <https://doi.org/10.33902/jpr.202531924>
- Wijaya, T. T., Su, M., Cao, Y., Weinhandl, R., & Houghton, T. (2024). Examining Chinese preservice mathematics teachers’ adoption of AI chatbots for learning: Unpacking perspectives through the UTAUT2 model. *Education and Information Technologies*, 30(2), 1387–1415.
- Wijaya, T. T., Yu, Q., Cao, Y., He, Y., & Leung, F. K. S. (2024). Latent profile analysis of AI literacy and trust in mathematics teachers and their relations with AI dependency and 21st-century skills. *Behavioral Sciences*, 14(11), 1008. <https://doi.org/10.3390/bs14111008>
- Yu, H. M., Kim, S. H., & Lee, H. (2025). AI-assisted integration of computational thinking: Pre-service teachers’ experiences in early childhood mathematics education. *International Journal of Early Childhood*, 57, 1–29. <https://doi.org/10.1007/s13158-025-00434-4>

The Role of Gesture in Determining Van Hiele Levels of Geometric Thinking of Young Children

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Children's gestures can be an important thinking and communication tool in geometry but were rarely considered in research on the development of geometric knowledge until recently. This study explored the differences between 4-8-year-old children's development in relation to van Hiele levels, with and without attending to their gestures made about a hidden 3D object. From 172 children, when verbal responses alone were considered, 74% of children aligned with Level 1. However, when verbal and gestural response were analysed, 40% of these children then met Level 2. This finding suggests the value of expanding the van Hiele model to include embodied representation modes.

Introduction

Nonverbal thinking is of special importance; all rational thinking has its roots in nonverbal thinking, and many decisions are made with only that kind of thought. We observe some things without having any words for them. (van Hiele, 1999, p. 311)

Van Hiele's seminal work (originally conducted in the late 1950's) on the development of geometric thinking has been used for decades in curriculum planning and for interpreting students' levels of thinking in research and education (e.g., Burger & Shaunessy, 1986; Clements, 2003) (See Pegg, 2020 for a summary of the whole model). Despite van Hiele's own evolving views on the nature of 'thinking' in relation to language (e.g., van Hiele, 1999), most research seeking to explicate the van Heile levels across various contexts, has focused on verbal and visual representations as both inputs and outputs in learning processes, with some inclusion of student drawing and actions on physical models such as sorting tasks (e.g., Burger & Shaunessy, 1986). Other studies have utilised test-style tasks that limit students' response modes (e.g. Wu & Ma, 2006). The dominance of a focus on two-dimensional shapes is also notable (Sinclair & Bruce, 2015), as is the focus on secondary students. In recent years, the rapid increase in research on embodied cognition and learning has broadened views on the role that non-verbal thinking, especially gesture, plays in children's spatial reasoning (e.g., Ehrlich et al., 2006; Thom, 2018). However, little research has explored how children express their perception of a three-dimension (3D) object in the absence of visual input, or how their gestural communications relate to the van Heile Model of geometric thinking. The aim of the present study is to explore the potential of adapting the descriptors that characterise the van Heile levels to better identify indicators of young children's progress in geometric thinking, particularly regarding 3D objects.

Various studies have advocated the importance of gesture in children's spatial reasoning (Ehrlich et al., 2006; Thom, 2018), in geometric communication in different contexts (Elia, 2018), and in revealing emerging geometric conceptions (Kim et al., 2011). However, the role of gesture is rarely explained in isolation, but rather as a component of multimodal experience linking action, speech, and/or visualisation in children's sense-making and progress towards abstraction (Elia et al., 2017; Nathan & Alibali, 2021). Both McNeill (1992) and Goldin-Meadow (2025) draw on substantial bodies of research to argue the co-evolution of language and gesture where the two modalities are complexly and dynamically linked, sometimes in harmony when underlying meanings coincide, and sometimes mismatched in a forecast of imminent conceptual change. Gesturing has many functions such as externalising thoughts, reinforcing language, filling gaps in available language or representing non-verbal thoughts and visualisations and therefore offers many avenues of research into children's spatial-geometric

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 74–81). MERGA.

reasoning (Elia et al., 2014; Thom, 2018), including developing “strategies for using children’s gestures to evaluate their knowledge development” in geometry (Kim et al., 2011, p. 235).

Conceptual Framework

It has been argued by some researchers that the van Heile model, with its sequential levels of geometric development and its emphasis the critical role of language in the manifestation of a geometric concepts, does not account for multiple ways in which children ‘come to know’ geometric ideas (Roth & Thom, 2009; Sinclair & Bruce, 2015). Rather than discard the model, for the exploratory purpose of this study, we propose three adaptations to the model’s application that acknowledge the intertwined nature of language and gesture development and enhance its suitability for reflecting early development in young children.

The first adaptation focuses on the first two van Hiele levels¹ and introduces the two sub-levels proposed by Pegg (1997) for Level 2. In *Level 1 Visualisation* figures are identified according to their overall appearance. In Level 2 Description, thinking becomes more analytical with recognition of the parts that compose a figure, beginning with *Level 2A Description – single feature* where figures are identified in terms of a single property, and *Level 2B Description – multiple features*, where more than one feature is recognised but the properties of a figure are considered as being independent of one another. The subdivision of Level 2 also acknowledges that levels appear to “be dynamic rather than static and ... continuous in nature” (Burger & Shaunessy, 1986, p. 45).

The second adaptation draws on the work of Ehrlich et al. (2006) that utilised scheme for recognising a broader range of strategies that children may reveal through speech and/or gesture while working on tasks involving combining and separating two-dimensional shapes. While the task context differs from this study, we incorporate Ehrlich et al.’s four strategy categories, *Movement, Perceptual Features, Perceptual Whole, and Alignment* as potential triggers for observing subtle signs of children’s attention to different aspects of geometric figures that might otherwise be missed.

The third adaptation focuses on hand movements produced by children “in effortful cognitive activity” (Elia et al., 2014. p.737), and whether the movements are co-speech gestures (aligned in meaning or mismatched), or gestures apparently detached from spoken language. We also incorporate four of McNeill’s (1992) gesture classifications that differentiate the function and meaning of gestures: a) *deictic*, pointing to real or visualised objects or locations; b) *iconic*, visual representation aligned with speech; c) *temporal*, highlighting gestures that might be repeated for emphasis; and d) *metaphoric*, representing an abstract idea or object with a more concrete or familiar with meaningful similarity.

The study focuses on the following research questions:

- *RQ1 Is there a connection between children’s gesture types and van Hiele levels?*
- *RQ2 Can children’s gestures be used as evidence for their developmental level of geometric thinking?*
- *RQ3 What is the impact of considering gestures on classification against van Hiele levels?*

Methodology

In exploring multiple representations children use to communicate their thinking, a qualitative approach allows for the collection of rich, descriptive data that provides insights into learning processes (Bikner-Ahsbahs et al., 2015). Qualitative data was collected from 172

¹ In this paper the first two van Hiele levels are described as Level 1 and 2 (Burger & Shaunessy, 1986)

preschool to Year 2 children (4 to 8-year-olds) from the four Control Schools in Phase 2 of the *Embodied Learning in Early Mathematics and Science* project. The four schools varied in location and student population contexts such as levels of socio-economic advantage, and percentage of students with English as an Additional Language or Dialect (EAL/D). This paper presents data from the Timepoint 1 task-based interviews where students responded to tasks that were specifically designed to elicit embodied modes of communication. The task is based on a task by Roth (2014) and required children to place both hands inside a bag to feel a mystery object (Figure 1). Once they removed their hands, children were invited to ‘tell me what you could feel’, and often further prompted by the question ‘what shape does it feel like’.

Figure 1

Feely bag object



The feely bag task removes visual exploration of the object, encouraging children to use other modes such as touch and hand movement, verbal responses to describe the object, and gestures—movements that ‘re-presents’ (Roth, 2014) the object independent of where it was originally, in the bag. Tasks such as this allow space for “embodied ways of knowing” (Roth & Thom, 2009, p. 46) to be shared. Each interview was video recorded to capture verbal and non-verbal behaviour, and later transcribed.

The analysis process was based on the application of the conceptual framework for the study as a multi-step analysis framework. A deductive micro-analysis approach was taken that followed four distinct analysis steps. First, video data were viewed, transcribing children’s verbal and gestural responses. Videos were often paused and rewatched several times to ensure transcription of nonverbal responses were detailed e.g., *hand/arm gesture* (whole hand or arm), *air drawing* (pointing) or *touch tracing* (pointing with surface contact e.g., table). During this process ‘everyday’ and mathematical vocabulary was noted as well as types of gestures aligned to McNeill’s (1992) gesture types (Table 1).

Table 1

Coding of gestures to McNeill’s categories

McNeill’s gesture types
Iconic—visually represent concrete entities or action, closely related to speech ‘ <i>curve [draws U]</i> ’
Deictic—pointing movement in space, may describe location ‘ <i>like this</i> ’ ‘ <i>over here</i> ’
Temporal—highlighting gesture, simple repeated movement ‘ <i>repeatedly air draws an arch shape</i> ’
Metaphoric—represents image of an abstract object or idea (story from real life) ‘ <i>walking on bridge</i> ’

Second, initial geometric thinking coding commenced where children’s verbal responses were coded to Ehrlich et al.’s (2006) representation strategies (Table 2). Responses were then aligned to the van Hiele levels.

Table 2

Representational strategies for geometric thinking

Ehrlich et al.’s Representation strategies
<i>Movement</i> – of figure or parts, location
<i>Perceptual Features</i> – a focus on a particular feature
<i>Perceptual Whole</i> – recognition of the whole figure
<i>Alignment</i> – emerging attention to the location of single feature in relation to the whole

Third, gesture, and co-speech gesture data were then considered “as a single integrated process” (McNeill, 1992, p. 35) coding gestures to Ehrlich et al.'s (2006) representation strategies (Figures 2a, b, and c). Fourth and finally, responses were examined again revising van Hiele level based on both verbal and gestural representations collectively.

Figures 2a, b and c

Examples of gesture types, description linked to Ehrlich et al.'s strategies



2a. Perceptual whole
Iconic gesture 'arch'



2b. Perceptual feature-movement
Deictic-temporal gesture 'like
this'



2c. Perceptual feature-alignment
Iconic gesture 'hole in the middle'

Results

Gesturers and non-gestures

Analysis revealed 102 of the 172 children used gesture and that 97% of these gesturers used co-speech gesture. Gesturing was spontaneous in 59% of children, whereas the other 41% only gestured after the prompt 'what shape does it feel like'. No distinct differences were noted across grades regarding the use of gesture; all were between 56-62%. Air drawing was the most common gesture noted, followed by hand/arm gesture, then touch tracing.

Verbal descriptions

The most frequent word used to describe the whole object was *bridge*, others included *block*, *ramp*, *arch*. Mathematical words were also used to describe the whole or parts (features) of the object e.g., *circle*, *curve*, *curved* or *curvy*, *rectangle*, *sides*. Several children also talked about the orientation and positioning of the object *upside down*, *top*, *around*. Others indicated 'missing' aspects such as *half* or *hole*. Children also mentioned the tactile feeling of the object such as *hard* or *soft*. These descriptive responses were later analysed as evidence for Ehrlich et al.'s (2006) representation strategies.

Types of gesture

Responses of children who used gesture were coded to McNeill's (1992) gesture types. Often children would use combinations of gesture e.g., hand movement then pointing. The most common type of gesture for describing the object was iconic gesture. These gestures were often congruent with the verbal responses during co-speech gestures, for example, a child may air draw an arch shape while saying the word *curve*. There were however nine children who has mismatched co-speech gesture. For example, child says *arch* but gestures an oval.

Evidence of Ehrlich et al.'s representation strategies and van Hiele levels

Initial coding to Ehrlich et al.'s strategies was based mainly on children's verbal descriptions as analysis of words to the strategies was less complex than the analysis of gestures. Examples include: Perceptual whole—'*It feels like a bridge*' '*it feels like a curve*'; Perceptual features—'*It has a curve in it, it is a small wooden block and if it didn't have the curve, it would be a rectangle*'; Alignment—'*It feels like there's a smooth bit on the bottom with no corners but corners at the top*'; and Movement—'*I could feel the block was like that then like that*'.

Ehrlich et al.'s strategies aligned well with the van Hiele levels. When children verbally describe the whole object, as one entity (perceptual whole) using either an 'everyday' word

such as *a bridge* or a mathematical term such as *a curve* this can be coded to van Hiele Visualisation Level 1. The other three strategies were all aligned to van Hiele Description Level 2 (inclusive of both single and multiple features mentioned). From 172 children, 165 could be placed on van Hiele's levels, 7 children could not be levelled as their verbal response was vague or unrelated to the task. At this analysis point 122 children were in Level 1 Visualisation, 6 were in Level 2A Description, and 37 were in Level 2B Description.

On a second round of coding, gesture and speech were considered collectively as one multimodal response. While analysing gesture cannot impact the levelling of the verbal only children, considerable change occurred when coding the gesturers. Of the 122 children at van Hiele Level 1, 40% of children could be shifted to Level 2A Description showing emerging geometric thinking that indicated one geometric feature. After this second analysis considering gesture alongside speech, 73 children were in Level 1 Visualisation, 54 were in Level 2A Description and 38 were in Level 2B Description, that is 75% of co-speech gesturers moved levels.

These shifts in level classification decisions were made based on the perceptual features observable in children's gestures, in particular their air drawing that was more geometric in nature compared with their verbal response. For example, many children who named the shape *a bridge* in their verbal response, used a pointing/tracing gesture accompanied by movement that highlighted a specific feature of the object such as the curve, right angle corners, or straight sides. If only attending to the verbal response 'a bridge', a child may have been classified at Level 1, as a bridge is a whole object. Yet once their gesture was considered, air drawing 'the curve' as a feature of the bridge, Level 2A classification is possible. Conversely, children whose gesture used static hands/arms to 'make a bridge' from the table, remained classified at Level 1. Children who indicated perceptual features using gesture, usually used an iconic representation or a temporal action to repeatedly highlight the feature.

Figure 3 displays a 'heat map' of overlapping representation strategies and gestures showing the prominence of iconic gestures to indicate features, not the whole. This connection indicates that gestures were a useful way for children to highlight one or multiple features of the object placing them beyond Level 1 of van Hiele. The majority of children who could identify features and were therefore placed at Description levels, used iconic or temporal gestures related to features, movement or alignment. The data in the heat map totals to more than 165 as children may have used more than one gesture.

Figure 3

Overlapping map of gesture type and geometric features

	Perceptual whole	Perceptual features	Alignment	Movement
Deictic	1	13	9	2
Iconic	14	78	25	14
Temporal	0	16	4	2
Metaphoric	1	4	1	2

Discussion

The task for this study was purposefully designed to activate embodied modes at the input (tactile object exploration) and to elicit embodied modes at the output (communication of complex features of unseen object). Relying on tactile information, the movement of hands, and the relationship between an embodied way of exploring objects and the use of gesture connects

our methods with our resulting data. When looking holistically at the findings, the majority (60%) of children preferred to use gesture or co-speech gesture to communicate. When analysing both verbal and gesture representations together, there was a distinct difference between the van Hiele levels obtained by verbal-only children compared with gesturers. This finding illustrates a connection between gesture types and the van Hiele levels where iconic air drawing aligned well with van Hiele Level 2's focus on features. Of the children placed at van Hiele Level 2 Description A or B, less than 10% of these children were verbal-only. This finding shows that when aligning children to van Hiele, there is evidence that children who incorporate gesture into their response are far more likely to show emerging recognition of geometric features at higher van Hiele levels. This finding also supports the use of the adapted van Hiele model (that includes gesture) in determining van Hiele level classifications. The inclusion of gesture may positively impact the way teachers gather geometric thinking data—attending to multiple representations provides a 'fuller' picture of children's development.

The important role of gesture

Attending to multiple representations children used to communicate geometric thinking provided further information and revealed higher levels of understanding expressed in co-speech gesture responses than in verbal responses for many children. This finding aligns with previous research (e.g., Roth & Thom, 2009). Our results revealed that some children produced gestures (Perceptual Features) whose meaning differed, and was 'more' mathematical, than their accompanying verbal response (Perceptual Whole). This mirrors findings from Ehrlich et al. (2006) about spatial transformation that children who referred to movement in their gestures, but not in their speech, were more likely to perform well on tasks. Gesture also benefitted children who did not have the mathematical language to describe known features. Gesture *was* language, non-verbal communication (Elia, 2018) for these children. Similarly, children who used gesture in place of speech are creating a clear point-of-intervention for teachers to explicitly make the connection to the missing terminology. Gesture in place of speech could be seen as an indication of readiness for a transitional stage in cognitive development to a higher level of understanding.

The importance of the type of gesture

Our findings also aligned to Elia et al.'s (2014) regarding the type of gesture, where iconic gestures were "more pertinent to the child's geometrical thinking" (p. 756) than other gesture types. From the heat map (Figure 3) the dominance of iconic gestures to describe a three-dimensional object is evident. This is an interesting finding that may suggest children found it easier to identify the edges of the object via the tactile nature of the task, features that may have been missed if visually examining the object. This conclusion requires further investigation.

Limitations and implications

Keeping in mind the limitations of a study based on a single task with a specific age group of children, our findings begin to address the research questions regarding gesture and their place in van Hiele's classification levels. By attending to children's verbal and gestural representations, researchers (and potentially teachers) can gain insights into how children might be visualising and thinking about geometric properties. Gestures have the capacity to provided evidence of children's thinking about (1) Features that seem most obvious to them, (2) orientation of the object and their ability/inability to mentally rotate the object, (3) structure and alignment of 'how features fit together'. We propose that the early van Hiele levels used in further research should include the types of gesture that children may use to process and communicate their geometric thinking (See Table 3). Further research should encompass a range of tasks with 3D objects, and upper primary grades to further 'road test' the adapted

model. The inclusion of older primary children could extend the adaptation to Level 3 Informal Deduction where relationships between properties are comprehended.

Table 3

van Hiele early levels with example gestures for three-dimensional geometry

van Hiele Level	Co-speech gesture representation aligned to Ehrlich et al. (2006) representation strategies and McNeill (1992) gesture types
Level 1 Visualisation (awareness and description of whole)	<u>Representation:</u> Perceptual Whole , gesturing with hands. Gesture may also indicate movement 'it's like that' [makes an arch with hands]; or alignment 'feels like there is one block here [air draws dot on left] and one block here' [air draws dot on right] <u>Gesture types:</u> Iconic e.g., 'square' [makes square with fingers and thumbs]; Deictic: 'it's like this' [makes a bridge with hands]; Metaphoric: 'I can jump across it' [jumps with fingers]
Level 2A Description (single feature)	<u>Representation:</u> Perceptual feature: Gesture may also indicate movement 'I could feel a bridge, there was a curve going over' [air draws curve]; or alignment 'it's like an upside-down, like a curve' [air draws large U-shaped curve] <u>Gesture types:</u> Deictic: 'it feels like a bridge upside-down' [air tracing whole outline of one face] 'like that, but it's more like this' [continues to air draw side view of object]; Iconic: It feels like a bridge [air draws three straight sides in one movement]; Temporal: 'It's like a ramp, one of these shapes probably' [repeatedly air draws U-shaped curve]; Metaphoric: [air draws curve] 'it's an arch, it's going to be a bridge that people can walk over'
Level 2B Description (multiple features)	<u>Representation:</u> Perceptual feature: 'I can show you [makes vertical sides with hands] there was two lines and there was one [makes horizontal top side] and there was a circle' [uses both hands to make circle]. Gesture may also indicate movement: 'I could feel two blocks like that [uses hands to make sides] then like that [makes curve with hands] curve like that' or alignment: 'Like a shape and it has a hole on the bottom; it has 2 rectangles at the sides' [makes right angles with arms] <u>Gesture types:</u> Deictic: '[Makes rectangle with hands] it's the same thing as this [air draws arch in rectangle] but has a hole in the middle' Iconic: 'It's like a cube [makes cube with hands] like a 3D shape it's got six sides' [air draws vertical and horizontal sides] Temporal: 'It feels like a semi-circle [repeatedly makes circle] ... smooth on the bottom, no corners but corners at the top' [moves fingers to top to make corners]

Conclusion

Although still widely used in education and research, the van Heile model of geometric thinking, like many other areas of mathematics education, is being challenged by increased awareness of the fundamental importance of embodied modes of representation and multi-modal views of learning. Our findings show that children's gesture provides additional data, therefore teachers should consider gesture in addition to speech to notice geometric thinking. Rather than discarding the model, we call for expanding the evidence used to classify student understanding against the model to reflect advances in learning theory such as embodied cognition. The study reported in this paper takes a small step towards achieving that possibility.

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References

- Bikner-Ahsbahs, A., Knipping, C., & Presmeg, N. (2015). Approaches to qualitative research in mathematics education. *Advances in Mathematics Education*, 10, 978-94. <https://doi.org/10.1007/978-94-017-9181-6>

- Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, 17(1), 31-48.
- Clements, D. H. (2003). Teaching and learning geometry. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.) *A research companion to principles and standards for school mathematics*. (pp. 151–178). NCTM
- Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in children's spatial reasoning. *Developmental Psychology*, 42(6), 1259. <https://doi.org/10.1037/0012-1649.42.6.1259>
- Elia, I. (2018). Observing the use of gestures in young children's geometric thinking. In *Contemporary research and perspectives on early childhood mathematics education* (pp. 159-182). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-319-73432-3_9
- Elia, I., Gagatsis, A., & van den Heuvel-Panhuizen, M. (2014). The role of gestures in making connections between space and shape aspects and their verbal representations in the early years: Findings from a case study. *Mathematics Education Research Journal*, 26(4), 735-761. <https://doi.org/10.1007/s13394-013-0104-5>
- Elia, I., van den Heuvel-Panhuizen, M., & Gagatsis, A. (2017). Geometry learning in the early years: Developing understanding of shapes and space with a focus on visualization. In *Forging connections in early mathematics teaching and learning* (pp. 73-95). Springer. https://doi.org/10.1007/978-981-10-7153-9_5
- Goldin-Meadow, S. (2025). The mind hidden in our hands. *Topics in Cognitive Science*, 17(3), 443-468. <https://doi.org/10.1111/tops.12756>
- Kim, M., Roth, W. M., & Thom, J. (2011). Children's gestures and the embodied knowledge of geometry. *International Journal of Science and Mathematics Education*, 9(1), 207-238. <https://doi.org/10.1007/S10763-010-9240-5>
- Nathan, M. J., & Alibali, M. W. (2021). An embodied theory of transfer of mathematical learning. In C. Hohensee, & J. Lobato (Eds.), *Transfer of learning: Progressive perspectives for mathematics education and related fields* (pp. 27–58). Springer. https://doi.org/10.1007/978-3-030-65632-4_2
- McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. University of Chicago press.
- Pegg, J. (1997). Broadening the descriptors of van Hiele's levels 2 and 3. In F. Biddulph & K. Carr (Eds.), *The proceedings of the 20th Mathematics Education Research Group of Australasia Conference* (pp. 391–396). New Zealand, Rotorua.
- Pegg, J. (2020). The van Hiele Theory, In *Encyclopedia of Mathematics Education* (pp. 896-900). Springer International. https://doi.org/10.1007/978-3-030-15789-0_183
- Roth, W. M. (2014). On the pregnancy of bodily movement and geometrical objects: A post-constructivist account of the origin of mathematical knowledge. *Journal of Pedagogy*, 5(1), 65. <https://doi.org/10.2478/jped-2014-0004>
- Roth, W. M., & Thom, J. S. (2009). The emergence of 3D geometry from children's (teacher-guided) classification tasks. *The Journal of the Learning Sciences*, 18(1), 45-99. <https://doi.org/10.1080/10508400802581692>
- Sinclair, N., & Bruce, C.D. (2015). New opportunities in geometry education at the primary school. *ZDM*, 47(3), 19–329. <https://doi.org/10.1007/s11858-015-0693-4>
- Thom, J. S. (2018). (Re)(con)figuring space: Three children's geometric reasonings. In I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank and C. Benz (Eds.), *Contemporary research and perspectives on early childhood mathematics education*. https://doi.org/10.1007/978-3-319-73432-3_8
- van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*, 5(6), 310-316.
- Wu, D., & Ma, H. (2006). The distributions of van Heile levels of geometric thing among 1st through 6th graders. In Novotná, J., Moraová, H., Krátká, M. & Stehlíková, N. (Eds.). *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 5, pp. 409-416. Prague: PME.

Australian Secondary Mathematics Teachers' Conceptions of Mathematical Challenging Tasks

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Despite their well-documented academic, motivational and affective benefits, challenging tasks remain underutilised in Australian secondary mathematics classrooms. This raises questions about whether Australian secondary mathematics teachers' conceptions of challenging tasks influence their decisions to implement them. In this exploratory study, 59 responses to one open-ended survey question were analysed thematically to understand these conceptions. Findings indicate that although most teachers' conceptions generally aligned with characteristics identified in the literature, some specific characteristics were often overlooked.

Challenging Task in Mathematics Education

Although mathematics education has traditionally entailed a passive acquisition of knowledge (Bird, 2023), this teaching and learning approach no longer meets the needs and demands of a world described by the OECD as “increasingly volatile, uncertain, complex and ambiguous” (2018, p. 3). These profound and ongoing changes are reshaping how we live, learn and work, prompting the OECD to deem it “appropriate that [both international and national] curricula should continue to evolve... in radical ways” (OECD, 2018, p. 3; OECD, 2023). Over the last few decades, mathematics education has responded to these changes through reforms in mathematics curricula both nationally and internationally to emphasise 21st century priorities such as reasoning, critical thinking and problem solving (Goos et al., 2020; Szabo et al., 2020). This has led to the emergence of and increased interest in teaching practices and pedagogies that address more contemporary needs and demands.

One mathematical task recognised for its ability to help foster 21st century skills in students is challenging tasks (or rich tasks). Challenging tasks are usually thought-provoking problems that are cognitively demanding and initially difficult to solve, requiring students to:

- plan their approach, especially sequencing more than one step
- process multiple pieces of information, with an expectation that they make connections
- engage with important mathematical ideas
- choose their own strategies, goals, and level of accessing the task
- spend time on the task
- explain their strategies and justify their thinking to the teacher and other students
- extend their knowledge and thinking in new ways (Sullivan, 2017)

This study draws on Sullivan and colleagues' pioneering model of challenging tasks because it is widely accepted and used in Australian primary classrooms and mathematics education research. It underpins the current priorities of Australian mathematics education by addressing national curricula and syllabi reforms to 21st century changes (ACARA, 2026; Sullivan & Davidson, 2014). Moreover, the model comprehensively outlines the definition and seven characteristics, clearly communicating what students engage in during challenging tasks. Therefore, the model offers a useful data analysis framework for cross-checking Australian secondary mathematics teachers' conceptions of challenging tasks with the published definition

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 82–89). MERGA.

and characteristics. In addition, for this study, Philipp's (2007) definition of conceptions is used to describe mental structures "encompassing beliefs, meanings, concepts, propositions, rules, mental images and preferences" (p. 259) that shape what a secondary mathematics teacher considers a challenging task to be and how it should be implemented.

Research findings have consistently demonstrated that challenging tasks in mathematics education are an effective medium for developing higher order thinking skills, while preparing students both academically and holistically for the workplace, world and life (English & Gainsburg, 2015). A possible reason for their effectiveness is that when tackling challenging tasks, students productively struggle and persist through the cognitive challenge (Sullivan et al., 2012). Struggle can support students to make mathematical connections and develop conceptual understanding, while failure can elicit hidden self-efficacy in students (Kapur, 2008). Moreover, challenging tasks require high cognitive demand and higher order skills, with students encouraged to explain, describe, and justify; make decisions, choices, and plans; formulate questions; apply existing knowledge and create new ideas; and represent their understanding in multiple formats (English & Gainsburg, 2015). Challenging tasks emulate both familiar and unfamiliar demands that can be found in the real world, so these tasks foster 21st century skills that students learn to apply beyond the mathematical classroom and in the real world (English & Gainsburg, 2015).

While there is strong evidence that challenging tasks are effective in helping to foster higher order thinking skills in students, their implementation is not widespread in secondary mathematics classrooms as their importance would seem to warrant (Wilkie, 2016). Research has identified enablers and barriers of challenging task implementation in both primary and secondary contexts, with reported barriers including student fear of taking risks and failing; classroom management; teacher knowledges; lack of support, resources and time; and student unwillingness to persist due to their cultural background, mathematical ability and/or language level (Ingram et al., 2020; Sullivan & Mornane, 2014). At a fundamental level, a teachers' decision to implement or not implement challenging tasks may be informed by their conceptions of what challenging tasks are. Yet, little research has investigated what mathematics teachers conceive a challenging task to be in terms of how it is defined and what its characteristics are, as studies have often been conducted within the context of an intervention involving teacher professional development that assumes correct conceptions already or that conceptions are influenced by participation already. Instead, some studies have focused on what teachers conceive the challenging elements of a challenging task to be or the elements that contribute to cognitive demand. For example, while Cheeseman et al.'s (2013) study found conceived challenging elements to be: level of mathematical reasoning demanded by the task; interpreting complex mathematical data; and creating multiple solution pathways, Parrish and Bryd's (2022) study found that the challenging task characteristics that contribute to cognitive demand level to be: multiple solution strategies; multiple representations; and incorporation of explanation and justification.

Evidently, there has been minimal focus on mathematics teachers' conceptions of what a challenging task is in an exploratory way outside of an intervention. Addressing this gap in the literature is important because conceptions may influence teachers' pedagogical decisions to implement or not implement challenging tasks (Lee, 2012; Schweig et al., 2020). Given the benefits of challenging tasks and the noted lack of implementation in high schools, this study aims to investigate Australian secondary mathematics teachers' conceptions of challenging tasks to understand whether their conceptions influence decision making around challenging task implementation. Hence, the research question is "What are Australian secondary mathematics teachers' conceptions of challenging tasks?"

Methodology

Participants were 59 Australian secondary mathematics teachers who opted in to participate in an online questionnaire about their conceptions of challenging tasks and use (or non-use) of challenging tasks. In this paper, the findings from one open-ended question “What do you know about challenging tasks?” are reported. Thematic analysis of this qualitative data was first conducted deductively, drawing on Sullivan’s (2017) definition and characteristics of a challenging task. Then, thematic analysis was conducted inductively using new codes on characteristics developed from the data. Each author independently reviewed and discussed the themes and codes prior to collaboratively refining the thematic structure through iterative comparison and discussion. This process supported credibility and dependability by allowing themes to be reviewed and refined. The aim of the analysis was to develop a comprehensive understanding of teachers’ conceptions of challenging tasks in relation to the Sullivan’s (2017) framework. This process is intended to be recursive rather than linear, as researchers can flexibly return to earlier steps given new data and/or themes. Moreover, themes can be coded regardless of the number of times the theme appears in the data and will contribute to the emerging conclusions. Three researchers including the PhD student and two supervisors contributed to the coding process for inter-rater reliability.

Results and Discussion

Alignment With Definition of Challenging Tasks

When responding to the question “What do you know about challenging tasks?”, a diverse range of responses were collected from teachers (Table 1).

Table 1

Higher to Lower Alignment With Sullivan’s (2017) Definition

Higher	“A challenging task in math education is a complex, absorbing problem designed to be accessible to students of all ability levels through enabling and extending prompts, requiring them to engage in sustained thinking, planning, and risk-taking to foster a deep understanding of mathematical concepts, problem-solving, and a growth mindset. These tasks encourage exploration, provide multiple pathways to solutions, and promote the productive struggle necessary for genuine learning and the development of resilience.” “Nothing really! I can assume what is meant by the term, but haven’t any particular knowledge on what makes a challenging task and the benefits of them.”
Lower	“I don’t know what you mean by this term ‘challenging tasks’”

The wording of the question meant that responses ranged from definitions, characteristics, benefits, enablers and barriers of challenging tasks. For this paper, only the responses that referred to the definition and characteristics were analysed. The responses gathered suggest that although many participants did not directly refer to Sullivan and colleagues’ model, their interpretations mostly aligned.

For the responses that were more closely aligned, teachers were able to state definitions and characteristics that emulated Sullivan and colleagues’ model even when no direct references to the researchers were made. Most teachers know that challenging tasks fall under the category of problem-solving tasks, by describing them using adjectives including “non-routine”, “complex”, “open-ended”, “multi-layered” and/or tasks that tackle “familiar and unfamiliar [situations]”. Two teachers were able to provide an alternative term for challenging tasks: they are “rich tasks” and “[cognitively] demanding and thought provoking [tasks]”. References were also made to other types of mathematical tasks such as “low floor high ceiling tasks” and

“inquiry tasks” which do overlap with the offerings of challenging tasks as those that enable inquiry and differentiation. As a result, challenging tasks were seen as distinctly different from textbook, fluency-focused questions characterised by “repetition” and “familiar situations”. One teacher commented that challenging tasks “often require students to probe further to understand concepts rather than simple mimicking”. Another teacher pointed out how challenging tasks are not immediately solvable because “they are tasks in which there are no clear immediate solutions and require thinking beyond ordinary classroom routine problems to solve”.

In terms of the responses that did not have significant alignment with the definition and characteristics of challenging tasks, teachers were either: (1) unfamiliar with the term completely and could not offer a definition; (2) unfamiliar with the term but could assume its meaning; or (3) familiar with the term but their definition did not really align. This shows that most teachers possess a broadly accurate conception of challenging tasks, but specific characteristics were often overlooked. This suggests that there are some gaps in teachers' understanding of challenging tasks, which may have implications for their decision making around challenging task implementation.

Teachers' Conceptions of Challenging Task Characteristics

In teachers' responses, key characteristics of challenging tasks were noted. When analysing the responses, characteristics can be divided into those that are listed as part of Sullivan and colleagues' seven published characteristics and those that are not but are still considered characteristic of challenging tasks. Overall, six out of seven characteristics by Sullivan and colleagues were directly stated by the participants, excluding the ‘Spend time on the task’ characteristic (Table 2).

Table 2

Alignment With the Seven Published Characteristics of Challenging Tasks

Plan their approach, especially sequencing more than one step ($n=1$)	“I’ve never used that term before. I assume it means tasks that... require students to develop their approach to solve the task... Tasks that require numerous interconnected steps”
Process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways ($n=3$)	“Highlight the connection between pieces of knowledge and concepts in topics, for example, the connection between graphical features and algebraic expressions” “Makes connections between strands and maths concepts”
Engage with important mathematical ideas ($n=12$)	“Tasks to challenge students’ cookie-cutter thinking to improve understanding” “Students engage with tasks that are non-routine, which despite being accessible extend their critical thinking of mathematical content”
Choose their own strategies, goals, and level of accessing the task ($n=16$)	“They are accessible to students i.e. not requiring unreasonable prior knowledge, but difficult enough to make students stop and think” “There are often many ways to approach a challenging task, so they are often cross-topic.”
Spend time on the task ($n=0$)	N/A
Explain their strategies and justify their thinking to the teacher and other students ($n=2$)	“My understanding is challenging tasks encourage approaching problems in unfamiliar contexts, generally requiring reasoning to be applied and conceptual understanding to find efficient solutions.”

	“These challenging tasks allowed students to start to work mathematically and explain their thinking of the mathematics they were implementing.”
Extend their knowledge and thinking in new ways (n=23)	“Developing critical thinking skills in students and push them to think out of the box”
	“They are often non-routine, so different from ‘textbook’ questions in that they require more creative or conceptual thinking.”

Sullivan and colleagues’ characteristic of ‘Spend time on task’ seems to be overlooked in the teacher responses and is typical of tasks requiring higher cognitive demand and higher order thinking skills, which were still mentioned by teacher participants (Table 3). One teacher response suggested this characteristic but did not explicitly refer to time as a factor, as they described challenging tasks as “complex tasks that requires higher skills, significant efforts, determination and strong focus in solving”. One reason for this oversight may be because the mathematics education landscape has come to value the correctness of answers in preparation for assessment and the product of the learning experience over the process. Challenging tasks, being a non-routine task that some participants describe as “not repetitive textbook questions”, appear different to the routine mathematics that students may be more familiar with, as students need to “draw on a variety of skills and apply knowledge to less familiar situations.”

In addition to these seven characteristics, other challenging task features were noted by the teachers. These features included higher cognitive demand, application to the real world, multiple solution pathways and solutions, and collaboration (Table 3 & 4).

Table 3

Characteristic of Challenging Tasks: Higher Cognitive Demand & use and Development of Higher Order Thinking Skills

Higher order thinking skills & Cognitive demand, with some references to educational theories (n=9)	“Providing students with non-routine problems that allow them to think beyond their regular questions. It allows them to treat numbers and concepts like puzzles and figure solutions out.”
	“Bloom’s taxonomy. How to create challenging tasks”
	“One that is at the very edge of the students’ zone of proximal development. In practice, this often includes some element of far transfer, where they need to apply their knowledge in a new setting or context.”

It is interesting to note that some teachers related their definition and characteristics of challenging tasks to educational research and literature such as Vygotsky’s *Zone of Proximal Development* (1978) and Bloom’s *taxonomy* (1956). With the *Zone of Proximal Development* related to challenging students and Bloom’s *taxonomy* related to engaging in higher order skills, this can be likened to the characteristic of “Extend their knowledge and thinking in new ways”, especially in both familiar and unfamiliar situations. These references to theories in educational research, although not explicit in the definition and characteristics of challenging tasks, show how some Australian secondary mathematics teachers are familiar with the theory and practice of challenging tasks, highlighting the teaching praxis they possess.

Table 4

Characteristic of Challenging Tasks: Application of Challenging Task Experience to Real World

Real life applications / connections (n=3)	“Allows for real world connections”

	“These are tasks where students are encouraged to apply maths to solve real life problems”
21 st century skills such as critical thinking, reasoning and problem solving ($n=16$)	“They are tasks in math that requires investigation and requires student to work collaboratively through critical thinking and discussions” “They promote the integration of topics, critical thinking, perseverance, and sometimes collaboration if structured well.”

The responses in Table 4 suggest that teachers recognise that challenging tasks foster real life connections and 21st century higher order thinking skills in students. Challenging tasks are often designed and implemented in ways that make the task more relevant and meaningful to students. This may be achieved by incorporating real life examples and modelling in the challenging task. More generally, with the 21st century skills developed from the challenging task experience, students learn to apply these mathematical skills to other disciplines and in the real world. They even learn skills such as productive struggle, persistence and collaboration as students practise thinking, exploring, taking risks and failing, often in a group setting. After all, challenging tasks require students to face a problem they cannot immediately solve so students must participate in the critical process of solving such tasks, which cultivates a persistent disposition underpinned by a growth mindset in students (Boaler, 2016). Application to the real world is thus a characteristic of challenging tasks that the teacher participants acknowledge.

Another characteristic that teachers recognised in challenging tasks is multiple solution pathways and solutions of challenging tasks, as challenging tasks are not immediately solvable.

Table 5

Characteristic of Challenging Tasks: Multiple Solution Pathways and Solutions

Multiple solution pathways and solutions ($n=10$)	“Challenging tasks in mathematics would require students to explore and utilise multiple strategies, drawing from a wide repertoire of mathematical knowledge, skills and techniques.” “They require critical thinking or problem solving, not immediately solvable by an algorithm, encourage creativity and possibly open-ended with multiple ‘correct’ answers.”

Lastly, a small group of teachers ($n=2$) characterised challenging tasks in ways that did not quite align with Sullivan and colleagues’ model. While many teachers conceive challenging tasks to be in-class tasks or activities, two teachers conceive challenging tasks to be investigative tasks for assessments or competition style problems (Table 6).

Table 6

Different Conceptions of Challenging Task Definition

Investigative tasks ($n=1$)	“For me it’s all in the rubric! How to structure and use them well”
Competitions ($n=1$)	“There are a lot of competitions to get included in.”

Further alternative responses are outlined below in Table 7.

Table 7

Different Conceptions of Challenging Task Characteristics

Teacher quote	Analysis / Comment
“I’ve never used that term before. I assume it means tasks that either lack scaffolding and have more of a modelling approach and require students to develop their approach to solve the task.”	While students should plan their approach when tackling challenging tasks as challenging tasks tend to be open-ended and non-routine, scaffolding should still exist to enable differentiation. For example, students can be supported through the task using enabling and extending prompts.
“Tasks that require numerous interconnected steps and associated	While the first half of this statement is true as challenging tasks tend to be multi-step, the second half of this statement

concepts that may extend slightly outside of the typical scope of the syllabus that is usually taught and assessed.	is debatable. Depending on the Australian state, different teaching strategies may be called for by curriculum, but the underlying skills remain applicable. For example, General Capabilities from the Australian Curriculum and Working Mathematically processes from the NSW mathematics syllabus target the higher order thinking and skills fostered by non-routine tasks like challenging tasks.
“HPGE, HSC, Practical Scenarios, Multiple Solutions, Cross Curriculum.”	Most of what is written here is correct but HPGE, which stands for High Potential and Gifted Education, seems to suggest that challenging tasks are limited to students excelling academically when they should be accessible by all students as challenging tasks support differentiation.
“Well-designed challenging tasks will have a low floor, generally allowing guess and check or simple applications of routine procedures for easy but potentially less accurate answers, also becoming a discussion point when analysing the problem”	Since challenging tasks are good for differentiation, the task should be accessible for all students, meaning it is low floor, high ceiling. This response suggests that challenging tasks tackle lower order skills and routine problems, which is not exactly the case.

Given some variability in teachers’ conceptions, these alternative responses offer insight into current teacher knowledges around challenging tasks and the implications this may have on professional learning. There is opportunity to create a discourse on challenging tasks and establish a consistent understanding of its definition and characteristics.

Conclusion

This study aimed to investigate Australian secondary mathematics teachers’ conceptions of challenging tasks. When teachers were asked about what they know about challenging tasks, a key finding in this paper is that most teachers held broadly accurate conceptions of challenging tasks, but specific characteristics of such tasks were often overlooked. Even if teachers are not familiar with Sullivan’s (2017) published model of challenging tasks, they are able to articulate the key concepts underpinning the definition and characteristics of challenging tasks in their responses. Many teachers summarise challenging tasks to be open-ended, non-routine tasks that foster different higher order thinking skills in students. However, teachers who were less familiar with challenging tasks had different conceptions of challenging tasks in terms of how they can be situated within the syllabus and curriculum, how accessible they are to all students and how much cognitive demand is required to tackle one.

The findings from this exploratory study contribute a richer understanding of what Australian secondary mathematics teachers conceive a challenging task to be. It is these teacher conceptions that may influence decisions surrounding challenging task implementation in secondary mathematics classrooms. Potential future directions include comparing teachers’ conceptions with their actual implementation of challenging tasks in classrooms. This could be achieved by analysing teachers’ classroom practice and challenging task examples. Another interesting avenue for research would be to investigate teachers’ responses to the similarities and differences between their own understandings of challenging tasks and Sullivan and colleagues’ model. This could be a professional development intervention to help enhance teacher knowledge and practice around challenging task implementation.

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References

- Australian Curriculum, Assessment and Reporting Authority. (2026). *Version 9.0 The Australian Curriculum*. <https://v9.australiancurriculum.edu.au/>
- Bird, E. (2023). Rich tasks for primary school mathematics. *Teaching Mathematics*, 48(2), 13-17.
- Bloom, B. S., Engelhart, M. D., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain*. David McKay.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching*. John Wiley & Sons.
- Cheeseman, J., Clarke, D., Roche, A., & Wilson, K. (2013). Teachers' Views of the Challenging Elements of a Task. In V. Steinle, L. Ball & C. Bardini (Eds.), *Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 154-161). MERGA.
- English, L. D., & Gainsburg, J. (2015). Problem solving in a 21st-century mathematics curriculum. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 313-335). Routledge.
- Goos, M., Stillman, G., Herbert, S., & Geiger, V. (2020). *Teaching secondary school mathematics: Research and practice for the 21st century* (2nd ed.). Routledge.
- Ingram, N., Holmes, M., Linsell, C., Livy, S., McCormick, M., & Sullivan, P. (2020). Exploring an innovative approach to teaching mathematics through the use of challenging tasks: A New Zealand perspective. *Mathematics Education Research Journal*, 32(3), 497-522. <https://doi.org/10.1007/s13394-019-00266-1>
- Kapur, M. (2008). Productive failure. *Cognition and instruction*, 26(3), 379-424. <https://doi.org/10.1080/07370000802212669>
- Lee, J. E. (2012). Prospective elementary teachers' perceptions of real-life connections reflected in posing and evaluating story problems. *Journal of Mathematics Teacher Education*, 15(6), 429-452. <https://doi.org/10.1007/s10857-012-9220-5>
- Organisation for Economic Co-operation and Development. (2018). *The future of education and skills: Education 2030*. OECD. <http://www.oecd.org/education/2030/oecd-education-2030-position-paper.pdf>
- Organisation for Economic Co-operation and Development. (2023). *Education at a Glance 2023: OECD Indicators*. OECD. <https://doi.org/10.1787/e13bef63-en>
- Parrish, C. W., & Bryd, K. O. (2022). Cognitively Demanding Tasks: Supporting Students and Teachers during Engagement and Implementation. *International Electronic Journal of Mathematics Education*, 17(1), em0671. <https://doi.org/10.29333/iejme/11475>
- Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.) *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: Information Age Publishing.
- Schweig, J. D., Kaufman, J. H., & Opfer, V. D. (2020). Day by Day: Investigating Variation in Elementary Mathematics Instruction That Supports the Common Core. *Educational Researcher*, 49(3), 176-187. <https://doi.org/10.3102/0013189X20909812>
- Sullivan, P. (2017). *Challenging mathematical tasks: Unlocking the potential of all students*. Oxford University Press.
- Sullivan, P., Clarke, D., Michaels, D., Mornane, A., & Roche, A. (2012). Supporting Teachers in Choosing and Using Challenging Mathematics Tasks. In *Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia*. <https://files.eric.ed.gov/fulltext/ED573381.pdf>
- Sullivan, P., & Davidson, A. (2014). *The Role of Challenging Mathematical Tasks in Creating Opportunities for Student Reasoning*. In *Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia*. <https://files.eric.ed.gov/fulltext/ED572597.pdf>
- Sullivan, P., & Mornane, A. (2014). Exploring teachers' use of, and students' reactions to, challenging mathematics tasks. *Mathematics Education Research Journal*, 26(2), 193-213. <https://doi.org/10.1007/s13394-013-0089-0>
- Szabo, Z. K., Körtesi, P., Guncaga, J., Szabo, D., & Neag, R. (2020). Examples of problem-solving strategies in mathematics education supporting the sustainability of 21st-century skills. *Sustainability*, 12(23), 1-28. <https://doi.org/10.3390/su122310113>
- Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press.
- Wilkie, K. J. (2016). Rise or Resist: Exploring Senior Secondary Students' Reactions to Challenging Mathematics Tasks Incorporating Multiple Strategies. *Eurasia Journal of Mathematics, Science and Technology Education*, 12(8), 2061-2083. <https://doi.org/10.12973/eurasia.2016.1260a>

Noticing Affordances of Tasks for Use with Low-Progress Learners in Mathematics: The Case of Mr Faizad

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Teaching through problem solving (TTPS) is an instructional approach that introduces new concepts by giving students opportunities to solve carefully crafted mathematics problems. However, many teachers struggle to design and implement TTPS lessons, especially when working with low-progress or emerging learners. In this paper, we compare how a teacher designed and implemented tasks to teach through problem solving for a Grade 3 class of 15 emerging learners, drawing data from two lessons to examine what he noticed about the task affordances.

In mathematics classrooms, a step-by-step approach to problem solving is often modelled by teachers, particularly for low-progress or emerging learners—those who require additional support or scaffolding. This is also the case for some Singapore teachers, who choose to first teach strategies or methods and then have these emerging learners apply them to solve less challenging questions (Kaur et al., 2019; Wang et al., 2014). Although such an “ends” approach (Lester, 2013, p. 246) provides a clear guideline for emerging learners to follow without requiring them to grapple with the problem, offering clearly articulated steps may hinder students’ development of problem-solving skills and dispositions (Goulet-Lyle et al., 2019). This approach stands in contrast to a “means” approach, through which “mathematical concepts, processes, and procedures are learnt” through problem solving (Lester, 2013, p. 246). Takahashi’s (2021) notion of teaching through problem solving (TTPS) is one such approach, in which problem solving goes beyond an “add-on” activity in a lesson and instead directs the flow of the lesson (Takahashi et al., 2013, p. 239).

A TTPS lesson begins with a carefully chosen problem, which is not only “low floor” (Resnick, 2008, p. 1) but also affords multiple solution pathways. Students solve the problem and present the different solutions to the whole class, which then sets the stage for the teacher to facilitate the comparison and discussion of these solutions. Such an approach provides opportunities for the teacher to revisit the solutions and connect these responses to the goal of deriving the intended facts, concepts, and procedures (Takahashi, 2021). Therefore, TTPS goes beyond show-and-tell, building on students’ ideas to explore and learn new mathematical content (Smith & Stein, 2011). The TTPS approach aligns well with the recent Singapore mathematics curriculum push towards “learning mathematics as a tool to solve problems and learning mathematics as a discipline” (Yeo & Choy, 2023, p. 125).

However, while many teachers believe that students should learn mathematics through problem solving, most of them may not make problem solving and discussions around problems part of their everyday mathematics lessons because they struggle with the idea of TTPS (Takahashi, 2021). These tensions are further exacerbated when teaching emerging learners (Kaur et al., 2019; Wang et al., 2014), as teachers may feel compelled to teach basic skills, even though these learners prefer lessons that engage them in thinking (Kaur & Ghani, 2012). A key lever for supporting teachers lies in helping them notice and harness the affordances of tasks (Choy & Dindyal, 2021) and students’ thinking during lessons (Goodwin, 1994; van Es, 2011).

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 90–97). MERGA.

Given that a well-designed task is the centrepiece of TTPS, it is important for us to understand what teachers perceive as the tasks' affordances to better support teachers in enacting TTPS.

This paper adds to ongoing conversations about supporting teachers in enacting mathematically productive teaching approaches, such as TTPS, for emerging learners. Here, we focus on Mr Faizad, a mathematics teacher who taught a Grade 3 class of 15 emerging learners at Zane Hall Primary School (all names are pseudonyms) in Singapore. We are interested in examining changes in his teaching as he learned to teach his emerging learners through the TTPS approach. The following question frames this paper: What did Mr Faizad notice about the affordances of tasks, and how did he harness them as he learned to teach through problem solving for his emerging learners?

Noticing Affordances of Tasks

In the context of mathematics tasks for use in TTPS lessons, we refer to their affordances in enabling teachers to orchestrate discussions with and between students to bring out the lesson's mathematical point. We follow Choy and Dindyal's (2021) argument that "the way tasks are designed and used" (p. 197) determines "the range of possibilities for human activity" (Brown, 2009, p. 20). Put in another way, this range of possibilities can be conceived as the *affordances* of the tasks. According to Gibson (1986), "the affordances of the environment are what it offers the animal, what it provides or furnishes, either for good or ill" (p. 127). Greeno (1994) explains affordance as "whatever it is about the environment that contributes to the kind of interactions that occur" (p. 338). While task affordances exist relative to teachers' actions and capabilities, their existence is independent of teachers' ability to perceive them, and these affordances do not change in response to changes in teachers' instructional goals (Choy & Dindyal, 2021). Therefore, for teachers to use tasks productively for TTPS, they will need to recognise the tasks' affordances and decide how to use them to craft instructional episodes aligned with the lesson's goals (Brown, 2009). The main challenge is for teachers to *notice* the task's hidden affordances to unlock its full potential when teaching by focusing on their students' thinking.

This places teacher noticing at the heart of the work of supporting teachers in enacting productive teaching practices (Choy, 2016; Mason, 2002; van Es, 2011). Broadly, noticing can be understood as the process of attending to and making sense of instructional moments to decide on instructional approaches (van Es, 2011), which can be seen as a form of "professional vision—consisting of socially organised ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (Goodwin, 1994, p. 606). Mason (2002) sees noticing as a discipline, highlighting it as "the heart of all practice" and offering it as a means by which teachers can "do something about it [teaching] in a practical and disciplined manner" (p. 1). Choy (2016) provides three focal points—mathematics, student thinking, and teaching actions—to direct teachers' noticing in ways that support their enactment of productive teaching practices, such as those needed in TTPS. Moreover, a teacher's ability to notice task affordances before lessons is critical (Choy & Dindyal, 2021). In this study, we focus on understanding how a teacher's noticing of task affordances changed as he learned to use TTPS tasks with emerging learners.

Methods

Data for this paper were collected as part of a project aimed at developing a pedagogical approach based on the notion of TTPS for emerging learners. The project involved three schools (Two secondary and one primary) and a total of 79 teachers. For each school, we conducted two professional development sessions, one at the beginning of the study and the other in the middle of the study (2 hours per session), to introduce TTPS ideas, practices related to orchestrating discussions (Smith & Stein, 2011), building thinking classrooms (Liljedahl,

2020), and universal design for learning (UDL) principles (Rose & Meyer, 2002). We then engaged extensively with one or two teachers from each school to develop and trial mathematics tasks for use with emerging learners in their TTPS lessons over another four sessions (1 to 2 hours per session). Mr Faizad is one of the teachers we worked with at Zane Hall Primary School, a typical public school whose student profile spans from emerging to talented. As an experienced teacher, Mr Faizad was assigned to teach a Grade 3 class of 15 emerging learners, who were specially pulled out as part of the school's Learning Support for Mathematics programme. Many of them struggled with English, the language of instruction, and had issues with factual fluency, resulting in very low achievement scores. Mr Faizad started out with a 'teach first, then apply' approach to problem solving, but his practices changed significantly towards the end of the project. The changes in his practices are of interest in this paper. For this paper, we will focus on the first and third TTPS lessons to examine what he noticed about the affordances and how he harnessed them.

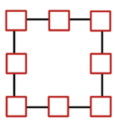
We collected his instructional materials and sampled students' work related to the lesson observed. We also conducted a post-lesson discussion with Mr Faizad, which was voice-recorded, to understand his thinking and instructional decisions after the lesson. To understand Mr Faizad's noticing of task affordances (Choy & Dindyal, 2021), we used Choy's (2016) three focal points to create snapshots of his noticing. First, we segment the lesson according to the stages described by Takahashi (2021)—introduction, posing of problem task, students working on the problem, whole-class discussion, and conclusion. Next, we analysed the task in terms of its mathematical point and possible students' responses (both correct and incorrect) to develop what we perceived to be the task's affordances. Third, we examined his teaching actions and his students' responses at different points in the lesson. We paid attention to how Mr Faizad used the tasks with his students, focusing on the kinds of talk moves (Van Zoest et al., 2023). Insights from these snapshots were developed by comparing them against Mr Faizad's responses during post-lesson interviews. More specifically, we want to highlight the changes in Mr Faizad's design and implementation of the problem task used in his TTPS lesson, focusing on what he noticed and how he harnessed the tasks' affordances.

Snapshots of Mr Faizad's First TTPS Lesson

The first TTPS lesson was centred on the problem of arranging a given number of cubes into a square while ensuring the same number of cubes on each side. Mr Faizad started with 8 cubes and asked his students to determine how many cubes were on each side of the square. See Figure 1 for the introductory problem and a visual representation of the solution.

Figure 1

Introductory Problem and its Visual Representation

Problem	Visual Representation
David arranges 8 cubes and ensures an equal number of cubes on each side. How many cubes did he arrange on each side of the square?	

This introductory problem served as a launchpad for the generalised problem of finding the number of cubes on each side, given the total number of cubes. An algebraic solution is not required at Grade 3. To support his emerging learners in the problem-solving process, Mr Faizad provided counting blocks for them to work out the answers to the cases of 12 cubes and 16 cubes, respectively. It appears that Mr Faizad wanted his learners to figure out the general method for obtaining the answers to any given number of cubes, as he tried to convince them of the need for a more general approach, beyond using the counting cubes:

Mr Faizad: During exams, did you get cubes? Then how?

Students: Draw!

Mr Faizad: But what if they say, David uses 144 cubes? Are you going to draw 144 cubes? So, what do we need? We need to transform what you put on the squares into a set of mathematical steps.

Although Mr Faizad circulated around the classroom and observed how his students worked on the problem, he did not call on them to share their answers, methods, or thinking processes. Instead, he was focused on sharing his approach to solving the problem:

1. Subtract 4 from the total number of cubes (take away the four cubes at each corner of the square)
2. Divide the remaining cubes by 4 (because there are four sides in a square)
3. Add 2 to the quotient (to account for the two cubes at the corners on each side of the square)

Algebraically, we interpreted this solution as $\frac{x-4}{4} + 2$, where x is the total number of cubes given (Figure 2). Note that the algebraic expression is not needed for this lesson.

Figure 2

Visualisation of Mr Faizad's solution.

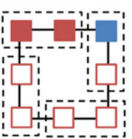
Note: The algebraic interpretation was ours and was not shared with the students.

Visual Image	Generalised Steps
 The students should exclude the red cubes first, before distributing the blue cubes equally to the 4 sides. Then add the 2 corner cubes to the answer to obtain the number of cubes on each side.	Minus 4: e.g., $8 - 4 = 4$ Divide by 4: e.g., $4 \div 4 = 1$ Plus 2: e.g., $1 + 2 = 3$ Number of cubes on each side = 3
Algebraic Interpretation	
Let the total number of cubes be x, Number of cubes on each side = $\frac{x-4}{4} + 2$	

When asked about his thinking behind his decision to focus solely on one solution, Mr Faizad reasoned, “I find that students need repetitive action to understand thoroughly, that same concept with the changed variable. I wanted students to build their muscle memory”. Mr Faizad’s approach to supporting his low-progress learners reflected the common belief that these emerging learners need a systematic, step-by-step approach to problem-solving (Goulet-Lyle et al., 2019; Kaur et al., 2019).

Figure 3

Other Possible Solutions to the Same Problem.

Method 2		Observing a Pattern by Listing	
Visual Image	Algebraic Interpretation	Total Number of Cubes	Number of Cubes on each side
	Let the total number of cubes be x, $\frac{x}{4} + 1$	8	3
		12	4
		16	5
		20	6
		...	...

However, focusing on a single solution pathway means that these learners would not have the opportunity to see other solutions and discuss the learning points afforded by the different solutions—elements that are critical to the TTPS approach (Smith & Stein, 2011; Takahashi,

2021). For example, students could have opportunities to discuss how Method 2 (Figure 3 left) is similar to or different from Mr Faizad’s solution (Figure 2) or highlight how systematic listing might lead students to see a pattern (Figure 3 right), which can provide another way of thinking about the problem. Here, we see Mr Faizad’s goal to “build their muscle memory” may have hindered him from perceiving the affordances of the task he used. Furthermore, it was evident that he did not fully harness the task’s potential in eliciting different generalisation strategies from his students.

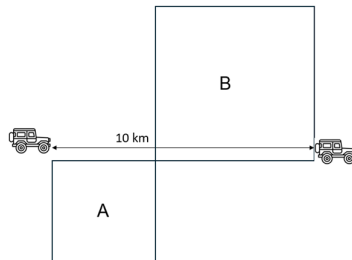
Snapshots from His Third TTPS Lesson

Towards the end of the project, Mr Faizad’s design and implementation of his TTPS task reflected a fundamental shift in how he identified and harnessed the task’s affordances. For this lesson, Mr Faizad designed a sequence of tasks leading to a challenging perimeter problem for his Grade 3 pupils, as shown in Figure 4. In Grade 3, perimeter is introduced as “distance around a figure” (Danielson, 2005, p. 31), and students are required to find the perimeter of rectilinear figures and rectangles given their side lengths. The problem is considered challenging even for some Grade 4 students because none of the squares’ dimensions is given.

Figure 4

A Challenging Perimeter Problem for Grade 3 Students

Farmer Kang has two square plots of land as shown below. He drove 10 km from the start of plot A to the end of plot B as shown. What would be the perimeter of the two square plots of land?

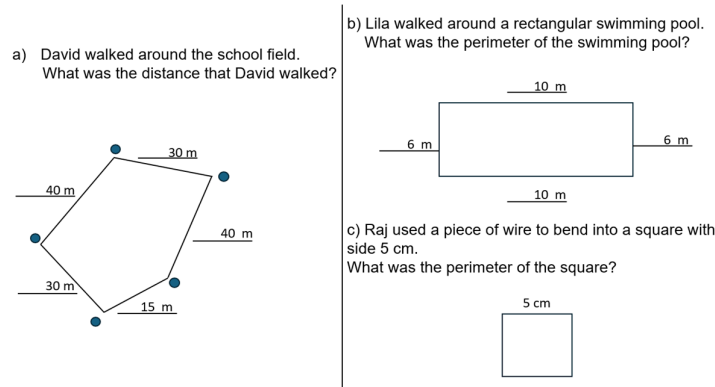


With the aim of supporting his low-progress students to understand perimeter, which is not always trivial (Danielson, 2005), Mr Faizad used contexts such as fences, walking around a field or a swimming pool, and bending wires to form rectilinear shapes (Figure 5). The sequence of tasks presented in Figure 5 was intentionally designed by Mr Faizad: Parts (a) and (b) shared the same context of “walking around”, but part (b) used the context of a rectangular pool for students to realise that the perimeter could be determined if the length and breadth are given; part (c) builds on the idea of a special rectangle and uses a new context of bending wires to form shapes. When put together, these three tasks gave students opportunities to apply their understanding of perimeter and provided Mr Faizad with a context to highlight the properties of rectangles and squares, which are necessary for solving the main problem.

Furthermore, Mr Faizad created animations (using movable dots) to show his students how David might have moved around the school field in part (a). Similarly, he created an animation showing how Farmer Kang drove from one end of the field to the other. These animations were created to support his students’ understanding of the problem by presenting information in a visual, dynamic way rather than in words alone (Rose & Meyer, 2002), so they can connect the concept of perimeter to the problem scenario (Danielson, 2005). Here, we see how Mr Faizad’s sequence of selected tasks and his presentation of the main problem reflect what he productively noticed and how he intended to harness the tasks’ affordances (Choy & Dindyal, 2021).

Figure 5

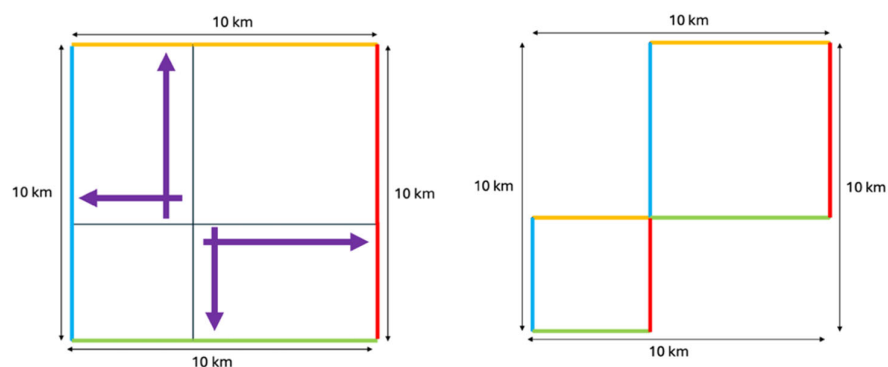
Sequence of Tasks Leading to the Perimeter Problem



Unlike the baseline study lesson, Mr Faizad asked his students to present their ideas and explain their thinking. In many ways, Mr Faizad's practices reflect the five practices for orchestrating productive discussions (Smith & Stein, 2011). While monitoring his students' work on the problem, Mr Faizad selected three students to present their solutions to the whole class in the following order. The first student "moved" selected lines parallelly to form a "big square of 10 km length" (see left of Figure 6), the second student coloured the lines to show that there are "four sets of 10 km" (see right of Figure 6), and the third student substituted 4 km and 6 km respectively for the sides of the smaller and larger squares respectively.

Figure 6

Two of the Solutions Presented by Mr Faizad's Students



Although one might argue that Mr Faizad could have sequenced the presentation differently to strengthen the mathematical connections, the opportunity given for students to share and comment on the different solutions is a critical feature of TTPS (Takahashi, 2021). For instance, although the third solution is a special case, Mr Faizad built on the answer to show that any pair of numbers that add up to 10 could be substituted, and the answer would still be 40 km, alluding to how ideas about invariance could be used to solve the problem. After making connections between the first two solutions, Mr Faizad presented a prepared animation of those solutions to consolidate the ideas learned from solving the problem, highlighting that he had *anticipated* the solutions when planning the TTPS lesson.

Discussion

Taken together, the two snapshots provided insights into what Mr Faizad noticed about the task affordances and how he harnessed them during the lessons. It was evident that Mr Faizad had expanded his range of possibilities for how the task could be solved and used for TTPS. His teaching stance had begun to shift slowly from a "teach first, then apply" approach towards

a TTPS approach. During the post-lesson discussion of the perimeter lesson, Mr Faizad said he was “personally surprised” by the solutions his emerging learners provided. We believe these “surprising” solutions from his students might be critical to Mr Faizad’s incremental changes in his teaching stance over the course of the year. The findings presented in this paper also suggest that emerging learners could solve problems when given the opportunities to do so. As highlighted by Wang et al. (2014, p. 22):

The right for LP [low progress] students to access high-quality teaching and education resources should not be deprived simply because of their current performance. To offer LP students an equal education opportunity, these students need to be encouraged and supported to engage with content knowledge and tasks that are above their current level of competencies or abilities.

As Mr Faizad worked with us through the project, it became evident that he became better at noticing and harnessing the affordances of tasks for TTPS. However, the challenge remains. How do teachers design and implement tasks that are not only accessible to emerging learners but also engaging and challenging at the same time? In this paper, snapshots from Mr Faizad’s lessons offer two main insights into how we can support teachers to do this ambitious work, especially when working with low-progress students.

First, it is crucial for teachers to notice the affordances of the problems used (Choy & Dindyal, 2021) and presented the problem in a way, which makes it a “low floor (easy to get started with)”, “high ceiling (opportunities for increasingly complex explorations over time)”, and “wide walls” task—multiple pathways from floor to ceiling (Resnick, 2008, p. 1). As highlighted in this paper, although the task used in the first lesson could be seen as a low-floor, high-ceiling, wide-walls task, its presentation and implementation did not fully harness its affordances. In addition to designing for the mathematical quality of the task, it is equally important to consider how the UDL principles—providing multiple means of engagement, representation, action, and expression—can be applied in its design and implementation (Rose & Meyer, 2002). Mr Faizad’s use of animations to present the problem and solution is a case in point, enabling his students to engage actively in problem solving.

Second, these task affordances can only be realised if teachers adopt a listening stance in their teaching: Mr Faizad, for instance, anticipated his students’ responses, monitored their work on the task, and then purposefully selected and sequenced their work for presentation in the third lesson (Smith & Stein, 2011; Takahashi, 2021). Doing this involves using talk moves such as eliciting ideas from students, asking for clarification, and the teacher’s revoicing of ideas to facilitate interactions (Van Zoest et al., 2023), while maintaining the accessibility of ideas for their students. These practices place great demands on a teacher’s attention to listen to students and notice the mathematical aspects of students’ ideas (Choy, 2016). This is critical because teachers will need to put in more effort to make sense of emerging learners’ ideas.

How do we support teachers in this ambitious form of teaching, where all learners, regardless of their current levels of competence, have opportunities to engage in mathematics? While Mr Faizad’s growth in his noticing of affordances is encouraging, the findings presented in this paper also suggest that more sustained professional development is needed. Perhaps it is important to consider ways for teachers to also engage in thinking and noticing task affordances as part of this professional development. As Mason (2002) has highlighted, noticing involves a shift in attention, which enables one to see new possibilities. For teachers, it is about adopting an open mind towards perceiving new possibilities in the tasks they use. For mathematics educators, it is critical to create opportunities for teachers to discuss and more carefully examine the hidden mathematical affordances embedded in these tasks (Choy & Dindyal, 2021). Focusing on enhancing teachers’ ability to notice affordances may be a critical area for future research, as we learn to support teachers to do different forms of ambitious teaching.

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References

- Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics Teachers at Work: Connecting Curriculum Materials and Classroom Instruction* (pp. 17-36). Routledge.
- Choy, B. H. (2016). Snapshots of mathematics teacher noticing during task design. *Mathematics Education Research Journal*, 28(3), 421-440. <https://doi.org/10.1007/s13394-016-0173-3>
- Choy, B. H., & Dindyal, J. (2021). Productive teacher noticing and affordances of typical problems. *ZDM – Mathematics Education*, 53(1), 195-213. <https://doi.org/10.1007/s11858-020-01203-4>
- Danielson, C. (2005). Perimeter in the curriculum. *For the learning of mathematics*, 25(1), 30-33.
- Gibson, J. J. (1986). The theory of affordances. In *The ecological approach to visual perception* (pp. 127 - 143). Lawrence Erlbaum Associates, Publisher.
- Goodwin, C. (1994). Professional vision. *American Anthropologist*, 96(3), 606-633.
- Goulet-Lyle, M.-P., Voyer, D., & Verschaffel, L. (2019). How does imposing a step-by-step solution method impact students’ approach to mathematical word problem solving? *Zdm*, 52(1), 139-149. <https://doi.org/10.1007/s11858-019-01098-w>
- Greeno, J. G. (1994). Gibson’s affordances. *Psychological Review*, 101(2), 336 - 342.
- Kaur, B., & Ghani, M. (2012). *Low Attainers in Primary Mathematics*. World Scientific Publishing Co. Pte. Ltd. <https://doi.org/10.1142/8332>
- Kaur, B., Toh, T. L., Lee, N. H., Leong, Y. H., Cheng, L. P., Ng, K. E. D., Yeo, K. K., Yeo, B. W. J., Wong, L. F., Tong, C. L., Toh, W. Y. K., & Safii, L. (2019). *Twelve Questions on Mathematics Teaching*. National Institute of Education, Nanyang Technological University.
- Lester, F. K. J. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, 10(1 & 2), 245 - 278.
- Liljedahl, P. (2020). *Building thinking classrooms in mathematics (Grades K–12): 14 teaching practices for enhancing learning*. Corwin Press.
- Mason, J. (2002). *Researching your own practice: The discipline of noticing*. RoutledgeFalmer.
- Resnick, M. (2008, 25 March). *Falling in love with Seymour’s ideas* American Educational Research Association Annual Meeting, New York. <https://web.media.mit.edu/~mres/papers/AERA-seymour-final.pdf>
- Rose, D. H., & Meyer, A. (2002). *Teaching every student in the digital age : universal design for learning*. Association for Supervision and Curriculum Development.
- Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. National Council of Teachers of Mathematics Inc.
- Takahashi, A. (2021). *Teaching mathematics through problem solving: A pedagogical approach from Japan*. Routledge.
- Takahashi, A., Lewis, C., & Perry, R. (2013). A US lesson study network to spread teaching through problem solving. *International Journal for Lesson and Learning Studies*, 2(3), 237-255. <https://doi.org/10.1108/IJLLS-05-2013-0029>
- van Es, E. (2011). A framework for learning to notice students’ thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers’ eyes* (pp. 134-151). Routledge.
- Van Zoest, L. R., Stockero, S. L., Peterson, B. E., & Leatham, K. R. (2023). (Counter)Productive Practices for Using Student Thinking. *Mathematics Teacher: Learning and Teaching PK-12*, 116(4), 244-251. <https://doi.org/10.5951/mtlt.2022.0307>
- Wang, L. Y., Teng, S. S., & Tan, C. S. (2014). *Levelling up academically low progress students* (NIE Working Paper Series, Issue 3). National Institute of Education.
- Yeo, J. B. W., & Choy, B. H. (2023). Fostering disciplinary thinking through mathematical inquiry. *The Mathematician Educator*, 4(2), 125-147. <https://ame.org.sg/2023/09/06/tme2023-vol-4-no-2-pp-125-147>

Risks and Opportunities of Online Platforms for Learning Mathematics

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New Zealand education policy is targeting the subject of mathematics in a series of rapid and extensive reform initiatives, including tutoring interventions and promoting online platforms for learning mathematics. Given this setting, this paper presents research into the mathematics instructional platforms *Matific* and *Education Perfect*. A multiple, embedded case study was conducted in three different classrooms that used these platforms, with a view to understand how students constructed mathematics learner identities in these contexts. Findings highlight various risks and opportunities for students using these platforms going beyond any affordances for mathematics learning.

Due in part to unsatisfactory results in recent TIMSS and PISA international tests (Education Counts, 2024; OECD, 2023), the New Zealand government has embarked on a period of rapid and extensive reform in mathematics education. The curriculum has been rewritten three times in the past three years, four different mathematics textbook programmes have been sourced from private, international providers and distributed to primary schools, and a mathematics intervention programme has been trialled in 2025 and expanded in 2026. This intervention programme was trialled with both in-person and online formats, using the privately operated platforms *Matific* and *Education Perfect*. Considering the known difficulties in resourcing extra mathematics teachers, it is likely that these online platforms will be relied on heavily as the intervention programme is rolled out across the country.

Initial trials showed the intervention programme to increase student progress, as widely announced by government media releases (Stanford, 2025). However, this progress was not without criticism, academics pointed out the lack of transparency in the methods used for assessment and argued that a ‘year’s progress’ could not be claimed from testing only a small portion of the curriculum (Gerritsen, 2025). Furthermore, those students who received the online intervention achieved similarly to the control group, raising the question of whether the costs of the programme justify the benefits in the case of the online option.

However, there is more than just achievement at stake when introducing a new programme of mathematics. The decisions we make in our mathematics teaching have implications for students’ mathematical learner identities (e.g., Darragh, 2021a). The experiences students have with the learning of mathematics impacts their development of positive relationships with mathematics, which in turn impact their future life choices (Black & Williams, 2013). Thus any nationwide promotion of a particular programme of mathematics, such as online platforms *Education Perfect* and *Matific*, should only be done with careful consideration of the wider implications.

In this paper we draw on case study research into the use of platforms *Education Perfect* and *Matific*. We interrogate the potential risks and opportunities regarding their use, with a focus on student learner identity.

Background

Online Mathematics Instructional Platforms (OMIPs) are subscription-based online platforms providing curricula material that students engage with on a computer or other digital device. They are more than a digital textbook as they may include interactive activities, video tutorials, and instant feedback on submitted responses. The platforms typically claim a

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 98–105). MERGA.

comprehensive coverage of mathematics curricula (adapted to different countries worldwide), and some include reward incentives, such as games or items for purchase with points received for mathematics work. OMIPs also incorporate assessment and data analytics that assign to the student a level and then identify their next learning steps, a process that uses hidden ‘black box’ algorithms (Boninger et al., 2019) and they are increasingly drawing on artificial intelligence, including large language models (Education Perfect, 2024).

Some research in mathematics education has explored the phenomenon of OMIPs. Studies have looked at individual OMIPs, including Mathletics, Matific, Khan Academy, or Sumdog (Attard, 2016; Day, 2014; Jablonka, 2017; Nansen et al., 2012). Other research has examined digital technology use in the classroom, of which OMIPs are a part (Attard & Holmes, 2020; Muir, 2014; Nicholas & Fletcher, 2017). Notwithstanding one critique of surveillance and gamification in the platform Sumdog (Jablonka, 2017) much of this body of research appears to see digital use as either inevitable or largely beneficial (Young, 2017), reflecting research on digital technologies in mathematics education more generally (Borba, 2021). A critical examination of these platforms remains limited within the context of our field (Darragh, 2021b).

Outside of mathematics education, by contrast, there is some critique of such platforms and of educational technology (EdTech) in general. This body of work notes the ‘hyper-individualism’ of EdTech which ‘magically’ determines pacing, path and content for the students (McRae, 2013), transferring power away from the teacher and to the private providers of the platforms (Roberts-Mahoney et al., 2016). Questions are raised about the impact of ‘Big data’ that harvests information from the student users to be resold (Knox et al., 2020; McRae, 2013). The literature also argues that EdTech is reductionist, as learning platforms are constrained by what is easily programmable (Boninger et al., 2019; McRae, 2013).

From the above literature it is clear that the mathematics education field could benefit from a critical look at the implications for mathematics learners of using OMIPs, going beyond achievement data and considering learner identity. Given the context of a future for New Zealand that may include ministerial promoted use of platforms such as Education Perfect and Matific, the research question we aim to address in this paper is:

- What are the risks and opportunities for student learner identity when using platforms such as Education Perfect and Matific as part of the mathematics programme?

Conceptual Framework

We use the concept of mathematics learner identity to understand the data produced in this study. Mathematics learner identity may be defined as:

A socially produced way of being, as enacted and recognized in relation to learning mathematics. It involves stories, discourses and actions, decisions, and affiliations that people use to construct who they are in relation to mathematics, but also in interaction with multiple other simultaneously lived identities. This incorporates how they are treated and seen by others, how the local practice is defined and what social discourses are drawn upon regarding mathematics and the self (Darragh & Radovic, 2020, p. 582).

Seeing identity as socially produced requires an understanding of the context in which an identity might be enacted. In the case of this study the online platform is the mathematics learning context, but it is also important to understand the context of the regular classroom to provide a comparison. In particular, we focus on how the “local practice” is defined in the context of OMIPs.

To operationalise mathematics learner identity in the context of OMIPs, we draw on Darragh’s (2024) *Identity Sites* framework, that includes the *scripts*, the *stage*, and the *audience*. Specifically, we attend to the ‘stage’, which is the context for learning mathematics (e.g., a classroom, or an online platform) and ask the following three questions during our data analysis: What is mathematics in this context? What roles are available for the mathematics learner? And

how does the learner access that mathematics, and take up those roles? The first two questions interrogate the context, in this case the online context of Education Perfect and Matific, whilst the final question considers an individual's identity within that context.

Methodology

The use of OMIPs in New Zealand is relatively widespread (Darragh & Franke, 2021) and relates to overall trends in the increase of digital technologies in mathematics education (Suripah et al., 2025). The data for this study comes from a wider project, in which we have been exploring the phenomenon of OMIPs from a variety of perspectives, including teachers' use, senior students' reflections, website analyses, and case study research (Amos et al., 2024; Darragh, 2021b; Darragh & Franke, 2021). For this paper we draw on three case studies in classrooms using OMIPs. We used a multiple, embedded case study approach (Yin, 2003). Three classroom cases were conducted, and we also recruited individual student cases within each of the classroom cases, a total of 12. The classroom case studies were: First, a Year 10 mathematics class (students aged approximately 14 years) at an innovative Year 7 to 10 middle school that used Education Perfect in conjunction with other educational applications and Apple iPads. We call this case *EP Classroom*. Second, a Year 5 and 6 modern learning environment classroom of 90 students (aged 9-10 years) and three teachers in an urban primary school using Matific. We call this case the *ILE Matific Classroom*. Third, a Year 7 and 8 Kura (students aged 11-12) that gave all instruction in Te Reo Māori and used Matific. We call this case the *Te Reo Matific Classroom*. In each classroom case, the students used the OMIPs as just part of their mathematics programme; that is, there were other types of mathematics learning, such as group work with the teacher or worksheets. The schools in the first two cases were situated in immigration catchment areas and extremely diverse. The third case classroom included only students who identified as Māori.

Data Collection and Analysis

In each case, we collected a variety of data. We observed mathematics lessons over a period of six-ten weeks, approximately 15 lessons in each classroom, and took field notes. We held semi-structured interviews with the teachers from each classroom at the beginning and end of the observational periods. We interviewed students from each class (five students in the first two cases and two students from the third case, a total of 12 students) and we interviewed the parents of these volunteer students (11 interviews), and finally we asked the students to take us on a 'tour' of their OMIP account which we screen-recorded and captured the audio as they spoke. Interviews were transcribed and sent to the participants for member checking.

Analysis involved three key steps. First, we identified the mathematics that was made available on the platform and compared it to the mathematics made available in the other contexts for mathematics learning in each classroom. Second, we identified the roles for mathematics learner that were available on the platform and again compared to the roles available in the classroom more generally. Third, we considered the ways in which each individual student accessed the mathematics and took on various available roles. This analysis enabled identification of risks and opportunities for mathematics learning and learner identity.

The findings we present below are organised according to each aspect of the research question; presenting various opportunities and risks of the OMIPs, illustrated with aspects of the individual cases, then a brief discussion of the potential impact on identity.

Findings and Discussion

Our analysis of the mathematics made available on the platforms and the roles available for the learner to take up revealed three key opportunities and three key risks in the use of the platforms for mathematics programmes. The opportunities included alternative access to

mathematics learning; an opportunity to learn mathematics in another language, specifically te reo Māori; and enhancing parental involvement in mathematics learning. The risks included: the promotion of a superficial and competitive version of mathematics; a loss of local and cultural relevance; and issues of dataveillance. Below we elaborate on each of these.

Opportunities

Whilst teachers allocated work to students on the platform, the platforms also contained other resources for learning mathematics that the students could independently access. For example, on Matific there was a ‘training zone’ (to practice skills), ‘arena’ (to compete with other users), and an ‘adventure island’ (with tasks assigned by the platform’s learner analytics), each offering different learning options. We noted occasions during observations across the case studies where some students accessed more mathematics learning than that which was assigned by the teacher.

I like the way [Education Perfect] explains it if you get something wrong like it has a good explanation and it is a good way to work on things that you are confused on because there are different tasks that you can do on your own without a teacher assigning it to you. (Sky, EP Classroom)

Thus, having the online platform available sometimes facilitated mathematics learning opportunities that would otherwise not have happened. For one student, David Bowi (all names are pseudonyms chosen by the students), he worked with his mother at home on extra Matific tasks, that she supervised from the kitchen. Another student (Sofia, ILE Matific Classroom) felt uncomfortable speaking to her teacher due to social anxiety and was thus excluded from much classroom learning. However, the Matific platform allowed her to engage in mathematics as it did not require oral participation.

Another aspect appreciated by students was the instant feedback provided:

“I think it takes a while to do each task [on EP] and especially if I don’t understand it then it is quite difficult because it is not like I can ask my iPad the question. But I feel good after because every time you do a correct answer it makes this noise and it makes me feel more smart and it is very nice.” (Claire, EP Classroom)

Altogether most students across all cases expressed a preference for mathematics learning with the teacher, however, there were some who found either the social context of learning interactions difficult to negotiate or the teacher difficult to access, and thus the online setting became an important mathematics learning resource for them.

In all three case study classrooms there were students for whom English was either not their first language or not their language of instruction. In the high immigration contexts of the first two case study schools, students often arrived in class with very little experience with English. Matific is available in many languages and can thus give mathematics learning access to students in their home language. In the ILE Matific Classroom case, some students had only been in New Zealand for a matter of days. The teacher found Matific extremely useful in being able to provide mathematics learning during the first months of settling into a new country and new language of instruction.

I think with Matific that was a big drawcard ... they had like 57 something languages that you could translate into. So, you could put it into their home language, and they would be able to [...] obviously you want them to learn English language, but also you want them to be able to access maths at the level that they are confident at. (Teacher, ILE Matific Classroom)

At the time, there were children in the class using Matific in Russian, Chinese, and Japanese.

Unique to Matific is that the available languages on offer include te reo Māori. There is an acknowledged dearth of resources, particularly digital resources for learning, within te reo (Allen & Trinick, 2021). Being able to learn mathematics in te reo confers not only the

advantage of learning the subject in a preferred language but it also upholds the cultural identity of the learner at the same time (Webber & MacFarlane, 2018).

[My past experiences with mathematics] haven't been that good. Because last year, I didn't like pāngarau [mathematics] at all, because I wasn't very good at it, and the whole class was good at it, except for me, that's why I didn't like it at all. But now that I've got to play like games, pāngarau games, it's helped me with my pāngarau experience, and I love pāngarau now. (Maia, Te Reo Matific Classroom).

However, whilst having a te reo Māori resource for mathematics (pāngarau) was useful, the classroom teacher had hesitations in that the language used on the platform was not 'everyday' reo, and the students required a fair amount of support to understand the mathematics vocabulary.

Finally, some parents expressed a liking for the online platforms due to them making the mathematics learning visible to them. For example, Sofia's mum commented that "Yes, every day, when I come home from work, she is already seated in her desk doing Matific. She says mum I have work to be done" (Mother of Sofia, ILE Matific Classroom). There is much research that explores the disconnect between home and school (Wadham et al., 2020). Having a platform that allowed for mathematics learning to enter the home invites the parent into the world of mathematics learning, as we have noted in previous research (Amos et al., 2024).

Risks

One risk of the OMIPS that we saw in our analysis was the promotion of a superficial and competitive version of mathematics. Because the platform design required instant feedback (and at the time of data collection did not incorporate large language model artificial intelligence) the questions were often in short answer or multiple-choice format. This generates a view of mathematics as being quickly or even randomly solvable. We observed instances of children clicking answers seemingly at random in order to gain access to the reward games when working on Matific. If students see mathematics as being about finding one right answer to short questions it may be difficult for them to cohere a sense of their creative self with the subject of mathematics.

Yeah, but you got to use your elimination, so you are going to be like because there are a bunch of houses on a hill which you will see and you will be randomly picking like is your door pink and if he says no you will basically tap all the ones with the pink doors and it will basically go off. [...] just more I'm saying it doesn't really help us with our learning because we don't know how to keep on doing it, after three tries it would tell us. [...] You will get zero stars which ... I don't think the teachers really check the stars. (Butter, ILE Matific classroom)

In this quote we see that Butter, a 9-year-old, is well aware that the random clicking of answers is not conducive to learning, however she is also aware that this is the way to achieve the task completion required by her teacher, who is unlikely to check the number of stars awarded.

A competitive notion of mathematics was promoted in Matific and Education Perfect. Both platforms awarded points to students as a reward for completion of work, arguably positioning mathematics as boring and in need of external motivators (see also Jablonka, 2017). These points could then be cashed in for various purchases - spent on one's avatar in the case of Matific or spent on stickers for friends in Education Perfect. Both thus position mathematics within a capitalist reward for work structure. Further, the points obtained were public. In every tour of the platform, we could see the students' ranking relevant to their classmates, and often students mentioned their ranking. In the case of Education Perfect we could also see a nationwide and global ranking.

Mannat Here we have the scoreboard. Right now I'm 16th place in the whole school wide.
 [...] The whole school yes, 1005 people.
Researcher: And what happens if you click on 'class'?

Mannat Class, I should be ... I'm 6th. So that's my whole maths class.
Researcher: And you can even see your ranking in the country and the world?
(Mannat, EP Class).

There was no way that students like Mannat could opt out of the competition, with the points always displayed on the screen, and the knowledge that her name and score is visible to others also. Furthermore, this competition was for the individual only, there was no possibility for engaging with mathematics as a group.

Whilst both platforms claimed to be relevant to the New Zealand mathematics curriculum, they were developed overseas and drew on item banks that were developed outside the country. This highlights the risks to a mathematics programme's local and cultural relevance if using the platforms. There is very limited opportunity to connect the mathematics learning developed on the platforms to the local communities in the school. Neither the multicultural and diverse contexts of two cases nor the indigenous context in the third case were reflected in the learning activities on the platforms. As noted in the second teacher interview "we've pulled back a little bit because I've gone back to the old school way" (Teacher, Te reo Matific Classroom). It may be difficult for students to align their cultural identity to that of mathematics learner when the mathematics does not reflect their cultural contexts, as supported in other local research (Hunter & Hunter, 2024).

One final risk that emerged from our analysis is likely to have increasing relevance in years to come. Each platform engaged in dataveillance to an extent the students were likely unaware.

I think with online it kind of like saves your progress like if you are working in a maths programme you lose it you have lost it type of thing, but on this you have log ins and stuff and it remembers what you've done so far and it kind of backs things up. If you ever want to look back you can. (Sky, EP Class)

So, when they revise everything and then they explain it to me like of course they explain to everyone whoever is doing the worksheet, but to me it is like they are explaining personally for me and so I understand it better. Yeah, it is basically like EP knows what I'm thinking so they explain it that way. (Mannat, EP Class)

These comments from the older students show an anthropomorphising of the EP platform. Yet giving it the human qualities of 'remembering' and 'personal explanations' generates an artificial separation from a machine that is capable of enormous data extraction and storage. Their parents too are likely unaware of the ways in which private corporations may mine the students' data for future use. Rather, parents spoke of digital technologies as inevitable, and unproblematic, for example, "It is just like having a pen and paper, it is a tool that they have to use so that is just how I see it, it's just a tool." (Claire's mother, EP Class).

Mathematics learner identity

The platforms presented a superficial version of mathematics that was, at times, reduced to short answers, with multi-choice questions that could be answered randomly. Yet the mathematics learning was available to the students anytime and anywhere, including in their own homes, and in many different languages. The roles available for the mathematics learner included the agentic and independent learner, who could access additional learning tasks for themselves, but it also positioned the learner as competitive and a compulsory player in the points and ranking systems of the platforms, and as a consumer, using the points to buy things in the world of the platform. These depictions of mathematics and the mathematics learner required the students to position themselves as learners of mathematics accordingly. Some students embraced the invitation to access additional mathematics learning opportunities and relished the ability to play mathematics games in their own language. Others looked at their teacher assigned tasks as simply another thing to complete and did so in the quickest way possible. Finally, there was no escape to being 'datafied,' all students were ranked, and all students provided data for these privately-owned platforms to use as they desired.

Conclusions

As the findings highlight, there are both opportunities and risks inherent in using OMIPs as part of a mathematics programme that go beyond achievement in mathematics. The opportunities include greater access to the learning of mathematics, for example in other languages, at home with parents, when the teacher is unavailable or the social context is challenging for the student. On the other hand, many of the risks reflect concerns shared in the literature outside of mathematics education. For example, the individualisation of learning (McRae, 2013; Roberts-Mahoney et al., 2016), reductionist mathematics (McRae, 2013), and dataveillance (Knox et al., 2020).

This study expands upon that literature by considering the potential impact of OMIPs for mathematics learner identity. Whilst there were mathematics learning opportunities that include access for diverse students (e.g., neurodiverse learners, non-English speaking learners), and for bringing school mathematics into the home, there are also considerable implications for mathematics learner identity. The students positioned themselves as learners of mathematics alongside their rankings and rewards in the points-based systems of the platforms. In some cases, this enabled them to see themselves as successful learners of mathematics, but at other times it positioned them as competitive capitalists and undermined the social context and value of human interaction (see also Roberts-Mahoney et al., 2016).

Given the context of reform in New Zealand, with the likelihood of an increase in the promotion of the platforms Matific and Education Perfect, the findings of this study will hopefully serve as a cautionary tale for the Ministry of Education. Whilst the learning opportunities may seem apparent, there is certainly a trade-off in the promotion of superficial, competitive and capitalist mathematics and a datafied mathematics learner.

Acknowledgements

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References

- Allen, P., & Trinick, T. (2021). Agency–structure dynamics in an indigenous mathematics education community in times of an existential crisis in education. *Educational Studies in Mathematics*, 108(1), 351–368
- Amos, M. M., Darragh, L., & Trinick, T. (2024). A kaupapa Māori critique of digital resources for pāngarau. *Set: Research Information for Teachers*, 3, 2–9. <https://doi.org/10.18296/set.1560>
- Attard, C. (2016). *Research evaluation of Matific mathematics learning resources: Project report*. <https://doi.org/10.4225/35/57f2f391015a4>
- Attard, C., & Holmes, K. (2020). “It gives you that sense of hope”: An exploration of technology use to mediate student engagement with mathematics. *Heliyon*, 6(1). <https://doi.org/10.1016/j.heliyon.2019.e02945>
- Black, L., & Williams, J. (2013). Contradiction and conflict between “leading identities”: becoming an engineer versus becoming a “good muslim” woman. *Educational Studies in Mathematics*, 84, 1–14. <https://doi.org/10.1007/s10649-013-9481-7>
- Boninger, F., Molnar, A., & Saldaña, C. M. (2019). *Personalized learning and the digital privatization of curriculum and teaching*. National Education Policy Center.
- Borba, M. (2021). The future of mathematics education since COVID-19: Humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics, online fir*. <https://doi.org/https://doi.org/10.1007/s10649-021-10043-2>
- Darragh, L. (2024). New Insights from Identity Sites. In T. Evans, O. Marmur, J. Hunter, & G. Leach (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education*. PME.
- Darragh, L. (2021a). Playing games for positive maths learner identities. *Set: Research Information for Teachers*, 1, 36–42. <https://doi.org/10.18296/set.0166>

- Darragh, L. (2021b). The promise of online mathematics instruction programs: Producing the mathematics learner and school mathematics. *Research in Mathematics Education*, 23(3), 262–277. <https://doi.org/10.1080/14794802.2021.1993977>
- Darragh, L., & Franke, N. (2021). Online mathematics programs and the figured world of primary school mathematics in the digital era. *Mathematics Education Research Journal*, online fir. <https://doi.org/https://doi.org/10.1007/s13394-021-00384-9>
- Darragh, L., & Radovic, D. (2020). Mathematics learner identity. In *Encyclopedia of mathematics education* (pp. 582–585). Cham: Springer International Publishing.
- Day, L. (2014). ICT the changing landscape. *Australian Primary Mathematics Classroom*, 19(2), 23–27.
- Education Counts (2024) *TIMMS 2023: Mathematics Year 5* <https://www.educationcounts.govt.nz/publications/series/2571/timss-trends-in-international-mathematics-and-science-study-2023/timss-2023-mathematics-year-5>
- Education Perfect (2024, May 14) Australian-first AI Feedback tool enhances teaching and learning outcomes <https://www.educationperfect.com/news/australian-first-ai-feedback-tool-enhances-teaching-and-learning-outcomes/>
- Gerritsen (2025, November 26) Maths teaching programme to be rolled out to 13,000 students nationwide. RNZ <https://www.rnz.co.nz/news/national/580046/maths-teaching-programme-to-be-rolled-out-to-13-000-students-nationwide>
- Hunter, J., & Hunter, R. (2024). Weaving together the threads of Indigenous knowledge and mathematics. *Educational Studies in Mathematics*, 116(3), 501–518. <https://doi.org/10.1007/s10649-023-10256-7>
- Jablonka, E. (2017). Gamification, standards and surveillance in mathematics education: An illustrative example. In A. Chronaki (Ed.), *Mathematics education and life at times of crisis: Proceedings of the ninth international mathematics education and society conference*. Mathematics education and society.
- Knox, J., Williamson, B., & Bayne, S. (2020). Machine behaviourism: Future visions of “learnification” and “datafication” across humans and digital technologies. *Learning, Media and Technology*, 45(1), 31–45.
- McRae, P. (2013). *Rebirth of the Teaching Machine through the seduction of data analytics: This time it's personal [Online resource]*. <https://nepc.colorado.edu/blog/rebirth-teaching-machine-through-seduction-data-analytics-time-its-personal>
- Muir, T. (2014). Google, Mathletics and Khan Academy: students’ self-initiated use of online mathematical resources. *Mathematics Education Research Journal*.
- Nansen, B., Chakraborty, K., Gibbs, L., Vetere, F., & MacDougall, C. (2012). “You do the math”: Mathletics and the play of online learning. *New Media & Society*, 14(7), 1216–1235. <https://doi.org/10.1177/1461444812442926>
- Nicholas, K., & Fletcher, J. (2017). What is happening in the use of ICT mathematics to support young adolescent learners? A New Zealand experience. *Educational Review*, 69(4), 474–489. <https://doi.org/10.1080/00131911.2016.1237476>
- OECD (2023) *Country Report for New Zealand Education GPS* <https://gpseducation.oecd.org/CountryReports?primaryCountry=NZL>
- Roberts-Mahoney, H., Means, A. J., & Garrison, M. J. (2016). Netflixing human capital development: Personalized learning technology and the corporatization of K-12 education. *Journal of Educational Policy*, 31(4), 405–420. <https://doi.org/10.1080/02680939.2015.1132774>
- Stanford, E. (2025, November 25) *Focus on basics delivers groundbreaking maths results*. Beehive <https://www.beehive.govt.nz/release/focus-basics-delivers-groundbreaking-maths-results>
- Suripah, Retnawati, H., Zetriuslita, Zafrullah, & Hidayat, R. (2025). No Title. *Research Trends in Scopus Database on Technological Innovation in the Process of Mathematics Learning: A Bibliometric Analysis*, 13(1), 97–116. <https://doi.org/10.23947/2334-8496-2025-13-1-97-116>
- Wadham, B., Darragh, L., & Ell, F. (2020). Mathematics home-school partnerships in diverse contexts. *Mathematics Education Research Journal*. <https://doi.org/10.1007/s13394-020-00357-4>
- Webber M., & MacFarlane, A. (2018). The Transformative Role of Iwi Knowledge and Genealogy in Māori Student Success In E. A. McKinley & L. Tuhiwai Smith (Eds.) *Handbook of Indigenous Education*, Springer <https://link.springer.com/referencework/10.1007/978-981-10-1839-8>
- Yin, R. K. (2003). *Case study research*. Sage Publications.
- Young, J. (2017). Technology-enhanced mathematics instruction: A second-order meta-analysis of 30 years of research. *Educational Research Review*, 22, 19–33.

Parental Involvement in Mathematics Learning in Multicultural School Communities in Australia

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This paper explores how parents in multicultural primary school communities describe their involvement in their children's mathematics learning in and across home and school spaces. Drawing on Bourdieu's concepts of field and capital, the analysis focuses on the practices parents enact rather than institutional rules. Using focus group data from two Australian schools, the study examines how parents organise routines at home, communicate with teachers, and decide when to intervene or step back. Findings highlight parental involvement as dynamic and negotiated, shaped by time, confidence, and access to relational resources across interconnected spaces of mathematics learning.

Parental involvement is an important influence on children's mathematics learning; however, less is known about how such involvement is enacted within the everyday realities of family life (Civil & Bernier, 2006; Hill & Tyson, 2009; Wu et al., 2022). In multicultural communities, parents often support learning while navigating diverse educational histories, expectations, and resources (Al-Mahdi, 2010; Civil, 2014; Yaro, 2023). These forms of involvement are shaped not only by school expectations but also by family routines, time constraints, confidence with mathematics, and children's responses (Bosire et al., 2025; Murphy et al., 2023). This study responds to the need for closer examination of how parental involvement is enacted in everyday practice, particularly beyond formal school spaces, by examining parents' own accounts of supporting mathematics learning between home-based routines and school-related expectations. It addresses the following research question: How do parents describe their involvement in their children's mathematics learning in and across home and school spaces?

Literature Review

Research has consistently established parental involvement as positively associated with children's mathematics achievement, engagement, and attitudes toward learning (Bosire et al., 2025; Hill & Tyson, 2009). At the same time, scholars have noted ongoing debate about how parental involvement is conceptualised and operationalised (Fiskerstrand, 2022; Hill & Tyson, 2009). Much of the literature has focused on parents' participation in school-based activities or their alignment with institutional expectations, privileging forms of engagement that are most visible within the school context (Fiskerstrand, 2022; Hornby & Lafaele, 2011).

Within mathematics education, this emphasis has been increasingly questioned, particularly in relation to culturally and linguistically diverse communities (Al-Mahdi, 2010; Civil & Bernier, 2006). Scholars have argued that parental involvement often takes informal, culturally embedded forms that may not align with dominant school norms but play an important role in supporting learning (Dediwalage et al., 2025; Yaro, 2023). Research demonstrates that immigrant parents draw on a range of resources, experiences, and relational networks to support their children's mathematics education, highlighting the significance of context and cultural histories in shaping involvement (Nicholas & Fletcher, 2017; Planas & Civil, 2013).

Recent research increasingly distinguishes between parental participation and parental practice in children's learning (Fiskerstrand, 2022; Yaro, 2023). *Parental participation* typically refers to visible forms of involvement in school contexts, such as attending meetings, volunteering, or assisting with school activities. In contrast, *parental practice* emphasises the

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 106–113). MERGA.

everyday ways families support learning within their own contexts. This includes routines, conversations, shared activities, and informal problem-solving that occur in daily family life (Bosire et al., 2025; Civil, 2014; Leyva et al., 2023). In this study, parental involvement is understood as parental practice, focusing on how families support learning through everyday interactions rather than through formal school participation. In mathematics learning, parents often engage in everyday conversations, decision-making, encouragement, and emotional support rather than direct instruction or the replication of school teaching (Al-Mahdi, 2010; Nicholas & Fletcher, 2017). Focusing solely on school-based participation can therefore obscure the substantial ways parents support mathematics learning at home.

The relational and distributed nature of parental involvement has also been highlighted, demonstrating that parents frequently mobilise relational resources, including siblings, extended family, and community networks, to support mathematics learning (Cahoon et al., 2024; Hunter et al., 2016). Such practices challenge individualised models of involvement that locate responsibility solely with parents and instead point to involvement as shared and negotiated across people and spaces. Hunter et al. (2016) and Al-Mahdi (2010) show that parents' social networks play a key role in shaping their engagement with schooling.

Despite these advances, comparatively less attention has been given to how parents themselves describe their involvement across interconnected home and school spaces, particularly in contemporary Australian multicultural primary settings (Dediwalage et al., 2025). Much of the existing literature prioritises barriers, expectations, or outcomes, offering limited insight into how parents organise, adapt, and negotiate involvement in their everyday lives over time (Fiskerstrand, 2022; Leyva et al., 2023). Addressing this gap requires closer attention to parents' own accounts of involvement as it is enacted in practice, rather than inferred through institutional measures or externally defined indicators.

In response, this study examines how parents in multicultural primary school communities describe their involvement in their children's mathematics learning in and across home and school spaces. Rather than evaluating involvement against predetermined standards, the analysis foregrounds parents' accounts of what they do, how they decide when to intervene or step back, and how involvement is sustained through relational engagement (Cahoon et al., 2024; Civil & Bernier, 2006). This study conceptualises involvement as situated practice to extend understandings of parental engagement beyond visible school participation.

Theoretical Framing

Bourdieu's concepts of *field* and *capital* (Bourdieu, 1977; Bourdieu & Passeron, 1977), inform this study and are used to sensitise attention to context and practice rather than to regulate behaviour. The concept of *field* highlights that parental involvement occurs across multiple spaces, including home, school, and the relational connections between them. Each space offers different possibilities for action, shaping how parents engage with mathematics learning. The concept of *capital* is used to understand the resources parents draw upon when supporting mathematics learning. These resources include embodied capital, such as parents' mathematical knowledge and confidence; social and relational capital, including support from siblings, extended family, tutors, and peer networks; and temporal capital, such as time availability and flexibility. Importantly, the analysis does not assess whether these forms of capital align with institutional expectations but rather examines how they shape parental practices and decisions.

Methodology

Research Design and Context

This study adopted a qualitative case study design to examine parents' experiences of supporting children's mathematics learning in multicultural primary school contexts. A case

study approach was appropriate because it enabled in-depth exploration of a phenomenon within its real-life context and supported investigation of the “how” and “why” underlying participants’ experiences (Simons, 2009; Thomas, 2020). In this study, the phenomenon of interest was parental involvement in mathematics learning, examined across four Year 5 classrooms situated within two multicultural primary schools in Melbourne, Australia.

Schools were identified using publicly available data from the My School website (MySchool, 2024), which provides information on school demographics, including the percentage of students classified as English as an Additional Language (EAL) learners. Schools with an EAL percentage exceeding 75% were considered eligible. As the My School website provides limited information on cultural or linguistic diversity beyond EAL classification, school principals were consulted to confirm the cultural composition of their communities and their willingness to participate.

Participants, Data Generation, and Analysis

Data were generated through four parent focus group interviews conducted across the two participating schools. Each focus group consisted of three parents of children enrolled in Year 5, resulting in a total of 12 parent participants across the four focus groups. Participants represented a wide range of cultural and linguistic backgrounds (Table 1), with most born outside Australia. To maintain anonymity, parents were invited to select their own pseudonyms, while the two schools were assigned pseudonyms by the researcher.

Focus group discussions explored parents’ experiences of supporting mathematics learning at home, their interactions with school mathematics practices, and the challenges they encountered across home and school. Parents reflected on their experiences through questions such as, “*What sort of mathematics-related activities do you do at home?*” and “*Are there any challenges when trying to help your child with mathematics?*” All focus group interviews were audio-recorded, transcribed verbatim, and analysed using reflexive thematic analysis (Braun & Clarke, 2019). Analytic attention focused on identifying patterns in how parents described practices of involvement as they were enacted within home and school spaces, rather than evaluating their engagement against school expectations or institutional norms.

Table 1

Participant information

School	Group	Participant	Born	Country	Home Language
Green Lakes Primary School	1	Kain (father)	Overseas	Vietnam	Vietnamese
	1	Mia (mother)	Overseas	Portugal	English
	1	Samantha (mother)	Australia	Mauritius	English
	2	Susan (mother)	Overseas	Sri Lanka	Sinhala
	2	Emily (mother)	Overseas	Serbia	Serbian
	2	Sarah (mother)	Overseas	South Sudan	Nuer
Blue Trees Primary School	3	Izzy (mother)	Overseas	India	Punjabi
	3	Kelly (mother)	Overseas	Romania	Romanian
	3	Lee (mother)	Australia	Lebanon	Lebanese
	4	Mark (father)	Overseas	Sri Lanka	Sinhala
	4	Betty (mother)	Overseas	South Korea	Korean
	4	Amy (mother)	Overseas	Samoa	Samoa

Researcher Positionality

This study was informed by my positioning as a researcher and former primary school mathematics teacher in multicultural communities. My experience supporting students and families from culturally and linguistically diverse backgrounds shaped my interest in how parents describe their involvement in children's mathematics learning across home and school contexts. Drawing on Bourdieu's theory of practice, I recognise that my own habitus informs my approach to the research process. Consistent with reflexive thematic analysis (Braun & Clarke, 2019), I maintained a reflexive stance throughout data generation and analysis to ensure parents' perspectives remained central.

Findings

Parents described involvement in mathematics learning as active and responsive rather than fixed or uniform. They reflected on different ways of supporting their children and how they adjusted them depending on what felt appropriate or helpful at the time. Although participants came from diverse cultural and linguistic backgrounds, many described similar forms of involvement. Parents commonly supported learning through everyday routines, informal activities, and efforts to align home practices with school expectations. Three overarching themes were identified.

Organising Mathematics Learning at Home

Parents described the home as a primary site of involvement, where mathematics learning was organised through routines and everyday activities. Many parents emphasised short, regular practices rather than extended instructional sessions. Betty described establishing a daily routine focused on developing habits of concentration, explaining, "I just want them to have a habit of sitting on the chair, concentrating on any question regarding the maths. So it takes about 10 to 15 minutes every day." This brief, routine-based approach was framed as a manageable way to support learning, particularly in the absence of formal homework.

Mathematics was frequently embedded in everyday activities. Mia and Emily described using cooking to support measurement, while Kain, Samantha, and Susan discussed shopping trips where children calculated spending and change. Others, including Izzy and Kelly, referred to games such as UNO or Monopoly as ways to reinforce number sense. These practices allowed parents to remain involved even when they felt uncertain about formal school methods and reflected deliberate choices about integrating mathematics into family life.

Parents' own mathematical confidence influenced how they structured involvement. Those who described themselves as "not good at maths," such as Lee, Amy, and Samantha, often focused on encouragement, repetition of basic skills, or creating opportunities for practice, rather than explaining formal procedures. Rather than withdrawing, these parents adjusted their involvement to align with what felt manageable and meaningful, demonstrating awareness of both their own limits and their children's learning dispositions.

Engaging with School Mathematics

Parents' involvement at school centred on communication and monitoring rather than direct participation in classroom activities. Many parents reported limited visibility of day-to-day mathematics instruction and relied on parent-teacher interviews, reports, or standardised assessments to understand their children's progress. These practices enabled parents to stay informed while reflecting on translating school information into support at home.

Several parents expressed a desire for clearer communication about what was being taught, not to challenge teaching approaches but to better support learning at home. Samantha suggested brief topic outlines or guidance "for the parents," while Kelly and Izzy said they wanted to know which topics were being covered so they could decide how to help at home.

Parents framed these requests as enabling alignment between home and school, indicating thoughtful consideration of their role in relation to teachers' expertise.

Opportunities to participate in school-based mathematics events were viewed positively when available. Lee, Izzy, and Kelly recalled school mathematics events they had attended and described them as valuable. However, parents such as Mark and Sarah noted that work commitments and scheduling constraints influenced whether they could attend. Involvement at school was therefore closely linked to time availability and flexibility rather than willingness, requiring parents to weigh possibilities for engagement.

Supporting Learning Across Home and School Spaces

A key feature of parental involvement was decision-making around when to intervene or step back. Parents described situations where children resisted help, expressed frustration, or preferred school strategies over parental explanations. Mia, Mark, and Betty recounted moments where children rejected parental methods, prompting parents to reduce direct instruction to avoid confusion or conflict. These decisions reflected parents' attentiveness to children's emotional responses and learning progress. For some immigrant parents, understanding school mathematics involved differences between their own schooling and their children's experiences in Australian classrooms. Stepping back sometimes reflected recognition that classroom approaches differed from the methods they had learned.

Older siblings, extended family members, and private tutors were frequently described as sources of support when parents felt unsure, time-poor, or less confident with mathematical topics. For example, Kain and Sarah described relying on older siblings, while Samantha, Lee, and Izzy discussed private tutoring as providing one-to-one support that was difficult to achieve at home. Tutoring was positioned as a strategic extension of involvement rather than a withdrawal from responsibility, with parents investing financially in the service while continuing to engage through encouragement, organisation, and monitoring. These accounts highlight involvement as layered and distributed across people and learning spaces.

Across the findings, parental involvement occurred in multiple learning spaces. The home functioned as a space for routines, encouragement, and everyday mathematics practices, while the school served as a reference point for curriculum expectations and progress monitoring. Relational learning spaces, including family networks and tutoring, enabled Kain, Kelly, and Samantha to sustain involvement when direct engagement was challenging. Shifts between these spaces were shaped by time, confidence, and access to resources, illustrating parental involvement as dynamic and responsive. In navigating these shifts, parents demonstrated awareness of their own capacities and roles, adjusting their involvement in response to children's feedback, emotional cues, and changing learning demands.

Discussion

Parental Involvement In and Across Home and School Spaces

This study examined how parents in multicultural primary school communities describe their involvement in their children's mathematics learning across home and school. By foregrounding parents' accounts, the findings reframe parental involvement as dynamic and flexible practices enacted across learning spaces (Cahoon et al., 2024; Civil, 2014; Hunter et al., 2016). Rather than a stable attribute or response to institutional expectations, parental involvement was organised and adjusted in response to children's needs, time constraints, confidence, and relational resources. (Bosire et al., 2025; Fiskerstrand, 2022; Leyva et al., 2023). This perspective shifts attention away from evaluating whether parents are "involved enough" toward understanding how involvement is sustained in everyday contexts.

Given the multicultural context of the participating schools, analytic attention was given to whether parents' accounts differed across cultural and linguistic backgrounds. While some parents reflected on differences between their own schooling experiences and those of their children in Australian classrooms, many described similar everyday practices for supporting mathematics learning at home. Overall, involvement appeared to be shaped more by shared efforts to understand school expectations and support children's learning than by cultural background. From a Bourdieuan perspective, this suggests that families mobilise different forms of cultural capital within the field of schooling, navigating its structures and expectations rather than acting solely from individual motivation.

A key contribution of this paper is the articulation of parental involvement as situated practice. At home, involvement was rarely characterised by formal instruction aligned with school mathematics; instead, it was embedded in routines, everyday activities, and short engagements. Activities such as cooking, shopping, games, or brief daily practice allowed parents to remain involved even when uncertain about contemporary approaches. These findings extend research on informal and culturally embedded mathematics engagement (Civil & Bernier, 2006; Nicholas & Fletcher, 2017). Reflexive analysis prioritised parents' accounts of practice over institutional benchmarks, recognising that forms of capital mobilised within the home may not always be visible or valued within the school field.

Relational, Distributed, and Responsive Forms of Involvement

The findings also illuminate the relational nature of parental involvement. Parents rarely acted alone; instead, they mobilised relational resources such as siblings, extended family members, tutors, and peer networks to support mathematics learning (Al-Mahdi, 2010; Cahoon et al., 2024; Hunter et al., 2016). These arrangements functioned as practical responses to constraints of time, confidence, and children's receptiveness. Rather than signalling disengagement, they represent strategic redistributions of involvement that allow learning to continue while preserving positive parent-child relationships. These relational arrangements highlight how parents draw on social and cultural capital to sustain involvement across contexts. When parents perceived limits in their own mathematical capital, they mobilised alternative resources that were more effective or better recognised within particular fields.

Parental involvement thus operated as a distributed practice across people and spaces, extending beyond the parent-child dyad. This challenges individualised models that locate responsibility solely with parents and aligns with sociocultural perspectives that view learning as relational and socially mediated. As a researcher with professional experience in primary mathematics education, I was attentive to how assumptions embedded within the school field might position such distributions as deficits. Reflexive analysis, therefore, focused on how parents themselves framed these decisions as purposeful and responsive.

Decision-making was a central dimension of involvement. Parents described navigating when to intervene or step back, and when to seek external support, particularly when children resisted help or experienced confusion. Stepping back reflected sensitivity to children's emotions and recognition that children also drew on different forms of capital across contexts. This complicates the linear assumption that increased intervention necessarily leads to better outcomes and aligns with research on autonomy-supportive parental engagement (Fiskerstrand, 2022; Hill & Tyson, 2009).

Understanding Home-School Spaces and Alignment

Parental engagement with school mathematics was primarily shaped through communication and monitoring rather than through direct classroom participation. Their desire for clearer information about topics or strategies reflected an orientation toward alignment rather than intervention. Parents did not seek to challenge pedagogical authority; instead, they

positioned themselves as supporters of learning at home in ways compatible with the school field. This suggests that involvement across home and school is mediated by information flows and relational trust, rather than by formal participation alone (Al-Mahdi, 2010; Dediwalage et al., 2025; Hunter et al., 2016).

Home, school, and relational spaces functioned as distinct but related fields through which parents adjusted their practices, mobilising different forms of capital in response to changing circumstances. Attending to how involvement is enacted across these spaces foregrounds continuity rather than fragmentation in families' support of mathematics learning. This invites reflexive consideration of how research itself may privilege forms of involvement that align more closely with the school field.

Implications for Practice

These findings have important implications for mathematics education within the Australian policy context, where strengthening partnerships between schools and families is recognised as a priority for improving student learning and equity. Although policy frameworks encourage family-school partnerships, school practices often privilege school-based involvement. The findings suggest the need to broaden institutional understandings of parental involvement by recognising how families support mathematics learning across home and community contexts, particularly within culturally and linguistically diverse communities.

For teachers and schools, this highlights the importance of making classroom mathematics practices more visible to families. Providing brief curriculum overviews, examples of strategies, or guidance through digital communication platforms may help parents understand classroom approaches. Such practices align with the Australian Professional Standards for Teachers (Australian Institute for Teaching and School Leadership, n.d.), which emphasise effective communication and collaboration with parents and carers while supporting connection without positioning parents as responsible for replicating classroom instruction.

Conclusion

This paper examined how parents in multicultural Australian primary school communities describe their involvement in their children's mathematics learning across home and school spaces. Drawing on focus group interviews and interpreted through Bourdieu's concepts of field and capital, the analysis foregrounded the practices parents enact rather than evaluating engagement through institutional measures. The findings show that parental involvement is dynamic and enacted across interconnected learning spaces, with parents organising support in response to school expectations and children's needs. Despite diverse cultural and linguistic backgrounds, parents described similar efforts to navigate schooling while supporting learning at home. Families mobilised everyday practices, relational networks, and external supports such as tutoring to sustain involvement across contexts. Recognising these practices broadens understandings of parental involvement and offers implications for strengthening home-school connections in mathematics education.

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References

- Al-Mahdi, O. (2010). A qualitative investigation of home-school relationships and children's mathematics learning in-and out-of-school in Bahrain. *Procedia, Social and Behavioral Sciences*, 8, 427–438.

- Australian Institute for Teaching and School Leadership. (n.d.). *Australian professional standards for teachers*. <https://www.aitsl.edu.au/standards>
- Bosire, J. P. O., Palermo, F., & Napoli, A. R. (2025). Early home literacy and math environment: Cross-domain associations between parental literacy and math beliefs. *Frontiers in Psychology*, 16, 1649713.
- Bourdieu, P. (1977). *Outline of a theory of practice*. Cambridge University Press
- Bourdieu, P., & Passeron, J. C. (1977). *Reproduction in education, society and culture*. Sage Publications.
- Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative Research in Sport, Exercise and Health*, 11(4), 589–597. <https://doi.org/10.1080/2159676X.2019.1628806>
- Cahoon, A., Campver, Y., Estévez, N., Jiménez Lira, C., Paz García, D. S., Benavides Pando, E. V., & Simms, V. (2024). Parents' views and experiences of the home mathematics environment: A cross-country study. *Infant and Child Development*, 33(4). <https://doi.org/10.1002/icd.2497>
- Civil, M. (2014). Why should mathematics educators learn from and about Latina/o students' in-school and out-of-school experiences? *Journal of Urban Mathematics Education*, 7(2), 9–20.
- Civil, M., & Bernier, E. (2006). Exploring images of parental participation in mathematics education: Challenges and possibilities. *Mathematical Thinking & Learning: An International Journal*, 8(3), 309–330.
- Dediwalage, M., Kalogeropoulos, P., Russo, J., & Vale, C. (2025). A systematic literature review of student, parent and teacher experiences of mathematics education in multicultural classrooms. *Review of Education*, 13(3), e70109. <https://doi.org/10.1002/rev3.70109>
- Fiskerstrand, A. (2022). Literature review – Parent involvement and mathematic outcome. *Educational Research Review*, 37, 100458. <https://doi.org/10.1016/j.edurev.2022.100458>
- Hill, N. E., & Tyson, D. F. (2009). Parental involvement in middle school: A meta-analytic assessment of the strategies that promote achievement. *Developmental Psychology*, 45(3), 740–763.
- Hornby, G., & Lafaele, R. (2011). Barriers to parental involvement in education: An explanatory model. *Educational Review*, 63(1), 37–52. <https://doi.org/10.1080/00131911.2010.488049>
- Hunter, J., Hunter, R., Bills, T., Cheung, I., Hannant, B., Kritesh, K., & Lachaiya, R. (2016). Developing equity for Pasifika learners within a New Zealand context: Attending to culture and values. *New Zealand Journal of Educational Studies*, 51(2), 197–209. <http://dx.doi.org/10.1007/s40841-016-0059-7>
- Leyva, D., Libertus, M. E., & McGregor, R. (2023). Math intervention targeting family routines increases parental math talk and math activities. *Journal of Applied Developmental Psychology*, 89, 101595.
- Murphy, S., Danaia, L., Tinkler, J., & Collins, F. (2023). Parents' experiences of mathematics learning at home during the COVID-19 pandemic: A typology of parental engagement in mathematics education. *Educational Studies in Mathematics*, 114(2), 455–474. <https://doi.org/10.1007/s10649-023-10224-1>
- MySchool. (2024). *Australian Curriculum, Assessment and Reporting Authority*. <https://www.myschool.edu.au/>
- Nicholas, K., & Fletcher, J. (2017). What supports 11- to 13-year-old Pasifika students in mathematics learning in New Zealand classrooms? *Education 3-13*, 45(1), 68–82. <https://doi.org/10.1080/03004279.2015.1048269>
- Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: Tensions in the bilingual mathematics classroom. *Mathematics Education Research Journal*, 25(3), 361–378.
- Simons, H. (2009). *Case study research in practice: Promoting positive relationships and wellbeing*. Sage.
- Thomas, G. (2020). Case study. In P. Atkinson, S. Delamont, A. Cernat, J. W. Sakshaug, & R. A. Williams (Eds.). *SAGE Research Methods Foundations*. Sage Publications Ltd.
- Wu, J., Barger, M. M., Oh, D., & Pomerantz, E. M. (2022). Parents' daily involvement in children's math homework and activities during early elementary school. *Child Development*, 93(4), 1347–1364.
- Yaro, K. (2023). Exploring race, children's mathematics learning and parental participation: Counternarratives of African immigrant families. *Journal of Mathematics and Culture*, 17(4), 198–214.

Advice for Aspiring School Mathematics Leaders

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Ten primary School Mathematics Leaders responded to a survey asking them to give advice at the beginning of the 2026 school year to teachers who were about to take on the responsibilities of leading the teaching and learning of mathematics. Leaders emphasised the importance of a clear purpose, building professional knowledge and fostering research informed pedagogy. They highlighted the value of professional learning communities grounded in trust, collaboration and resilience. The findings offer practical guidance for aspiring mathematics leaders and highlight the personal qualities essential for effective leadership.

The mathematics leadership role has long been recognised as being complex in nature, having many demands, and involving tensions (Cheeseman & Clarke, 2005). More recently with changes in emphasis and direction in some aspects of mathematics education this complexity has increased, and some school mathematics leaders find themselves conflicted. Many of these changes have created clashes between what school mathematics leaders' value and believe to be effective practice and what has been advocated or even mandated. There is no doubt that school mathematics leaders continue to play a critical role in supporting teachers' professional development and that their work is integral to the success of mathematics learning in schools (Driscoll, 2021).

While many aspects of mathematics leadership practice have previously been reported, few studies have reported the advice experienced school mathematics leaders who are presently leading mathematics in primary schools would provide to others. We believed it was timely to get a sense of mathematics leadership from those who have an 'ear to the ground' and those who are currently leading mathematics in schools. By exploring in detail, the work of these highly effective mathematics leaders, we believe the information they share can inform us about the nature of the work they do and support future mathematics leaders.

In this paper we report the perspectives and insights of ten experienced school mathematics leaders who responded to a survey. The research question we sought to answer was: *What advice would you give to teachers who are just about to take on the responsibilities of leading the teaching and learning of mathematics?*

Research literature

Ultimately, the objective for primary school mathematics leaders is to create conditions that support improved learning of mathematics by students through continuous professional learning of teachers resulting in improved classroom practice. It is through the actions of school mathematics leaders that "positive practical and sustainable change" (Grootenboer et al., 2015, p. 277) can occur. The key to improvement is developing teachers' mathematical knowledge for teaching (Ball et al., 2008). While mathematics leaders recognise the importance of teacher knowledge, they also understand the importance of looking at the bigger picture to develop a shared vision for school improvement, defined as "a set of images of ideal classroom practice for which teachers' strive" (Hammerness, 2001, p. 143)

Research suggests that developing a shared purpose of high-quality teaching can influence teachers improved practice and guide professional learning (Cobb, 2019). Cobb (2019) proposed a theory of action for improving mathematics teaching which included a sophisticated vision of high-quality instruction that specified several instructional practices to support the achievement of learning goals. Similarly, Vale and colleagues (2010) found that school

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 114–121). MERGA.

mathematics leaders who “had a clear vision of school and classroom practice and set directions for improved student learning,” together with “clearly stated goals and targets” played a vital role in affecting change that led to improved student learning (Vale et al., 2023, p. 242). Connected to a shared vision, mathematics leaders frequently establish measurable goals focused on student achievement and teacher practice. A recent study that focused on mathematics leaders’ goals found that while the leaders held a range of goals related to the improvement of mathematics teaching and learning, they often relied on teachers’ willingness to change to achieve these goals (Downton et al., 2022).

It is well recognised that teacher change can be slow and difficult (e.g., Fullan, 2001). When making changes to practice, teachers need to feel supported in an environment that offers both trust and challenge (Eden, 2018; Timperley et al., 2020). Relational trust is critical and underpins the work of school mathematics leaders as they demonstrate interpersonal skills, such as interpersonal respect, personal regard for others, role competence and personal integrity (Bryk & Schneider, 2003). This point is reinforced with the recent addition to the suite of Australian Professional Standards of the *Australian Professional Standards for Middle Leaders*. Under the banner of ‘Enabling knowledge and skills’ one of the focus areas - Standard 2c is ‘Building relational trust’ which is described as “The ability to build and sustain a culture of trust in which teaching and learning improvement is shared, well supported and competently led” (Australian Institute for Teaching and School Leadership [AITSL], 2025, p. 17). Although these standards are not specific to mathematics, they are intended to be inclusive of all middle leader roles, of which mathematics leadership is one. The standards reinforce the point that developing constructive relationships and trust with colleagues is critical to leading mathematics successfully in schools (Driscoll, 2021).

While “relationships make the difference” (Fullan, 2001, p. 51) and effective leaders constantly work on building relationships, creating opportunities for teacher learning through knowledge creation and sharing is also an important element of effective leadership (Fullan, 2001). This is achieved through consistency, coherence and an appreciation of effective pedagogical approaches, collaborative planning, and shared assessment practices with a focus on coherence-making, capacity building, and sustained professional learning rather than fragmented initiatives (Fullan, 2016). The way that the components of leadership come together was theorised by Michael Fullan as a Framework for Leadership (Fullan, 2001). A diagram of this framework is shown in Figure 1. This framework provided a theoretical underpinning of this research and a structure for our analysis and discussion of the open-ended qualitative data offered by mathematics leaders. We connected our coding of the data to Fullan’s components: moral purpose, understanding change, coherence building, knowledge creation, and relationship building. We also considered what Fullan called “the core mind and action sets” (2003, p. 92) of energy, enthusiasm and hope.

Method

An email was sent to the school email addresses of fourteen experienced school mathematics leaders. We introduced ourselves in the email and provided background information related to our research. Data were collected via a one-page pro-forma survey with one open question asking each school mathematics leader, what advice they would give to teachers who were about to take on the responsibilities of leading the teaching and learning of mathematics in their school. The leaders were asked to write spontaneously in answer to the question and elaborate on this advice then return the pro-forma to the first author by email.

The purpose of the survey was to collect leaders’ impromptu thoughts at the start of a school year when they would likely be thinking ahead about the nature of their role. The format was chosen to be quick to complete and brief. In this way we intended the “top of the head” responses to be priority advice. From the data we wanted to build a picture of what leaders

believed was important for a novice school mathematics leader to think about when beginning their leadership role. Ten leaders responded.

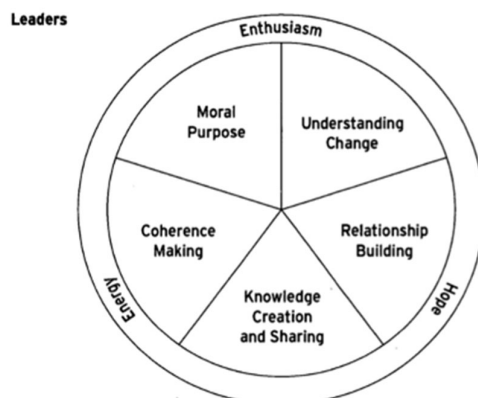


Figure 1. A Framework for Leadership (Fullan, 2001, p. 4)

The data were analysed as follows: each survey was read carefully by each of the researchers as they were received. Independently each author identified each idea a leader offered and noted how the advice was reflective of a finding in the literature. For example, Emily (pseudonym) wrote “Start with relationships – spend time building connections and trust.” While this was only part of the sentence Emily wrote, it was noted initially in the Black Book as two separate codes: relationship building I (Fullan) and trust building I (Bryk & Schneider, 2003). Each idea was recorded and added to a list that expressed leaders’ responses as a first pass of the data. Categories of response were refined as surveys were returned. Clusters of categories were created to search for patterns and an overview of the responses. The results of coding independently were compared, and a sample of responses were checked for inter-rater reliability.

Broadly our analysis methods could be described as coding thematically using an open-ended approach (Creswell, 2014) to identify themes that emerged from the data. Themes became the categories listed in the tables that follow. Upon further examination of the data sub-categories were evident. An illustrative quote that matched each sub-category was included and corresponding boxes were shaded to indicate a response that matched.

Findings

The main findings of the survey from the ten respondents are summarised and presented in three two-way tables showing six categories and linked sub-categories. The broad categories identified were Vision and purpose, Improvement of mathematics learning, Knowledge making and sharing, Consistency and coherence and Relationship building as well as Personal qualities. Each category will be presented in turn in the next section.

Vision and purpose and Improvement of mathematics learning

The categories of *Vision and purpose* and *Improvement of mathematics learning* are shown in Table 1. Further sub-categories are also included. A shaded box indicates that a School Mathematics Leaders (SML) response matched the category and sub-category.

Each SML (n=10) considered having a clear sense of purpose to be essential and acknowledged that student success in mathematics is their vision and that building students’ and teachers’ knowledge and confidence is fundamental to achieving that vision. The SMLs believed that success involves productive pedagogies which can lead to improvement in the

mathematical outcomes for their schools. Seven of the ten SMLs advised setting short-term and long-term goals as can be seen in Table 1.

Table 1

Categories of advice with illustrative quotes reported by individual respondents

Category	Illustrative quote	Initials representing the SMLs names									
Vision and purpose		L	A	H	E	J	S	A	E	J	M
Student success	A clear vision that all students can succeed	✓			✓		✓		✓		
Building knowledge	A commitment to building teacher confidence	✓				✓	✓	✓	✓	✓	✓
Strong Pedagogy	A commitment to building strong pedagogy	✓			✓	✓	✓	✓	✓	✓	✓
Improvement of mathematics learning		L	A	H	E	J	S	A	E	J	M
Short term outcomes	a laser sharp focus on just one (sometimes two) skill(s) for each Year level	✓	✓		✓	✓	✓		✓		✓
Long term goals	work towards instructional consistency across classrooms	✓	✓	✓	✓	✓	✓		✓		

Knowledge making and sharing and Consistency and coherence

The categories of *Knowledge making and sharing*, *Consistency and coherence* are shown in table 2 along with sub-categories.

Table 2

Categories of advice with illustrative quotes about knowledge creation and coherence

Knowledge (making and sharing)		L	A	H	E	J	S	A	E	J	M
Mathematical concepts	I recommend that leaders first and foremost upskill their mathematical content knowledge	✓		✓	✓		✓	✓	✓	✓	✓
Pedagogy (teaching practices)	focus on high-impact teaching practices rather than relying solely on programs	✓		✓	✓		✓	✓	✓	✓	✓
Co-teaching	If you need to enact change, the best way is co-teaching							✓			
Use of data	Ensure assessment and data analysis is consistent informing teaching and learning	✓	✓		✓		✓	✓		✓	✓
Prioritise concepts/focus	having a laser sharp focus on one skill, element or big idea for each year level	✓	✓	✓	✓		✓		✓		
Consistency/coherence making		L	A	H	E	J	S	A	E	J	M
Plan for improvement	It's just a dream without a plan. Planning helped to map out goals I wanted to achieve		✓			✓	✓				
Provide visible support	remain visible and supportive in practice	✓				✓		✓	✓		
Ensure time on task	Year 2 teachers took every opportunity to practice this with students in maths	✓	✓		✓		✓				
Cooperate when planning	collaboratively plan ... encourage teachers to come prepared with ideas and insights	✓		✓	✓	✓	✓	✓	✓		

Judging from these results it could be assumed that the advice for future leaders would be to focus on knowledge making and sharing as an important part of the role. There was a dual emphasis on mathematical concepts as a priority and effective teaching practices as their delivery. Knowledge of the students' understanding of concepts was advocated. Table 2 also indicates that ($n=3$) SMLs would advise new and future leaders to plan for improvement with their teachers, something achievable to start with and others ($n=7$) suggested to focus on collaborative planning.

Relationship building and Personal qualities.

The categories of *Relationship building and Personal qualities* are shown in Table 3 with sub-categories of collaboration, trust, shared purpose and many personal qualities.

Table 3

Categories of advice with illustrative quotes relationship building and personal qualities

Relationship building		L	A	H	E	J	S	A	E	J	M
Collaboration	Collaboration is one of the most important things you can do to ensure success	✓	✓	✓	✓	✓	✓	✓			
Trust	the key to successful leadership is often based on the relational trust	✓	✓		✓	✓	✓	✓		✓	✓
Shared purpose	Build relational trust, offering in classroom support, beginning informally then moving toward a co-constructed goal focus	✓	✓	✓		✓	✓		✓		
Personal qualities		L	A	H	E	J	S	A	E	J	M
Open Minded	having an open mind to what else is out there					✓	✓				✓
Critical	be critical of any resource and decide how you can make it fit your school context	✓	✓	✓		✓				✓	
A learner	I still position myself as a learner, I can't imagine that ever changing		✓	✓	✓	✓	✓	✓			✓
Enthusiastic	Let your passion be visible.		✓				✓	✓	✓		✓
Risk-taker	Take risks! Risks are what move your leadership practice forward, change doesn't happen in the comfortable		✓				✓	✓		✓	
Listener	Listen carefully to teachers					✓	✓	✓		✓	✓
Be authentic	Walk the walk - don't just talk	✓					✓	✓			
Vulnerable	It's ok to learn alongside your colleagues		✓				✓	✓			

Table 3 indicates that the SMLs believe that collaboration and particularly trust are important. Relationship building was present in the responses of every SML ($n=10$). Three essential aspects of mathematics leadership are mentioned: building a team of collaborators who have a shared purpose and trust each other. The personal characteristics listed by respondents were interesting and comprised an impressive list of qualities. Open minded, Critical, A learner, Enthusiastic/enjoy mathematics, Risk-taker, Listener, Reasonable, Be authentic, Vulnerable. These qualities resonate with the characteristics of enthusiasm, hope and energy suggested by Fullan (2001).

Four more themes emerging from the data will be discussed: Vision and purpose, Knowledge making and sharing, Relationship building, and Personal qualities.

Discussion

Begin with a vision and purpose

Each school mathematics leader surveyed, advised novice leaders to have a clear vision of high-quality classroom practice to improve student learning. This high priority echoes the findings of Vale and colleagues (2023). As stated by Lyn “leading the teaching and learning of mathematics begins with a clear vision that all students can succeed and a commitment to building teacher confidence alongside strong pedagogy.” A similar point was made by Jane who pointed out that you need to “make a plan of what you want to achieve and break it down year by year and then ... term by term” and think about “What is actually achievable?” and “What will make a difference?” In her words, Jane said that leaders need to “Dream big but start small.” Likewise, Julie suggested that new leaders need to “focus on the big picture.” To achieve the big picture, as was pointed out in the literature, mathematics leaders need to have clear visions and goals related to student achievement and teacher practice (Vale et al., 2023).

Sue was aware of the need to develop goals and made the point that it was necessary “to map out the goals that you want to achieve over the course of the whole year” and “prioritise what matters most,” otherwise, she revealed “it’s just a dream without a plan.” She described the steps involved, stating that it is necessary to “break them down in manageable chunks.” Abi also made the point that leaders need to be “realistic about the goals” they choose to work on. While another leader, Alex, suggested that teachers and leaders should work on developing co-constructed goals. These leaders were describing the importance of having a clear vision of the purpose of leadership. It is interesting to note that little was advised about the importance of aligning your colleagues and the other leaders in the school (such as the Principal) to leaders’ views which was an important finding by Downton et al. (2022).

Develop mathematical knowledge for teaching

While knowledge making and sharing and effective teaching practices were a focus of all but one school mathematics leader, seven leaders positioned themselves as learners in this survey. It is not unusual for school mathematics leaders to lack the confidence to lead when they do not possess the knowledge and skills they feel are important (Driscoll 2021). One leader, Amy, spoke about being “open to learning better ways of doing things” and suggested that “it is okay to learn alongside your colleagues”. She also pointed out that “you don’t need to be the fountain of all knowledge.” While Ann’s advice was to “be vulnerable to other teachers” and at the same time she suggested becoming involved in co-teaching. Alex made the point that he “would recommend that leaders first and foremost upskill their mathematical content knowledge to support teachers in using data to inform planning.”

When offering advice to new and prospective mathematics leaders Sue reflected on her experiences after five years of being a mathematics leader and made the point that “you don’t have to be an ‘expert’ to be a great leader.” Initially in her role she pointed out that she “felt like an imposter” and explained that “as soon as I focused my decision making onto teacher practice and student learning, that’s when I started to feel less like an imposter and more like a leader.” Sue also shared this advice:

Often when taking on the responsibility of leading, you feel like you have to know everything about everything and the whole curriculum inside out. I think this was the biggest mistake I made in my first-year leading numeracy, and I just got so burnt out. My advice would be position yourself as a learner. It’s okay not to know everything right off the top of your head. Learning side-by-side with teachers is one of the best ways to build trust and rapport.

Sue also explained that positioning herself as a learner was her ticket into teachers' classrooms where she could see what was happening and learn how to best support teachers.

Build relational trust

Interestingly in the survey responses all ten mathematics leaders commented on the importance of building relationships with an emphasis on trust. While there are many ways to develop relational trust amongst teachers, the importance of these relationships cannot be over emphasised. Building constructive relationships is very much part of being an effective leader (Fullan, 2001). In fact, Ann believed that "building respectful relationships with ALL teachers is the foundation of a successful mathematics program." Emily suggested that you "start with relationships [and] spend time building connections and trust with staff." While building "relationships with staff, on a personal and professional level, (build trust before change)" was also emphasised by Julie. Amy reiterated this point with her comment that "teachers who respect you, listen to you in planning, ask for advice, take risks and try new things, and allow you into their classrooms can be vulnerable around you, which can lead to positive change." Feeling supported to make change in an environment where there is trust and a certain amount of risk is reflected in the work of Eden (2018) and Timperley et al., (2020).

Jane's initial advice was to build relationships suggesting that "the key to successful leadership is often based on the relational trust." She also stressed that it is important to:

Spend time with people, listen to their needs, value their input and show them that they matter. The more they see you, feel you are walking alongside them and think you understand what is happening in their day-to-day work lives, the more they are likely to support you as a leader. This support and trust is critical if you want to make changes.

Consider personal qualities

There was an emphasis on a variety of personal qualities throughout the survey responses. In her responses Mel highlighted many of the qualities that we also found common, such as being "open to different perspectives" and "listening carefully to other teachers and learning from their experiences and ideas." She suggested that it was important "to learn yourself, stay approachable and transparent, and create a safe space where teachers feel comfortable asking questions, trying new strategies, and reflecting together." Emily's response summed up the focus on personal qualities nicely in this quote, "Let your passion be visible. Your enthusiasm for mathematics is contagious. When teachers see that maths can be joyful, intellectually rich and meaningful, it shifts the culture of the school" and the students benefit in the long run. As emphasised by Fullan (2001) personal traits continue to be an important consideration when leading others and "the core mind and action sets" (2003, p. 92) of energy, enthusiasm and hope continue to be inherent qualities of effective mathematics leaders that others aspire to.

In summary, although our surveyed sample was small, these school mathematics leaders provided a comprehensive range of advice. Our research question was deliberately designed to be open as it was intended to gather responses that came quickly to mind or 'top-of-the-head ideas.' Had there been further prompts the leaders might have included more detail. This could be a focus for future research. However, this short survey could also be seen as an advantage, as these were the things that the leaders felt were most salient at the time. Our findings contribute to building a picture of what leaders believed was important for a novice school mathematics leader to think about when beginning their leadership role.

Conclusion

The mathematics leadership role seems to be as complex as ever. The experienced school mathematics leaders surveyed when considering teachers who are new to leadership responsibilities had two main areas of advice: to have a clear sense of direction and purpose and to think of leadership as knowledge-building. The importance of imagining a school where

every student learns mathematics enthusiastically and where research-informed pedagogy is in place was foremost. The school mathematics leaders acknowledged that to do so would require sharing knowledge and skills in a learning community built on trust, collaboration and resilience. Building personal professional knowledge and that of others through collaboration and shared experiences seems to be fundamental to being an effective school mathematics leader which leads us to understand some of the tensions created by educational authorities when prescribed mathematics content and teaching approaches are instituted. The advice of the experienced leaders also highlighted personal elements of the role. For example, to continue to learn, listen carefully to colleagues, embrace mistakes as opportunities for growth and sustain a passion for mathematics and its teaching. By capturing these insights, the study provides practical guidance for aspiring mathematics leaders and highlights the personal and relational qualities essential for effective leadership.

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References

- Australian Institute for Teaching and School Leadership (AITSL) (2025). *Australian Professional Standards for Middle Leaders*. <https://www.aitsl.edu.au/teach/standards>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal for Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Bryk, A. S., & Schneider, B. (2003). Trust in schools: A core resource for school reform. *Educational Leadership*, 60(6), 40-45.
- Cheeseman, J., & Clarke, D. (2005). Early numeracy coordinators in Victorian primary schools: Components of the role, highlights and challenges. *Building connections: Theory, research and practice*, 225-232.
- Cobb, P. (2019). Investigating What It Takes to Improve the Quality of Mathematics Teaching and Learning on a Large Scale. In G. Hine, S. Blackley, & A. Cooke (Eds.). *Mathematics Education Research: Impacting Practice* (Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia) pp. 1-14. Perth: MERGA.
- Cresswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approach* (4th ed.) Thousand Oaks, CA: Sage.
- Downton, A., Cheesman, J. & Roche, A. (2022). Goals and challenges of school mathematics leaders. *Mathematics Teacher Education and Development*, 24(1), 96115. <https://mtd.merga.net.au/index.php/mtd/article/view/678>
- Driscoll, K. (2021). *An investigation of the ways in which School Mathematics Leaders support primary teachers' professional learning*. Thesis, Monash University. Melbourne, Australia.
- Eden, R. (2018). Opening classroom practice to challenge: The role of trust in mathematics teachers' collaborative inquiry involving co-teaching. In Hunter, J., Perger, P., & Darragh, L. (Eds.). *Making waves, opening spaces* (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia) pp. 282-289. Auckland: MERGA.
- Fullan, M. (2001). *Leading in a culture of change*. Jossey-Bass.
- Fullan, M. (2016). *The New Meaning of Educational Change* (5th ed.). New York: Teachers College.
- Fullan, M. (2003). *The moral imperative of school leadership*. Thousand Oaks, CA: Corwin Press.
- Grootenboer, P., Edwards-Groves, C., & Rönnerman, K. (2015). The practice of 'middle leading' in mathematics education. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia) pp. 277-284). Sunshine Coast: MERGA.
- Hammerness, K. (2001). Teachers' visions: the role of personal ideals in school reform. *Journal of Educational Change*, 2, 143-163.
- Timperley, H., Ell, F., Le Fevre, D. M., & Twyford, K. (2020). *Leading professional: Practical strategies for impact in schools*. Melbourne, Victoria: Australian Council for Educational Research.
- Vale, C., Roche, A., Cheeseman, J., Gervasoni, A., Livy, S., & Downton, A. (2023). The practices of middle leaders of mathematics: alignment of their goals and activities. *School Leadership & Management*, 43(3), 238-260. <https://doi.org/10.1080/13632434.2023.2179982>

Developing Mathematics Registers in Australian Indigenous Languages: Classification and Comparison

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This paper reports on a multi-site initiative aiming to improve mathematics teaching for Indigenous language-speaking students. The Mathematics in Indigenous Languages Project is developing a framework to identify and use mathematical registers in Australian Indigenous languages. After three years, we provide a cross-site update, highlighting insights from a functional typological approach. Focusing on classification and comparison, we show how this method helps navigate conceptual, semantic, and structural non-equivalence when mapping mathematical expressions across languages.

This paper reports on progress in the Mathematics in Indigenous Languages Project (MiIL), a multi-site, multi-language initiative developing a framework for identifying and using mathematics registers in Australian Indigenous languages to support the mathematical learning of Indigenous language-speaking students. While Indigenous Australian languages have been used in a variety of bilingual school programs for mathematics instruction over the past few decades (Devlin et al., 2017), there has been little investigation and documentation of the structural affordances of Australian languages for formal school mathematics programs. More broadly, little systematic cross-linguistic research has examined how languages vary in their capacity to express mathematical ideas. After three years of implementation, this paper provides a cross-site project update and outlines emerging theoretical implications of applying a functional typological approach developed for cross-linguistic mathematical investigation (Edmonds-Wathen, 2019). We present examples showing how this approach helps address conceptual, semantic, and structural non-equivalence that arises when mathematical expressions are mapped directly between languages, focusing on the language of classification and comparison.

Literature Review

Mathematics education in students' first language can improve the mathematics outcomes of Indigenous language-speaking students because it provides a strong foundation for mathematical thinking and further learning. The use of a language in school also contributes to its vitality, which is significant given the endangered status of most Australian languages (Edmonds-Wathen et al., 2026). However, most Indigenous language-speaking students currently learn mathematics in English, even in some bilingual schools (Devlin et al., 2017).

To teach and learn school mathematics, there needs to be a mathematics register which includes the vocabulary used for mathematics concepts, and the language structures to express mathematics processes and relations (Halliday, 1978). Mathematics is often regarded as difficult to teach in Australian Indigenous languages due to a lack of formal mathematics registers (e.g., ACARA, 2014), thus a key issue being how to develop such registers. Previous international and Australian research that explores theoretical and methodological aspects of language development include Lunney Borden (2011); Meaney et al. (2012) and Wilkinson and Bradbury (2013). Key points of interest relevant to this study include mathematical uses of verbs: in Mi'kmaq (Canada) for describing shapes (Lunney Borden, 2011) and in

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 122–129). MERGA.

Djambarrpuyngu for comparison (Wilkinson & Bradbury, 2013). Considering grammatical diversity is important for developing mathematics registers and pedagogy (Barton, 2009).

Early mathematics learning involves foundational relational concepts such as classification and comparison. Classification involves recognising that objects belong to the same category or type because they share common properties. Sorting and classifying activities are central to many early childhood mathematics programs, and the development of these abilities has been widely studied (Sarama & Clements, 2009). Early classification is often based on salient perceptual features such as shape or colour (Gentner & Namy, 1999). However, most research on early mathematical development has been conducted in Western contexts, and less is known about how linguistic and cultural variation may influence these learning trajectories.

Comparison is another foundational mathematical concept. Research suggests that comparison of visual features such as size generally precedes comparison of quantity (Gentner, 2005). Cross-linguistic studies show considerable variation in how languages express comparison (Beck et al., 2009). In Australian mathematics education, comparison in Indigenous languages has sometimes been framed as a deficit. For example, a resource for teachers of students with English as an Additional Language suggested that “many Aboriginal and Torres Strait Islander languages don’t have words of comparison” (ACARA, 2014, p. 7). However, comparison remains an understudied area in the linguistics of Australian languages, with only a few detailed descriptions available (e.g., Bowler, 2016; Kapitonov, 2019). Investigating how comparison is expressed in these languages is therefore important both for mathematics education and for linguistic research. Systematic research is needed to examine how mathematical concepts such as classification and comparison can be expressed and taught through Indigenous languages.

Theoretical framework

There are two main elements to the theoretical framework that we apply in this project: it is typological and functional. Typology is the field of linguistics that investigates similarity and diversity amongst the world’s languages. Nichols (2007) describes it as a framework-neutral field which enables languages to be contrasted on equal terms, rather than from analytic perspectives that privilege one language over another. While some typological studies involved large numbers of languages, many focus on only a few; Nichols (2007) states that hypothesis-raising is generally done with a small sample of languages, whereas typological hypothesis-testing more likely involves larger numbers of languages and is more data driven. A wide variety of linguistic phenomena are investigated typologically, from phonology (sound systems), morphosyntax (how words are constructed and put together in phrases), and semantics (word meanings).

Language performs communicative functions; functional linguistics take these communicative functions as the basis for analysis (Butler, 2003). Functional linguistics here refers to a core understanding of communicative function: language allows us to make and respond to requests, to develop and maintain relationships and to describe the world and our thoughts, for example. We apply this functional approach to mathematical language and mathematics education, investigating how different languages describe mathematical objects, express mathematical relations and can be used for the teaching and learning of mathematics in school. Because we are looking at languages that have limited history of having been used for school mathematics, this requires a broad definition of mathematics. We delineate the lesson planning for the project by the Australian Curriculum: Mathematics, but we also bring to our investigations a view of mathematics that varies across languages and cultures (Barton, 2009).

A functional typological approach to mathematics register development takes mathematics as the semantic and functional field of interest (Edmonds-Wathen, 2019): we look at how

different languages function to communicate mathematical meanings, and what the similarities and differences are. This approach avoids positioning any language from a deficit perspective.

Project Design and Implementation

The project applies a design-based research approach, where a model based on theory and research is iteratively designed, improved and validated (Collins et al., 2016). It focuses on three main case sites, with each case study taking a participatory action research approach (McTaggart & Kemmis 2005). Activities were co-designed with each community and school to reflect Indigenous research partners' ways of knowing and priorities (Kendall et al., 2011), while remaining aligned with the broader project scope.

Sites and Participants

The project involves three main cases: (1) Areyonga School in Areyonga/Utju community (Pitjantjatjara); (2) Umbakumba School, Groote Eylandt and Milyakburra School, Bickerton Island (Anindilyakwa); and (3) Murrupurtiyanuwu Catholic Primary School at Wurrumiyanga (Tiwi). These cases represent diverse languages and geographic locations, and align with the project team's linguistic expertise and community connections. A project linguist specialising in the relevant language was appointed for each site. Utju is a small community approximately 200 km west of Alice Springs. Areyonga School's long running bilingual (Pitjantjatjara and English) program is closely supported by the Utju Kutju Nintitinganyi cultural leadership team of Anangu elders. Philanthropic funding has assisted Areyonga to continue project activity since its inception. Sasha Wilmoth is the project linguist for this site. On the Groote Eylandt and Bickerton Island Archipelago, we partnered with Groote Eylandt Bickerton Island Primary College Aboriginal Corporation (GEBIPCAC) which is developing a bilingual, bicultural curriculum in Anindilyakwa. MiIL implementation commenced at Umbakumba School and Milyakburra School. Although partners were committed to continuing the project, damage to mining infrastructure from Tropical Cyclone Megan (March 2024) disrupted funding streams, and activities at this site are currently paused. James Bednall is the project linguist for this site. The third site is Murrupurtiyanuwu Catholic Primary School (MCPS) at Wurrumiyanga, Bathurst Island. MCPS was previously bilingual and aims to become formally bilingual again; the project aligns with that goal. Activities were paused during 2024, but are due to resume following new funding for another three years. Kate Charlwood is the project linguist for this site.

Elicitation

Across sites, key activities included language elicitation, development of lesson sequences, assessment design, educator professional learning, and teaching trials. Elicitation was mostly conducted with elders in groups, facilitated by the project lead (Cris Edmonds-Wathen) and the project linguist for that site. Sessions typically focusses on one main topic using concrete resources as stimuli. Elicitation methods included asking for translations from English, asking for judgments ('does it make sense to say this?'), as well as more open-ended story-telling and discussion based on the stimuli. For example, when shown drawings of lines intersecting at different angles, some elders related this to their experience of travelling through Country, and the discussion followed naturally into how they would describe different types of paths and landscapes, and the language for giving directions. Between these elicitation sessions, Cris developed outlines of sequenced lessons on the respective topic in English, sometimes adapting externally available lesson plans, sometimes developing lesson ideas responsive to the context. Local language versions were then developed, checked with elders, and refined with Indigenous educators before classroom trials. Bilingual assessment tools were also developed in English and the relevant local language.

Mathematical Foci

The project focuses on the early primary years (Foundation to Year Two), although in smaller schools it extended into Year Three due to multi-age classes. The mathematical content was aligned with the Australian Curriculum: Mathematics v9.0. While mathematics can be understood more broadly than school curricula (e.g., Trinick et al., 2014; Xu & Ball, 2024), alignment with the curriculum ensured that teaching programs could be integrated into existing school planning. Initial work prioritised the Space strand, which includes the sub-strands of Shape and Location (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2022). Australian languages have precise ways of describing spatial relations that are used widely in everyday speech, and exhibit considerable diversity between different languages (Palmer et al., 2022). These features made spatial concepts a productive starting point for developing mathematical language. Subsequent work also addressed Measurement, particularly comparison concepts relevant to early mathematical learning. These topics allowed the project to develop foundational mathematical language that could later support learning across other areas of the mathematics curriculum.

Findings

We report on selected findings regarding terminology identified in each of the languages, focussing in turn on the language of classification and comparison. Examples come from lessons sorting and classifying 2D geometric shapes and the creation of attribute paths based on the Patterns: Attribute Trains lesson from *reSolve: Maths by Inquiry* (2020).

Classification

Classification involves the ability to recognize that objects belong to the same category because they share similar characteristics, that is, understanding that something is the same type of thing as another thing. Key language includes naming things or objects, naming groups and the process of putting things in groups and describing similarity and difference.

Difference and similarity

Pitjantjatjara uses adjectives *kutjupa* ‘different’ and *lipula* ‘same.’ It is quite straightforward to say: *Walka kutjupa. Puntu lipula* ‘The colour is different. The shape is the same.’ *Lipula* is a loan word from English ‘level’ but now has quite a different range of meanings, including ‘same’. In Anindilyakwa *adidirbura* ‘same’ is commonly used to describe people or things that share one or more of the same characteristics. However, an alternative term of *ediyarra* ‘twin or same birth sibling’ was chosen as a more precise term for mathematical usage in sorting and classifying activities to describe two or more identical objects (1):

- (1) *Ena alida eminingka-kiya. Ena amurndakajika ediyarra(-manja)*
 The colour different-two the shape same-locative
 ‘The colour is different. The shape is the same.’

Anindilyakwa *eminingka* ‘different’ takes a dual marker *kiya* ‘two’ which specifies that two things are different to each other. *Ediyarra* ‘twin/same.birth.sibling’ takes the optional locative case marker *-manja*, reflecting the primary sense of same sibling relationship showing connection through land/Country.

Sameness in Tiwi is indicated using the verb *-wuraparri* ‘be same, be alike, resemble’, as in (2), while togetherness is expressed with the collective pronoun *wutiyati* ‘they.as.one’:

- (2) *Kapinaki yuwurrara kurlamari, wutiyati yuwuraparri*
 these two circle they.as.one be.same/resemble
 ‘These two are circles, they are the same.’

Tiwi has a rich repertoire of different pronouns with subtle semantic differences related to sameness and difference. Seeking to find the word for ‘together; through direct translation would fail to recognise this important mathematical resource. There is no specific Tiwi term that means ‘different’; difference is usually indicated by negating *-wuraparri* ‘be the same, resemble.’ In some contexts where difference is being described, the contrastive determiner *nyoni* ‘other/another’ can be used, as in (3).

- (3) *Wuta sheyip karlu yuwuraparri. Ngarra nyoni arramikamini*
 those shape not be.same/resemble It another thing
 ‘The shape is different (not the same). It’s a different thing.’

Nyoni ‘other/another’ modifies nouns but is not a regular adjective. It occupies the same linguistic slot as quantifiers. Distinguishing between “being other” and “being different” is subtle but important.

Grouping

Anindilyakwa uses complex verbs with many affixes to describe putting things into groups (4):

- (4) *Wurru-murndukwa-yi-ji-na ena amurndakajika.*
 you.all(command)-come.together-each.other-cause-present these things
 ‘You are going to group these things together.’

The root verb *murndukwa* ‘come together’ is modified by the reciprocal suffix *-yi* and the causative *-ji* which with the imperative pronominal prefix *wurru-*, package an instruction that grouping should occur and it should be done by the addressees by bringing things together.

For a similar instruction, Tiwi also uses verbs (5):

- (5) *Nyirrapamukuri sheyip ngini purruwuraparri.*
 put.them.together shape that.is they.are.same
 ‘Put the same shapes together.’

Unlike English which uses a basic clause, Tiwi uses a complex clause to express the instruction. Further work will continue to investigate how complex clauses are used to express relations in Tiwi, something which is under-documented in Australian languages but important for expressing mathematical concepts.

Grouping in Pitjantjatjara can make use of a complex system of noun markers including the semblative *purunypa* (meaning ‘similar to’) and the locative case suffix (*-ngka*) as in (6):

- (6) *Puntu palu purunypa-ngka parara tjura*
 shape it to.be.like-locative dividing put-command
 ‘Divide up and put down the ones with the same shape together’

A more literal translation might be ‘Divide (them) up and put (it) with (the ones which have a) body like it.’ While there is no term for ‘object’ or ‘thing’, Pitjantjatjara can very freely omit nouns where they are clear from context. E.g. in Pitjantjatjara you say something like “Put with the similar shape” to mean “Put each shape with the ones which have a similar shape (to it).” This instruction also makes use for a pervasive feature of Pitjantjatjara grammar known as clause-chaining, where multiple verbs can be linked together to describe complex events. In this example, “divide and put” are more closely linked as a single complex event than the English translation would suggest. While Pitjantjatjara does not have direct equivalents of many words used in the English mathematics classroom (such as sort, measure, compare), the use of clause chains allows educators to give instructions that explicitly describe the component parts of complex activities.

Comparison

Languages vary with respect to the structure of sentences that express comparison as well as the subtle semantics of those sentence types (Beck et al., 2009). Many Australian languages, including the three languages involved in the MiIL project, express comparison using a ‘conjoined construction’, that is, two separate clauses with antonymic adjectives. An example from Anindilyakwa is given in (8).

- (7) *Ena aruma, umba ena eyukwujiya*
 this big, but this small
 ‘This one is big, but this one is small’

Unlike the literal English translation, these do not mean that X is objectively big and Y is objectively small, but rather express a difference in size, that X is bigger than Y. This type of conjoined construction can be used even when this difference is very small, or when both objects are objectively very big or very small. This difference in how the conjoined construction functions form a similar English construction appears not to have been understood in mathematics education previously (e.g. ACARA, 2014). For example, in (8), the Anindilyakwa sentence expresses a superlative height despite there being no explicit superlative marker:

- (8) *Wurring-al-akina wurring-ambilyuma wurring-alingajirra,*
 she.dual-dual-that she.dual-two she.dual-tall
umba d-akina ding-alingajirra.
 but she-that she-tall
 ‘They are both tall, but she is the tallest’

There may also be additional sentence structures used for comparison. While the English preposition *than* is used to mark the standard of comparison, Pitjantjatjara draws on its complex system of grammatical case-marking, and may also recruit spatial language. Example (9) shows the use of the locative case suffix on the standard.

- (9) *Nyangatja pulka pala-ngka*
 this.one big that-location
 ‘This one is bigger than that one’

A similar use of the locative case also appears to be permissible in Anindilyakwa (10), although how widespread this is requires further investigation.

- (10) *Ngayuwa ng-alingajirra d-akina-manja.*
 I I-tall she-that-location
 ‘I am taller than her’

The conceptual differences between languages go deeper than finding equivalents for individual words. A sentence such as (11) may superficially appear similar to the English translation provided below (and not simply because the English words are used for classroom items such as ‘block’ and larger numbers such as ‘eleven’).

- (11) *Ngayuku tjina wara block eleven-tjara*
 my foot long block eleven-having
 ‘My foot is eleven blocks long’

However, a better translation would be ‘my foot is long, it has eleven blocks’. The measure phrase *block eleven-tjara* ‘having eleven blocks’ does not actually modify *wara* ‘long’ in the same way that it does in ‘eleven blocks long’ in English. We can draw on other linguistic evidence to show this difference. In English, if we are measuring a toddler’s foot, we might say ‘her foot is 4 blocks long’ even though her foot is definitely not ‘long’, whereas it would be odd to say ‘her foot is 4 blocks short’. The opposite is true in Pitjantjatjara, where a sentence like (12) is not acceptable (indicated with *) when describing a very small foot, but (13) is.

- (12) **Palumpa tjina warablock kutjara-kutjara-tjara*
his/her foot long block two-two-having
Intended meaning: ‘His/her foot is four blocks long’
- (13) *Palumpa tjina mutumuṯu block kutjara-kutjara-tjara*
his/her foot short block two-two-having
‘His/her foot is four blocks short’

This is not just a difference in individual words, but in the fundamental conceptual structure of the languages, and how adjectives such as ‘long’ and ‘short’ work; in the linguistics literature, languages with similar properties to Pitjantjatjara are called ‘degreeless’, and research over recent years has expanded our knowledge of how different languages conceptualise and express comparison (Beck et al., 2009). This type of difference has a range of other effects in precisely how comparison/measurement is spoken about in the English and Pitjantjatjara mathematics classrooms, at a deeper level than individual words.

Discussion and Conclusion

This paper has presented selected findings from the Mathematics in Indigenous Languages (MiIL) project, focusing on how concepts of classification and comparison are expressed in three Australian Indigenous languages. The examples illustrate that developing mathematics registers in these languages is not simply a matter of translating English terminology. Rather, it requires careful attention to the structural and semantic differences between languages and the ways in which mathematical meanings are conventionally expressed.

Across the three languages examined, classification is expressed through a range of linguistic resources including adjectives, verbs, noun modifiers, and clause chaining. While English mathematics classrooms often rely on specialised verbs such as “sort” or “group,” the languages examined here frequently express these processes through descriptions of actions such as “put together,” “divide,” or “place with similar ones.” These constructions demonstrate that the conceptual work required for classification is linguistically available, although it may be packaged differently from English classroom language. In practice, developing mathematical registers therefore involves identifying how existing linguistic structures can be used or extended to express these classroom activities.

Comparison provides further evidence of cross-linguistic diversity. We give examples from two languages of how comparison can be expressed through constructions that differ from English comparative forms. For example, conjoined clauses with antonymic adjectives (e.g., “this one big, but this one small”) can convey relative comparison rather than absolute properties. Case-marking constructions may be used to compare. Careful attentions to how comparison operates semantically and grammatically in each language is important. Superficially similar expressions may function differently, and direct translation may obscure important conceptual distinctions, as in the example of measure phrases in Pitjantjatjara.

Our findings support the usefulness of a functional typological approach to mathematics register development. The findings demonstrate that Australian Indigenous languages possess rich resources for expressing relational concepts central to early mathematics learning. By examining how languages function to communicate mathematical meanings, rather than assuming direct equivalence with English terminology, the project avoids framing Indigenous languages from a deficit perspective. Instead, typological comparison reveals both common communicative goals and diverse linguistic strategies for expressing mathematical relations. Throughout these investigations, the importance of collaborative and iterative processes in register development remains crucial, as appropriate terms and constructions are negotiated with speakers, educators, and community authorities. Typological diversity should be understood not as an obstacle to mathematics education, but as a resource for developing more

linguistically responsive approaches to teaching mathematics in multilingual contexts, particularly to support the mathematical learning of Indigenous language-speaking students.

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References

- Australian Curriculum Assessment and Reporting Authority (ACARA). (2014). *English as an additional language or dialect teacher resource: Annotated content descriptions Mathematics F to Year 10*.
- Barton, B. (2009). *The language of mathematics: Telling mathematical tales*. Springer.
- Beck, S., Krasikova, S., Fleischer, D., Gergel, R., Hofstetter, S., Savelsberg, C., Vanderelst, J., & Villalta, E. (2009). Crosslinguistic variation in comparison constructions. *Linguistic variation yearbook*, 9(1), 1-66.
- Bowler, M. (2016). The status of degrees in Warlpiri. *Proceedings of the semantics of African, Asian and Austronesian languages 2*, Universität Potsdam.
- Butler, C. S. (2003). Structure and function: a guide to three major structural-functional theories. *Structure and Function*, 1-593.
- Collins, A., Joseph, D., & Bielaczyc, K. (2016). Design research: Theoretical and methodological issues. In *Design-based research* (pp. 15-42). Psychology Press.
- Devlin, B. C., Disbray, S., & Devlin, N. R. F. (Eds.). (2017). *History of bilingual education in the Northern Territory: People, programs and policies*. Springer.
- Edmonds-Wathen, C. (2019). Linguistic methodologies for investigating and representing multiple languages in mathematics education research. *Research in Mathematics Education*, 21(2), 119-134.
- Edmonds-Wathen, C., Alimankinni, M., Bara, M., Bednall, J., Bennett, C., Carrol, S., Charlwood, K., Goldsworthy, L., Lalara, R., Nipper, L., Nyhuis, P., Peters, S., Puantulura, R. L., Wilmoth, S., & Yantarrnga, H. (2026). Strengthening language through first language mathematics education. *Australian Review of Applied Linguistics*, 49(2). <https://doi.org/10.1075/aral.25034.edm>
- Gentner, D. (2005). The development of relational category knowledge. In L. Gershkoff-Stowe & D. H. Rakison (Eds.), *Building object categories in developmental time* (pp. 263-294). Psychology Press.
- Gentner, D., & Namy, L. L. (1999). Comparison in the development of categories. *Cognitive Development*, 14(4), 487-513.
- Halliday, M. A. K. (1978). Sociolinguistic aspects of mathematical education. In *Language as social semiotic: The social interpretation of language and meaning* (pp. 194-204). Edward Arnold.
- Kapitonov, I. (2019). Degrees and scales of Kunbarlang. In M. R. Bochnak, M. Butt, E. Meertens, & M.-M. Zymła (Eds.), (pp. 91-105). Universität Tübingen. <https://doi.org/10.15496/publikation-36875>
- Kendall, E., Sunderland, N., Barnett, L., Nalder, G., & Matthews, C. (2011). Beyond the rhetoric of participatory research in Indigenous communities: Advances in Australia over the last decade. *Qualitative Health Research*, 21(12), 1719-1728. <https://doi.org/10.1177/1049732311418124>
- Lunney Borden, L. (2011). The ‘verbification’ of mathematics: using the grammatical structures of Mi’kmaq to support student learning. *For the learning of mathematics*, 31(3), 8-13.
- McTaggart, R., & Kemmis, S. (2005). Participatory Action Research. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (3rd ed. ed., pp. 559-603). Sage Publications.
- Meaney, T., Trinick, T., & Fairhall, U. (2012). *Collaborating to meet language challenges in indigenous mathematics classrooms*. Springer.
- Nichols, J. (2007). What, if anything, is typology? *Linguistic Typology*, 11(1), 231-238.
- Palmer, B., Hoffmann, D., Blythe, J., Gaby, A., Pascoe, B., & Ponsonnet, M. (2022). Frames of spatial reference in five Australian languages. *Spatial Cognition & Computation*, 22(3-4), 225-263.
- reSolve: Maths by Inquiry. (2020). *Patterns: Attribute trains*.
- Sarama, J. A., & Clements, D. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.
- Trinick, T., Meaney, T., & Fairhall, U. (2014). Teachers learning the registers of mathematics and mathematics education in another language: an exploratory study. *ZDM*, 46(6), 953-965.
- Wilkinson, M., & Bradbury, J. (2013). Number and two languages in the early years: Report on a project with paraprofessional Indigenous teachers in two NT northeast Arnhem Yolŋu schools. *Australian Review of Applied Linguistics*, 36(3), 335-354.
- Xu, H., & Ball, R. (2024). Indigenous mathematics: From mainstream misconceptions to educational enrichment. *Canadian Journal of Science, Mathematics and Technology Education*, 24(2), 160-175.

Integration of Education for Sustainable Development in Mathematics Education: A Scoping Review

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This review explores research on integrating Education for Sustainable Development (ESD) into mathematics education. Significant findings indicate that mathematics education can enhance ESD competencies. However, challenges persist, including limited teacher preparation, curriculum constraints, and tensions between mathematical rigour and sustainability contexts. The review emphasises four key instructional methods: problem-based learning on sustainability issues, mathematical modelling for real-world sustainability problems, STEM education, and Place-Based Education (PBE).

Introduction

The integration of Education for Sustainable Development (ESD) into formal education systems has seen significant growth after the United Nations Decade of Education for Sustainable Development (2005-2014) and the later adoption of the Sustainable Development Goals (SDGs) in 2015. ESD aims to equip students with the knowledge, skills, values, and attitudes necessary to address complex sustainability challenges, including climate change, resource depletion, social inequality, and environmental degradation (Vorhölter et al., 2025). ESD has been widely implemented in science and social studies; however, its integration into mathematics education remains relatively underexplored (Li et al., 2021; Su et al., 2023).

Mathematics education has a unique position in the ESD landscape. On one hand, mathematical literacy is essential for understanding and addressing sustainability challenges, from analysing climate data to modelling resource consumption patterns (Tesfamichael et al., 2024). On the other hand, the abstract nature of mathematics and its traditional emphasis on decontextualised problem-solving have created barriers to meaningful ESD integration (Alsina et al., 2023). Recent scholarship has begun to address this gap, exploring how mathematics education can contribute to sustainability goals while maintaining mathematical rigour and coherence (Semiz et al., 2020; Hopkinson et al., 2010).

The rationale for integrating ESD in mathematics education rests on several foundations. First, mathematics provides powerful tools for analysing, modelling, and solving real-world sustainability problems (Vásquez et al., 2023a). Second, contextualised mathematics instruction grounded in sustainability themes can enhance student engagement and demonstrate the relevance of mathematical concepts (Wiegand & Borromeo Ferri, 2023). Third, mathematics education can contribute to the development of critical sustainability competencies, including systems thinking, critical analysis, and collaborative problem-solving (Vásquez et al., 2022).

This scoping review synthesises current research on ESD integration in mathematics education, examining pedagogical approaches, curriculum strategies, implementation challenges, and learning outcomes. The review addresses the following research questions: (1) What pedagogical approaches are employed to integrate ESD in mathematics education? (2) What challenges and barriers affect implementation? (3) What learning outcomes and effectiveness evidence exist for ESD-mathematics integration?

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 130–137). MERGA.

Methodology

This review drew from a comprehensive search of scholarly articles on ESD integration in mathematics education conducted in SciSpace and Google Scholar databases. The initial search resulted in 220 papers (200 from SciSpace and 20 from Google Scholar). After removing duplicates and applying inclusion criteria, 77 peer-reviewed journal articles were identified. Papers were filtered to include only journal articles from reputable international publications and sorted by relevance to the research questions.

The review focuses on the top 21 most relevant papers, which were analysed to identify key themes, pedagogical approaches, implementation challenges, and reported outcomes. Data extraction focused on study context, theoretical frameworks, pedagogical methods, curriculum integration strategies, reported challenges, and learning outcomes. Thematic analysis was employed to synthesise findings across studies and identify patterns, convergences, and gaps in the literature.

Theoretical Foundations and Frameworks

The integration of ESD into mathematics education is grounded in several theoretical frameworks. The UNESCO framework for ESD competencies provides the foundation, emphasising systems thinking, anticipatory thinking, normative competence, strategic competence, collaboration, critical thinking, self-awareness, and integrated problem-solving (Vorhölter et al., 2025). These competencies align with mathematical practices, particularly in modelling, problem-solving, and reasoning.

Mathematical modelling serves as a central theoretical bridge between mathematics and sustainability. Modelling involves formulating, solving, validating, and communicating mathematical representations of real-world phenomena, making it particularly suitable for addressing complex sustainability issues (Wiegand & Borromeo Ferri, 2023; Bulut & Borromeo Ferri, 2025). The modelling cycle, which transitions between real-world contexts and mathematical representations, incorporates systems thinking and critical analysis, the core of ESD.

Place-Based Education (PBE) theory provides another important framework, emphasising connections between mathematical learning and local environmental and social contexts (Teise, 2025). This approach grounds abstract mathematical concepts in students' lived experiences and local sustainability challenges, enhancing relevance and engagement while developing place-based knowledge and action competence.

Science, Technology, Engineering, Arts, and Mathematics (STEAM) integration frameworks offer interdisciplinary approaches to ESD-mathematics integration (Alsina et al., 2023). These frameworks emphasise connections across disciplines, authentic problem-solving, and creative thinking, aligning with the holistic, interdisciplinary nature of sustainability challenges.

Key Themes and Findings

Pedagogical Approaches and Practical Implications

Efforts to integrate ESD into mathematics education have led researchers to explore a range of pedagogical approaches. Rather than identifying a single preferred method, the literature suggests that sustainability can be meaningfully incorporated into mathematics classrooms through various forms of contextualised, inquiry-oriented learning. Across these approaches, mathematics is positioned not only as a set of abstract procedures but also as a tool for understanding and responding to complex environmental and social issues. Importantly, the studies reviewed indicate that sustainability contexts can enrich mathematical learning when

they are authentically connected to real-world problems rather than added superficially to routine exercises.

One of the most frequently discussed approaches is the use of sustainability-related contexts in mathematical problem-solving and inquiry activities. In these learning environments, students engage with issues such as climate change, resource consumption, waste production, or social inequality while applying mathematical concepts and reasoning. Research suggests that these contexts can make mathematical tasks feel more meaningful and relevant to students while still maintaining mathematical rigour (Domínguez-González & Delgado-Martín, 2022; Li et al., 2021). However, scholars caution that simply inserting environmental terminology into traditional word problems is insufficient for meaningful ESD integration. Instead, tasks need to involve genuine sustainability challenges that encourage critical thinking, interpretation, and discussion. For teachers, this finding suggests that effective ESD integration does not necessarily require a complete curriculum redesign, but rather thoughtful adaptation of existing mathematical topics using authentic sustainability contexts.

Mathematical modelling has received particular attention in the literature as a powerful means of connecting mathematics and sustainability. Modelling activities typically require students to represent real-world phenomena mathematically, test assumptions, interpret results, and evaluate possible solutions. Sustainability-related issues, such as population growth, energy consumption, transportation systems, and carbon emissions, provide particularly rich contexts for this work. Studies indicate that modelling activities can support not only mathematical understanding but also competencies commonly associated with ESD, including systems thinking, critical analysis, and decision making (Wiegand & Borromeo Ferri, 2023; Bulut & Borromeo Ferri, 2025; Vorhölter et al., 2025). One example presented by Wiegand and Borromeo Ferri (2023) asked students to analyse the energy consumption of aluminium production in Brazil and Germany and to evaluate how relocating production might contribute to sustainable development. Such tasks demonstrate how mathematical modelling can encourage students to consider environmental, economic, and social dimensions simultaneously. The literature also suggests that teachers can begin with accessible and locally relevant datasets, such as community waste statistics, water usage data, or energy consumption records, to make modelling activities more manageable and meaningful for students.

Another approach highlighted in the literature involves linking mathematics lessons explicitly to the SDGs. Researchers describe SDG-linked problem design as an accessible entry point for teachers new to ESD integration, as it allows sustainability themes to be embedded within existing curriculum topics (Domínguez-González & Delgado-Martín, 2022; Lestari et al., 2024). For example, percentage calculations can be connected to discussions about gender pay gaps (SDG 5), while exponential functions can be used to model population growth, plastic accumulation, or resource depletion (SDG 12). Studies suggest that this approach can increase student motivation because mathematical concepts are connected to issues students recognise from everyday life and global discussions. At the same time, the literature emphasises the importance of maintaining mathematical depth so that sustainability contexts support rather than overshadow conceptual understanding.

Interdisciplinary learning environments provide another pathway for integrating sustainability into mathematics education. Because sustainability challenges are inherently complex and cross-disciplinary, projects combining mathematics with science, technology, or engineering appear frequently in the literature. Examples include analysing environmental data, designing technologies for monitoring ecosystems, or investigating renewable energy systems. Research suggests that these interdisciplinary experiences can strengthen students' awareness of sustainability issues while also deepening conceptual understanding of mathematics (Gerretson et al., 2008; Hopkinson & James, 2010). However, scholars also note that interdisciplinary work requires careful planning; otherwise, mathematics risks being reduced to

a supporting tool rather than remaining a subject of inquiry in its own right. This highlights the importance of designing tasks where mathematical reasoning remains central to the investigation.

A somewhat different perspective emerges from research on PBE. Here, the emphasis shifts from global sustainability concerns to the local communities and environments in which students live. Mathematical investigations may involve analysing local biodiversity data, measuring patterns of waste production, examining water usage, or designing sustainable community spaces. Studies suggest that these locally grounded approaches can increase student engagement by helping students see how mathematics connects to their everyday experiences and communities (Teise, 2025; Kadji-Beltran et al., 2017). PBE also appears to encourage civic awareness and participation by positioning students as active contributors to understanding local sustainability challenges.

Beyond classroom practice, the literature also highlights several institutional and systemic factors that influence the successful integration of ESD into mathematics education. Teacher professional development emerges as one of the most important conditions for effective implementation, particularly programs that combine sustainability knowledge, collaborative lesson design, classroom experimentation, and opportunities for reflection on balancing mathematical rigour with sustainability contexts (Su et al., 2023; Alsina & Silva-Hormazábal, 2023; Olawale et al., 2025). Similarly, pre-service teacher education programs are encouraged to embed ESD principles directly within mathematics methods courses rather than treating sustainability as a separate topic (Bulut & Borromeo Ferri, 2025). The literature also points to the need for high-quality curriculum resources, as existing textbooks often include sustainability only superficially (Kim and Pang 2022). At the systemic level, curriculum frameworks such as Norway's LK20 curriculum demonstrate how sustainability can be embedded as a cross-curricular theme that explicitly includes mathematics (Tesfamicael & Enge, 2024), while initiatives such as the CoDesignS ESD Framework provide models for integration within university mathematics teacher education programs (Ahmad et al., 2023).

Taken together, the reviewed studies illustrate that there is no single pathway for integrating ESD into mathematics education. Instead, the literature reflects an ongoing effort to identify pedagogical practices that allow students to develop mathematical understanding while critically engaging with authentic sustainability issues. At the same time, the findings suggest that successful implementation depends not only on classroom strategies but also on broader institutional support, teacher preparation, curriculum resources, and policy alignment.

Curriculum Integration Strategies

Researchers have proposed several ways of incorporating ESD into mathematics curricula. These approaches vary widely, from relatively small adjustments to existing lessons to more comprehensive curriculum changes. However, studies of mathematics textbooks suggest that sustainability is still only marginally represented in many curricular materials. Analyses across countries show that references to sustainability are infrequent and unevenly distributed (Kim and Pang 2022).

Even when sustainability contexts are included, they are often used simply as the setting for routine mathematical problems rather than as opportunities for students to engage more deeply with sustainability issues.

Some education systems have attempted to address this gap through curriculum reform. For example, Norway's recent curriculum explicitly identifies sustainability as a cross-cutting theme that should be integrated across subjects, including mathematics (Tesfamicael et al., 2024). Such policy frameworks can provide important structural support for integrating ESD. Nevertheless, the extent to which sustainability is actually incorporated into classroom practice depends on several factors, including teachers' familiarity with ESD, the availability of

appropriate teaching resources, and the alignment between curriculum goals and assessment practices.

In the literature, several practical strategies for curriculum integration have been discussed. One common approach is to introduce sustainability-related examples into existing mathematical topics so that students can see how familiar concepts apply to real-world issues. Other initiatives develop dedicated instructional units that focus more explicitly on the use of mathematics to analyse sustainability challenges. Project-based learning has also been explored, allowing students to investigate sustainability problems over extended periods. At the most ambitious level, some scholars advocate for broader curriculum redesign in which sustainability becomes a central theme throughout the mathematics program (Ahmad et al., 2023). Each of these approaches requires different levels of effort and support, and they vary in how deeply sustainability becomes embedded in students' mathematical learning.

Implementation Challenges

Despite growing interest in integrating ESD and mathematics, significant challenges impede widespread implementation. Teacher preparation emerges as the most frequently cited barrier. Many mathematics teachers lack training in ESD principles, pedagogical approaches for integration, and confidence in addressing sustainability topics (Su et al., 2023; Olawale et al., 2025). Teachers report uncertainty about how to maintain mathematical rigour while incorporating sustainability contexts and concerns about their own knowledge of sustainability issues.

Curriculum constraints present another major challenge. Crowded curricula, standardised testing pressures, and traditional mathematical scope and sequence create barriers to incorporating sustainability contexts and extended modelling activities (Semiz et al., 2020). Teachers report tension between covering required mathematical content and dedicating time to sustainability-focused activities, particularly when assessments do not value ESD competencies.

Conceptual tensions between mathematics and sustainability create additional challenges. Some educators question whether sustainability contexts compromise mathematical abstraction and generalisation, while others worry that superficial integration may trivialise either mathematics or sustainability (Meyer, 2026). Balancing mathematical learning goals with ESD objectives requires careful pedagogical design and ongoing reflection.

Resource limitations, including a lack of appropriate teaching materials, assessment tools, and professional development opportunities, further constrain implementation (Vásquez et al., 2023b). Many teachers lack access to high-quality, curriculum-aligned resources that authentically integrate mathematics and sustainability.

Learning Outcomes and Effectiveness

Research on learning outcomes from ESD-mathematics integration has yielded promising yet mixed results. Studies report that well-designed integration can enhance both mathematical understanding and sustainability competencies without compromising either domain (Wiegand & Borromeo Ferri, 2023). Students engaged in sustainability-focused mathematical modelling demonstrate improved systems thinking, critical analysis, and the ability to work with complex, ill-defined problems.

Affective outcomes include increased student engagement, enhanced perceptions of the relevance of mathematics, and greater awareness of sustainability issues (Domínguez-González & Delgado-Martín, 2022; Li et al., 2021). Students report finding sustainability contexts more meaningful than traditional textbook problems and appreciate opportunities to apply mathematics to real-world issues they care about.

However, outcomes vary significantly based on implementation quality, teacher expertise, and pedagogical approach. Superficial integration that merely changes problem contexts without substantive engagement with sustainability issues has a limited impact on both mathematical and sustainability learning (Kim and Pang 2022). Effective integration requires sustained engagement, authentic tasks, and explicit attention to both mathematical and ESD learning goals.

Assessment of ESD competencies in mathematics contexts remains underdeveloped. Few studies employ validated instruments to measure sustainability competencies, and most rely on self-report measures or qualitative analyses of student work (Holst et al., 2024). The development of robust assessment frameworks that capture both mathematical and ESD learning is an important research need.

Discussion

The synthesis of the literature reveals that mathematics education has significant potential to contribute to ESD goals, but realising this potential requires addressing substantial pedagogical, curricular, and systemic challenges. The field has moved beyond questioning whether mathematics should engage with sustainability toward exploring how to do so effectively while maintaining mathematical integrity and achieving meaningful sustainability learning.

Several key insights emerge from the review. First, effective ESD-mathematics integration requires more than simply changing problem contexts; it demands pedagogical approaches that engage students in authentic mathematical practices applied to genuine sustainability challenges. Mathematical modelling emerges as particularly promising because it naturally incorporates systems thinking, critical analysis, and decision-making, which are central to both mathematics and ESD.

Second, teacher education represents a critical leverage point for advancing ESD-mathematics integration. Pre-service and in-service teacher education must address both content knowledge (understanding of sustainability issues and ESD principles) and pedagogical content knowledge (how to design and implement effective integration) (Su et al., 2023; Alsina et al., 2023). Teachers need opportunities to experience ESD-mathematics integration as learners, develop appropriate pedagogical approaches, and reflect on tensions and challenges.

Third, curriculum and assessment systems must evolve to support and value ESD-mathematics integration. When standardised assessments focus exclusively on decontextualised mathematical procedures, teachers face disincentives for investing time in sustainability-focused activities. Curriculum frameworks that explicitly mandate and support ESD integration, coupled with assessment systems that value sustainability competencies, create enabling conditions for implementation (Tesfamicael et al., 2024).

Fourth, the field needs more rigorous research on learning outcomes and effectiveness. While existing studies provide promising evidence, many rely on small samples, short interventions, and limited outcome measures. Longitudinal studies examining sustained impacts on both mathematical achievement and sustainability competencies, using validated assessment instruments, strengthen the evidence base.

Finally, attention to equity and justice dimensions of ESD-mathematics integration remains limited in the literature. Sustainability challenges disproportionately affect marginalised communities, and mathematics education can either reproduce or challenge these inequities. Future work should explicitly address how ESD-mathematics integration can promote social justice and ensure that all students develop the mathematical and sustainability competencies needed for informed citizenship.

Conclusion and Future Directions

This comprehensive literature review demonstrates that integrating ESD in mathematics education represents both a significant opportunity and a complex challenge. Mathematics provides powerful tools for understanding and addressing sustainability issues, and appropriately designed instruction can develop both mathematical competencies and critical sustainability competencies. However, effective integration requires substantial shifts in pedagogy, curriculum, teacher education, and assessment systems.

Future research should prioritise several directions. First, developing and validating assessment frameworks that capture both mathematical and ESD learning outcomes would strengthen the field's ability to evaluate effectiveness. Second, longitudinal studies examining the sustained impacts of ESD-mathematics integration on student learning, attitudes, and behaviours would provide crucial evidence. Third, research on teacher learning and professional development for ESD-mathematics integration would inform efforts to build teacher capacity. Fourth, an investigation of how ESD-mathematics integration can promote equity and social justice would address critical gaps in current literature.

For practice, the review suggests several recommendations. Teacher education programs should incorporate ESD principles and pedagogical approaches for integration into mathematics methods courses. Curriculum developers should create high-quality resources that authentically integrate mathematics and sustainability while maintaining mathematical rigour. School systems should provide professional development, collaborative planning time, and supportive assessment policies that enable teachers to implement ESD-mathematics integration effectively. Policymakers should ensure that curriculum frameworks and standards explicitly support and value ESD integration across all subjects, including mathematics.

As sustainability challenges are unprecedented, education systems must evolve to prepare students for informed, responsible citizenship. Mathematics education, with its emphasis on logical reasoning, quantitative analysis, and problem-solving, has essential contributions to make. By thoughtfully integrating ESD principles and practices, mathematics education can help develop the next generation of citizens equipped with both mathematical competencies and sustainability consciousness needed to build a more just and sustainable world.

References

- Ahmad, N., Zainuddin, N., Wahab, N. A., & Rahim, M. B. (2023). CoDesignS Education for Sustainable Development: A framework for embedding education for sustainable development in curriculum design. *Sustainability*, 15(23), 16460. <https://doi.org/10.3390/su152316460>
- Alsina, Á., Batllori, R., Falgàs, M., & Vidal, I. (2023). Promoting mathematics teacher education for sustainability through a STEAM approach. *Avances de Investigación en Educación Matemática*, 23, 73-95. <https://doi.org/10.35763/aiem23.5402>
- Bulut, N., & Borromeo Ferri, R. (2025). Bridging mathematical modelling and education for sustainable development in pre-service primary teacher education. *Education Sciences*, 15(2), 248. <https://doi.org/10.3390/educsci15020248>
- Domínguez-González, R., & Delgado-Martín, L. (2022). Arousing early strategic thinking about SDGs with real mathematics problems. *Mathematics*, 10(9), 1446. <https://doi.org/10.3390/math10091446>
- Gerretson, H., Howes, E., Campbell, S., & Thompson, D. (2008). Interdisciplinary mathematics and science education through robotics technology: its potential for education for sustainable development (a case study from the USA). *Journal of Teacher Education for Sustainability*, 10(1), 32. <https://doi.org/10.2478/V10099-009-0023-4>
- Holst, J., Singer-Brodowski, M., Brock, A., & De Haan, G. (2024). Monitoring SDG 4.7: Assessing Education for Sustainable Development in policies, curricula, training of educators, and student assessment (input-indicator). *Sustainable Development*, 32(4), 3908–3923.
- Hopkinson, P., & James, P. (2010). Practical pedagogy for embedding ESD in science, technology, engineering and mathematics curricula. *International Journal of Sustainability in Higher Education*, 11(4), 365–379. <https://doi.org/10.1108/14676371011077586>

- Kadji-Beltran, C., Christodoulou, N., Zachariou, A., Lindemann-Matthies, P., Barker, S., & Kadis, C. (2017). An ESD pathway to quality education in the Cyprus primary education context. *Environmental Education Research*, 23(7), 1015–1031.
- Kim, J., & Pang, J. (2022). *An Analysis of Sustainable Activities in Japanese, Korean, and Singaporean Elementary Mathematics Textbooks*. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(2), em2080. <https://doi.org/10.29333/ejmste/11651>
- Li, M., Lam, C. C., & Chik, P. M. (2021). Education for sustainable development in mathematics education: What could it look like? *International Journal of Mathematical Education in Science and Technology*, 52(7), 1109–1119. <https://doi.org/10.1080/0020739X.2021.1941361>
- Meyer, M. (2026). Sustainable Development in Mathematics versus Mathematics in Sustainable Development: Elaboration of Challenges and Tensions in Curriculum Design and Implementation. *Research in Integrated STEM Education*, 1(aop), 1–23.
- Olawale, B. E., Hendricks, W., & Saddiq, K. (2025). Education for sustainable development and its implications for the preparation of pre-service mathematics teachers. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 88–104). ERRCD Forum.
- Semiz, K., Tican, C., & Aksu, Z. (2020). Middle school pre-service mathematics teachers’ opinions related to mathematics education for sustainability. *Eurasian Journal of Educational Research*, 20(89), 175–200. <https://doi.org/10.14689/EJER.2020.89.6>
- Su, Y., Liang, S., & Xie, N. (2023). Sustainable development education for training and service teachers teaching mathematics: A systematic review. *Sustainability*, 15(10), 8435. <https://doi.org/10.3390/su15108435>
- Teise, K. L. (2025). Place-based mathematics education: An education for sustainable development pedagogy to enhance mathematics teacher training. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 72–87). ERRCD Forum.
- Tesfamicael, S. A., & Enge, O. (2024). Revitalising Sustainability in Mathematics Education: The Case of the New Norwegian Curriculum. *Education Sciences*, 14(2), 174. <https://doi.org/10.3390/educsci14020174>
- Vásquez, C., Alsina, Á., Pincheira, N., Gea, M. M., & Chandía, E. (2022). What challenges does the 21st century impose on the knowledge of primary school teachers who teach mathematics? An analysis from a Latin American perspective. *Mathematics*, 10(3), 391. <https://doi.org/10.3390/math10030391>
- Vásquez, C., García-Alonso, I., Seckel, M. J., & Alsina, Á. (2023a). Integrating sustainability in mathematics education and statistics education: A systematic review. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(12), em2373. <https://doi.org/10.29333/ejmste/13809>
- Vásquez, C., Ruz, F., Martínez, M. V., & Alsina, Á. (2023b). Connecting statistics education and sustainability: Beliefs of primary education teachers. *Avances de Investigación en Educación Matemática*, 23, 97–118. <https://doi.org/10.35763/aiem23.5415>
- Vorhölter, K., Krüger, A., & Wendt, L. (2025). Education for sustainable development in mathematics classrooms. *Environmental Education Research*. <https://doi.org/10.1080/13504622.2025.2565461>
- Wiegand, S., & Borromeo Ferri, R. (2023). Promoting pre-service teachers’ professionalism in STEAM education and education for sustainable development through mathematical modelling activities. *ZDM–Mathematics Education*, 55(7), 1269–1282. <https://doi.org/10.1007/s11858-023-01500-8>

Developing and Piloting a Responsive Mathematics Teaching Observation Scale (RMTOS)

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This study reports the development and piloting of the Responsive Mathematics Teaching Observation Scale (RMTOS), a structured classroom observation instrument designed to capture teachers' responsiveness to students' mathematical thinking. We trialled the instrument across 24 mathematics lessons observed twice weekly over six weeks across two Year 6 classrooms in a Catholic primary school. The findings provide preliminary evidence that responsive mathematics teaching can be operationalised through structured observation, and the refined instrument offers a promising tool for future research on mathematics instruction and teacher professional development.

Introduction

Responsive teaching in mathematics has become an important focus in mathematics education because it foregrounds teachers' attention to students' ideas and their capacity to adapt instruction in response to those ideas. Rather than treating teaching as the delivery of predetermined procedures, responsive teaching emphasises noticing the substance of students' mathematical thinking, interpreting the significance of that thinking, and using it to shape subsequent instructional moves (Jacobs et al., 2010; Sherin et al., 2011; Robertson et al., 2016). In mathematics classrooms, such responsiveness is especially important because students' explanations, strategies, partial understandings, and misconceptions provide critical resources for advancing conceptual understanding and collective sense-making (Kazemi & Stipek, 2001; Lampert, 2001). The aim of the study, along with the guiding research questions, is presented following the literature review.

Literature Review

Responsive teaching is defined here as teachers' capacity to notice, interpret, and build on students' mathematical thinking during instruction, drawing on research on professional noticing in which teachers recognise students' strategies, interpret their mathematical meanings, and decide how to respond instructionally (Jacobs et al., 2010; Robertson et al., 2016; Sherin et al., 2011). This conception aligns with scholarship on ambitious and discussion-based mathematics teaching, where classroom discourse is organised not simply to elicit answers, but to make reasoning public, compare strategies, and press for explanation and justification (Chapin et al., 2009; Lampert, 2001; Kazemi & Stipek, 2001). In this sense, managing cognitive load is also important, as it helps teachers reduce unnecessary task demands so that students can focus more fully on understanding, reasoning, and explaining their mathematical thinking (Sweller, 2024; van Gog et al., 2011). Across this body of work, responsive teaching is understood as an interactional practice in which teaching decisions are shaped by students' developing ideas rather than by a fixed script (Robertson et al., 2016). It is important to note that responsive mathematics teaching, as theorised in this tradition, is conceptually distinct from culturally responsive pedagogy (Gay, 2010) and culturally relevant pedagogy (Ladson-Billings, 1995). Whereas culturally responsive pedagogy concerns the use of students' cultural knowledge and experiences as resources for learning, responsive mathematics teaching concerns the use of students' content-specific mathematical thinking as the basis for instructional decision-making. Both are important dimensions of effective teaching; they are

not, however, the same construct. This distinction is elaborated in Ersozlu (under review) and is foundational to the measurement purposes of the present instrument.

Despite substantial theoretical and empirical interest in responsiveness, measuring responsive teaching in classroom settings remains challenging. Widely used observation frameworks have contributed important ways of studying instructional quality, including the Mathematical Quality of Instruction framework and the Teaching for Robust Understanding framework, both of which foreground the intellectual quality of classroom mathematics and students' opportunities to engage meaningfully with mathematical ideas (Hill et al., 2008; Schoenfeld, 2013). In the Australian context, the Quality Teaching framework (NSW Department of Education and Training, 2003; Gore & Bowe, 2017) represents an established approach to observation-based teaching quality research, though it was similarly not designed to isolate responsiveness to students' mathematical thinking specifically. However, these frameworks were not designed specifically to isolate how teachers elicit, interpret, and build on students' mathematical thinking moment by moment. Their primary concern is with the mathematical richness of instructional tasks and explanations rather than with the interactional quality of teacher responses to student ideas as they emerge during a lesson (Hill et al., 2008; Schoenfeld, 2013). Therefore, a teacher could score reasonably well on general instructional quality measures while rarely probing student reasoning, rarely building on student contributions, and rarely adjusting instruction in response to what students actually say or do. A scale designed specifically to capture responsiveness is therefore needed to make these distinctions visible. Furthermore, achieving reliable scores from established observation instruments requires well-developed scoring designs, extensive rater training, and often video-based coding procedures, creating resource demands that make such instruments difficult to deploy in school-based research or teacher professional learning contexts (Hill et al., 2012; Worthington & Whittaker, 2006). There is a practical need for instruments that experienced teachers can use in real time during live classroom observation, with sufficient structure to support reliable ratings and sufficient accessibility to be usable outside specialist research settings. As a result, there remains a need for observation instruments that more directly operationalise responsive mathematics teaching as an observable set of classroom practices, and that are designed with practical feasibility as well as conceptual precision in mind which requires careful pilot work. Responsive teaching, as conceptualised here, is applicable across classroom contexts in which teachers engage with student reasoning during whole-class instruction, small-group work, and individual conferencing. The present study focused specifically on Year 6 mixed-ability primary classrooms, contexts in which teachers must navigate diverse student thinking across multiple instructional phases.

The aim of this study was to develop and pilot a classroom observation instrument for capturing responsive mathematics teaching practices. The quality of instructional responsiveness has implications beyond classroom discourse; classroom conditions characterised by limited mathematical engagement and evaluative feedback have been identified in the literature as antecedents of mathematics anxiety in primary students (Ersozlu & Karakus, 2019; Ersozlu, 2024). The research questions examined: (1) how responsive mathematics teaching can be operationalised in a classroom observation scale, (2) what underlying dimensions emerge from pilot observation data, and (3) how those data can inform refinement of the scale for future research and classroom analysis.

Methodology

Research Design

This study employed a pilot instrument development design in which a responsive teaching observation scale was trialled in authentic classroom settings and subsequently refined based on observation evidence and statistical analysis. The study combined quantitative analysis of

observation ratings with qualitative insights from observer field notes and moderation discussions.

Context, Participants and Procedure

The participants were two Year 6 mathematics classrooms teachers within the same Catholic primary school in term three. The two classrooms comprised 21 and 20 students respectively and were mixed-ability classes, representing a typical cross-section of Stage 3 learners. Each classroom had a regular classroom teacher, with one classroom also observed under casual teachers on five of the twelve observation sessions. We conducted observations twice per week over six weeks, yielding a total of 24 classroom observations across the two classrooms (12 observations per classroom). The first author also conducted a subset of classroom observations and facilitated the weekly moderation meetings with the two observers. Each observation session ran for approximately one hour, generally from 11:20 am to 12:30 pm, after the students' first morning break. Observed lessons spanned several mathematical topics across the six weeks, including area of two-dimensional and composite shapes, decimal operations, unit conversions, fractions, and percentage calculations. Lessons generally followed a four-phase structure: an orientation phase in which the teacher introduced the learning intention and activated prior knowledge; an explicit teaching phase involving modelling and demonstration; an exploration phase in which students worked independently or in small groups; and a consolidation phase in which answers were reviewed. Observer field notes were recorded for each lesson in a structured format addressing the orientation, explicit teaching, exploration, and consolidation phases of lessons, as well as observations of responsive pedagogy practices including prioritising student ideas, probing student thinking, and providing feedback. We had the opportunity to review and compare observation notes, discuss scoring decisions and interpretation of scale items, and identify issues requiring instrument refinement in these meetings.

Observation Instrument

A pilot Responsive Mathematics Teaching Observation Scale was developed to capture observable features of responsive pedagogy identified in prior research. Item development was informed by scholarship on professional noticing, responsive teaching, mathematically productive classroom discourse, and instructional quality in mathematics classrooms (Jacobs et al., 2010; Sherin et al., 2011; Hill et al., 2008; Schoenfeld, 2013). The initial 13-item version of the instrument represented practices associated with eliciting student thinking, interpreting mathematical ideas, building on student contributions, and supporting classroom participation. Each item was initially rated on a five-point scale (1–5) reflecting the degree to which the practice was evident during the lesson. However, during the pilot, observers consistently raised the neutral midpoint (3) as problematic: in practice, it was difficult to determine whether a rating of 3 indicated that the practice was partially observed or that the observer was genuinely uncertain. Following moderation discussions, the scale was revised to a four-point format (1 = No evidence, 2 = Limited evidence, 3 = Moderate evidence, 4 = Strong evidence) removing the neutral option to require a directional judgement for each item. Our revisions are consistent with recommendations in the scale-development literature that neutral midpoints can reduce score variance and obscure meaningful differences in observed practice (DeVellis & Thorpe, 2021; Worthington & Whittaker, 2006). One additional item (RP5) was included as a global anchor rating, inviting observers to make a holistic overall judgement of the lesson's responsiveness. This item was treated separately from the 12-dimensional items and was not included in the dimensional total score, serving instead as a criterion for convergent validity checking.

Data Analysis

Data analysis proceeded in three stages. First, internal consistency reliability was examined using Cronbach's alpha as an initial indicator of the coherence of the item set (Cronbach, 1951). Second, an exploratory factor analysis using Principal Component Analysis with Varimax rotation was conducted to examine the underlying structure of the scale and explore whether items clustered around interpretable dimensions of responsive teaching. EFA was selected because no prior factor structure existed for this instrument, making it the appropriate approach for emerging scale development (Costello & Osborne, 2005). Given the pilot nature of the study and the limited sample size, these analyses were treated as exploratory and heuristic rather than confirmatory, in line with guidance on early-stage scale development and factor-analytic interpretation (Costello & Osborne, 2005; DeVellis & Thorpe, 2021). Third, item-level statistics were examined alongside observer field notes and moderation discussions to identify items requiring removal, rewording, or conceptual clarification.

Results

Internal Consistency

The pilot observation scale demonstrated strong internal consistency. The 14-item instrument yielded a Cronbach's alpha of .847, indicating good reliability across the observation items. Inspection of item-total correlations indicated that most items showed moderate to strong alignment with the overall construct of responsive mathematics teaching. However, one item relating to general classroom encouragement demonstrated a negative item-total correlation, indicating that it did not align with the core construct of responsiveness to students' mathematical thinking. This item was therefore removed from the refined version of the instrument.

Exploratory Factor Analysis

Prior to factor analysis, the factorability of the data was assessed. Given the small pilot sample, these diagnostics should be interpreted with caution although item intercorrelations and the pattern of communalities supported the suitability of the data for factor extraction. We used Exploratory Factor Analysis (EFA) using Principal Component Analysis with Varimax rotation was then conducted to examine the underlying structure of the observation instrument. Four components with eigenvalues greater than one were extracted, together explaining 87.6% of the total variance in the observation data. Inspection of the rotated component matrix suggested that the items clustered around several meaningful dimensions reflecting key aspects of responsive teaching practice. These dimensions broadly reflected practices associated with *Attending to Student Thinking*, *Interpreting Student Ideas Mathematically*, *Taking up and Pursuing Student Thinking*, and *Structuring Classroom Participation and Feedback*. These dimensions are consistent with key features of responsive pedagogy described in the literature, particularly work on teacher noticing and instructional responsiveness to students' mathematical thinking (Jacobs et al., 2010; Sherin et al., 2011; Robertson et al., 2016; Sweller, 2024). Factor loadings are presented in Table 1. These findings address RQ2 by identifying the underlying dimensional structure of responsive teaching practices as they emerged from pilot classroom observation data.

Table 1

Exploratory Factor Loadings for the Responsive Mathematics Teaching Observation Scale

Dimension	Item	Factor 1	Factor 2	Factor 3	Factor 4
Taking up student thinking	RP3.3: Teacher encourages students to elaborate on or clarify their thinking.	.853	.108	.004	-.135
Supporting participation	RP4.3: Teacher encourages students to respond to or build on one another's ideas.	.823	.327	.125	-.258

Supporting participation	RP4.4: Teacher structures opportunities for multiple students to contribute ideas	.809	.100	.403	.315
Supporting participation	RP4.1: Teacher provides feedback that helps students refine or extend their mathematical thinking.	.763	.158	-.029	.552
Overall responsiveness	RP5: Overall, instruction in this lesson was shaped by and responsive to students' mathematical thinking.	.662	.637	.173	.067
Attending to student thinking	RP1.1: Teacher invites students to explain their strategies or reasoning.	.420	.867	.062	.194
Attending to student thinking	RP1.3: Teacher makes student thinking visible to the class (e.g., revoicing or recording ideas).	.350	.823	.362	.174
Interpreting mathematical ideas	RP2.1: Teacher connects student ideas to important mathematical concepts, relationships, or principles.	-.274	.809	.208	-.297
Taking up student thinking	RP3.2: Teacher asks follow-up questions that probe students' reasoning.	.483	.738	.146	-.038
Interpreting mathematical ideas	RP2.2: Teacher highlights important mathematical relationships or patterns emerging from student ideas.	-.058	.131	.837	-.251
Taking up student thinking	RP3.4: Teacher facilitates discussion of different student strategies or solutions.	.509	.319	.737	-.024
Attending to student thinking	RP1.2: Teacher attends to and responds to the mathematical content of students' ideas, not just whether their answers are correct.	.133	.495	.707	.090
Supporting participation	RP4.2 (this item is removed)	.026	-.084	-.021	.991
Taking up student thinking	RP3.1: Teacher builds on student ideas or strategies to guide the next step of instruction.	.198	-.207	.542	-.714

Note. Factor loadings above .40 were considered meaningful. Item communalities ranged from .75 to .99, indicating that the extracted components explained a large proportion of item variance.

Scale Refinement

Based on the pilot analyses, the observation instrument was refined in four ways. First, one item relating to general classroom encouragement (RP4.2: “Teacher provides positive reinforcement and encouragement”) was removed due to a negative item–total correlation ($r = -.109$) and independent loading on a separate factor. Because this item reflected general classroom climate rather than responsiveness to student mathematical thinking, its inclusion weakened the conceptual coherence of the scale. Although encouragement and positive reinforcement are important features of classroom climate, they do not necessarily indicate that a teacher is responding to the substance of students' mathematical reasoning. The refined instrument therefore distinguishes general supportiveness from mathematical responsiveness, which is more specifically concerned with how teachers notice, interpret, and build on student thinking during instruction (Hill et al., 2008; Robertson et al., 2016).

Second, two items were reworded to improve clarity and ensure that they captured observable instructional actions associated with building on and interpreting student ideas. Item RP3.1 was revised from “Teacher builds on student ideas to advance the lesson” to “Teacher builds on student ideas or strategies to guide the next step of instruction,” to more precisely emphasise the observable instructional action of using student contributions to shape subsequent moves. Item RP2.1 was revised from “Teacher connects student ideas to relevant mathematical concepts and principles” to “Teacher connects student ideas to important mathematical concepts, relationships, or principles,” to better align the item wording with the interpretive dimension of responsive teaching in the literature (Jacobs et al., 2010; Robertson et al., 2016). Third, the remaining 12 items demonstrated satisfactory statistical performance (Cronbach's alpha of .868) and conceptual alignment and were retained without change in the refined scale. Fourth, the response scale was revised from five points to four points by removing the neutral midpoint. Observers raised this issue consistently during moderation meetings, noting that a rating of 3 made it difficult to distinguish between partial observation and genuine uncertainty about whether the practice had occurred. Removing the neutral option requires

observers to make a directional judgement for each item, improving the discriminant utility of the scale.

Observer field notes further supported the quantitative patterns identified in the ratings. Across both classrooms, notes consistently recorded that teachers acknowledged student responses but rarely pursued the reasoning behind them. One observer noted that a student used a calculator during a lesson and discovered the teacher's solution on the board was incorrect, yet the teacher responded only with "Oh!" and moved on without using the moment as an opportunity for mathematical discussion or collective reasoning. In other lessons, teachers asked students to explain a step in a procedure but redirected or completed the explanation themselves rather than allowing the student's reasoning to be fully explored. Teachers were also noted to provide predominantly generic feedback phrases such as "good job" and "yes, you got it" rather than feedback tied to the specific mathematical substance of student contributions. These patterns were consistent across the two classrooms and corroborated the quantitative finding that items relating to probing, extending, and interpreting student thinking were among the lowest-rated dimensions of the scale.

Discussion

Interpreting the Structure of Responsive Mathematics Teaching

The exploratory factor analysis suggested that responsive teaching practices observed in the pilot lessons could be organised into several interrelated dimensions. These dimensions broadly reflected attending to student thinking, interpreting students' ideas mathematically, taking up and extending student thinking, and structuring participation in ways that made reasoning visible. Together, these dimensions align with prior theory that conceptualises responsive teaching as more than eliciting participation; it involves attending, interpreting, and responding in ways that advance mathematical sense-making (Jacobs et al., 2010; Sherin et al., 2011; Robertson et al., 2016). The first factor, attending to student thinking, resonates with research on teacher noticing, which argues that instruction becomes more responsive when teachers focus on the substance of students' strategies rather than only on answer correctness (Jacobs et al., 2010). The factor concerning interpretation of student ideas reflects the second component of noticing and making sense of what students' contributions reveal about their understandings. The factor involving instructional uptake or extension of student thinking connects with responsive teaching scholarship that emphasises using students' ideas to shape subsequent instructional moves (Robertson et al., 2016). Finally, the factor related to participation and feedback reflects the role of discourse structures in making students' mathematical reasoning available for collective examination, comparison, and refinement (Kazemi & Stipek, 2001; Chapin et al., 2009). The pilot findings therefore provide initial empirical support for conceptualising responsive mathematics teaching as a set of observable classroom practices, while acknowledging that the factor structure should be treated as provisional given the small pilot sample.

Insights from Classroom Observations and Implications for Mathematics Classroom

The classroom observations help situate the scale within known tensions in mathematics teaching. Across both classrooms, lessons typically followed a structured routine in which students worked independently on differentiated tasks while the teacher provided guided support to a small group of three to five students on the floor. Within this structure, observer field notes recorded that teachers frequently acknowledged student responses during whole-class interactions but rarely pressed for explanation or reasoning. In one classroom, the consolidation phase was observed to be consistently abbreviated or absent entirely, meaning that students had little structured opportunity to compare strategies, reflect on their learning, or

hear peers' mathematical reasoning. In the other classroom, consolidation was more regularly observed but tended to involve reading out answers from a displayed PDF rather than facilitating substantive mathematical discussion. This pattern reflects a distinction noted in the literature between participation that is answer-oriented and discourse that is reasoning-oriented. Studies of mathematics classrooms have shown that conceptual engagement is more likely when teachers press for explanation, justification, and the comparison of strategies rather than moving quickly to confirmation or demonstration (Kazemi & Stipek, 2001; Chapin et al., 2009; Lampert, 2001).

Although teachers frequently acknowledged student responses, the pilot observations suggested that acknowledgement alone did not consistently lead to instructional uptake of students' mathematical reasoning, a distinction highlighted in prior research on responsive and discussion-based mathematics teaching (Kazemi & Stipek, 2001; Robertson et al., 2016). Observer notes indicated that during explicit teaching phases, questions directed at students were predominantly of a recall or procedural nature, following an initiate–respond–evaluate (IRE) pattern in which correct answers were accepted and the lesson moved on. During exploration phases, teacher-student interactions in the guided group were more substantive, with observers noting moments where students were encouraged to explain their strategies or check their reasoning; however, these interactions were confined to the small guided group and did not extend to the broader class. Observer notes across both classrooms also documented high rates of student disengagement during independent work, with students frequently off task, and several instances where the pace of lessons was driven by time pressure rather than student understanding. The RMTOS was developed within a broader research program concerned with the relationship between teaching quality and students' mathematical experience; these observed conditions provide a meaningful context for understanding why measuring instructional responsiveness matters beyond classroom discourse research alone.

Limitations, Future Research and Conclusion

Several limitations should be acknowledged. First, the pilot dataset consisted of 24 observations across two classrooms observed over six weeks, and the factor analysis results should therefore be interpreted with caution. For that reason, the present study should be understood as identifying a promising conceptual and empirical structure rather than establishing a final validated instrument. Second, although the observers were experienced primary teachers and participated in moderation discussions to support consistent interpretation of the scale items, the observations were conducted within a single school context across a limited range of Stage 3 content topics including decimals, fractions, area, and order of operations. Future studies should examine the applicability of the instrument across a wider range of mathematical topics, year levels, and classroom contexts. Further research with larger samples, shared video-coded lessons, and multiple raters will be necessary to test the stability of the factor structure, strengthen inter-rater reliability evidence, and examine whether the scale predicts important outcomes such as the quality of classroom discourse or student learning opportunities. Incorporating qualitative methods such as teacher interviews or post-lesson conversations would also allow future research to investigate the reasons behind observed instructional patterns, including teachers' reported challenges in pursuing individual students' thinking during whole-class instruction. Examining the relationship between RMTOS ratings and established frameworks for instructional quality would provide important evidence of convergent validity and situate the instrument within the broader landscape of observation-based research on teaching quality in Australian schools.

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reported here represents a distinct contribution being extended in subsequent validation studies. Ethics approval for this study was granted by the University of Newcastle College Human Ethics Advisory Panel (Approval No. H-2024-0153), and all participants provided informed consent prior to participation. Approval to conduct the research in the relevant Catholic education setting was also obtained from the Catholic Schools Office, Diocese of Maitland-Newcastle.

References

- Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K–6* (2nd ed.). Math Solutions.
- Costello, A. B., & Osborne, J. W. (2005). Best practices in exploratory factor analysis: Four recommendations for getting the most from your analysis. *Practical Assessment, Research & Evaluation*, 10(7), 1–9. <https://doi.org/10.7275/jyj1-4868>
- Cronbach, L. J. (1951). Coefficient alpha and the internal structure of tests. *Psychometrika*, 16(3), 297–334. <https://doi.org/10.1007/BF02310555>
- DeVellis, R. F., & Thorpe, C. T. (2021). *Scale development: Theory and applications* (5th ed.). Sage.
- Ersozlu, Z. (2024). The role of technology in reducing mathematics anxiety in primary school students. *Contemporary Educational Technology*, 16(3), ep517. <https://doi.org/10.30935/cedtech/14717>
- Ersozlu, Z. & Karakus, M. (2019). Mathematics anxiety: Mapping the literature by bibliometric analysis. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(2), em1673. <https://doi.org/10.29333/ejmste/102441>
- Ersozlu, Z. (under review). Responsive mathematics teaching: Conceptual distinctions, measurement challenges, and a framework for the Australian context. Manuscript submitted for publication.
- Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, 26(4), 430–511. <https://doi.org/10.1080/07370000802177235>
- Hill, H. C., Charalambous, C. Y., & Kraft, M. A. (2012). When rater reliability is not enough: Teacher observation systems and a case for the generalizability study. *Educational Researcher*, 41(2), 56–64. <https://doi.org/10.3102/0013189X12437203>
- Gay, G. (2010). *Culturally responsive teaching: Theory, research, and practice* (2nd ed.). Teachers College Press.
- Gore, J. M., & Bowe, J. M. (2017). Reassembling teacher professional development: The case for Quality Teaching Rounds. *Teachers and Teaching: Theory and Practice*, 23(3), 352–366. <https://doi.org/10.1080/13540602.2016.1206522>
- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202. <https://doi.org/10.5951/jresmetheduc.41.2.0169>
- Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, 102(1), 59–80. <https://doi.org/10.1086/499693>
- Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal*, 32(3), 465–491. <https://doi.org/10.3102/00028312032003465>
- Lampert, M. (2001). *Teaching problems and the problems of teaching*. Yale University Press.
- NSW Department of Education and Training. (2003). *Quality teaching in NSW public schools: A classroom practice guide*. NSW Department of Education and Training, Professional Support and Curriculum Directorate.
- Robertson, A. D., Scherr, R. E., & Hammer, D. (Eds.). (2016). *Responsive teaching in science and mathematics*. Routledge.
- Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM Mathematics Education*, 45(4), 607–621. <https://doi.org/10.1007/s11858-012-0483-1>
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. Routledge.
- Sweller, J. (2024). Cognitive load theory and individual differences. *Learning and Individual Differences*, 111, 102434.
- Worthington, R. L., & Whittaker, T. A. (2006). Scale development research: A content analysis and recommendations for best practices. *The Counseling Psychologist*, 34(6), 806–838. <https://doi.org/10.1177/0011000006288127>
- van Gog, T., Kester, L., & Paas, F. (2011). Effects of worked examples, example-problem, and problem-example pairs on novices' learning. *Contemporary Educational Psychology*, 36(3), 212–218.

Seven Barriers (Perceived or Real) to Inclusive Secondary Mathematics

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The study of secondary mathematics has been largely inaccessible for students with intellectual disability and options for studying mathematics reduce as school years increase. In this paper we discuss seven barriers that secondary mathematics teachers indicated they faced in teaching year level adjusted mathematics curriculum in the junior secondary years. These teachers were part of a research project undertaken in Queensland schools over three years. We address these barriers and offer approaches for enabling inclusive mathematics practice.

The study of mathematics at secondary school level has been largely inaccessible for students with intellectual disability². Indeed, for many low attaining students, options for studying mathematics reduce as school years increase (Marchant & Kennedy, 2025). Two approaches are typical: a simpler curriculum with narrower content; or a functional “life skills” curriculum (Faragher, 2019). Tan et al. (2019) found that teaching approaches that focussed on following set steps in mathematical procedures were common for students with mathematics learning difficulties and disabilities, with few opportunities to develop mathematical thinking.

An alternative approach is to make adjustments to the year level curriculum to make mathematics accessible, thus allowing students to be successful. This approach to curriculum design, called Learning Year Level Adjusted Curriculum (LYLAC) is not new (for a review, see Faragher, 2023a); however, it is also not commonly adopted teaching practice.

In this paper, we explore barriers that might exist that restrict the uptake of LYLAC, even though there is evidence that it can lead to student success with year level content (Browder et al., 2012; Browder et al., 2020; Monari Martinez & Benedetti, 2011). It would appear little previous work specifically on barriers to inclusive secondary mathematics has been published.

Our study exploring inclusive secondary mathematics adjustments

Here we report findings from a three-year, qualitative study which was conducted in partnership with five secondary schools in Southeast Queensland. The study focussed on the work of secondary mathematics teachers as they adjusted year level mathematics from the Australian Curriculum (AC:M) for students with intellectual disability. The project was longitudinal and used a codesign approach where the Heads of Mathematics departments (HODs) collaborated with the research team to plan annual (term 1) professional learning (PL) workshops for all mathematics staff from the participating schools over the three years of the project. The aim of the PL was to develop expertise in adjusting mathematics teaching and assessment to support the learning of low attaining students. Lesson observations and interviews with classroom teachers were designed to explore inclusive mathematics education

² Defined as a lifelong condition that includes significant limitations in both intellectual functioning and adaptive behaviours that begins in the developmental period. Definition was included in Workshop 1 professional learning.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 146–153). MERGA.

in practice. In this paper, we explore the research question: What are the factors that appear as barriers to teachers making adjustments to mathematics lessons for low attaining learners?

Participants

The schools (Table 1) were located in Brisbane, Moreton and Ipswich cities. Three of the five enrolled a significant proportion of students with disability. All mathematics teachers (including those “out of field”, such as qualified Health and Physical Education teachers who taught junior mathematics), HODs and learning support staff participated, a total of 104.

Table 1

Participating schools

School affiliation	Type	Average Enrolment 2022-2024
State	Co-education years 7-12*	814
Independent Catholic	Co-education 5-12	595
Systemic Catholic	Boys 5-12	1715
Independent Anglican	Girls Prep-12	986
Independent	Girls Prep-12	1198

*This school opened in 2020 – enrolment years during the project: 7-10; 7-11; 7-12

Professional learning workshops

The PL workshops enabled participants to collaborate in understanding students’ specific mathematics learning support needs and build the conceptualisation of reasonable adjustments of mathematics, through focus on partner school contexts. The workshops also offered a rare opportunity for teachers from different school systems to work together over time.

Workshop survey

As part of the workshops, teachers completed a survey that was divided into two parts. Part 1 contained Likert scale responses and are not reported in this paper. Part 2 asked five open response questions: two were procedural about the project, and three (Table 2) asked about participants’ perspectives on the barriers to enacting inclusive mathematics practice.

Table 2

Open response questions

Question number	Question
1	What skills do you have that will enable you to include students with an intellectual disability in your classroom?
2	If you have anxieties about including students with an intellectual disability in your classroom, what are they?
3	What do your colleagues feel about the inclusion of students with an intellectual disability in their classrooms?

Classroom observations

Partner schools were asked annually to nominate up to two teachers to take part in lesson observation cycles. There were four cycles of co-design work, resulting in 18 observations (one of the schools was able to nominate staff for only two cycles). Each observation involved three parts: pre-lesson planning; the lesson observation; and a post-lesson discussion. Audio recordings were made of interviews and video recordings (two cameras, one recording the work of nominated students with intellectual disability, the other the whole class) were made and transcribed.

Data analysis

The classroom observation cycles gave depth to the survey responses and enabled the university team to more deeply understand the contexts in which teachers worked. We analysed data from the survey questions and observation cycles, particularly the interview transcripts.

In undertaking the data analysis, we were mindful of our backgrounds as researchers and how that influences our work. The first two authors of this paper have been secondary mathematics teachers, HODs, researchers and teachers of pre-service mathematics education teachers. The third author is an educational psychologist; the fourth a teacher and disability-focused researcher. Our backgrounds necessarily have influenced what we noticed while we were closely working with our teaching colleagues over the three years of field work for this project. This aspect of noticing has been called “wonder” by MacLure (2013) who discussed the potential for data to provoke thought for researchers.

This potentiality can be felt on occasions where something—perhaps a comment in an interview, a fragment of a field note, an anecdote, an object, or a strange facial expression—seems to reach out from the inert corpus (corpse) of the data, to grasp us. These moments confound the industrious, mechanical search for meanings, patterns, codes, or themes; but at the same time, they exert a kind of fascination, and have a capacity to animate further thought. (p. 228)

In deeply considering our engagement with our teaching colleagues and the data that emerged, it struck us that certain experiences did indeed cause us to wonder, playing on our minds and our conversations. Because our research involved working with some of the participants over the three years of the study, we were able to share our wonderings in various conversations: in steering committee meetings, at the PL workshops, in lesson observation interviews and other similar opportunities. We were hearing their perspectives on our thinking and having our ideas challenged and enriched.

In the results to follow, we take these “wonderings” and provide illustrative quotations from our data that elucidate the issues that emerged and the nature of our deliberations on them.

Barriers to Inclusive Secondary Mathematics Classroom Practice

Teachers indicated strong commitment to inclusion of all learners, and especially those with intellectual disability. They also indicated a belief that their colleagues shared this commitment. It was clear that they wanted all learners to succeed in mathematics. It was also clear that they experienced barriers in supporting their learners with intellectual disabilities to do so. We have discerned seven barriers to enacting inclusive secondary mathematics teaching. The seven barriers have been categorised into three groups: Organisational constraints; Teachers’ mindsets; and Teachers’ capacity to do their work.

Organisational constraints

1. Time and workload – impact on a teacher’s work

Time pressures were raised in participants’ responses to the surveys with comments including: “*Not enough time to adjust effectively*”, “*Time to organise alternatives*”, “*Whether I have all the time available to provide the resources required*” and “*Extra workload to provide access and UDL activities*”. A prevalent view was that students with intellectual disability require additional resources or activities to be developed and undertaken by teachers. The underpinning philosophy of Universal Design for Learning (UDL; CAST, 2024) as planning for all through provision of multiple approaches to learning, did not appear to be understood by teachers, even though it was often described as an approach promoted by their schools. Rather, their comments suggest that planning for students with intellectual disability was seen as extra work they would not otherwise have to do.

2. Crowded curriculum – constraints on teaching

Closely related to the sense of time pressure is the commonly expressed barrier that can be described as a “crowded curriculum”. This refers to notions that there is too much to cover in the specified curriculum. This perception leads to teachers feeling that coverage of the curriculum overrides a focus on student achievement. Some responses by teachers on the survey are illustrative: *“In Mathematics we have no room in our class plans for any consolidation so always a new topic for each lesson (unfair for all students)”*, and *“Fast paced nature of mathematics – a lot of content to cover”*.

The AC:M specifies content to be taught, embedding proficiencies of understanding and fluency. It acknowledges that some students will need more time than others to build conceptual understanding and fluency with the content. Guidance on the implementation of the AC reminds teachers that they are required to provide learning opportunities and choices for students with disability and that the AC “can be used flexibly to meet the needs of all students” (ACARA, 2026). Even so, the pressure teachers feel to get through content was expressed by many in our study. Some schools responded to this barrier by designing alternative mathematics programs that modified the AC:M to reduce the content for low attaining learners. Others removed or minimised lessons that engage students in use of manipulative materials or open-ended tasks, approaches intended to support the learning of students who find abstraction difficult and foster mathematical thinking.

While we did not specify teaching approaches, some schools appeared to design lessons as special events for our observations. This created unintended pressures on teachers and HODs who felt the already crowded mathematics program needed to be put on hold to make room for a special observation lesson. One HOD in a post lesson interview stated:

At the moment the Australian curriculum is so jam-packed that the big thing in maths is we're so time poor, and most teachers have removed a lot of the physical nature of activities, because we can't fit it all in. If we spend time like we did for you, especially to come in, we wouldn't cover all of the content. And so sometimes you have to make the decision is the benefit going to outweigh the time loss, because, you know, [teacher] only got through maybe one type of question and didn't get through you know the multiple types that could be assessable.

Comments from the project participants expressed the sentiment that they would like to do more lessons where multiple approaches were offered (as per the principles of UDL) as they recognised the benefits to students, particularly those who were low attainers; however, they believed they could not afford the time. The technical imperative of curriculum coverage overrode the imperative of students achieving learning outcomes.

3. External assessment regimes – and assessment adjustments

In the PL workshops, we discussed the use of adjustments in assessment. Australian law requires the provision of approved assessment adjustments (Commonwealth Department of Employment Education and Training, 2005). In our study, we observed the impact of Queensland's recently implemented external examination scheme on the teachers' approach to assessment. Even though the external examinations only applied to students in Year 12, teachers were modifying their assessment practices across all year levels. As an example, one year 12 external examination paper assessing a mathematics course designed for students intending to study university mathematics, does not permit the use of calculators. This led one school to restrict the use of calculators on all tests, even in the lower secondary years, in order to prepare students for future assessment practices.

Teachers' mindsets

The next two barriers relate to how teachers view learners' capacity to do mathematics and how they should be taught.

4. Mindset of inclusion vs integration for students with intellectual disability

The distinction between the terms “inclusion” and “integration” was clarified in General Comment Number 4 of the Committee on the United Nations Convention on the Rights of People with Disability (Committee on the Rights of Persons with Disabilities, 2016) as:

Integration is the process of placing persons with disabilities in existing mainstream educational institutions with the understanding that they can adjust to the standardized requirements of such institutions. Inclusion involves a process of systemic reform embodying changes and modifications in content, teaching methods, approaches, structures and strategies in education to overcome barriers with a vision serving to provide all students of the relevant age range with an equitable and participatory learning experience and the environment that best corresponds to their requirements and preferences. (Paragraph 11)

When teachers adopt an inclusion mindset, they focus on supporting the learning needs of all students in their class, as they arise and from whatever source. Conversely, viewing students with disabilities as distinct from other students is an integration mindset where students need to be able to cope to be part of the class. Seeing students with difficulties with learning as fundamentally different and requiring special provisions that may or may not be able to be provided portrays an integration mindset. This was evident in the concerns raised by some of our participants in their survey responses: *“Ensuring that they are enjoying and are engaged as I am teaching the mainstream class”, “I don’t believe I have the training/skills to teach these students”, “Ensuring other students are still in a learning environment that will ensure success”, “Other students’ learning being disrupted” and “Causing disruption/holding back other kids”.*

5. Mindset of student ability

Ability is a contested term (Mighton, 2003). Once thought to be fixed, measurable and reported by a single number, the Intelligence Quotient, it is now understood to be malleable and affected by many factors. Dweck’s (2006) work has introduced the terms “fixed mindset” and “growth mindset” into the lexicon of Queensland teachers, with many schools even proclaiming on school advertising that they are “a school with a growth mindset”. This should indicate a view that intelligence and ability can change with effort, good teaching and the right support.

The mathematics teachers in our project were dedicated professionals who had a keen interest in providing the best learning opportunities for their students. While it was clear the teachers wanted the best for all their students, it was also clear that their view of the ability of their students with difficulties with learning did not always align with a growth mindset. At the start of one lesson observation, the HOD at a school that proclaimed a growth mindset gave us an overview of the class pointing out three students who “were E students”. This referred to the A to E achievement rating scale with E being the lowest grade. By being designated an “E student”, there was a sense that this was a fixed characteristic of the student. There was no indication in the conversation that change was likely or even possible.

In responses to the survey given in the PL workshops, teachers expressed concerns about: *“Mixed ability classrooms and large range of abilities trying to cater for all”, “How will they cope in class? My interactions with them?” and “Greatest concern in managing the range of abilities. How do I support those with ID while still supporting the other learners?”.* These concerns suggest a fixed ability mindset that learning outcomes cannot be ameliorated by good teaching, support or student effort.

Teachers’ capacity to do their work

6. Lack of resources

Some survey responses indicated a perceived need for specific resources that the teacher did not have, e.g. *“I cannot think of, or find, resources that promote open-ended tasks”* and

“Funds for resources”. These comments suggest a link to the barriers about time and a crowded curriculum, with a view that something additional or special is required to support learners who are low attainers.

A second category of responses related to provision of teacher aide support, with a number reporting a lack of such support. One Head of Learning Support did make an alternative point when raising concerns about inclusive practice: *“too much use of TA”*. This is an issue that has been addressed in previous research which we take up in the discussion.

7. Lack of confidence or skills

The final barrier combines two concerns that are difficult to disentangle. Is it the skills that are lacking or the confidence to apply them? Teachers shared their insecurities, honestly, writing, *“feeling of judging whether I’ve done it successfully, anxiety about including students with ID: That I am doing a good job.”*, *“Anxieties about how to offer students with an intellectual disability with alternative explanations”* and *“What parents think about me, Meeting HOD’s expectation”*. Teachers also raised some specific concerns such as: *“Lack of experience, therefore unsure how effective strategies will be”*.

Specifically, teachers wrote about their insecurities about their abilities to design learning adjustments. *“I don’t believe I have the training/skills to teach these students”*, *“Not knowing the student (level of need; type of need)”*, and *“How best to support a child working below level in a way that doesn’t highlight their support.”* The comment, *“Not making work too hard/easy; Correctly adjusted assessments; Pushing students”* gives insight into the challenge teachers felt about pitching the level of adjustments. Heads of Learning Support also wrote about colleagues’ needs for support with making adjustments, and more training, and also that they believed some teachers thought, *“It is the responsibility of the “Inclusive Learning Department”*.

It is clear that teachers want to do the best for all their students and worry that they are not. Skills and confidence are closely linked. Some may perceive a lack of skills, though they may actually have the skills and lack confidence to use them or to try new approaches.

Discussion

Teachers perceived a number of barriers preventing them from effectively including students with intellectual disability in their mathematics classrooms. These perceived barriers acted to prevent teachers from implementing inclusive practices and reflect three overarching areas of concern.

The first, Organisational Constraints, is significant for teachers. The perception of a crowded curriculum results in a real barrier to teachers that affects the approach they take to the learning they plan for their students. However, data from this and other projects we have undertaken, lead us to challenge the perception of a crowded curriculum. In previous studies, we have recorded three different lessons, each focussing on the rule for finding the volume of a prism: the first a year 3 and 4 composite class, the second a year 7 class and the final one, a year 9 class. The year 9 class did not include any student with an intellectual disability and included at least one student excelling in mathematics. The spiral curriculum in evidence here does offer opportunities for learners to revisit concepts and consolidate their understanding. However, there can be a view that all concepts mentioned in the AC:M need to be taught, rather than finding out whether and which students have already mastered concepts in previous years. The time on a particular topic in successive revisits could likely be reduced. Also, if the emphasis of teachers moved from a focus on curriculum coverage to student understanding of the learning goals, less time repeating concepts is likely to be required.

The Organisational Constraint of external assessment requirements serves as a warning about the impact of decisions misapplied to other contexts. Many of the students of interest in this study, that is, those with intellectual disability, realistically are unlikely to study advanced

mathematics courses in their final years of secondary school, nor would many other low attaining students. Yet teachers in two participating schools were making decisions to require all mathematics examinations to be undertaken without calculators, therefore denying junior secondary students the opportunity to show what they had learnt in their current courses. In addition, the use of calculators is an essential adjustment for many learners with mathematics learning difficulties (Faragher, 2023b).

Teachers' Mindsets, the second category of barriers, influences teachers' perceptions of learners and their ability to learn. The "othering" (Mik-Meyer, 2020) of students with difficulties with learning is evident in the comments of the teachers reported above. There is a striking sense that in some way, these students do not belong (Swinton, 2012) and are being at best, tolerated, and at worst, seen as a potential source of disruption to the class. This is a long way from a mindset of inclusion which welcomes all students, values their gifts and meets their needs (Thousand & Villa, 2000). In an inclusive classroom, diversity is expected and planned for.

The final two barriers in the third category, Perception of Teachers' Capacity to do their Work, relate to teachers' views of their ability to teach students with intellectual disabilities in inclusive mathematics classrooms. The two barriers: lack of resources and lack of skills or confidence are closely related. Sometimes the lack can be viewed by teachers as someone else's fault or responsibility, or sometimes perceived as a personal limitation. In inclusive education, this is a well-known phenomenon where teachers rarely feel completely confident to teach learners who require additional learning support (Mónicoa et al., 2020). This can manifest in a reliance on teacher aide support, often abdicating learning responsibility to an unqualified teacher. Teacher aide support can be invaluable, but requires careful attention to best practice guidelines (Butt, 2016).

Conclusion

We wish to be clear that the teachers in our study were not averse to the inclusion of students with intellectual disability in mathematics classes and were supportive of the aspirations for them to succeed. We also wish to acknowledge that our teachers had many comments about what enabled their inclusive practice. However, they felt constrained by barriers that impeded their ability to make adjustments for learners.

We leave as an open question whether these barriers are objectively real or instead, perceived barriers. The reliance on teachers' perceptions does introduce some limitations, including potential bias if attitudes towards inclusion or those with intellectual disability are negative, though this was not evident in our data. In addition, perceptions are recognised as contributing to behaviours, although it is also clear they are not the only determinant (Granados Samayoa & Albarracin, 2025).

In practical terms, if a teacher experiences an issue as a barrier to inclusive mathematics practice, it will be real for them, even if another teacher in the same system does not experience the same issue as a barrier. These barriers need to be identified and removed so that students with intellectual disabilities and low attainment in general can engage in high quality inclusive mathematics education.

Acknowledgments

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References

- ACARA. (2026). *Meeting the needs of students with disability*. <https://www.australiancurriculum.edu.au/student-diversity/students-with-disability>
- Browder, D. M., Jimenez, B., & Trela, K. (2012). Grade-Aligned Math Instruction for Secondary Students with Moderate Intellectual Disability. *Education and training in autism and developmental disabilities*, 47(3), 373-388.
- Browder, D. M., Spooner, F., & Courtage, G. R. (2020). *Teaching students with moderate and severe disabilities*. (2nd ed.). Guilford Press.
- Butt, R. (2016). Teacher assistant support and deployment in mainstream schools. *International Journal of Inclusive Education*, 20(9), 995-1007. <https://doi.org/10.1080/13603116.2016.1145260>
- CAST. (2024). CAST Universal Design for Learning Guidelines version 3.0. <https://udlguidelines.cast.org>
- Committee on the Rights of Persons with Disabilities. (2016). *General Comment No.4, Article 24: Right to inclusive education*. <https://documents-dds-ny.un.org/doc/UNDOC/GEN/G16/263/00/PDF/G1626300.pdf?OpenElement>
- Commonwealth Department of Employment Education and Training. (2005). *Disability standards for education 2005 plus guidance notes*. Author. <https://www.education.gov.au/disability-standards-education-2005>
- Dweck, C. (2006). *Mindset. How you can fulfil your potential*. Random House.
- Faragher, R. (2019). The new 'functional mathematics' for learners with Down syndrome: Numeracy for a digital world. *International Journal of Disability, Development & Education*, 66(2), 206-217. <https://doi.org/10.1080/1034912X.2019.1571172>
- Faragher, R. M. (2023a). Individual student characteristics, abilities and personal qualities and the teacher's role in improving mathematics learning outcomes. In A. Manizade, N. Buchholtz, & K. Beswick (Eds.), *The evolution of research on teaching mathematics. International perspectives in the digital era*. (pp. 227-253). Springer Nature. <https://doi.org/https://link.springer.com/book/10.1007/978-3-031-31193-2>
- Faragher, R. M. (2023b). *A practical guide to educating learners with Down syndrome: Supporting lifelong learning*. Routledge.
- Granados Samayoa, J. A., & Albarracin, D. (2025). Understanding belief-behavior correspondence: Beliefs and belief-to-behavior inferences. *Psychological Inquiry*, 36(1), 1-22. <https://doi.org/10.1080/1047840X.2025.2482343>
- MacLure, M. (2013). The wonder of data. *Cultural Studies. Critical Methodologies*, 13(4), 228-232. <https://doi.org/10.1177/1532708613487863>
- Marchant, T., & Kennedy, S. (2025). *Year 12 Mathematics Participation Report Card. Numbers remain at record lows and female participation declines*. AMSI. <https://amsi.org.au/?publications=year-12-mathematics-participation-report-card-numbers-remain-at-record-lows-and-female-participation-declines>
- Mighton, J. (2003). *The Myth of ability*. House of Anansi Press Inc.
- Mik-Meyer, N. (2020). Symbolic interactionism, stigma and othering. In M. Järvinen & N. Mik-Meyer (Eds.), *Qualitative analysis: Eight approaches for the social sciences* (pp. 51-72). Sage. https://search.library.uq.edu.au/permalink/61UQ_INST/c60qab/cdi_proquest_ebookcentralchapters_31568997_9_62
- Monari Martinez, E., & Benedetti, N. (2011). Learning mathematics in mainstream secondary schools: experiences of students with Down's syndrome. *European Journal of Special Needs Education*, 26(4), 531-540. <https://doi.org/10.1080/08856257.2011.597179>
- Mónica, P., Mensah, A. K., Grünke, M., Garcia, T., Fernández, E., & Rodríguez, C. (2020). Teacher knowledge and attitudes towards inclusion: a cross-cultural study in Ghana, Germany and Spain. *International Journal of Inclusive Education*, 24(5), 527-543. <https://doi.org/10.1080/13603116.2018.1471526>
- Swinton, J. (2012). From Inclusion to Belonging: A Practical Theology of Community, Disability and Humanness. *Journal of Religion, Disability & Health*, 16(2), 172-190. <https://doi.org/10.1080/15228967.2012.676243>
- Tan, P., Lamber, R., Padilla, A., & Wieman, R. (2019). A disability studies in mathematics education review of intellectual disabilities: Directions for future inquiry and practice. *The Journal of Mathematical Behavior*, 54(June), 100672. <https://doi.org/10.1016/j.jmathb.2018.09.001>
- Thousand, J., & Villa, R. A. (2000). Inclusion. *Special Services in the Schools*, 15(1-2), 73-108. https://doi.org/10.1300/J008v15n01_05

Exploring Mathematical Justifications in Collaborative Problem Solving of a Heterogeneous Group

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Mathematical justification, the act of giving arguments to defend a claim, strongly supports students' Collaborative Problem Solving (CPS). Yet studies exploring justifications in a heterogeneous-achievement group are still limited. This study describes mathematical justifications of such a group in CPS by identifying the levels of justification and the role of each student. The findings showed that the group used perceptual demonstration to generalisable arguments level of justifications. Higher-achievement students were found initiating justifications, while lower-achievement students triggered or merely accepted them.

Mathematical justification, an activity of providing arguments that support or refute a mathematical claim, is at the core of reasoning activities in problem-solving and improves reasoning skills (Brodie, 2010). In collaborative contexts, students who engage in justification activities with peers learn more as they defend their positions and resolve cognitive conflict (Fuchs et al., 1998). In a classroom problem-solving environment, teachers often group students based on their mathematics achievement to promote such activity. A heterogeneous setting was deliberately created to scaffold high-achieving students to their lower-achieving peers (Wyman & Watson, 2020). In the case of justification, this action might be preceded by the findings from the context of individual problem solving, i.e., students with high mathematics achievement tended to carry out more theoretical mathematical justifications, while students with low abilities justified perceptually more often (relying on what they saw or experienced) or did not justify at all (Chua, 2016). This is expected to lead students with lower achievement to question their claims and justify them in a more theoretical manner than their high-achieving peers. However, in the context of collaborative problem-solving, whether heterogeneous grouping mediates the quality of justification and problem-solving work remains debatable. For instance, it was found that a high-low achieving pair interacted less, as shown by offering fewer conceptual explanations, richer arguments, and more cognitive conflict resolutions than the homogeneous pairs (Fuchs et al., 1998).

We believe that investigating the quality of justification within heterogeneous grouping will inform teachers about the dynamics of heterogeneous grouping in the context of collaborative problem-solving and thus allow them to design the necessary support for the expected learning experience (e.g., arguing for claims, resolving cognitive conflict) for such a group. In this paper, we explore mathematical justifications within the problem-solving process of a group of heterogeneous achievers. Specifically, we want to answer the following questions: (1) How are the levels of the mathematical justifications in such a group? and (2) what is the role of each student in the mathematical justifications?

Theoretical Considerations

We referred to mathematical justification as an activity in which students compile arguments to support an existing mathematical claim, so that the claim is accepted by the group in which the argument is presented (Staples et al., 2012). In a collective setting, mathematical justification involves “acceptance” from the individuals involved in it (Bieda et al., 2022). This acceptance is based on the individual's agreement that a particular claim or mathematical answer makes sense. Thus, mathematical justification also considers the social context in which mathematical activities are carried out.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 154–161). MERGA.

Justification within heterogenous groups

Research indicates that grouping students based on mathematics achievement can expose them to activities such as justifying ideas and negotiating with peers. This exposure yielded various findings regarding the dynamics of achievement-heterogeneous grouping in producing and justifying arguments during discussions. Some studies seemed to agree that the high-achieving members of the groups showed dominance in exploring ideas and using claims and data to create arguments, while their low-achieving partners relied on these peers (Ekawati et al., 2025; Fuchs et al., 1998). These studies also found that, in heterogeneous groups, high-achieving students accommodated their lower-achieving peers by providing clearer explanations tailored to them. Yet, despite the dominance and contributions of the higher-achieving members, there was evidence that the group members, including the high-achievers, did not fully understand the quality of some justifications for the claim or how they made sense for the claim (Lin & Mintzes, 2010). It shows that there are more issues to be investigated besides the contributions of each group member and the way they respond to their peers. Two of such issues are the quality of the justification they made and how their interaction led to or otherwise hampered the quality of justifications of the constructed claims.

When analysing students' interactions during mathematical justification, it is essential to focus on those specifically directed toward claims and their justifications (Cobb et al., 2000). In that way, the analysis will focus on students' utterances toward each other that are directed to claims or mathematical justification supporting those claims. To allow for such analysis, Cobb et al. (1992) introduced the notion of a situation for justification, a situation where a mathematical justification is demanded. This situation is initiated by questions demanding reasons, clarification, or explanation of the truth of claims. The person who questions or asks for reasons acts as a trigger for the justification. For example, Fredriksdotter et al. (2022) studied justifications during the group's interactions while solving a problem: "There is a queue at the bus stop. In how many ways can 3 persons form a queue?" In their study, a discussion began when Jason claimed the answer was six ways. Milton then simply asked, "Are you sure?", which triggered other members of the group to justify their position. In response to triggers, students might develop mathematical justifications to convince others.

Mathematical justifications levels

Several studies identified levels of mathematical justification carried out by students. Simon and Blume (1996) divided students' mathematical justification into five levels, i.e., level 0 (without justification), level 1 (appeal to authority), level 2 (empirical demonstration), level 3 (deductive justification expressed in terms of example), and level 4 (deductive justification independent of example). Sowder and Harel (1998) also identified several types of mathematical justifications. They started at the externally based justification level, where students justified using an outside source. Students with better justification might give an empirical justification that was made solely based on examples. Based on Sowder and Harel (1998), the highest level of justification was analytical justification. In this level, students might try to defend a claim by reasoning toward generalising a conjecture, which was a transformational analytic justification. They also might use definitions, theorems, or logical mathematical rules, which are called axiomatic justification. Carpenter et al. (2005) identified that students' arguments to justify a claim progressed through three forms, i.e., appeal to authority, using examples, and generalisable arguments. In appeal to authority level, students used others with more authority to justify a claim. In the level of using examples, students did not think in general terms. They defended that a certain claim was true based on some specific examples. Carpenter et al. identified this level as using examples, but we considered this level as a combination of levels 2 and 3 of what other experts described. In the highest example, students gave a generalisable argument to justify a claim. This justification went beyond

examples as students started to think in general terms. They used mathematical principles independent of examples. Summarising from these works (Carpenter et al., 2005; Simon & Blume, 1996; Sowder & Harel, 1998), we used the levels in Table 1 to help us describe the quality of mathematical justifications.

Table 1

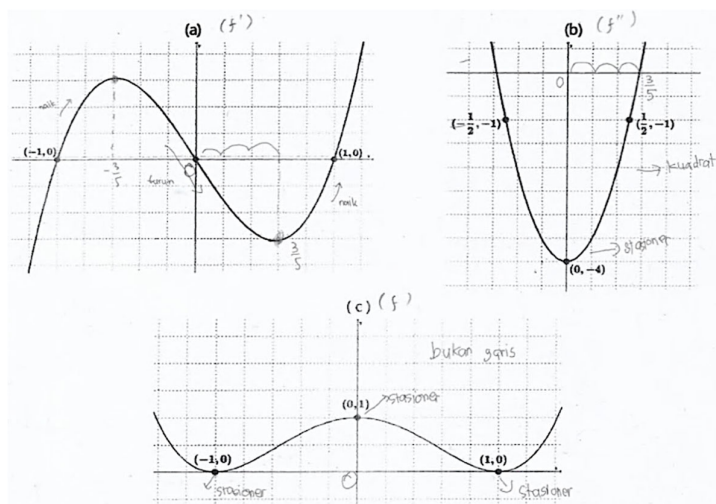
Components of Mathematical Justifications

Component	Description	Code
Trigger	Trigger of a situation for justification	T
Claim	The claim in the problem-solving process	Ci*
<i>Levels of mathematical justifications</i>		
No justification	Not justifying or accepting a claim without explaining why a claim is reasonable	L0
Appeal to authority	Stating the truth of claims using the authority of other people or other sources such as teachers, books, or friends	L1
Empirical or perceptual demonstration	Stating the truth of claims based on the demonstration of perceptual examples, e.g., objects, pictures, or gestures	L2
Symbolic example without generalization	Stating the truth of a claim by using some symbolic examples without generalizing those examples	L3
Generalisable argument	Stating the truth of a claim by using definitions, theorems, or rules of mathematical logic; or generalizations that do not depend on a particular example	L4

Figure 1

The task and the group work

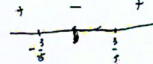
Andi is given three graphs as follows. Among the three graphs, he is asked to determine which one is the graph for function f , f' , and f'' . Andi stated that a is the graph of f , b is the graph of f' , and c is the graph of f'' . Is Andi's statement true? Justify your answer.



Translation:

Increasing
decreasing

naik $x < -\frac{3}{5}$ $\vee$ $x > \frac{3}{5}$
turun $-\frac{3}{5} < x < \frac{3}{5}$
 $x = -\frac{3}{5}$, $x = \frac{3}{5}$
 $(3x+5)(x-5) \rightarrow 9x^2 - 25$



$f'(0) = g(0) - 25$
 $= -25 < 0$

$\int 9x^2 - 25 dx = \frac{9x^3}{3} - \frac{25x}{1} + C$
 $= 3x^3 - 25x + C$

Methods

This paper was part of a larger study exploring justifications of four achievement-based triads, i.e., homogenous-high, homogenous-medium, homogenous-low, and achievement-heterogenous, in an eleventh-grade class of a public school in Malang, Indonesia. The class has just finished an 8-week lesson on the derivative topic without explicit instructions on justifications. The participants' selection began with administering an expert-validated mathematics test to all students and categorising them as high, medium, or low achievers based on their test scores. This paper reported the work of one male achievement-heterogeneous triad,

consisting of one high-, one medium-, and one low-achievement student (pseudonyms: Hans, Mike, and Leo, respectively), on the task in Figure 1. The group worked on the task outside of the classroom schedule, and a task-based interview was done afterwards. Conversations during the group work and interviews were video-recorded, and paperwork was collected. The data analysis was carried out on group work data collected while completing the mathematical justification task, as well as on transcripts of group discussions and group interviews. Initial data condensation was conducted by coding the mathematical justification activities performed by the groups (see Table 1). Coding related to mathematical justification included the mathematical claim that appears (Ci), the trigger for the emergence of mathematical justification (T), and the level of mathematical justification (L0 to L4). The coding process began by identifying the emergence of justification situations throughout the problem-solving process, then coding the mathematical claims (Ci) that arose in these situations. The level of mathematical justification that emerged was then analysed around the claims being identified.

Results

To find the solution to the problem, the group's general strategy was to determine the degree of the polynomial function that each curve represents and their relationship (see Figure 1). First, they claimed that function a is a degree-3 polynomial by identifying its stationary points. After observing the shape of graph b and c , they also claimed that graph b showed a quadratic function and that c is not the first derivative of b . Combining these claims, they agreed that b is the first derivative of a and that Andi's statement in the task is incorrect. Due to the limited space in this paper, we presented examples of the group's mathematical justifications at different justification levels and discussed the student's role in justifying claims during the problem-solving process.

Mathematical justification levels

The use of empirical or perceptual demonstrations (L2)

We observed the group's use of level 2 justification in justifying the claim that b is a quadratic function. The group's discussion focused on the shape of the graph being concave up and that it has one stationary point as shown below.

Hans: What about this (pointing at graph b)? Andi said that b is the derivative of a . It is true, right? [T]

Mike: is a three degree? The derivative is two degree. Two degree (function) has one (stationary point), just like this (pointing at graph b) quadratic function [L2]. It is correct.

Hans: I am tired (laughing)

Mike: Two degree, right.

Leo: How is it? [T]

Hans: Yes, it is two degree (gesturing the curve shape), it is the same (agreeing).

Another example was when they observed graph c after agreeing that b is a quadratic function. The group claimed that c is not the first derivative of b since the shape of c is not a line as written in the transcript below.

Leo: Are there three stationary points (for curve c)? [T] Is its derivative b ?

Mike: No [C].

Hans: If yes, c is supposed to be a line. [L2]

Mike: Why is that so?

Hans: Because it is only x . But this (curve c) is not a line. [L2]

Leo: So, it is not the derivative of b .

The mathematical justification that led to the claim that the function represented by c was not a derivative of b was put forward by Hans. He stated that if it was a linear function, it should

be in the shape of a straight line. This mathematical justification was level 2 because it used an empirical or perceptual demonstration in the form of a graph shape. Mike questioned this mathematical justification and then explained again by Hans that a linear function is a polynomial function of degree one. The group agreed that the function represented by c was not the derivative of b .

The use of symbolic examples without generalisation (L3)

We observed the group's use of level 3 justification for the claim that a is a three-degree polynomial function. The group constructed an equation of $f'(x) = 9x^2 - 25$ as the first derivative of a . Hans started this discussion by marking the stationary points on graph a and he stated that the x coordinate of the point is $\frac{3}{5}$ (see Figure 1, graph a). The group then explored on this data to infer that the function was stationary at $x = \frac{3}{5}$ and $x = -\frac{3}{5}$, increasing on $x < -\frac{3}{5}$ or $x > \frac{3}{5}$, and decreasing on $-\frac{3}{5} < x < \frac{3}{5}$. This idea was confirmed by Mike and Leo by checking that the points were located three scales from O (note: five scales represented 1 unit on the graph). The group escalated the idea into constructing an equation representing the derivative of function a and finding its integral (see Figure 1, bottom part). We considered that there was also a part of perceptual reliance (L2) in this case as they relied on their identification of the exact locations of the stationary points on the graph. Nevertheless, to support the claim, the group went beyond this and moved into symbolic examples (L3) by formulating the equation of the function derivative. They formulate the equation by multiplying the factors $(3x - 5)$ and $(3x + 5)$, resulting on $9x^2 - 25$. Further, the group connected the fact that if the derivative function is two-degree, then the function is of three-degree.

Hans: It means that this $(9x^2 - 25)$ is the derivative of this (pointing at function a). [C]

Mike: Yes.

Hans: Wait a second.

Mike: So, the function is three degree? [T]

Leo: This $(9x^2 - 25)$ is the derivative? [T]

Hans: Wait (thinking).

Mike: Yes, right. It means that this one (pointing at a) is $3x^3$. [C]

Hans: Oh, I see. What did we do? I forget the name of this method.

Mike: Let's try finding the derivative (deriving $3x^3$). Yes, right. It is $9x^2$.

Hans: Do you think we find the integral? (writing $\int (9x^2 - 25)dx = \frac{9}{3}x^3 - \frac{25}{1}x + c = 3x^3 - 25x + c$).

The claim that the function represented a polynomial function of degree three was put forward by Hans. This claim was then triggered for mathematical justification by Mike and Leo. Then, Mike realized that the function was a polynomial function of degree three (i.e. with the first term of $3x^3$) by determining the integral of its derivative function. Mike reconfirmed this claim by redefining the derivative of $3x^3$ that yielded $9x^2$. This claim was confirmed by Hans, which also determined the integral of $f'(x) = 9x^2 - 25$.

The use of generalisable argument (L4)

The group's use of generalisable argument was shown during their discussion about the claim that b is the first derivative of a . In the previous discussions, they had agreed that a is a three-degree polynomial function and b is a quadratic function. After exemplifying that the derivative of a is $f'(x) = 9x^2 - 25$ (L3), they claimed that b is the first derivative of a , which directed their attention to graph b . We observed a further effort using generalisable argument

when they identified that the x -intercepts on graph b are the same with the stationary points of graph a , which they argued to be at $x = \frac{3}{5}$ and $x = -\frac{3}{5}$.

Hans: It means that the stationary points are here (marking the stationary points in curve a) (see Figure 1, graph a)

...

Mike: Oh, yes (agreeing that the x -coordinate of the stationary points are $\frac{3}{5}$ and $-\frac{3}{5}$).

Hans: It is the same here, though (on graph b , scaling from O to the x -intercept and wrote $\frac{3}{5}$) (see Figure 1, graph b)

Mike, Leo: (looking at graph b)

Hans: The stationary here (pointing at graph a), intercept of x here (pointing at graph b). [L4]

Leo: Does it matter? [T]

Hans: I think it matches, right?

Mike: How? [T]

Hans: Like here is the function (pointing at graph a) and 0 here in the derivative (pointing at graph b). [L4]

Mike: But it does not change anything, right?

Hans: Yeah.

Hans associated the coordinates of stationary points of a function with the x -intercepts of its derivative function. We considered that somehow Hans understood that if f was stationary at x , then $f'(x) = 0$, which he tried to offer to his peers. Yet we felt that this justification, initiated by Hans, was accepted but not shared by his peers as they went back to the function equation for the justification later.

Student's role within group

By tracing who offered initial justifications and who triggered them, we see that each student played a different role while interacting during the problem-solving process (Table 2). We saw that the initial justifications for claims 1, 3, and 5 were offered by Hans, while Mike offered them for claims 2 and 4, and Leo repeated Hans's justification for claim 1. By questioning claims or confirming their validity, we observed that Mike and Leo often trigger others' justifications for each claim. During the problem-solving process, Leo only interacted by triggering or continuing the mathematical justifications made by others. Hans and Mike interacted by both offering and triggering mathematical justifications.

Discussions and Conclusions

The group managed to agree on levels 2 to 4 of justification during the problem-solving process. They tried to connect the data to the intended claim by considering a plausible explanation linking the data and the claim. Tracing back to the case, Hans was mostly found to initiate discussions leading to level 3 or 4 justifications (e.g., for claims 1 and 4). This finding reiterated other studies that found students of high achievement tended to offer higher-level mathematical justifications (Chua, 2016). We also found that students with low achievement offered mathematical justifications at lower levels, repeated others' justifications, or did not offer mathematical justifications at all, as reported in other studies (Whitacre et al., 2017). We also observed that mathematical justifications were mostly initiated by Hans. These mathematical justifications were then met with triggers from Mike or Leo. The triggers were then followed by either an acceptance of the initial justifications or the refinement of them by either Hans or Mike. However, for some cases, the justification offered by high achiever which was failed to be understood would be merely accepted as long as it did not change the agreement on the claim. For instance, when Hans offered the justification that if $a(x)$ was stationary at x ,

then $a'(x) = 0$ to support claim 3, his peers were more inclined towards the equation example related to claim 1. As triggers arose from doubts or confusion, we observed efforts to accommodate achievement differences within the group (An & Zhang, 2024) and acceptance of other students' explanations by lower-achievement students (Wu, 2024). We also noted a possibility that the lower-achievement students put their high-achievement peer as a source of authority (Bleiler-Baxter et al., 2023), which led us to reconsider our coding for level 1 for future studies, especially in the context of CPS. The group dynamics suggest that the teachers should provide scaffolding that encourages each student to construct and refine justifications, rather than relying on the high achiever in the group. Instructions should integrate prompts or structures that distribute opportunities for justification more equitably, regardless of achievement level.

Table 2

Examples of student's justifications and triggers

Claim	Sample utterances of justifications (L0-L4) or triggers (T) by		
	Hans	Mike	Leo
Claim 1: Function a is a polynomial function of degree three	(L3) using an example that a derivative function of a is $9x^2 - 25$, which is of two-degree (L3) using $x = 0$ as an example that $f'(x) < 0$	(T) "There is no function (written) on the task", "the derivative is $18x$ ". (T) "is the function three degree?"	(L3) using $x = 0$ as an example that $f'(x) < 0$ (T) "is $9x^2 - 25$ the derivative?"
Claim 2: The derivative of function a is a quadratic function	(L4) determining the integral of $9x^2 - 25$ (T) "Is this true?"	(L4) using derivative rule to determine function a	
Claim 3: Function b is the first derivative of function a and b is quadratic	(L2) using the shape of graph b and b has a stationary point (L4) stated that if $a(x)$ was stationary at x , then $a'(x) = 0$	(T) How?	(T) Does it matter?
Claim 4: The derivative of function b is a linear function		(L4) using derivative rule to determine function b (T) "For x^2 , the shape is like this (of the graph, pointing at curve b), what about only x ?"	(T) " c has three stationary points, is c the derivative of b ?"
Claim 5: Function c is not the first derivative of function b	(L2) the graph of function c is not a line	(T) "Why is that so?"	
Final claim: Andi's statement in the task is incorrect	(L4) stated that Andi's statement, "graph b is f' , graph c is f " is not supported by the data		

While this study has described mathematical justifications of students in heterogeneous groups in terms of the level of mathematical justifications and students' interactions within each group during CPS, we acknowledge some limitations. First, the notably small participant in one group limited the generalizability of the findings. Second, despite no explicit instruction to promote justification being observed prior to this study, no thorough observation was made on whether the justification norm was implicitly instilled or existed. Third, the study was set outside the classroom, which might have constrained the interpretation of the findings toward a natural classroom setting.

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References

- An, S., & Zhang, S. (2024). Effects of ability grouping on students' collaborative problem solving patterns: Evidence from lag sequence analysis and epistemic network analysis. *Thinking Skills and Creativity*, 51(August 2023), 101453. <https://doi.org/10.1016/j.tsc.2023.101453>
- Bieda, K. N., Conner, A. M., Kosko, K. W., & Staples, M. (2022). *Conceptions and Consequences of Mathematical Argumentation, Justification, and Proof* (K. N. Bieda, A. Conner, K. W. Kosko, & M. Staples (eds.)). Springer International Publishing. <https://doi.org/10.1007/978-3-030-80008-6>
- Bleiler-baxter, S. K., Kirby, J. E., & Reed, S. D. (2023). Understanding authority in small-group co-constructions of mathematical proof. *Journal of Mathematical Behavior*, 70(November 2022), 101015. <https://doi.org/10.1016/j.jmathb.2022.101015>
- Brodie, K. (2010). *Teaching Mathematical Reasoning in Secondary School Classrooms* (K. Brodie (ed.); Vol. 148). Springer US. <https://doi.org/10.1007/978-0-387-09742-8>
- Carpenter, T. P., Levi, L., Berman, P. W., & Pligge, M. (2005). Understanding Mathematics and Science Matters. In T. A. Romberg, T. P. Carpenter, & F. Dremock (Eds.), *Understanding mathematics and science matters*. Routledge. <https://doi.org/10.4324/9781410612618>
- Chua, B. L. (2016). Examining Mathematics Teachers justification And Assessment Of Students justifications. *Proceedings of the 40th Conference of the International, August*.
- Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of Classroom Mathematics Traditions: An Interactional Analysis. *American Educational Research Journal*, 29(3), 573–604. <https://doi.org/10.3102/00028312029003573>
- Cobb, P., Yackel, E., & McClain, K. (2000). *Symbolizing and Communicating in Mathematics Classrooms: Perspectives on Discourse, Tools, and Instructional Design*. Routledge.
- Ekawati, A., Siswono, T. Y. E., & Lukito, A. (2025). Collective Argumentation through Scaffolding: Homogeneous and Heterogeneous Groups in Solving Mathematics Tasks. *Mathematics Education Journal*, 19(2), 217–240. <https://doi.org/10.22342/mej.v19i2.pp217-240>
- Fredriksdotter, H., Norén, N., & Bråting, K. (2022). Investigating grade-6 students' justifications during mathematical problem solving in small group interaction. *Journal of Mathematical Behavior*, 67(August), 100972. <https://doi.org/10.1016/j.jmathb.2022.100972>
- Fuchs, L. S., Fuchs, D., Hamlett, C. L., & Karns, K. (1998). High-achieving students' interactions and performance on complex mathematical tasks as a function of homogeneous and heterogeneous pairings. *American Educational Research Journal*, 35(2), 227–267. <https://doi.org/10.3102/00028312035002227>
- Lin, S.-S., & Mintzes, J. J. (2010). Learning Argumentation Skills Through Instruction in Socioscientific Issues: The Effect Of Ability Level. *International Journal of Science and Mathematics Education*, 8(6), 993–1017. <https://doi.org/10.1007/s10763-010-9215-6>
- Simon, M. A., & Blume, G. W. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior*, 15(1), 3–31. [https://doi.org/10.1016/S0732-3123\(96\)90036-X](https://doi.org/10.1016/S0732-3123(96)90036-X)
- Sowder, L., & Harel, G. (1998). Types of Students' Justifications. *The Mathematics Teacher*, 91(8), 670–675. <https://doi.org/10.5951/mt.91.8.0670>
- Staples, M. E., Bartlo, J., & Thanheiser, E. (2012). Justification as a teaching and learning practice: Its (potential) multifaceted role in middle grades mathematics classrooms. *Journal of Mathematical Behavior*, 31(4), 447–462. <https://doi.org/10.1016/j.jmathb.2012.07.001>
- Whitacre, I., Azuz, B., Lamb, L. L. C., Bishop, J. P., Schappelle, B. P., & Philipp, R. A. (2017). Integer comparisons across the grades: Students' justifications and ways of reasoning. *Journal of Mathematical Behavior*, 45, 47–62. <https://doi.org/10.1016/j.jmathb.2016.11.001>
- Wu, J. (2024). Research on Individual Authority and Group Authority Relations in Collaborative Problem Solving in Middle School Mathematics. In Y. Cao (Ed.), *Students' Collaborative Problem Solving in Mathematics Classrooms* (pp. 75–99). https://doi.org/10.1007/978-981-99-7386-6_4
- Wyman, P. J., & Watson, S. B. (2020). Academic achievement with cooperative learning using homogeneous and heterogeneous groups. *School Science and Mathematics*, 120(6), 356–363. <https://doi.org/10.1111/ssm.12427>

An Australian-Ghanaian Comparison of STEM Education

Contexts: Supporting STEM Reasoning

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This paper compares STEM contexts in two Australian and Ghanaian education studies to examine how they supported children's mathematical and scientific reasoning. Analysis was guided by a sociocultural perspective and Gilbert's (2006) attributes of context. Findings show that coherent, activity-focused contexts enabled children to engage with mathematical and scientific reasoning through shared inquiry. Across both settings, context mediated participation and shaped which forms of reasoning became visible and available for pedagogical response. This research highlights the importance of selecting meaningful STEM contexts that foreground explanation and justification.

STEM education can be siloed, integrated and multidisciplinary, and transdisciplinary. In a siloed approach for example, a teacher in a primary science lesson may rely upon their students' knowledge of mathematical concepts to measure temperature. In an early childhood setting, teachers may present a multidisciplinary approach through scientific exploration of crab colonies while making counting experiences explicit to support students to count crabs hiding on a leaf (English, 2025). Informed by a transdisciplinary approach, students may explore area as a measurement concept to better understand how plants generate sugars through the scientific process of photosynthesis (Fry & English, 2023). In each of these examples, choice of context is significant and frames the STEM teaching and learning experience. This paper explores STEM education contexts framing research in Australia and Ghana, to capture a comparative view of possibilities for promoting scientific and mathematical reasoning in early years settings (ages 4-8) and middle-years primary classrooms (ages 8-10).

Introduction

Selecting suitable contexts for STEM learning presents a persistent challenge for teachers as they plan meaningful mathematics and science classroom experiences. In Australia, STEM education has been criticised for frequently neglecting real-world contexts (ACARA, 2025). A large-scale study by Jones et al. (2025), examining STEM learning contexts across 47 Australian schools found that where the dominant vision of STEM was to promote general familiarity with the discipline, over half of the reported teaching-learning experiences relied on coding and robotics or were largely devoid of meaningful context. The authors called for a shift towards contexts that incorporate localised action and socially oriented or activist dimensions of STEM learning. Jones and colleagues (2025) further argued that engagement with authentic and sometimes controversial issues, such as climate change, may help address declining enrolments in STEM-related subjects. While the Australian Curriculum authority acknowledges the importance of purposeful connections across learning areas, it cautions that forced or superficial integration can diminish learning impact (ACARA, 2025). Consequently, careful attention is required when selecting STEM contexts to ensure that they authentically ground learning experiences and support students' engagement with disciplinary ideas involving mathematics and science.

In Ghana, attention to context is explicitly articulated at the level of curriculum policy through the Standards-Based Curriculum (SBC), which positions kindergarten and lower (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 162-169). MERGA.

primary learning as inquiry-oriented, learner-centred, and grounded in children's everyday experiences (National Council for Curriculum and Assessment [NaCCA], 2019). The SBC encourages teachers to draw on familiar home, school, and community contexts to support conceptual understanding, problem-solving, and reasoning across subject areas, including mathematics and science. This orientation reflects a deliberate shift away from transmission-focused pedagogies towards constructivist and sociocultural approaches that conceptualise learning as socially mediated through interaction, language, and culturally situated activity. Drawing on Bourdieuan perspectives, Agbenyega (2018) further argued that transformational early childhood pedagogy in Ghana requires learning experiences embedded within children's social worlds and attentive to cultural capital, habitus, and power relations.

Despite these policy aspirations, research across both countries highlights challenges in teachers' capacities to enact integrated and transdisciplinary, context-rich STEM learning. In Australia, limited professional preparation in integrated STEM has contributed to teachers' lack of confidence in designing and implementing authentic STEM experiences (Ku et al., 2025). Ku and colleagues (2025) identified context as a critical stage within an integrated STEM instructional model, emphasising authentic, problem-based, and student-centred contexts grounded in learners' lived experiences. Parallel concerns are well documented in Ghanaian early childhood and basic education research. Empirical studies show that many kindergarten and early grade teachers feel insufficiently prepared to design and implement integrated, context-rich learning experiences, citing limited pre-service training and inadequate continuous professional development aligned with the Standards-Based Curriculum reforms (Agbosu, 2022; Ntumi et al., 2023). Ghanaian studies consistently report gaps between curriculum intentions and classroom enactment. Although learner-centred and inquiry-based pedagogies are emphasised in policy, classroom practices are often constrained by teacher preparedness, instructional routines, and limited professional learning opportunities (Agbosu, 2022; Cobbold, 2017; Ntumi et al., 2023; Sofu et al., 2019).

Against this backdrop, we examined how STEM context choices supported young children in sharing their mathematical and scientific reasoning in Australian and Ghanaian classrooms. We saw the role of context in STEM educational experiences as an entry point into better understanding how young children can be supported to reason, with the view that context frames the educational experience. In the research presented below, STEM contexts are conceptualised as transdisciplinary and integrated learning settings in early to middle primary years classrooms in which mathematical reasoning (e.g., measurement, comparison, statistics), scientific reasoning (e.g., observation, prediction), and engineering reasoning (e.g., design, testing) are simultaneously elicited. Adopting a sociocultural perspective that foregrounds context as a mediator of learning opportunities, this paper explores how two STEM education experiences were framed in both countries to support reasoning. By analysing how contexts are represented and mobilised across the two settings, the study seeks to identify focal characteristics of STEM contexts that can inform teachers' planning, noticing, and pedagogical responses to children's emerging STEM ideas, thereby contributing to comparative and classroom-based STEM education research.

Context

Context is defined in this study as an event or setting that weaves together and gives meaning to words, actions, and concepts, providing a frame for teaching and learning. In classroom practice, teachers commonly use context to support meaningful exploration, guide discourse, and shape representations, positioning context as a potential leverage point for reasoning. Early work by Sullivan, Zevenbergen, and Mousley (2002) highlighted the importance of relevant and realistic contexts in mathematics education, while also cautioning that context selection is complex. They argued that teachers must consider mathematical

suitability, relevance to learners, motivational impact, and the risk of exclusion when choosing problem-solving contexts.

In Ghanaian early years classrooms, STEM education contexts that foreground mathematics and science are typically drawn from children's everyday social and cultural experiences, including market activities, household routines, environmental exploration, and the use of locally available materials (Agbenyega, 2018). Such contexts provide opportunities for children to engage in counting, measuring, comparing quantities, observing change, and explaining outcomes, supporting the integration of mathematical and scientific reasoning within shared activity (Fleer, 2019). Within the Ghanaian Kindergarten Curriculum, learning is organised around integrated thematic strands rather than discrete STEM subjects (NaCCA, 2019). However, research indicates that the educational potential of context depends on how it is pedagogically mobilised; when contexts are used merely to organise activity rather than to support sustained inquiry and dialogue, opportunities for deeper reasoning may be limited (Cobbold, 2017; Ntumi et al., 2023; Sofo et al., 2019).

Conceptual bridge via Gilbert's (2006) attributes of context

Building on Gilbert's (2006) conceptualisation of context as a focal event that gives coherence and meaning to learning, four attributes of context were adapted in this study to support analysis of STEM education contexts. Rather than serving as a passive backdrop, context is positioned as an active mediator of participation, meaning-making, and disciplinary sense-making within STEM learning experiences (Gilbert, 2006). Collectively, these attributes provide a lens for examining how mathematical and scientific reasoning is enabled, constrained, and made visible through classroom activity. Significantly, Gilbert (2006) frames the four attributes of a context by relevance (for students) and supporting meaning-making. We have adapted to add a specific STEM emphasis. The first attribute (a), the value of the setting as a social, spatial, and temporal framework, highlights how contexts organise participation and shape which ideas are taken up during STEM tasks. The context becomes the setting for a community of practice that values student participation. Socioculturally, these settings shape interaction patterns and the social distribution of knowledge, influencing learners' opportunities to reason and what teachers can notice during instruction (Mousley, 2003; Robbins, 2005). The second attribute (b) refers to the behavioural environment of the activities that take place, emphasising the kinds of tasks or encounters that would take place and framing the talk that students need to use. The third attribute (attribute c), the promotion of mathematical and scientific language and reasoning, aligns directly with the focus of this paper. Contexts that foreground reasoning as a shared classroom practice support the articulation of relationships, testing of ideas, and interpretation of evidence, shaping how reasoning becomes visible through talk, representation, and action (Mousley, 2003). Tasks requiring explanation, justification, comparison, or interpretation create opportunities for reasoning to emerge and moments for teachers to notice learner's thinking and respond instructionally (Gilbert, 2006; Sullivan et al., 2002). The fourth attribute (d) highlights how contexts build on learners' existing experiences as resources for sense-making. When contexts resonate with learners' lived experiences, they support richer explanation and justification, while providing teachers with entry points for noticing emerging reasoning and scaffolding movement toward more disciplinary forms of understanding (Gilbert, 2006; Robbins, 2005).

Theoretical background

Using a sociocultural perspective, a comparative lens offers the chance to examine the researchers' selections of STEM contexts to frame their work in Australia and Ghana, to generate characteristics of STEM contexts that could shape children's mathematical and scientific sense-making, as well as to consider pre-emptively how STEM contexts may support teachers to notice and respond to children's emerging STEM ideas. Through a sociocultural

perspective, the social and the person are inextricably interwoven, and we argue, the context contributes and shapes the social distribution of knowledge. Social activity seemed to have the strongest bearing on what mathematical ideas were made available to children in Mousley's (2003) study grounded in Situated Cognition. With a knowledge focus, Mousley (2003) discussed the important role of the social in raising mathematical concepts to a level of consciousness. Resnick and Stieff (2024) examined the theoretical roles of cognition and context in mathematics education research, arguing that these roles are intertwined by positioning context as a mediator of cognition—even in the learning of abstract mathematics. They note that qualitative studies must provide rich description in order to identify “the kinds of relevant contexts which exist and why they are functionally meaningful” (Resnick & Stieff, 2024, p.282). Similarly, we consider the selection of STEM contexts important for framing social activity and mediating discussion, and for emphasising the teaching and learning of scientific and mathematical knowledge.

Research Design

The research question guiding this exploratory study is: *how are young children's STEM education experiences framed in Australia and Ghana to support scientific and mathematical reasoning?* This research explores Australian and Ghanaian early- and middle-years STEM education contexts within the researchers' bodies of work (science- and mathematics-focused). Selected cases were relevant to addressing the research question. Both studies were conducted in classrooms, generating rich and diverse data including video recordings and detailed field notes. Qualitative meta-analysis (Ulloa & Schwerer, 2024) supported a cross-study interpretation of these data and research experiences, and existing research, to generate new findings focused specifically on the contexts framing each study. We conducted a deductive, framework-guided analysis, using the attributes of Gilbert's (2006) framework as coding categories in one study by each of the first two authors. Braun and colleagues (2018) refer to such an approach as *codebook Thematic Analysis*. Findings can support teachers in planning, noticing, and scaffolding both mathematical and scientific reasoning simultaneously, supporting teachers to respond to children's emerging STEM ideas during inquiry-based tasks.

Results

Australia: Context-Mediated Mathematical Reasoning in Agricultural Science Activity

Fry's (2025) study examined STEM exploration in a Year 4 Australian classroom using a sociocultural approach, focusing on how shared mathematical practices emerged within a scientific context of plant growth (Table 1, column 2). The classroom activity centred on data collection and interpretation, with students engaging with mathematical ideas embedded in a YouTube clip depicting the growth (time lapse) of a capsicum plant (see Fry, 2025). Students were challenged to graph plant growth, requiring them to encode and represent multivariate features of growth.

The shared scientific context structured collective engagement and supported the coordinated use of cultural tools, including language, representations, and classroom routines. The teacher framed the task around the meaningful challenge of representing plant growth in agricultural science, which supported sustained engagement and sense-making in science (attribute a). Students' activity mirrored practices used by agricultural scientists and data scientists, as they made decisions about which growth features to attend to and how to represent them graphically (attribute b). The context supported the use of both mathematical and scientific language, with students relationally describing plant height, growth patterns, leaves, and life-cycle stages, while also attending to statistical graphing conventions such as axes, scaling, and appropriate conventions for displaying statistical information (attribute c). Without explicit instruction, students produced graphs on A3 paper and incorporated visual features inspired by

infographics to encode salient aspects of plant growth. Extra-situational background knowledge was drawn from concurrent science studies on plant life cycles and energy transfer within food chains, situating the activity within broader understandings of agricultural science (attribute d).

Ghana: Context-Mediated Mathematical Reasoning in Engineering Construction Activity

Amo-Darko's (2025) study examined STEM activity in early and primary years classrooms in Ghana through a sociocultural lens, focusing on how children's mathematical reasoning emerged within engineering construction contexts (Table 1, column 3). Classroom activities centred on constructing houses, household items (e.g. tables and chairs), and wheeled artefacts using manipulatives. Through these activities, children engaged with mathematical ideas embedded in the selection, arrangement, and testing of materials.

Engineering construction tasks functioned as shared, time-bound classroom events that elicited mathematical reasoning (attribute a). While building and material use were familiar to children through everyday experiences, the classroom focus was on designing stable structures and objects capable of movement, providing a purposeful setting for exploring mathematical ideas. Children's engagement mirrored engineering design practices, requiring them to construct, measure, compare, and test structures as they worked toward design goals (attribute b). Decisions about the number of blocks, placement of wheels, and relative size and length of components supported sustained reasoning related to quantity, measurement, comparison, and spatial arrangement. The construction context also supported the use of specific mathematical and scientific language during interaction, with children using mathematical language related to quantity, measurement, and spatial relations (e.g., more, less, longer, same size, next to), alongside scientific language used to explain stability, movement, and cause-and-effect relationships (e.g., balance, fall, move, stay up) (attribute c). Teachers' prompts drew attention to these forms of reasoning as constructions were tested and revised. Children's reasoning was further supported by connections to extra-situational background knowledge drawn from everyday experiences with building, balance, and movement (attribute d). Teacher interviews indicated that these familiar experiences were intentionally leveraged to support learning. In this way, engineering construction functioned as a STEM education context that mediated mathematical reasoning, with science providing explanatory meaning and engineering activity structuring opportunities for engagement.

Table 1 summarises the key attributes of the STEM education contexts identified in the Australian and Ghanaian studies, drawing on Gilbert's (2006) framework to highlight how context mediated mathematical and scientific reasoning across both settings.

Table 1

Attributes of STEM education contexts used in the researchers' studies in Australia and Ghana

Attributes of context (Gilbert, 2006), framed by relevance and meaning-making	Australian Study (Author, 2025)	Ghanaian Study (Author, 2025)
a) Value of the setting as a social, spatial, and temporal framework	YouTube clip depicting plant growth	Classroom engineering construction tasks using manipulatives as shared, time-bound building and testing events
b) Behavioural environment of the activity	Mirroring the work of agricultural scientists and data scientists in agriculture	Mirroring engineering design practices through constructing, measuring, comparing, and testing
c) Use of specific language (supporting mathematical and scientific reasoning)	Encoding multivariate features depicting plant growth	Mathematical language of quantity, measurement, comparison, and spatial

		relations, with scientific language of stability and motion
d) Relationship to extra-situational background knowledge	Life and living studies in science, life stages of a plant	Everyday experiences with building, balance, and movement support informal measurement and engineering concepts

Cross-Context Patterns in Reasoning and Noticing

Across the Australian and Ghanaian studies, the findings highlight how STEM contexts functioned as mediating structures that shaped the visibility of both mathematical and scientific reasoning within classroom activity. In both settings, student reasoning emerged through participation in coherent, activity-focused contexts; in data representation within a scientific inquiry in Australia; and in engineering construction activity in Ghana, rather than as isolated individual responses. These contexts enabled learners to engage with mathematical ideas (e.g., representation, measurement, comparison) and scientific ideas (e.g., growth, stability, cause–and–effect) through shared interaction, representation, and talk, replicating professional STEM professions in an informal manner. In both cases, the STEM context influenced which forms of mathematical and scientific reasoning became observable and available for pedagogical response. Context supported shared engagement, and clear purposes enabled teachers (and the researchers) to more readily notice, interpret, and extend learners’ reasoning (Amo-Darko, 2025; Fry, 2025).

Discussion

Context, Task Framing, and the Social Organisation of Reasoning

This study examined how STEM education contexts, in the researchers’ own studies, mediated mathematical and scientific reasoning in Australian and Ghanaian classrooms, drawing on a sociocultural perspective to interpret how participation, interaction, and pedagogical action were organised within activity. We draw on Resnick and Stieff’s (2024) work to substantiate our finding that the contexts framing the STEM research across both nations, curricula, and learning settings, functioned to support mathematics learning. Across both studies, the findings show that context functioned not merely as a backdrop for learning, but as a mediating structure that shaped which forms of reasoning became visible, attainable, and pedagogically supportable.

The STEM contexts organised the social conditions in each classroom, shaping patterns of interaction and access to disciplinary practices. In the Australian study, the scientific focus on plant growth provided a coherent frame for collective engagement, enabling students to appropriate mathematical practices related to data representation through dialogue, representation, and negotiated meaning. Similarly, in the Ghanaian study, engineering construction activities structured classroom interaction around shared material activity, with reasoning related to measurement, comparison, stability, and movement emerging through talk, gesture, and action. Across both cases, context functioned as a mediating structure that shaped participation and made reasoning available for pedagogical response, consistent with sociocultural perspectives on learning (Mousley, 2003). Contexts supported clear purposes, sustained interaction, and appropriate cultural tools enabled reasoning to emerge through talk, action, and representation rather than through isolated task completion. The findings suggest that, despite differences in curricular structures and classroom conditions, principles related to context coherence, interactional organisation, and the integration of mathematics and science within STEM activity were relevant across diverse early years and primary middle-years settings.

Task framing further influenced the visibility of reasoning. When activities were framed as shared inquiry tasks requiring interpretation, justification, or testing of ideas, mathematical and

scientific reasoning became observable through collective interaction and representation. In contrast, procedurally framed or time-constrained tasks can limit learners' opportunities to articulate and extend their thinking. These findings suggest that meaningful reasoning depends on the alignment of context and task design rather than context alone (Gilbert, 2006; Sullivan et al., 2002).

In the Australian study, Fry (2025) demonstrated that mathematical reasoning related to data representation emerged through collective engagement in a scientifically framed inquiry on plant growth. The use of a time-lapse video as a shared artefact structured students' attention to multivariate features of growth and supported the coordinated use of mathematical and scientific language. Through graphing and discussion, students appropriated statistical conventions and life science concepts via interaction, representation, and negotiated meaning, illustrating how a coherent scientific context organised shared participation and supported the social construction of mathematical reasoning.

Similarly, in the Ghanaian context, Amo-Darko (2025) showed that engineering construction activities functioned as focal STEM contexts through which children's mathematical reasoning became visible as children interacted with shared materials during construction. Mathematical ideas related to quantity, measurement, comparison, and spatial arrangement were enacted as children selected materials, tested constructions, and revised designs, while scientific reasoning supported explanations of stability, movement, and cause-and-effect. As in the Australian study, reasoning was mediated through socially organised activity, with context shaping opportunities for noticing, discussion, and pedagogical response.

The findings highlight the importance of positioning context as a core pedagogical resource in early years and middle-primary years STEM education rather than as a motivational add-on. Teachers should be supported in selecting socially meaningful contexts and in framing tasks that foreground explanation, comparison, prediction, and justification, thereby increasing the visibility of children's mathematical and scientific reasoning. These findings suggest that strengthening teachers' capacity to design and adapt STEM contexts that foreground reasoning may be a productive pathway for supporting noticing and responsive pedagogy in early years and primary years STEM classrooms. The findings also demonstrate the value of examining STEM contexts as units of analysis. Future research could investigate how attributes of context interact with task framing and teacher noticing over time, particularly through classroom-based and video-supported methodologies. Comparative studies across diverse settings would further clarify which principles of context-based STEM learning are transferable and which require local adaptation.

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References

- Agbenyega, J. S. (2018). Examining early childhood education system in Ghana: How can Bourdieusian theorisation support a transformational approach to pedagogy? In M. Fleer & B. van Oers (Eds.), *International handbook of early childhood education* (pp. 1–18). Springer. https://doi.org/10.1007/978-94-024-0927-7_32
- Agbosu, D. A. (2022). *Challenges kindergarten teachers face in the implementation of the new kindergarten curriculum in the Agona East District* [Doctoral dissertation]. University of Education, Winneba, Ghana.
- Amo-Darko, B. (2025). *Context-mediated mathematical reasoning in engineering construction activities in early childhood classrooms in Ghana* [Manuscript in preparation]. University of Education, Winneba.
- Australian Curriculum, Assessment and Reporting Authority (ACARA). (2025). *Approaches to STEM learning*. <https://www.australiancurriculum.edu.au/resources/stem-connections/approaches-to-stem-learning>.
- Braun, V., Clarke, V., Hayfield, N., & Terry, G. (2018). Thematic Analysis. In P. Liamputtong (Ed.), *Handbook of research methods in health social sciences*. Springer Nature Singapore Pte Ltd. https://doi.org/10.1007/978-981-10-2779-6_103-1
- Cobbold, C. (2017). Moving from page to playground: The challenges and constraints of implementing curriculum in Ghana. *Research on humanities and social sciences*, 7(4).
- Fleer, M. (2019). Scientific playworlds: A model of teaching science in play-based settings. *Research in Science Education*, 49, 1257–1278. <https://doi.org/10.1007/s11165-017-9653-z>
- Fry, K. (2025). Classroom data science: data and dataing for making sense of multivariate plant growth. *ZDM—Mathematics Education*, 57(1), 75–86. <https://doi.org/10.1007/s11858-025-01655-6>.
- Fry, K., & English, L. D. (2023). How big is a leaf? Mathematical modeling through STEM inquiry. *Mathematics Teacher: Learning and Teaching PK-12*, 116(2), 99–107.
- English, L. D. (2025). Spatial modelling in second grade: Creating crab colonies. *ZDM Mathematics Education* 57, 259–273. <https://doi.org/10.1007/s11858-025-01667-2>
- Gilbert, J. K. (2006). On the nature of “context” in chemical education. *International Journal of Science Education*, 28(9), 957–976.
- Jones, M., Geiger, V., Falloon, G., Fraser, S., Beswick, K., Holland-Twining, B., & Hatisaru, V. (2025). Learning contexts and visions for STEM in schools. *International Journal of Science Education*, 47(3), 337–357. <https://doi.org/10.1080/09500693.2024.2323032>
- Ku, C.-J., Lin, K.-Y., Kwon, H., & Kelley, T. (2025). A six-stage instructional design model for collaborative implementation of integrated STEM education. *Journal of Technology Education*, 36(2), 25–60. <https://doi.org/10.21061/jte.v36i2.a.3>
- Mousley, J. A. (2003). *Mathematical and pedagogical understanding as situated cognition*. [Research paper]. International Group for the Psychology of Mathematics Education, Hawaii. <https://files.eric.ed.gov/fulltext/ED501036.pdf>
- National Council for Curriculum and Assessment (NaCCA). (2019). *Kindergarten Curriculum (KG 1 & KG 2)*. Ministry of Education, Ghana.
- Ntumi, S., Agbenyo, S., Tetteh, A., Yalley, C. E., Yeboah, A., & Nimo, D. G. (2023). Teacher preparedness and implementation of the national pre-tertiary education curriculum framework in Ghana. *Journal of Educational Research and Practice*, 13(1), Article 16.
- Resnick, I., & Stieff, M. (2024). The role of theory in research and practice: Characterizing cognition and context within mathematics education. In M. A. (Ken) Clements, B. Kaur, T. Lowrie, V. Mesa, & J. Prytz (Eds.), *Fourth International Handbook of Mathematics Education, Springer International Handbooks of Education* (4 ed., pp. 271–292). Springer. https://doi.org/10.1007/978-3-031-51474-6_11
- Robbins, J. (2005). Contexts, collaboration, and cultural tools: a sociocultural perspective on research children’s thinking. *Contemporary Issues in Early Childhood*, 6(2), 140–149.
- Sofo, S., Asola, E. F., & Ocansey, R. (2019). An assessment of Ghana’s enacted kindergarten curriculum. *African Journal of Teacher Education*, 8, 86–110.
- Sullivan, P., Zevenbergen, R., & Mousley, J. (2002). Contexts in mathematics teaching: Snakes or ladders. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.) *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland), pp. 649–656.
- Ulloa, I. C., & Schwerer, J. (2024). Qualitative meta-analysis. In A. J. Heinrich, S. Marguin, A. Million, and J. Stollmann (Eds.). *Handbook of Qualitative and Visual Methods in Spatial Research*. (1st ed., pp. 325–335). Transcript Verlag. <https://doi.org/10.1515/9783839467343>

Assessment Contexts: Oral vs Written Performance in Mathematics Education

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Generative AI has intensified concerns about assessment integrity in both face-to-face and online education, prompting interest in oral examinations as an alternative to written assessments. While oral assessment research is established in universities, little is known about enabling programs. This study compares written and oral mathematics assessments in an enabling cohort. Although group-level performance was similar across delivery modes, individual results differed markedly. Weak, non-significant correlations suggest the two formats capture complementary aspects of student understanding, supporting the value of multimodal assessment for diverse learners.

Introduction

Generative Artificial Intelligence (Gen AI) has significantly disrupted higher education, prompting academics to reassess assessment design due to growing concerns about authenticity, integrity, and graduates' preparedness for the workplace (Dawson, 2024; Nallaya et al., 2024). As online education expands, these problems intensify. These challenges are especially salient in enabling programs, pre-university pathway programs created to help students who do not meet standard university admission criteria to transition into higher education. Oral assessments are gaining renewed attention as they can support academic integrity, identity verification, communication skills, and relational engagement, particularly for diverse and EAL (English as an additional language) learners (Fenton, 2025; Iannone & Vondrová, 2024). However, they also raise concerns around anxiety, fairness, and reliability. The present study offers a timely and novel contribution by directly comparing written and oral mathematics assessments in an enabling cohort, providing new empirical insight into assessment design, student experience, and the diverse ways in which enabling students demonstrate mathematical understanding.

Oral and written assessments

The concept of assessment has always been central to education, but contemporary conditions have introduced new and complex challenges. Generative AI is not the only force driving the need for new assessment approaches. The speedy expansion of online education has also created challenges for assessment modes that should adhere to the principles of validity, reliability and fairness. Akimov et al. (2020) argue that assessment should also have multiple parameters, and its components maintaining academic integrity should be both formative and summative. The authors broaden the requirements for online assessments' design to include considerations about issues of accessibility and legality, identity security and, inevitably, academic integrity. Assessing mathematical understanding is challenging because students' reasoning is not always evident in written work. Since mathematical proficiency involves both procedural skill and conceptual understanding (Videnovic, 2017), oral assessment can offer valuable insight by allowing students to explain their thinking, address misconceptions, and demonstrate understanding in real time.

Online format of Higher Education obliged respective Institutions to find a way to prevent academic fraud and to stop or identify possible violations of the exam rules. Many universities worldwide started to use the technology of online proctoring (the Latin word "proctor" – follow, observe) which includes supervising an exam or its preparation remotely, using a webcam and/or specialised proctoring software. These systems typically collect data such as webcam

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 170–177). MERGA.

images, microphone audio, the computer's screen display, keystrokes entered during the session and conduct some sort of identity verification, such as checking that the student looks like the person on their identity (ID) card (Dawson, 2024).

According to Puchkov et al. (2020), modern proctoring systems can automatically detect various violations, including opening unauthorized applications (some systems block these entirely), running programs that allow remote access, using external devices or books, looking away from the screen, absence or change of the examinee's face, presence of other individuals, or unauthorized sounds and non-verbal communication with people outside the camera's view. In this context, as oral assessments involve co-temporal interaction, they naturally verify the student's identity and comprehension without employing intrusive proctoring solutions. Oral assessments allow lecturers to observe students' real-time reasoning, error correction, and decision-making strategies, elements that are often hidden in written work. Such interactions reveal the student's conceptual frameworks, misconceptions, and strengths, providing rich diagnostic information. The oral assessment format offers a notable time-saving advantage for examiners as well. Although that type of examination demands considerable time and effort from instructors for preparation, scheduling, and administration, it offers a distinct practical advantage: grading occurs immediately and transparently during the session (Theobold, 2021).

As students learn in diverse ways, a uniform approach to mathematics assessment is insufficient. While educators widely acknowledge the importance of differentiated instruction to address student individuality and variability, they must likewise recognise the need for differentiated assessment practices to accurately represent the learning of a heterogeneous student population (Videnovic, 2017). Some researchers argue that written assessment formats are preferable in situations when it is necessary to assess higher-order cognitive skills such as problem solving and critical thinking. The linguistic complexity of mathematical test items unrelated to the content being assessed may contribute to the performance disparities between native English speakers and EAL learners, potentially compromising the validity of achievement tests. In fact, oral assessments can be particularly advantageous for students who speak English as a second language or experience challenges with speech processing/comprehension. In particular, oral examinations provide an opportunity for examiner involvement, facilitating students' ability to interpret concepts, reach shared understanding, and demonstrate conceptual mastery, even when their written English proficiency is limited.

Alongside its benefits, oral assessment entails potential drawbacks. Videnovic (2020) suggests that there is a common perception that oral assessments may provoke greater anxiety in students compared to other assessment formats. In addition, the author attributes this anxiety to two main factors: the unfamiliar nature of oral examinations and the belief that such tasks demand a deeper level of understanding, along with the ability to clearly explain concepts to others. Nevertheless, Iannone & Vondrova (2024) suggest that the novelty of the assessment method can encourage deeper learning. Because students perceive the unfamiliar assessment as more demanding, they may engage in study practices that promote stronger conceptual understanding. However, Ferdoush et al. (2025) express their disappointment, noting that oral assessment can be susceptible to errors such as halo effects, central tendency errors, a general leniency bias, and contrast errors. These biases can impact the accuracy and fairness of the assessment process. The authors also emphasise that studies have shown that scores are directly proportional to the number of words spoken by the examiner and the time taken. Overall, oral assessments are time-consuming and can become a resource-intensive exercise.

Nallaya et al. (2024) note that another major concern for the academic community regarding oral assessment is fairness. The authors refer to the opinion that without interrater reliability, oral assessments risk being perceived as subjective and potentially biased, which undermines their credibility. Moreover, even involving multiple assessors does not necessarily resolve the

challenges associated with achieving consistent scoring across evaluators. That said, Moothadeth (2024) underscores that the use of a rubric system can significantly enhance both the effectiveness and fairness of oral assessments, providing a structured mechanism to ensure objectivity and consistency in evaluation. Rubrics serve as a structured instrument that ensures objectivity and consistency in scoring, reducing the risk of subjectivity and bias often associated with oral evaluations.

Crawford et al. (2022) stress that the enabling environment can be seen as a collaborative community of teachers and learners. Learners arrive at this setting with aspirations for a transformed and enriched life, while teachers share a commitment to supporting these changes. However, working within enabling education also implies significant emotional labour. Teaching diverse and often vulnerable students imposes considerable emotional demands on educators, particularly in assessment contexts where students' anxieties are more visible. The oral assessment format is one such context: students' vulnerability frequently becomes evident, and teachers' capacity to provide emotional reassurance is brought to the forefront. The novelty of the oral assessment format can also amplify feelings of anxiety and discomfort, which in turn may influence student performance outcomes.

Although oral examinations and viva-voice assessments have been explored in undergraduate mathematics contexts, particularly in relation to academic integrity, reliability, and student anxiety (Fenton, 2025), there is currently no published research examining oral mathematics assessment within Australian enabling programs. Existing studies of enabling mathematics focus on curriculum, learner characteristics, critical pedagogy impacts, preparedness, and numeracy development rather than on assessment modality (Baker et al., 2022). Likewise, the enabling education literature acknowledges that oral presentations and performance-based tasks can heighten student vulnerability and anxiety (Crawford et al., 2022). Yet, these discussions are not discipline-specific and do not compare oral and written assessment outcomes. In other words, research offers little insight into enabling students' performance across different mathematical assessment types or the extent to which these modalities draw on separate competencies. This study explores the benefits and limitations of oral assessment as a context-sensitive response to contemporary assessment challenges.

Methodology

Oral dialogic assessments can be organised in several ways. In some cases, specific questions are not negotiated with students; instead, the examiner selects questions based on material previously covered in class. Alternatively, students may not know the exact questions in advance but could be asked to choose one from a list or draw a question from a set, with varying amounts of preparation time allowed (Iannone, 2023). Another approach involves providing students with a bank of theoretical questions, such as theorems and proofs, beforehand. The examiner then arranges these into "tickets," each containing two theoretical questions and one practical question unknown to the student. The tickets are numbered and placed face down on a table, with the student selecting one and being given time to prepare before responding. This paper discusses assessment based on interrogations, in which the student is quizzed by one or more examiners, and the short interview of students to confirm their authorship of a written paper (Joughin, 2010).

This is a cross-method study that compares the performance of 64 students on written tests and quizzes with their performance in oral interviews conducted as part of the assessment tasks in an Introductory Mathematics and Statistics unit offered through an enabling program. The program provides an alternative pathway into undergraduate degrees by equipping prospective students with the academic skills necessary for successful Higher Education in one of the regional Universities in Australia. The authors, both lecturers, are actively involved in teaching the unit. These raw scores were subsequently normalised to proportional values ranging from 0

to 1. Although results were expressed as percentages when drawing parallel boxplots to compare written and oral scores, other statistical analyses were performed using the normalised decimal scores.

Rahman (2011) emphasises that any well-structured examination demands considerable time and effort from examiners. The key challenge is to identify assessment instruments that justify this investment by delivering meaningful educational value. So, before implementing the oral interview, researchers carefully considered several key factors:

- Which skills and proficiencies should be assessed (e.g., conceptual knowledge, practical mastery, analytical reasoning)?
- What should be the size of the question bank?
- Should students receive the exact list of questions in advance?
- How many questions should each student answer, and how many attempts should be permitted?
- Should the assessment include theoretical questions (definitions, key concepts) or focus solely on practical ones?
- Should students be allowed to use calculators or consult notes?
- Should preparation time be provided, and should the interview be timed?

A 15-minute timed interview was ultimately selected, consisting of three questions (both theoretical and practical) drawn from previous assignments and coursework. Students were given one minute of preparation time, permitted to use notes but not calculators. The question bank comprised 30 questions, and students selected their own by choosing three numbers between 1 and 30 “on the spot”. The answers were assessed against an elementary rubric shown in Table 1. Each question was scored out of four points. Zero marks were given to students who were unable to answer any questions. The interviews were conducted in two formats: face-to-face and online via Teams. All the interviews were recorded for reference.

Table 1

Rubric used for oral assessment

Mark	Descriptor
4	Answered the question straight away, clearly and accurately. No prompt required.
3	Answered correctly after one prompt or hint. Response is mostly accurate with adequate reasoning.
2	Answered only after several prompts or provided a partially correct/vague answer that shows incomplete understanding.
1	Required extensive prompting to get an answer.
0	Unable to give any answer.

The current study was primarily concerned with the following research questions:

- Do oral and written assessments’ results correlate? If not, what does that discrepancy indicate?
- To what extent do the quantitative results differ between face-to-face and online students?

Data Analysis

To analyse the dataset, a combination of statistical methods was utilised. Spearman rank correlation coefficient (r_s) was used to examine the strength and direction of monotonic relationships between the written assignment and oral assignment scores, given its suitability for the data. This coefficient is a nonparametric, distribution-free statistic that measures both the strength and direction of a monotonic relationship between two ranked variables, or between one ranked variable and one continuous measurement variable (Xiao et al, 2015). Researchers frequently examine this association, as its value ranges from -1 (strong negative monotonic

relationship) to +1 (strong positive monotonic relationship). Values near zero suggest a weaker association.

Additionally, independent-samples t -tests were carried out to compare group means, with degrees of freedom calculated to determine the appropriate distribution for hypothesis testing. Statistical significance was assessed using p -values, which provided evidence about the null hypothesis. According to Jankowski et al. (2017), the modern t -test consists of two components: mathematical calculations of the t -statistic and a reference table that provides the distribution of possible t -values. The calculations quantify the difference between the sample means while accounting for variability within the data. The reference table represents all potential t -statistics and their associated probabilities (critical values or p -values) based on sample size.

The p -value represents the probability of rejecting or not rejecting the null hypothesis (H_0). The null hypothesis assumes that there is no effect, no relationship, or no difference between the two groups for a given variable. The “ p ” stands for probability, and the p -value is calculated as the likelihood of observing an effect as extreme as, or more extreme than, the one found, assuming H_0 is true. In fact, the p -value indicates the evidence weight against the Null Hypothesis, the smaller the p -value, the stronger the evidence suggesting H_0 is incorrect (Thiese et al., 2016). In this case, the null hypothesis (H_0) is the assumption that there is no relationship between oral and written assessment scores. The null hypothesis is the one that is test against.

No proctoring software was used to invigilate the written assignments submitted by online students; only time restrictions were imposed. Whereas all face-to-face students completed their written work in class under supervision. For this reason, the written assessment results of online and face-to-face students were analysed separately to avoid distortion related to supervision conditions.

The above multi-method approach ensured a rigorous examination of both associations and differences within the data. With all this in mind, the researchers have tried to determine the degree of convergence or divergence between the two assessment formats and to identify and qualitatively analyse cases where students’ performances diverge, potentially revealing differences in theoretical insight, operational proficiency, test anxiety, verbal processing, or issues related to academic integrity.

Results and Discussion

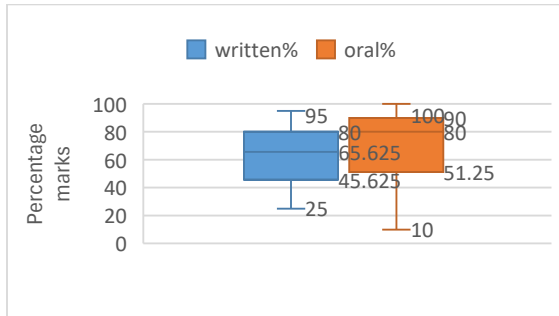
Written versus oral (face-to-face students)

The Spearman correlation (r_s) examination of the relationship between written and oral assessment scores has shown that the correlation is negligible and not statistically significant, with $r_s = 0.05788$ (essentially zero) and the t -statistic, $T(38) = 0.35741$. Because the p -value is 0.72276 (a very large value, indicating that the observed correlation does not differ statistically from zero), the null hypothesis of no association between the two assessment formats cannot be rejected. Statistically, this means that written and oral tasks behave as separate measures: students who perform well in the written task are not necessarily the ones who perform well in the oral task, and vice versa. There is no predictable pattern between the two.

To investigate the measures of spread, the researchers have also utilised parallel box plot analyses. For the face-to-face students, the overall oral scores are slightly higher than the written scores while written and oral median scores are 65.625% and 80% respectively (see Figure 1). Oral scores show a greater spread, especially toward the lower end. Written scores are a bit more tightly clustered, especially in the lower quartile. The middle 50% of oral scores sit slightly higher than the middle 50% of written scores. Oral scores also have a higher upper quartile, median and lower quartile, suggesting that more students scored relatively better on oral. Written scores are more compact, suggesting more uniform performance. This means that the oral task elicited a slightly higher but more variable level of achievement.

Figure 1

Written versus oral (face-to-face students) comparison



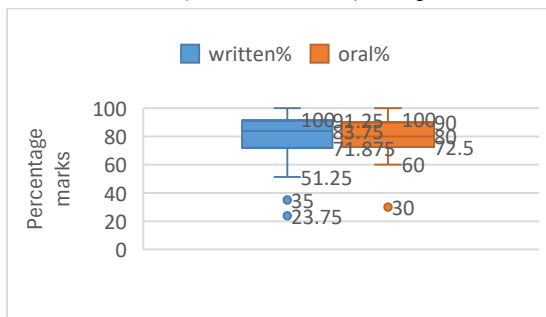
Written versus oral (online students)

A similar statistical analysis of the online students' outcomes has shown a stronger correlation than that observed in the face-to-face cohort, suggesting a somewhat more coherent performance pattern across modalities for online learners. The Spearman correlation for online students is positive, $r_s = 0.350677$, indicating a weak-to-moderate association in which stronger written performance tends to align with stronger oral performance. However, the evidence remains insufficient to reject the null hypothesis. The t -statistic, $T(19) = 1.632219$, indicates that the observed correlation, although positive, might not be large enough relative to the sample size to reach statistical significance. This is confirmed by the p -value of 0.119097, which again has shown that the null hypothesis of no association cannot be rejected.

For the online cohort, the parallel boxplot demonstrates that students performed comparably on written and oral tasks, with nearly identical medians and distributions as shown in Figure 2. The results do not contradict the outcomes of the Spearman correlation scrutiny. Although written and oral scores have similar medians, spread (IQR), and ranges and distributions of written and oral scores look alike overall, the boxplots do not tell the readers whether individual students who score high on one task score high on the other.

Figure 2

Written versus oral (online students) comparison



The comparison between oral and written performance reveals a clear pattern when the distributional characteristics and the interrelationship of paired scores are considered together. The oral scores display a slightly lower central tendency (median = 80%) than the written scores (median = 83.75%), suggesting that students tend to perform marginally better in the written task. In addition, the written assessment exhibits greater variability, with a wider range extending from 23.75% to 100%, compared to 30% to 100% in the oral task. The interquartile ranges of the two assessments overlap substantially, indicating broadly similar central distributions. Both formats reach the same maximum score of 100%, while the written assessment displays greater dispersion at the lower end of the distribution due to the presence of multiple low outliers.

Despite these written and oral scores differences of face-to-face cohort and similarities in central tendency and spread of online cohort, the degree to which written and oral performance align for the same student must be evaluated through correlation. In this dataset, there was no correlation and a weak correlation between written and oral scores of face-to-face students and online students respectively. In other words, the students who performed strongly in the written assessment were not consistently the same students who performed strongly in the oral assessment. Thus, while the oral task yielded slightly higher overall performance, the pattern of who succeeds and who struggles differs across modalities. Collectively, these findings point to the fact that the oral and written assessments are not interchangeable measures: although group-level outcomes show broadly similar performance, individual-level outcomes diverge. Such discrepancies may stem from the distinct demands of each medium. Written mathematics assessment emphasizes organized, formal solutions, whereas the oral format allows for interactive mathematical reasoning. Consequently, oral formats can reveal aspects of students' thinking that written solutions may not show. This implies that the two modalities activate skill repertoires and competencies that are partially distinct. The marginally higher performance scores observed in the oral assessment can be attributed to certain structural features of the task that differentiate it from the written format. First, the oral assessment generally evaluates a different subset of skills and competencies, such as swift information recall, oral clarification, and superficial analytical reasoning, rather than the extended problem-solving, structured argumentation, and deeper reasoning strategies required in written tasks. Second, unlike the written assessment, the oral task was not designed to be as score-heavy or cognitively demanding, leading examiners to select relatively straightforward questions that were intended to elicit quick responses rather than sustained analytical work. As a result, students could achieve higher performance without demonstrating the same level of reasoning required in the written assessment. Third, it is possible that examiners unknowingly provided guiding cues through facial expressions, tone, or body language, which may have supported or reassured students during their responses. This phenomenon is well-documented in the psychological literature, most famously in the case of Clever Hans, the "counting horse," whose apparent mathematical ability was later understood to arise from subtle, unconscious signals from human observers. As previously noted, such interpersonal unintentional hints can influence oral assessments despite examiners' best efforts to remain neutral, leading to inflated performance compared to written tasks.

Conclusion

From an assessment-design perspective, these results underscore the pedagogical importance of employing multimodal assessment strategies. Because the two formats cover complementary aspects of students' mathematical proficiency, relying exclusively on a written or oral assessment mode may misrepresent the competencies of particular learners. Using both written and oral components allows educators to obtain a more comprehensive view while ensuring that students with differing strengths, including neurodivergent learners who may prefer systematic written processing or dynamic oral reasoning, have equitable opportunities to demonstrate understanding. The results suggest that written assessments by themselves do not adequately capture diverse students' conceptual knowledge and relational understanding; consequently, the inclusion of oral assessments is an alternative assessment modality in mathematics. Further investigation is required into the use of oral assessments in contemporary educational contexts, particularly with respect to the perceptions of both students and educators. Additionally, the capacity of oral assessments to support academic integrity in the context of artificial intelligence may be regarded as a further advantage.

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References

- Akimov, A., & Malin, M. (2020). When old becomes new: A case study of oral examination as an online assessment tool. *Assessment & Evaluation in Higher Education*, 45(8), 1205–1221.
- Baker, S., Irwin, E., Hamilton, E., & Birman, H. (2022). What do we know about enabling education as an alternative pathway into Australian higher education, and what more do we need to know? A meta-scoping study. *Research Papers in Education*, 37(3), 321–343.
- Crawford, N., Olds, A., Lisciandro, J. (2022). *Emotional labour demands in enabling education: A qualitative exploration of the unique challenges and protective factors*. University of the Sunshine Coast. Retrieved from <https://research.usc.edu.au/esploro/outputs/journalArticle/Emotional-labour-demands-in-enabling-education/991122151902621>
- Dawson, P. (2024). Remote proctoring: Understanding the debate. In S. Eaton (Ed.), *Second handbook of academic integrity* (pp. 1511–1526). Springer. https://doi.org/10.1007/978-981-99-9374-8_77
- Fenton, A. (2025). Reconsidering the use of oral exams and assessments: An old way to move into a new future. *Educational Researcher*, 54(5). <https://doi.org/10.3102/0013189X251333638>
- Ferdoush, F., Alam, S. S., Jannifa, F. T., Haque, M. J., Johora, F., Parveen, M. F., & Rahman, M. S. (2025). Written versus oral assessment: Are both equally effective in pharmacology? *Bangladesh Journal of Medical Education*, 16(1), 1–8. <https://doi.org/10.3329/bjme.v16i1.79095>
- Iannone, P. (2023). Collaborative evaluation of teaching and assessment interventions: Ideas from realistic evaluation. In *Mathematicians' reflections on teaching* (pp. 139–151). Springer. https://doi.org/10.1007/978-3-031-34250-9_10
- Iannone, P., & Vondrová, N. (2024). The novelty effect on assessment interventions: A qualitative replication study of oral performance assessment in undergraduate mathematics. *International Journal of Science and Mathematics Education*, 22(2), 375–397. <https://doi.org/10.1007/s10763-023-10399-4>
- Jankowski, K. R. B., Flannelly, K. J., & Flannelly, L. T. (2017). The *t*-test: An influential inferential tool in chaplaincy and other healthcare research. *Journal of Health Care Chaplaincy*, 24(1), 30–39. <https://doi.org/10.1080/08854726.2017.1335050>
- Joughin, G. (2010). *A short guide to oral assessment*. Leeds Met Press in association with University of Wollongong.
- Moothadeth, M. (2024). Reliability of rubrics in the assessment of clinical oral presentation: A prospective controlled study. *Journal of Education and Health Promotion*, 13, 182.
- Nallaya, S., Gentili, S., Weeks, S., & Baldock, K. (2024). The validity, reliability, academic integrity and integration of oral assessments in higher education: A systematic review. *Issues in Educational Research*, 34(2). Institutes for Educational Research in NSW, SA and WA.
- Puchkov, E., Knyaginina, N., Novoselov, I., & Jankiewicz, S. (2020). COVID-19 and final (state) examination: Strategies of international and Russian universities. *Białostockie Studia Prawnicze*, 25(4), 257–274. <https://doi.org/10.15290/bsp.2020.25.04.17>
- Rahman, G. (2011). Appropriateness of using oral examination as an assessment method in medical or dental education. *Journal of Education and Ethics in Dentistry*, 1(2), 46–51. <https://doi.org/10.4103/0974-7761.103674>
- Theobald, A. S. (2021). Oral exams: A more meaningful assessment of students' understanding. *Journal of Statistics and Data Science Education*, 29(2), 156–159. <https://doi.org/10.1080/26939169.2021.1914527>
- Thiese, M. S., Ronna, B., & Ott, U. (2016). P value interpretations and considerations. *Journal of Thoracic Disease*, 8(9), E928–E931. <https://doi.org/10.21037/jtd.2016.08.16>
- Videnovic, M. (2017). Oral vs. written exams: What are we assessing in mathematics? *IMVI Open Mathematical Education Notes*, 7(2017). <https://doi.org/10.7251/OMEN1701001V>
- Videnovic, M. (2020). *Mathematics professors' views on written and oral assessment in mathematics* (Doctoral dissertation, Simon Fraser University). Summit Research Repository. <https://summit.sfu.ca/item/20438>
- Xiao, C., Ye, J., Esteves, R. M., & Rong, C. (2015). Using Spearman's correlation coefficients for exploratory data analysis on big dataset. *Concurrency and Computation: Practice and Experience*, 28(14), 3866–3878.

Challenging One-Size-Fits-All Contexts: Primary Teachers Experiencing Culturally Responsive Mathematics Teaching as Learners

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Contextualisation is expected to bridge school mathematics with real life, yet classroom contexts are often treated as universal, assuming that one context can fit all learners. Culturally Responsive Pedagogy (CRP) challenges this assumption by positioning students' cultural and lived experiences as central resources. This paper examines how Indonesian primary teachers, engaged as learners in a culturally responsive mathematics professional development (PD) workshop on whole numbers and place value, came to reframe mathematical meaning as culturally situated rather than universal.

Contextualisation is widely suggested as a means of making school mathematics meaningful and relevant (Boaler, 1993; Palm, 2008). Yet many students are continuing to struggle to connect school mathematics with their lived experiences (Civil, 2016). A persistent problem lies in how contexts are selected: classroom tasks frequently draw on situations assumed to be universally familiar, yet what counts as “everyday” is itself culturally situated. When instructional contexts privilege particular social groups while rendering others' experiences invisible, contextualisation can inadvertently deepen rather than bridge cultural disconnects (Sullivan et al., 2003; Nasir et al., 2008). De Lange (1995) distinguishes between *relevant and essential contexts*, which actively support mathematisation by requiring genuine reasoning, and *camouflage contexts*, which dress tasks in superficial narratives while eliminating the cognitive demand that authentic contextualisation is meant to produce. The overuse of camouflage contexts, particularly those drawn from culturally narrow or socioeconomically exclusive situations, limits opportunities for meaningful mathematical learning (Boaler, 1993).

Nasir et al. (2008) distinguish two levels of context in mathematics classrooms: the *task context* - the real or imagined setting framing a mathematical problem, and the *pedagogical context* - the broader learning environment shaping how students participate and whose knowledge is valued. One-size-fits-all approaches typically operate at the level of task context, deploying standardised situations assumed to be meaningful across diverse learners. This is illustrated in Indonesian mathematics textbooks, where problems situated in swimming extracurricular activities (Fitrianawati et al., 2022) may reflect experiences that are more accessible to students from higher socioeconomic backgrounds. Such contexts can transmit implicit messages about whose experiences are valued in school, giving advantages to students whose home culture aligns more closely with school culture, not because of inherent ability, but because of cultural familiarity. This study is concerned specifically with the *pedagogical context* of professional development (PD). Although effective mathematics PD is characterised by content focus, active learning, coherence, sustained duration, and collective participation (Desimone, 2009), PD programmes often treat pedagogical context as a delivery channel for techniques rather than as a cultural learning environment for its participants. When PD itself operates within universalised pedagogical context, teachers' tacit assumptions about whose mathematics counts and whose contexts matter remain unexamined. This study therefore foregrounds how the design of PD as a cultural learning environment shapes what teachers come to understand about mathematics, context, and their students.

Culturally Responsive Mathematics Teaching (CRMT) addresses the problem of pedagogically universalised contexts by centring students' cultural identities and funds of knowledge as foundations for mathematical learning (Aguirre & Zavala, 2013). When the same task contexts are deployed across all classrooms, students whose lived experiences fall outside those contexts may disengage or participate only superficially, while students whose home cultures align with the chosen contexts gain unearned advantages (Boaler, 1993; Sullivan et al., 2003). Grounded in sociocultural theory (Vygotsky, 1978), CRMT recognises that mathematical meaning is neither culturally neutral nor universal, but is shaped by linguistic practices, representational systems, and social contexts. CRMT is not limited to classrooms serving specific minority groups; its principles enhance learning for all students by fostering relevance, engagement, and belonging in diverse educational settings (Aronson & Laughter, 2016). In mathematics, CRMT connects learning to students' lived experiences, affirms diverse problem-solving approaches, and employs mathematics as a tool for reasoning about social justice (Gutstein et al., 1997; Aguirre & Zavala, 2013). Viewed through a CRMT lens, the problem of universal contexts is reframed: rather than seeking a single context that works for everyone, teachers come to recognise that any context is perceived differently across learners, and that this diversity is the resource through which mathematical meaning is constructed.

Yet transforming teaching towards CRMT requires more than professional awareness. Teachers' understandings of mathematics and context are deeply shaped by their own schooling within culturally neutral or universalised systems. Meaningful teacher learning must surface and reconstruct these assumptions through experience rather than transmission alone (Ball et al., 2008). Prior research has documented how teacher identity and positionality shape beliefs about equity and participation in mathematics (Bonner & Adams, 2012; Gutstein et al., 1997). However, fewer studies examine what teachers learn when they engage *as learners* in CRMT environments, rather than merely reflecting on CRMT principles as observers (Abdulrahim & Orosco, 2020; Gutiérrez, 2007). Guided by Vygotsky's (1978) view that knowledge is constructed through participation in meaningful social practice, this study positioned teachers as mathematics learners within a CRMT-focused PD programme. The programme comprised five sequential workshops moving teachers from cultural engagement with mathematics (Workshops 1–2), through explicit study of CRMT principles (Workshop 3), to designing their own CRMT lessons (Workshops 4–5). This paper focuses on Workshop 2, in which teachers engaged as learners with whole numbers and place value, because it offered the clearest evidence of how mathematical meaning is constructed when teachers themselves occupy the learner position within a CRMT environment. The study is guided by the research question: "How does engaging as learners in a CRMT-focused professional development reshape primary teachers' understanding of mathematical context and meaning in concept of whole numbers and place value?"

Background Literature

Professional Development in Culturally Responsive Mathematics Teaching

Teachers' understandings of mathematics, learning, and context are deeply shaped by their own experiences as students within traditional, often culturally neutral or universalised mathematics systems (Ball, 1988; Mewborn & Tyminski, 2006). Shifting these assumptions becomes a central task of PD for mathematics teachers. In general, effective mathematics teacher PD is generally characterised by content focus, active learning opportunities, coherence with teachers' existing knowledge and policy contexts, sufficient duration, and collective participation (Desimone, 2009). Yet syntheses of CRMT-oriented PD show that even well-designed programmes frequently leave teachers' underlying assumptions about what counts as mathematics and whose contexts matter unexamined, focusing instead on awareness of cultural

diversity or surface-level adaptation of curriculum materials (Abdulrahim & Orosco, 2020; Aguirre & Zavala, 2013; Gutiérrez, 2007). Meaningful teacher learning therefore requires opportunities to surface, question, and reconstruct these assumptions through experience rather than transmission (Ball et al., 2008). Drawing on Vygotsky's (1978) account of learning as participation in social practice, this study takes the position that teachers learn most powerfully about CRMT not by being told its principles, but by participating in mathematics learning environments designed around them, encountering the norms, expectations, and cultural messages of CRMT first-hand.

Culturally Responsive Mathematics Teaching (CRMT)

Culturally Responsive Mathematics Teaching (CRMT) is grounded in sociocultural theory, which asserts that cognitive development is mediated through sociocultural practices and the use of culturally constructed tools and symbols (Vygotsky, 1978). CRMT addresses the lack of cultural relevance in mathematics education by drawing on Culturally Responsive Pedagogy (CRP), articulated by Ladson-Billings (1995) and Gay (2018), which rests on three fundamental pillars: (1) a commitment to academic success; (2) a commitment to cultural competence; and (3) the empowerment of students' critical socio-political consciousness. In the mathematics classroom, Gutstein et al. (1997) specify these commitments by connecting mathematics to social justice, respecting diverse methods of problem-solving, using culturally relevant contexts, and treating mathematics itself as a tool for social-justice reasoning.

Methodology

Participants and Professional Development Design

The study draws on data from a CRMT-focused PD programme involving 25 upper primary teachers (Grades 4–6) from 12 public primary schools in Indonesia. Participants held varied teaching experience, with the majority having more than five years of classroom practice and primary education qualifications. The PD comprised five workshops, each lasting approximately 5–6 hours, facilitated by two mathematics educators experienced in CRMT research and curriculum design. PD design drew on the CRMT lesson analysis framework articulated by Aguirre and Zavala (2013) and principles from Ellis et al. (2019), emphasising high cognitive demand, affirmation of diverse mathematical identities, use of students' linguistic and experiential resources, and connections to social justice. Workshop 1 introduced relationships between mathematics and culture; Workshop 2 positioned teachers as learners in a CRMT lesson on whole numbers and place value; Workshop 3 examined CRMT principles directly; and Workshops 4–5 focused on designing CRMT lessons.

This paper concentrates on Workshop 2, in which teachers engaged with whole numbers and place value. While shifts in teachers' thinking were observed across all five workshops, the illustrative evidence presented here is drawn specifically from Workshop 2. Two task types were used: a personal name-tag activity anchoring number meaning in participants' own lived experiences, and a grouping-and-packaging task designed to surface the conceptual foundations of place value. Facilitators used participants' informal language and task artefacts as instructional resources throughout, introducing formal mathematical terminology only after conceptual engagement with concrete and pictorial representations.

Data Sources and Analysis

Data sources for this paper were selected to address the research question by capturing both teachers' in-the-moment engagement and their retrospective sense-making of that engagement. They include teachers' written reflections (completed individually at the end of Workshop 2), completed task artefacts (name tags and grouping worksheets), and facilitator field notes. Written reflections responded to three structured prompts asking teachers to describe what they

noticed, what surprised them, and how their thinking about mathematics teaching may have shifted, each prompt directly soliciting evidence of conceptual reshaping. Task artefacts provided a record of teachers' mathematical activity as learners, while field notes documented facilitator observations of group discussion and conceptual shifts. Qualitative thematic analysis (Braun & Clarke, 2006) was conducted across these sources to identify patterns in how teachers reframed mathematical context and meaning through the workshop experience.

Findings

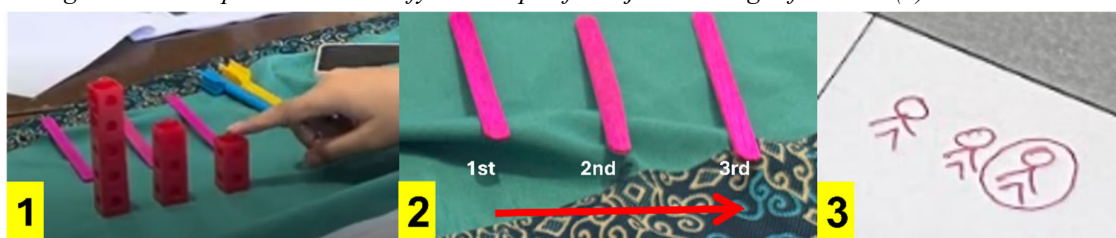
Theme 1: Reframing Mathematical Meaning Through Language and Lived Experience

A key shift during Workshop 2 was teachers' reconceptualisation of mathematical meaning as grounded in their own language and lived experience, rather than in universalised contextual examples and abstract definitions. Activities on whole-number meaning began with teachers constructing personal name tags: their nickname at the centre, surrounded by four personally chosen numbers at the corners, each carrying a different story. These artefacts became instructional resources for distinguishing cardinality (numbers representing quantity) from ordinality (numbers representing position), anchored in participants' own experiences. The personal name-tag task made visible the diverse meanings numbers carry across different lives and contexts, directly challenging the assumption that number concepts are experienced uniformly.

As teachers discussed their chosen numbers in groups, they came to know one another through these personal number stories, and a recurring conceptual confusion emerged. When asked to represent "the third" using concrete manipulatives, most groups initially constructed a podium arrangement—three objects in descending height (Figure 1, left), rather than identifying a single object in a sequence. This response reveals a common conflation of ordinal and cardinal meaning: treating ordinality as a property of a collection rather than as a property of a position within it.

Figure 2

Conceptual shift in the ordinal meaning of whole numbers: from a podium-based ranking (1), to a reference-point model in which the number names a position in a sequence relative to a starting point (2), to using that ordinal position to identify which specific object is being referred to (3).



The facilitator followed up with the question, "Which one is the third?", and offered an everyday illustration: a mother whose three children all come home together, contrasted with a mother who calls only her third child, with that single child coming alone. This contrast prompted a visible conceptual shift across most groups: in the podium arrangement, the number three referred to a cardinal collection of objects, whereas in the mother-and-children scenario, three identified the ordinal position of a specific child relative to a starting point. Facilitator field notes recorded that this moment generated extended discussion across all six groups, with teachers recognising that, in showing ordinality, the number of objects does not directly represent the ordinality itself; the reference point from which the sequence is counted is equally important. The shift from conflation to differentiation was confirmed through the colouring assessment task and a categorisation task on cardinal and ordinal meanings of numbers, drawing

on each group member's personal story (see Figure 2), in which the majority of teachers correctly applied both cardinal and ordinal interpretations.

Figure 3

Task on Differentiating the Meaning of Whole Numbers (Cardinality/Ordinality) using their own personal story (left) and a Colouring Activity (Right).

Try to look again at the number on your name tag. Now, let's distinguish: which number indicates cardinality/ordinality.

Numbers	Tick the appropriate one		Situation / Context / The story behind the numbers	Picture
	Cardinality	Ordinality		
3	☒	☐	Bu Arsyi has ever lived in three cities in west java	
4	☐	☒	I am Azimah, was born on the fourth month	
3	☒	☐	Hi, I am Maspupah, I have three children	



Assesmen

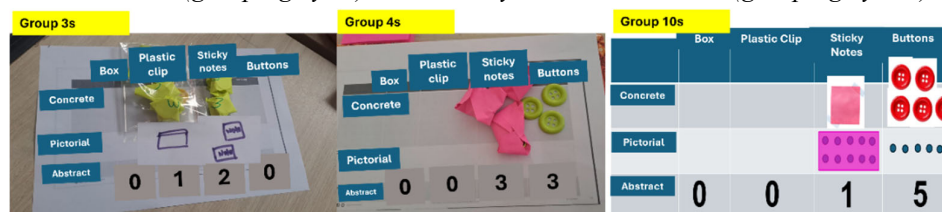
1. Color the 3rd shoe from the bottom with green.
2. Color 3 shoes with red.
3. What is the difference between the number 3 in the two questions above?

Written reflections reinforced this reorientation toward language as a resource. One participant wrote: "I used to think students had to learn the correct mathematical terms first. Now I realise their everyday stories are actually the doorway to understanding the concept." Another noted: "When I worked with the objects first, the symbols finally made sense. It reminded me how students might feel." These responses, evident across more than two-thirds of the written reflections, indicate a substantive shift away from the assumption that formal mathematical language must precede or replace informal expression. Rather than treating everyday language as something to be corrected, participants increasingly recognised it as a legitimate and productive entry point into conceptual understanding. This reorientation repositioned students' linguistic and cultural resources, often treated as deficits in traditional mathematics instruction, as pedagogical assets central to CRMT. In doing so, it directly challenged the one-size-fits-all assumption that mathematical meaning must be transmitted through a single "correct" linguistic or contextual frame: when teachers' own everyday words and lived stories produced legitimate mathematics, the necessity of a uniform context for all learners dissolved.

Another line of evidence showed how CRMT experiences enabled teachers to see place value as the transformation of informal language into the formal language of mathematics. The place-value activity used a packaging-and-grouping task: teachers chose their own grouping base (e.g., groups of 3, 4, or 10) and applied it consistently to a set of concrete objects, developing informal language to describe their groupings before formal mathematical terminology was introduced. Figure 3 shows sample teacher artefacts from this activity, illustrating how different grouping decisions produce visually and linguistically distinct representations of the same quantity.

Figure 4

Sample teacher artefacts from the Grouping and Packaging Activity on place value using fifteen buttons. Informal linguistic representations show: 1 plastic, 2 sticky notes, and 0 buttons (grouping by 3s); 3 sticky notes and 3 buttons (grouping by 4s); and 1 sticky note and 5 buttons (grouping by 10s)



Facilitator field notes recorded that most groups initially struggled to articulate why this activity was necessary, but through the activity came to see the ones/tens/hundreds structure as

the result of agreed-upon grouping decisions rather than an intrinsic property of numbers. Task artefacts showed teachers using informal language, describing groups as “homes” or “containers” for objects, as a conceptual bridge to formal place-value terminology. Written reflections showed this framing to be generative: one participant wrote, “I always taught place value as tens and hundreds because that is what the book says. I never questioned why it works that way.” Another noted that the activity helped her understand “where the words actually come from,” rather than accepting formal terms as given. Facilitator field notes also documented that teachers who chose non-standard grouping bases (e.g., base 3 or base 4) found the subsequent introduction of base-ten more meaningful, precisely because they had experienced the *choice* that underpins any grouping system. Because each group could choose its own container and grouping rule, comparison of artefacts across groups also made the relational nature of place value visible: the digit “1” in a base-3 grouping does not represent the same quantity as the digit “1” in a base-10 grouping. Through this contrast, teachers experienced place value as a system whose meaning depends on agreed-upon contextual conventions rather than as a single fixed structure, again undermining the one-size-fits-all framing.

Theme 2: Experiencing Mathematics as a Tool for Social Justice

The second theme captures how teachers experienced mathematics as a means for reasoning about social justice, rather than as a set of skills applied to neutral or superficial contexts in word problems. This theme bears directly on the research question: by foregrounding contexts in which whose lives, whose access, and whose fairness were explicitly at stake, the workshop offered an alternative to one-size-fits-all contextualisation. Rather than situating mathematics within a generic “real-world” setting, tasks were grounded in contexts that named the cultural and social specifics of the people they concerned. One task is set to Eid Al-Adha celebration when neighbourhood committees typically distribute qurban meat coupons to local families: students were asked to analyse how a community leader’s transcription errors, recording 210 as 21 for RW 03 and 160 as 106 for RW 04, would affect the just distribution of meat between the two neighbourhoods. The task invited teachers to use place-value concepts to analyse and respond to an unequal distribution scenario drawn from a realistic social context.

Several participants reported that this was their first experience using foundational mathematics concepts to reason about social justice. Written reflections showed that many teachers initially approached these tasks as standard word problems, focusing on calculation strategies, before recognising the embedded social justice context and shifting to questions of fairness. One teacher wrote: “I thought social justice was only for older students. I didn’t realise basic number concepts could help children think about fairness.” Another reflected: “I started solving the numbers and then stopped. I realised the question wasn’t asking me to just calculate, it was asking me to decide what was fair.” Facilitator field notes recorded that reflection prompts asking teachers to consider whose perspectives were represented in the mathematical problems generated the most extended discussions of the workshop, with participants drawing connections to their own students’ diverse socioeconomic backgrounds.

Discussion and Conclusion

This study contributes to CRMT professional development research by demonstrating how positioning teachers as learners within CRMT-designed mathematical environments makes the cultural dimensions of mathematical meaning visible and experientially grounded. Prior syntheses note that CRMT-oriented PD often addresses culture at the level of awareness, reflection, or curriculum context without showing how culture operates within mathematical meaning-making itself (Abdulrahim & Orosco, 2020; Gutiérrez, 2007), leaving teachers to value cultural responsiveness in principle while continuing to experience mathematics content as culturally neutral and procedurally fixed. The findings respond to this gap by showing that

experiential engagement—encountering culture through mathematical activity rather than as a separate topic—shifts how teachers make sense of mathematical concepts, with contextual diversity functioning as a resource that supports rather than complicates sense-making.

The central contribution of the first theme is that engaging teachers as learners in CRMT-designed mathematical tasks restructures their understanding of mathematical meaning itself. Teachers experienced linguistic and cultural resources as constitutive of mathematical content rather than backdrop, consistent with Vygotsky's (1978) account of language as a primary mediating tool for thought. Across both activities, teachers moved from accepting formal mathematical language as given to recognising it as emerging from lived cultural experience—a trajectory mirroring the informal-to-formal progression. The packaging-and-grouping task illustrated this most clearly: by constructing their own grouping systems and informal terminology before encountering base-ten, teachers experienced the cultural contingency of place value as a felt insight rather than an abstract claim. This experiential basis is consistent with Prediger and Wessel's (2013) intervention evidence that scaffolding from informal to formal registers produces significantly stronger conceptual gains than mathematics-only instruction. The present findings extend this evidence from learners to teachers-as-learners, addressing long-standing assumptions about mathematics as neutral and universal (Bishop, 1988).

The second theme demonstrates that foundational number concepts can serve as substantive vehicles for social justice reasoning from the early primary grades. This extends Gutstein's (2003) demonstration of mathematics as a tool for “reading and writing the world” into the under-researched terrain of foundational arithmetic, where the connection between place-value reasoning and distributive justice is rarely made explicit. That many teachers described the social-justice framing as genuinely new, and reported that it transformed their engagement with the mathematical task itself, resonates with Bartell's (2013) finding that teachers learning to teach mathematics for social justice must continually negotiate between mathematical and social-justice goals. The Eid Al-Adha task made this visible: a transcription error of 210 as 21 was no longer a careless slip but a misrepresentation affecting whose family received qurban meat, so place value became a representational system whose accuracy carries fairness weight. In direct relation to the research question, this indicates that the contextual diversity foregrounded by CRMT—whose lives, whose fairness, whose mathematics—is not an extension added after content learning, but a vehicle through which content learning is itself constructed (Gutstein et al., 1997).

This study demonstrates the potential of experience-based professional learning to support CRMT. When teachers engage as learners in CRMT-focused PD, encountering whole numbers and place value through their own languages and lived experiences, they are better positioned to challenge one-size-fits-all contextualisation and to envision mathematics as a site for equity and social justice. Future studies should examine how these shifts are enacted and sustained in the classroom.

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References

- Abdulrahim, N. A., & Orosco, M. J. (2020). Culturally responsive mathematics teaching: A research synthesis. *The Urban Review*, 52(1), 1–25. <https://doi.org/10.1007/s11256-019-00509-2>
- Aguirre, J., & Zavala, M. D. (2013). Making culturally responsive mathematics teaching explicit: A lesson analysis tool. *Pedagogies: An International Journal*, 8(2), 163–190. <https://doi.org/10.1080/1554480X.2013.768518>

- Aronson, B., & Laughter, J. (2016). The theory and practice of culturally relevant education. *Review of Educational Research*, 86(1), 163–206. <https://doi.org/10.3102/0034654315582066>
- Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, 8(1), 40–48.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407. <https://doi.org/10.1177/0022487108324554>
- Bartell, T. G. (2013). Learning to teach mathematics for social justice: Negotiating social justice and mathematical goals. *Journal for Research in Mathematics Education*, 44(1), 129–163. <https://doi.org/10.5951/jresmetheduc.44.1.0129>
- Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics*, 19(2), 179–191.
- Boaler, J. (1993). The role of contexts in the mathematics classroom: Do they make mathematics more “real”? *For the Learning of Mathematics*, 13(2), 12–17.
- Bonner, E. P., & Adams, T. L. (2012). Culturally responsive teaching in the context of mathematics: A grounded theory case study. *Journal of Mathematics Teacher Education*, 15(1), 25–38. <https://doi.org/10.1007/s10857-011-9198-4>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Civil, M. (2016). This is nice but they need to learn to do things the US way. In M. Civil & E. Turner (Eds.), *The mathematics education of immigrant students* (pp. 219–232). Springer.
- De Lange, J. (1995). Assessment: No change without problems. In T. A. Romberg (Ed.), *Reform in school mathematics* (pp. 87–172). SUNY Press.
- Desimone, L. M. (2009). Improving impact studies of teachers’ professional development: Toward better conceptualizations and measures. *Educational Researcher*, 38(3), 181–199. <https://doi.org/10.3102/0013189X08331140>
- Ellis, M. W., Larson, M., & Berry, R. Q. (2019). Equity-based mathematics teaching. In I. Goffney & R. Gutiérrez (Eds.), *Rehumanizing mathematics for Black, Indigenous, and Latinx students* (pp. 1–14). National Council of Teachers of Mathematics.
- Fitrianawati, M., Hartono, Y., Indaryanti, I., & Hapizah, H. (2022). Analysis of contextual problems in Indonesian primary mathematics textbooks. *Journal of Mathematics Education*, 13(1), 45–58.
- Gay, G. (2018). *Culturally responsive teaching: Theory, research, and practice*. Teachers College Press.
- Gutiérrez, R. (2007). (Re)defining equity: The importance of a critical perspective. In N. S. Nasir & P. Cobb (Eds.), *Improving access to mathematics* (pp. 37–50). Teachers College Press.
- Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, 34(1), 37–73. <https://doi.org/10.2307/30034699>
- Gutstein, E., Lipman, P., Hernandez, P., & de los Reyes, R. (1997). Culturally relevant mathematics teaching in a Mexican American context. *Journal for Research in Mathematics Education*, 28(6), 709–737. <https://doi.org/10.2307/749639>
- Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal*, 32(3), 465–491. <https://doi.org/10.3102/00028312032003465>
- Mewborn, D. S., & Tyminski, A. M. (2006). Lortie’s apprenticeship of observation revisited. *For the Learning of Mathematics*, 26(3), 23–32.
- Nasir, N. S., Hand, V., & Taylor, E. V. (2008). Culture and mathematics in school: Boundaries between “cultural” and “domain” knowledge in the mathematics classroom and beyond. *Review of Research in Education*, 32(1), 187–240. <https://doi.org/10.3102/0091732X07308962>
- Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*, 67(1), 37–58. <https://doi.org/10.1007/s10649-007-9083-3>
- Prediger, S., & Wessel, L. (2013). Fostering German-language learners’ constructions of meanings for fractions—Design and effects of a language- and mathematics-integrated intervention. *Mathematics Education Research Journal*, 25(3), 435–456. <https://doi.org/10.1007/s13394-013-0079-2>
- Sullivan, P., Zevenbergen, R., & Mousley, J. (2003). The contexts of mathematics tasks and the context of the classroom: Are we including all students? *Mathematics Education Research Journal*, 15(2), 107–121.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.

Mathematical Creativity Among (Preservice) Teachers: An Assessment Approach

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Many nations today include the goal of creativity in their curriculum. Goals on their own, however, are insufficient without support by teachers for students to become creative themselves. This paper describes the development of an assessment rubric, that includes three characteristics of creativity (*fluency*, *flexibility*, and *novelty*), to analyse mathematical creativity in written work. It then applies the rubric to responses gathered from 19 preservice secondary mathematics teachers (PSMTs) in the context of completing multiple solution problems. The overall distribution of ratings indicates that level of creativity among the solutions of PSMTs was generally modest to moderate.

Creativity in mathematics has long been recognised as an essential component of mathematical activity, not only for mathematicians, but also for students and teachers of mathematics across the entirety of educational levels (e.g., Sriraman, 2005). Earlier works conceive creativity as an innate quality (which one is born with), but according to most current mathematics education works, creative abilities can be gained through one's effort, commitment (Bicer, 2021a) and are supported by instruction and curriculum materials (Hadar & Tirosh, 2019). While most research has focused on students, there is a growing recognition that teachers' own mathematical creativity is a crucial part of the puzzle (Bicer et al., 2023; Hadar & Tirosh, 2019). Given that there is a current international focus in educational policy and curricula to develop creative thinking skills in students (see Wilkie, 2024), this component is particularly important as teachers design and select tasks, anticipate and respond to students' solution strategies, and model what counts as creative mathematical work (Bicer et al., 2023; Bolden et al., 2010). Against this background, understanding and assessing the mathematical creativity of teachers (both practicing and preservice) becomes a key part of preparing them to support creativity in their students. To date, research on teachers' mathematical creativity is scarce in the literature, especially at secondary school levels (Suherman & Vidákovich, 2022). In this paper, we describe the development of an assessment rubric to analyse mathematical creativity among preservice secondary mathematics teachers (hereafter PSMTs). We apply this rubric to PSMTs' written responses to a division related multiple solution (MS) problem consisting of the approximation of a quotient of two fixed, co-prime positive integers.

Mathematical Creativity and Its Assessment

There are different definitions for mathematical creativity in the mathematics education literature, and these largely come from the psychology and educational psychology literatures (Sriraman, 2005; Wilkie, 2024). At the school level in particular, mathematical creativity can be characterised as the ability to produce unusual or novel and insightful solutions to a given problem, even if not original in a historical context (Bicer, 2021a; Sriraman, 2005). The relevant literature largely has conceived mathematical creativity compose of the following main criteria: *flexibility*, *fluency*, and *novelty*. According to the systematic review conducted by Suherman and Vidákovich (2022) onto assessment of mathematical creative thinking, *fluency* is about if one can produce multiple solutions or ideas. *Flexibility* is associated with one's ability to chance thinking pathways or to generate different types of solutions or ideas. *Novelty* refers to one's

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 186–193). MERGA.

ability to seek out a solution pathway that is particularly original, distinctive, or uncommon for one's existing or expected knowledge level. Wilkie (2024) argued that students' mathematical creativity, in terms of the identified characteristics, can be investigated through processes they use and products they create in the content of problem-solving or problem-posing.

Substantial work has been done on ways to assess mathematical creativity, particularly in regard to school students, using a variety of approaches (Suherman & Vidákovich, 2022). Early approaches adapted general creativity tests, such as the Torrance Tests of Creative Thinking to mathematical content, but more contemporary studies have favoured tasks with mathematics embedded in them, such as open-ended questions, MS tasks, problem-posing activities (Leikin, 2009), and tasks that give students opportunities to develop creative thinking types (Wilkie, 2024). More recently, multiple representations have been proposed as both a didactical and psychometric tool for assessing and boosting mathematical creativity. In a study with elementary preservice teachers (PSTs), Bicer (2021b) developed a rubric to score the *fluency*, *flexibility*, and *originality* of representations used in fraction multiplication problems, demonstrating the use of multiple representations as a creativity measure. Within PST education, assessment work is less developed. Some studies have focused on PSTs' conceptions of creativity (e.g., Bolden et al., 2010), while others report on the creativity score of PSTs on MS tasks or problem-posing tests. Bicer (2021b) in particular showed that a methods course emphasising multiple representations and creativity can increase PSTs' performance on creativity tasks. However, very few studies have developed and validated assessment approaches specifically targeting teachers' mathematical creativity as measured through their own solutions to tasks that align with prescribed curricula.

The Current Study

The current study is an exploratory pilot study designed to identify preliminary patterns in mathematical creativity among secondary mathematics teachers. It is situated in a larger body of research focused on the knowledge and understandings of PSMTs in teaching algebra, wherein solving a set of MS problems were examined (Hatisaru et al., 2024). The first goal of this paper is to develop an assessment rubric for the analysis of mathematical creativity, and we ask: *What kinds of tools can be used in assessing mathematical creativity among mathematics teachers?* The second goal is the application of this rubric to the written responses of a sample of PSMTs to a MS problem centred on the approximation of a quotient of two fixed, co-prime positive integers. We ask: *When the rubric is applied, what does it reveal about PSMTs' mathematical creativity with respect to fluency, flexibility, and novelty?*

Methods

Data for the larger study was collected from 19 PSMTs enrolled in a mathematics pedagogy unit at a metropolitan university in Australia, completing a mathematics minor or major. These PSMTs would be expected to teach mathematics to students in Years 7-10 (ages 12-16) upon graduation. At the time of the study, all participants had prior experience supporting school students in mathematics, either as PSTs on placement or as private tutors. Within this unit, PSMTs engaged in problem-solving activities as part of their learning for five weeks during Semester 2, 2023, with a focus on Number and Algebra. Participation in the research was voluntary, and informed consent was obtained from all participants. At the beginning of each class, PSMTs completed a reflection form containing a purposefully selected MS problem. In total, five reflection forms (detailed below) were administered across the five weeks, each featuring a different problem. PSMTs were asked to complete each form within 20–25 minutes.

Research Instrument and Task

The reflection forms comprised two pages. The first page presented a MS problem and required PSMTs to solve the problem in as many different ways as they could (Item #1). This

approach aligned with the broader study's aim of examining PSMTs' capability to generate multiple solutions and provided a suitable context for exploring variations in *fluency*, *flexibility*, and *novelty* in their written responses. The second page contained reflective items that extended Item #1 and situated the task within a teaching context; however, these items fall outside the scope of this paper. Among the five problems, we selected the Division problem stated below. This choice was due to the fact that data related to the other problems have been reported elsewhere (Hatisaru et al., 2024).

The Division Problem

Divide 800 by 303 and round your answer to the nearest tenths, without using a calculator. Think and explain as many different possible solutions to the problem as you can.

The problem entails approximating, to the nearest tenth, the division of two co-prime, positive integers, namely 800 divided by 303. It is a typical type of problem in school algebra that is both simple to state, is of a form that is both familiar to the PSMTs and their future students, but one that is also complex enough that it provides multiple avenues of approach to solving. We envision that this is a suitable style of problem that could be used to demonstrate a teacher's own creativity, as well as reflecting the type of creativity they could potentially model for their own students.

The Meaning of Division

According to Siemon et al. (2020), there are two meanings for division: *partition* and *quotition*. They are distinguished by the meaning attached to the divisor. For instance, in the problem '128 divided by 8', 8 can be interpreted as the number of groups (*quotition*) or the number in each group (*partition*). A more generalised conception of division is *division as the inverse of multiplication*, and this conception (i.e., *multiplication and division as inverse operations*) supports work with fractions, algebraic texts, and more complex problems.

The Division problem can be approached in a variety of ways depending on one's conception of division, as well as one's knowledge of the field of mathematics. Traditional, algorithmic approaches to the problem include less novel methods such as long/short division, which is a partitional approach to the problem. More novel approaches include: (i) estimation approaches wherein a simpler, but related problem is created and solved, to infer a solution to the original problem, (ii) considering division as the inverse of multiplication, which can be used to help refine bounds established in estimation methods, (iii) establishing connections to other areas of mathematics, which involves the use of more complex mathematical topics, such as calculus, or number theory, or (iv) a combination of these approaches. On this basis, the problem offered a rich and authentic context for examining mathematical creativity. The multiple solution nature of the problem (Leikin, 2014), grounded in the varied conceptions of division encourages PSMTs to move beyond routine procedures and engage in diverse strategies. That has made the problem particularly suitable for understanding mathematical creativity in this study.

The Development of the Assessment Rubric

We first undertook an inductive review of all 19 responses to the problem in order to identify the solution methods produced by PSMTs. We started with open coding to identify all solution methods and then grouped them into standard/conventional versus novel, which supports openness to unexpected creativity. Our approach was primarily data-driven. That is, we examined each response to determine the methods used, considered how these methods could be grouped, and explored similarities and distinctions among them. This process revealed three most common solution methods, referred to here as Methods A, B, and C. While these methods are distinct, they are not entirely independent; in particular, Methods B and C share notable interconnections.

Method A: Algorithmic division (long or short). An iterative process of determining how many times 303 can be repeatedly subtracted from the remainder, and then subtracting 303 that many times to get the next remainder. This is a standard/conventional approach to division, used in many mathematics classrooms.

Long division

$$\begin{array}{r} 2.\overline{6402} \\ 303 \overline{)800.00000} \\ \underline{-606.00000} \\ 194.00000 \\ \underline{-181.80000} \\ 12.20000 \\ \underline{-12.12000} \\ 0.08000 \end{array}$$

Short division

$$\begin{array}{r} 2.\overline{6402} \\ 303 \overline{)800.194012208000} \end{array}$$

The decimal is recurring as we are back to dividing 800 by 303.

The decimal is recurring as we are back to dividing 800 by 303.

Method B: Estimation. Estimating the numerator or denominator in a way that makes the division easier to do (e.g. via inspection). In its most basic form, it does not answer the question fully. To arrive at more accurate solutions would typically involve multiple estimations (e.g. upper bound/lower bound) and/or refinement using division as the inverse of multiplication (see below).

Given that $303 \approx 300$ it follows that

$$\frac{800}{303} \approx \frac{800}{300} = \frac{8}{3} = 2.\overline{6} \approx 2.7$$

Furthermore, since $303 > 300$ it follows that

$$\frac{800}{303} < \frac{800}{300} = 2.\overline{6}$$

However, further investigation would be necessary to confirm whether this would result in an answer of 2.6 (or less) to the nearest tenth. A further refinement would be to proceed as follows

$$\begin{aligned} \frac{800}{303} &= \frac{606 + 194}{303} \\ &= \frac{606}{303} + \frac{194}{303} \\ &= 2 + \frac{194}{303} \\ &\approx 2 + \frac{194}{300} \\ &= 2 + \frac{194/3}{100} \\ &= 2 + \frac{64.\overline{6}}{100} \\ &= 2 + 0.64\overline{6} \\ &= 2.64\overline{6} \end{aligned}$$

In a similar way to the previous estimation, we can conclude that

$$\frac{800}{303} < 2.64\overline{6}$$

which in turn means that the answer to the nearest tenth will be less than or equal to 2.6. Obtaining a lower bound is less neat but could be achieved (for example) by estimating 303 as 320. This would yield

$$\frac{800}{303} \approx \frac{800}{320} = \frac{8 \times 100}{8 \times 40} = \frac{10}{4} = 2.5$$

Given that $303 < 320$ it follows that

$$\frac{800}{303} > \frac{800}{320} = 2.5$$

Hence, we now know that to the nearest tenth the answer is either 2.5 or 2.6. Further refinement of the lower bound and/or the use of a multiplication as the inverse of division argument could lead to the correct answer (below).

Method C: Division as the inverse of multiplication. This approach is in some ways a 'guess and check' approach. An initial guess is required which can then be refined. An estimation approach is a good starting point.

From the first estimation above we know that the answer is at most 2.7 to the nearest tenth. We can calculate $2.7 \times 303 = 818.1$ (confirming that 2.7 is an over-estimate). Next, we try $2.6 \times 303 = 787.8$, from which we conclude that 2.6 is a lower bound. We could evaluate $2.65 \times 303 = 802.95$, from which we conclude that 2.65 is an overestimate; hence 2.6 is the correct answer to the nearest tenth. That is, the answer is bound by 2.6 and 2.65 and this entire interval would round to 2.6

Unconventional and less common, and therefore somewhat more novel methods, involving slightly more sophisticated or rigorous estimation were as follows. *Method D*: Establishing an initial estimate and then refining using multiplication (e.g., initial estimate 2.7, so try 2.6×303 to see if it is closer). *Method E*: Applying the estimate of $303 \approx 300$ only to the remainder part of the division to improve overall accuracy. *Method F*: Using the Maclaurin series of $\frac{1}{1+x}$ to first order.

Next, we drafted the rubric for assessing mathematical creativity presented in Table 1. This process was partly deductive as we used the three characteristics of mathematical creativity defined in the existing literature. And we established a rating for each characteristic along with its description, making interpretation of each rating straightforward and reproducible. The procedure was transparent about its inductive approach to identifying solution methods followed by a deductive approach. This blended reasoning strengthens how accurately the rubric measures the actual mathematical creativity of the participants.

Table 1

Three Characteristic of Mathematical Creativity and Their Ratings

Characteristic	Rating and description
<i>Fluency</i>	0: No correct solution method
Number of different non-repeating solution method	1: A correct solution method containing at least one error and/or lacking in rigor
	2: A well explained solution method with rigorous justification
	3: At least one well explained and rigorous method, plus a method lacking rigor or containing an error/s
	4: At least two well explained and rigorous methods
<i>Flexibility</i>	0: No correct solution method
Variety of solution methods or shifts in methods based on distinct / different meanings of division	1: One correct solution method
	2: Two correct solution methods based on distinct / different meanings of division
	3: Three correct solution methods based on distinct / different meanings of division
<i>Novelty</i>	0: No correct solution method
Aspect of unconventionality in a solution method	1: A correct solution method containing at least one error and/or lacking in rigor, or the only solution method being standard/conventional algorithmic division
	2: The most basic estimation ($303 \approx 300$) without any systematic / justified refinement
	3: A solution based on one of the least common correct approaches (Methods D, E, or F in this case)

Application of the Rubric and Findings

The first two authors of this paper applied the rubric to the responses of 19 participating PSMTs independently. Here, a response refers to a reflection form (19 in total); in a response, there can be more than one solution method. Across 57 rating decisions (19 responses evaluated against three creativity characteristics), the agreement was 45, which corresponds to approximately 80% inter-rater reliability. Figure 1 illustrates the distribution of consolidated creativity ratings for *fluency*, *flexibility*, and *novelty*.

The Ratings of Mathematical Creativity for Each Characteristic

Fluency was observed across all five ratings. Three responses were rated 0, ten were rated 1, three rated 2, two rated 3, and one rated 4. This pattern suggests that while most responses

demonstrated limited fluency (ratings 0 and 1), a small number showed moderate fluency (ratings 2 and 3), and only one response reached the higher rating. *Flexibility* was grouped at the limited to moderate end of the rating. Three responses were rated at 0 and eight at 1 (limited flexibility), with the rest were rated at 2 (moderate flexibility). This indicates that the majority of responses relied on one or two methods, and none exhibited the level of variation expected at the higher rating for this characteristic (producing more than two solution methods). *Novelty* spanned the four ratings. Three responses were rated at 0, two at 1, eight at 2, and six at 3. This distribution suggests that while six responses incorporated elements of novelty beyond the most standard/conventional methods, relatively original methods (Methods D, E, or F) were seen less.

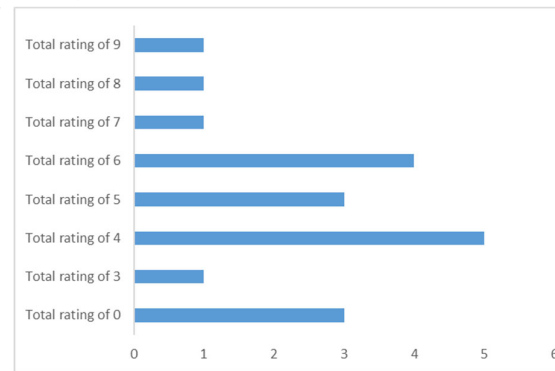
Figure 1

Distribution of Ratings Across Three Characteristics of Mathematical Creativity



Figure 2

Distribution of Total Mathematical Creativity Ratings



The Overall Ratings for Mathematical Creativity

Figure 2 presents the total creativity ratings for the 19 responses, where each total is the sum across three characteristics (*Fluency*: from 0 to 4; *Flexibility*: from 0 to 3; *Novelty*: from 0 to 3), yielding a possible range from 0 to 10 or above. In the current sample, observed totals span 0 to 9. The distribution across observed totals is as follows: one response rated 3; five rated 4; three rated 5; four rated 6; one rated 7; one rated 8; and one rated 9. All counts together account for the 19 responses. Three responses rated 0, as there was any correct solution in these responses. Overall, the ratings are grouped in the lower-to-mid range (3 to 6), with fewer responses attaining totals at the higher end (7 to 9). This pattern suggests that, among these PSMTs, and for the Division problem, many responses combined the three characteristics to achieve moderate totals, while only a small number achieved higher totals indicative of stronger creativity across multiple characteristics simultaneously.

Example Responses Illustrating Mathematical Creativity

To illustrate what mathematical creativity looks like in the responses to the Division problem, we now present two representative examples from the two ends of the creativity rating: one response with low total creativity rating (PSMT #18; Figure 3, left) and another with a higher rating (PSMT #14, right). These examples provide concrete insight into how PSMTs approached the problem and demonstrate the variation in creativity observed in this study.

Figure 3

Examples from the Two Ends of the Mathematical Creativity Rating

Solution A

$$\frac{8}{3} = 2\frac{2}{3}$$

$$\frac{80}{30} = 2\frac{2}{3}$$

$$\frac{800}{303} \approx 2.66$$

Solution B: Straight Division

$$800 : 303 = 2.63 \Rightarrow 2.6$$

Solution B: 303 close to 300

$$\Rightarrow 800 : 303 \approx 800 : 300 = \frac{8}{3} = 2.\bar{6}$$

Problem: This rounds to 2.7, but the actual solution is smaller because $\frac{800}{303} < \frac{800}{300}$, and it is inconclusive whether or not this will round down the digit.

Workaround: The deviation is expected to be around 1%, which is 0.027, therefore

$$\frac{2.667}{-0.027} = \frac{2.64}{\leftarrow \text{better estimate}} \Rightarrow \frac{2.6}{\text{Page 11}}$$

Remark: This approach works because for small x , $\frac{1}{1+x} \approx 1-x$

PSMT #18; *fluency* rated at 1, *flexibility* at 1, and *novelty* at 2. Total rating of 4.

PSMT #14; *fluency* rated at 3, *flexibility* at 2, and *novelty* at 3. Total rating of 8.

PSMT #18 rated low on *fluency* and *flexibility* as they only presented a single estimation method, and there was no explanation provided to explain their solution. Furthermore, the solution is not complete as they do not attempt to establish the correct answer to the nearest tenth, as stated in the problem. The *novelty* rating is in the mid-range as the use of an estimation method (estimating 303 as 300) was considered more unconventional than a routine algorithmic method (e.g., long division). PSMT #14 rated well on *fluency* as they presented two reasonably complete solution methods. The long division method (Solution A) unfortunately contained an error, so did not allow for a rate of 4 on *fluency*. That said, the classification of Solution B as complete was generous in the sense that the PSMT did not confirm that 2.6 was correct to the nearest tenth, with the generosity justified by the unique sophistication of using the first two terms of the Maclaurin series of $\frac{1}{1+x}$ to estimate the solution. The rating for *flexibility* was 2; that is, two correct methods provided, while the rating for *novelty* was 3 in acknowledgement of the sophistication of the Maclaurin series method.

Discussion

The application of the rubric to the analysis of preservice secondary mathematics teachers' (PSMTs') solutions has shown that the rubric effectively captures the mathematical creativity among PSMTs with respect to the three characteristics of creativity. *Fluency* showed the widest variation, with a few responses reaching higher rates, whereas *flexibility* was consistently limited to one or two solution methods. This may be a reflection of the time students were allocated to spend on the task (25 minutes), with higher scores in *fluency* requiring more detailed and rigorous explanations, this may have been a barrier to providing more than two solutions. Student familiarity with different areas of mathematics would also be a determining factor here, and it may be of interest for future studies to examine the outcome when certain methods, such as the less novel approaches, are prohibited, or provided to students, to understand the full depth of their creativity. These results are consistent with existing literature wherein in general those assessed score higher on *fluency* than *flexibility* (e.g., Bicer, 2021b). *Novelty* was present in some responses, while standard/conventional methods were more common. This is possibly a result of the previous experience of the PSMTs themselves and the opportunities provided for the development of their own creative abilities through instruction wherein more traditional, algorithmic approaches would be more familiar and widely taught.

The overall distribution of ratings indicates that level of creativity was generally modest to moderate, with the majority of responses received a total rating of 4, 5, or 6, which broadly aligns with a previous study as to the total creativity score of elementary PSTs (Bicer, 2021b).

In this study, we sought to develop an assessment approach and a rubric for evaluating mathematical creativity, based on empirical data from a PSMT sample. The rubric should be applied to similar other mathematical problems and contexts, offering a pathway for future research. It is also important to extend the analysis into the classroom to understand how teachers' mathematical creativity, with respect to its identified characteristics, influence their instructional decisions, and their students' mathematical creativity. The rubric used in this study shows that teachers may display varying levels of mathematical creativity, some strong, others moderate or limited. Mathematics teacher educators can use this rubric to decide what to emphasise in initial teacher education and ongoing professional development. For researchers, the rubric can be used in studies of the impact of teacher education or teacher professional development on mathematical creativity. For teachers, it offers a tool to understand students' mathematical creativity to support them adequately in the development of their own creative ability. We argue that more detailed information on the levels of mathematical creativity among mathematics teachers is needed (Suherman & Vidákovich, 2022), which opens the door for future research.

Acknowledgments, Author Contribution

Ethics approval 2022-03252-HATISARU was granted by Edith Cowan University, and all participants gave informed consent. All authors conceived of the work and contributed to writing the paper. Investigation: VH and SR. Conceptualisation and Methods: VH. Literature review: AM and VH. Data analysis: SR and VH. Findings: VH and SR. Discussion and Conclusions: VH and AM. Critical review of the paper: AM.

References

- Bicer, A. (2021a). A systematic literature review: Discipline-specific and general instructional practices fostering the mathematical creativity of students. *International Journal of Education in Mathematics, Science and Technology*, 9(2), 252–281. <https://doi.org/10.46328/ijemst.1254>
- Bicer, A. (2021b). Multiple representations and mathematical creativity. *Thinking Skills and Creativity*, 42, 100960. <https://doi.org/10.1016/j.tsc.2021.100960>
- Bicer, A., Bicer, A., Capraro, M., & Lee, Y. (2023). Mathematical connection is at the heart of mathematical creativity. *Creativity: Theories – Research – Applications*, 10(1), 17-40.
- Bolden, D. S., Harries, T., & Newton, D. P. (2010). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics*, 73(2), 143-157.
- Hadar, L. L., & Tirosh, M. (2019). Creative thinking in mathematics curriculum: An analytic framework. *Thinking Skills and Creativity*, 33, 100585. <https://doi.org/10.1016/j.tsc.2019.100585>
- Hatisaru, V., Stacey, K., & Star, J. (2024). Mathematical connections in preservice secondary mathematics teachers' solution strategies to algebra problems. *Advances of Research in Mathematics Education*, 25, 33-55. <https://doi.org/10.35763/aitem25.6354>
- Leikin, R. (2014). Challenging mathematics with multiple solution tasks and mathematical investigations in geometry. In: Li, Y., Silver, E., Li, S. (Eds.), *Transforming Mathematics Instruction. Advances in Mathematics Education* (pp. 59-80). Springer, Cham. https://doi.org/10.1007/978-3-319-04993-9_5
- Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129-145). Sense.
- Siemon, D., Warren, E., & Beswick, K. (2020). *Teaching mathematics: foundations to middle years*. Oxford University Press.
- Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? *Journal of Secondary Gifted Education*, 17(1), 20-36. <https://doi.org/10.4219/jsge-2005-389>
- Suherman, S., & Vidákovich, T. (2022). Assessment of mathematical creative thinking: A systematic review. *Thinking Skills and Creativity*, 44, 101019. <https://doi.org/10.1016/j.tsc.2022.101019>
- Wilkie, K. J. (2024). Creative thinking for learning algebra: Year 10 students' problem solving and problem posing with quadratic figural patterns. *Thinking Skills and Creativity*, 52, Article 101550.

Maths Self-Concept Change in Makerspaces

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Makerspaces are learning environments with the potential to engage students in STEM and broaden their career aspirations. This explanatory mixed-methods study investigated how students' academic self-concept in maths and their future possible selves changed after participating in a high school makerspace subject. The analysis indicated no self-reported change in students' academic self-concept in maths, with students finding makerspaces and maths classrooms and pedagogies very different. However, participation in a maker space subject decreased the number of students who feared a future that included maths, reducing the effect of maths as a gatekeeper subject.

Introduction

Australia is one of many countries experiencing a long-term decrease in enrolments in “maths-intensive” subjects like advanced mathematics, physics and chemistry (Department of Industry Science and Resources, 2025; Marchant & Kennedy et al., 2025). There are many factors driving this participation issue, including societal influences, factors within schools, and the Australian university entrance system (Cook, 2025; Finkel et al., 2020). While this is true for all students, the underrepresentation of girls in maths has remained stubbornly consistent since the early 90s, despite numerous interventions (Marchant & Kennedy, 2025).

The maker movement in education has the potential to engage students in STEM learning (Kidman & Tan, 2025), particularly for underrepresented groups (Fasso & Knight, 2019). Maker education provides a context where learning occurs through design and construction with physical and digital tools. Grounded in the principles of constructionism (e.g. Papert & Harel, 1991), and popularised by the maker movement (Dougherty, 2012), makerspaces have proliferated in schools as well as in universities and public settings such as libraries. Makerspaces, unlike more traditional spaces for making in schools such as art and woodworking rooms, are not constrained by connection to a defined curriculum, and instead focus on integrating knowledge, project-based learning, collaboration and community.

While makerspaces have obvious applications in the science, technology and engineering aspects of STEM (Kidman & Tan, 2025), the role and prominence of maths within makerspace initiatives can vary significantly (LeBlanc et al., 2022). Integrating maker practices and pedagogies can augment maths learning and teaching (Namukasa et al., 2023) and, specifically in maths, makerspaces have been proposed as a means to move beyond the focus on procedural techniques and support the development of critical and creative problem-solving skills (Doorman et al., 2019).

This paper explores how participation in a makerspace elective in high school can affect students' academic self-concept in maths and their perceptions of how maths relates to their future possible selves in terms of hope for and feared careers and work characteristics.

Literature review

Researchers have employed various approaches to understand what factors influence students towards and away from STEM careers, including examining attitudes, identity, goals, (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 194–201). MERGA.

attainment value, self-efficacy and self-concept (Wigfield & Eccles, 2000). This study selected self-concept as a predictor of participation in maths. Self-concept was chosen for the normative standards it uses and its long-term predictive links to career choices, as opposed to self-efficacy which is goal-referenced and measured in relation to a specific task; e.g. “how confident are you that you can solve this specific equation?”. These task-specific scales allow researchers to establish a causal relationship between self-efficacy and achievement behaviours, but reduce the predictive power for broader outcomes like course selection and career aspirations (Bong & Clark, 1999). In contrast, self-concept is a multifaceted construct encompassing students’ perceptions of their abilities as well as their desire to engage in specific activities (Brunner et al., 2010), thus self-concept is similar to self-efficacy, but also includes affective factors such as enjoyment of a subject (Bong & Skaalvik, 2003). Self-concept has been shown to correlate with academic engagement and motivation (Green, 2012) and is a precursor to attainment value in maths (Arens et al., 2019). Self-concept is also normative; e.g. “compared to others my age I am good at maths classes”. This relates to feelings of enjoyment, interest and pride (Bong & Clark, 1999) resulting in self-concept being a better predictor of future aspirations than self-efficacy (Marsh et al., 2019).

Self-concept is a multifaceted construct encompassing students’ perceptions of their abilities as well as their desire to engage in specific activities (Brunner et al., 2010). In this way self-concept is similar to self-efficacy, however self-concept also includes affective factors such as enjoyment of a subject (Bong & Skaalvik, 2003). Self-efficacy is goal referenced, measured in relation to a specific task; e.g. “how confident are you that you can solve this specific equation?”. These task-specific scales allow the researchers to establish a causal relationship between self-efficacy and achievement behaviours, but reduce the predictive power for broader outcomes like course selection and career aspirations (Bong & Clark, 1999). In contrast, self-concept is normative; e.g. “compared to others my age I am good at maths classes”. This relates to feelings of enjoyment, interest and pride (Marsh et al., 2005), resulting in self-concept being a better predictor of future aspirations than self-efficacy (Marsh et al., 2019).

Maths self-concept serves as a gatekeeper for STEM career aspirations (Goldman & Penner, 2016). Students with lower maths self-concept are less likely to aspire to STEM careers, even in fields that are not maths-intensive (Ceci et al., 2014). There are also gendered aspects (Sax et al., 2015), with girls reporting significantly lower perceptions of their maths abilities than their boy counterparts, despite demonstrating equal or better academic performance (Shapka et al., 2007). Additionally, Watt et al. (2016) found that for Australian girls, perceived maths ability was a much stronger predictor of STEM-related career aspirations than for boys, so girls both underestimate their maths abilities, and this lower self-perception has a disproportionate influence on their career aspirations. While previous research has explored the development of self-concept in traditional single subject disciplines, self-concept development in newer non-traditional learning areas such as makerspaces has been less studied. Maker education could offer an alternative pathway into higher level maths through the development of different aspects of self-concept.

Lyons and Quinn (2015) found that the most common reason Australian students gave for not choosing science was that they were unable to picture themselves as a scientist. For all students, having an accurate perception of who a STEM professional is and what they do is an important step towards seeing themselves in such a role in the future. Thus, it is necessary to capture this future oriented aspect of the students’ self-concept, called Future Possible Selves (FPS). FPS provides a link between an individual’s self-concept and their future orientated motivations (Molina et al., 2017). These possible selves encompass various dimensions of an individual’s potential future, such as personal characteristics, professional roles, lifestyle choices, and desired future achievements. Examples might include working with maths, owning a house, or being well-travelled. FPS can be further categorised based on their

desirability: hoped-for possible selves (highly desirable outcomes), expected possible selves (anticipated outcomes), and feared possible selves (undesired outcomes). It is important to note that the term “feared possible selves” does not necessarily imply dread or horror, but rather refers to possible selves that individuals simply wish to avoid (Molina et al., 2017).

This study aims to determine the influence participation in a makerspace has on students’ mathematical self-concept and the career-orientated aspects of future possible selves.

Methodology and Data Collection

This study, which was part of a larger study exploring eight subject areas, employed a mixed-methods, quasi-experimental design, comparing data from students taking makerspace electives (n=76) to those in control classes (n=71) across four schools. Quantitative data measuring academic self-concept and future possible selves was collected through pre- and post-surveys. Qualitative data from teacher interviews and student focus groups was used to triangulate the quantitative data and provide deeper insights into the findings.

Academic self-concept was measured using the Academic Self-Description Questionnaire (ASDQ) (Marsh, 1990), which uses an eight-point Likert scale to measure eight subject specific items. The ASDQ-2 was used, as it has been tested for validity for years 7-10, across different genders and within the Australian context (Byrne, 2002). The survey statements for this study are:

- Compared to others my age I am good at maths classes.
- I get good marks in maths classes
- Work in maths classes is easy for me
- I am hopeless when it comes to maths classes
- I learn things quickly in maths classes
- I have always done well in maths classes
- It is important to me to do well in maths classes
- I am satisfied with how well I do in maths

The survey instrument for Future Possible Selves provided options for students to select career groups from the thirty-three listed on the Career Bullseye posters (one of which was maths). Career Bullseye is a resource used with students in all the participating schools as part of their careers education and are well regarded by careers teachers and used widely in Australia (Firth, 2020). Students were asked to drag and drop the careers into boxes labelled either “I want a job in” or “I don’t want a job in”. This questionnaire was used with schools 2,3 and 4 but not school 1.

To triangulate with the two quantitative surveys, qualitative data was also collected in the form of student focus groups conducted after the subjects had been completed.

Results

Participant demographics

Table 2*Characteristics of participating schools and classes*

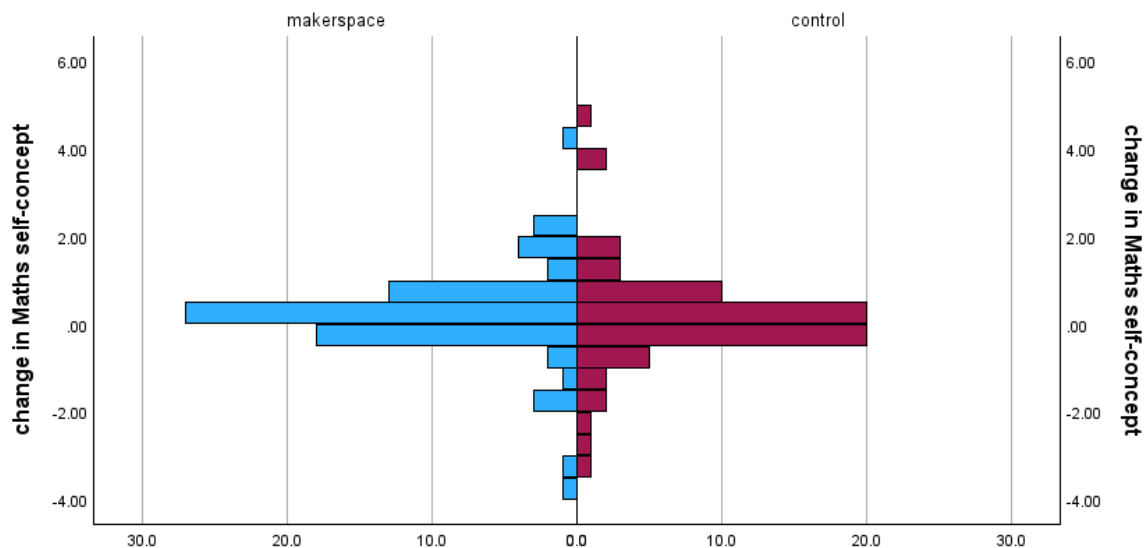
Characteristic	School 1	School 2	School 3	School 4
Sector	Independent	Independent	Catholic	Independent
Gender	Co-ed.	Girls' school	Girls' school	Girls' school
Year level	Year 9	Year 10	Year 10	Year 10
Control Class	Science	Drama	Creative Writing	Printmaking
Control participants	23	22	18	8
Makerspace participants	18	27	21	10

Academic self-concept

Academic self-concept change was calculated by subtracting students' average initial score from their average final score. To determine if there was a difference in self-concept change between makerspace students, an independent t-test was conducted. The 76 students in the makerspace classes ($M = 0.15$, $SD = 1.08$) and the 71 students in the control classes ($M = 0.09$, $SD = 1.23$), did not demonstrate a significant difference in performance ($t[145] = -0.32$, $p = .77$). The majority of students in both classes did not have a change in their self-concept from the start of the semester to the end of the semester, as can be seen in the back-to-back frequency chart in Figure 1.

Figure 1

Comparison of academic self-concept change in maths for students in makerspace (left) and control (right) classes.



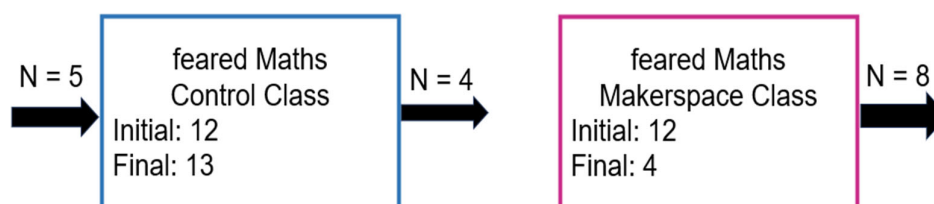
The analysis showed that there was no significant change in maths self-concept before and after participation in a makerspace subject compared to the control classes.

Future Possible Selves

The control class students consisted of 48 participants, while the makerspace students had 58 participants as school 1 was not included in the analysis. Among the 33 career group options available, maths emerged as by far the most frequently chosen feared career group in the pre-intervention data in both makerspace and control groups, with 24 out of 106 students saying they did not want a job involving maths. The next most common option was selected by only four students, so the other subjects were not included in further analysis as due to low numbers. Figure 2 shows how the number of students choosing maths as a feared future career changed after participating in a makerspace subject, with eight fewer students selecting this option, and no students adding maths to their list of feared future careers. In comparison, four students in the control classes removed maths from their feared list, but five added it, leading to an overall increase of one student selecting this option post intervention.

Figure 2

The change in numbers of students selecting maths as a feared future career.



Despite small sample sizes preventing any statistical significance in these results, participation in a makerspace subject appears to have had an effect on reducing the fear of maths compared to other creative electives, even though it did not increase students self-reported self-concept in maths.

Qualitative Data

Mathematical Content

When asked what subjects the students associated with the makerspace, art was most frequently mentioned, with maths used as a counterexample: “Not maths. Maybe art is probably like the closest”. While students recognised the presence of maths in their makerspace activities, they perceived it as different from the maths encountered in traditional classrooms. This was exemplified in this quote from a student in school 3: “I think the maths we were doing, wasn’t like maths-maths, because it’s kind of different to what we’re obviously doing in maths class. I found I was doing much better in the maths here than in our other normal class.”

This differentiation suggests a difference in how the context and application of maths influences students’ perception and experiences of maths. This was also seen in school 2: “I suppose I use maths in here. Maybe in a much less “using brain” type way, more using my common knowledge... What I already knew. And then also learning new things along the way.” Her characterisation of the maths used in the makerspace as “common knowledge” suggests that it may be more relevant or practical than the maths of the traditional classroom. These students made it clear that the maths they were doing in the makerspace was different to the maths in the maths classroom, and that they can more easily do the maths in the makerspace. It might be expected that this ability to apply maths in the makerspace would result in an increase in the students’ maths self-concept, yet this was not the case. One explanation for this is that,

as the students see the maths used in the makerspace as being a different kind of maths they do recognise their abilities and development as being related to “maths maths”.

Pedagogies, Engagement and Mistakes

When describing why the makerspace is different to their other classrooms, one student describes the levels of active participation, “science and maths aren’t that hands-on”. Another difference was their engagement in the learning and the value they placed on the work: “Sometimes I don’t even check if I got the answer right. I just don’t care really [in maths class]. But with my project [in the makerspace] I really wanted it to work”.

One feature that students discussed as being different in the makerspace, particularly in relation to science and maths classes, is the way mistakes were handled. In the makerspaces, mistakes were viewed as constructive and as a natural part of the design process. For example: “In the Makerspace the feedback seems more for change” (school 2), and “I think we found the mistakes kind of good because we could change for next time, for our next class, so our mistakes were kind of helpful” (school 3). Both these students recognise the role that mistakes played in moving their design forward. Students again used maths as a counter example: “I think in maths and science like there’s less room for error” (school 2), and “I feel like [mistakes are] not as helpful as in maths.” (school 3).

The contrast of mistake making was more than procedural, students also described an emotional component. “In math class if you’ve got something wrong, you might feel a bit discouraged. Like, oh no, I did it wrong or I don’t know what to do now” (school 2). Again, this is contrasted to the makerspace. “I know [mistakes] happen for everyone so I just fixed them, but in maths it means I wasn’t paying attention.”. This suggests that mistakes made in maths classes are perceived as a deficit, rather than the learning opportunity they are viewed as in a makerspace.

Conclusion

Maths classrooms were seen by students as being different from makerspaces, with maths often being used as a contrasting example when describing their experiences in makerspaces. Students were able to identify that they used maths knowledge used in the makerspace, but made a distinction between maths in the makerspace and maths in their maths classroom. Differences in “makerspace maths” compared to “maths maths” included increased persistence when the correct answer led to an improvement in project, a reduced fear of failure and a shift from mistakes being perceived as a result of lack of attention or ability to being accepted as a natural part of the design process. This indicates a desired shift from fixed a mindset towards a mathematical mindset (Boaler, 2015). Pedagogies also differed, with the makerspace being seen as “hands-on” compared to maths classrooms.

The effects of participation in makerspace subjects were mixed, with no significant change apparent in students’ academic self-concept in maths, but with a clear reduction in fear of careers containing maths. The lack of gain in self-concept may be due to students not connecting maths with what they did in the makerspaces, as the classroom experiences were so different. What is interesting is that, despite not connecting maths explicitly to making or improving their self-concept, students who had participated in the makerspace classes grew much more open to careers that included maths compared to the control groups who had taken other elective subjects. This indicates makerspaces have the potential to present maths as a subject with real-world applications, making it less daunting for students.

These results highlight the nuanced relationship between academic self-concept, future possible selves, and educational experiences in makerspaces, with makerspaces

potentially helping students to see maths as less of a barrier to future careers, even if their perceived math abilities remain unchanged. Noting the small sample numbers in this study, this connection between maths and makerspaces requires future exploration.

Acknowledgments

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References

- Arens, A. K., Schmidt, I., & Preckel, F. (2019). Longitudinal relations among self-concept, intrinsic value, and attainment value across secondary school years in three academic domains. *Journal of Educational Psychology, 111*(4), 663-680.
- Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.
- Bong, M., & Clark, R. E. (1999). Comparison between self-concept and self-efficacy in academic motivation research. *Educational Psychologist, 34*(3), 139-153. https://doi.org/10.1207/s15326985ep3403_1
- Bong, M., & Skaalvik, E. M. (2003). Academic self-concept and self-efficacy: How different are they really? *Educational Psychology Review, 15*(1), 1-40. <https://doi.org/10.1023/A:1021302408382>
- Brunner, M., Keller, U., Dierendonck, C., Reichert, M., Ugen, S., Fischbach, A., & Martin, R. (2010). The structure of academic self-concepts revisited: The nested Marsh/Shavelson model. *Journal of Educational Psychology, 102*(4), 964-981. <https://doi.org/10.1037/a0019644>
- Byrne, B. M. (2002). Validating the measurement and structure of self-concept: snapshots of past, present, and future research. *American Psychologist, 57*(11), 897-909. <https://doi.org/10.1037/0003-066x.57.11.897>
- Ceci, S. J., Ginther, D. K., Kahn, S., & Williams, W. M. (2014). Women in Academic Science: A Changing Landscape. *Psychological Science in the Public Interest, 15*(3), 75-141. <https://doi.org/10.1177/1529100614541236>
- Cook, E. J. (2025). Influences on engineering students' choices of higher-level mathematics. *Australasian Journal of Engineering Education, 30*(2), 95-108. <https://doi.org/10.1080/22054952.2025.2563936>
- Department of Industry Science and Resources. (2025). *Year 12 subject enrolment in STEM and other fields*. Canberra: Australian Government Retrieved from <https://www.industry.gov.au/publications/stem-equity-monitor/primary-and-secondary-school-data/year-12-subject-enrolment-stem-and-other-fields>
- Dougherty, D. (2012). The maker movement. *Innovations, 7*(3), 11-14.
- Doorman, M., Bos, R., de Haan, D., Jonker, V., Mol, A., & Wijers, M. (2019). Making and implementing a mathematics day challenge as a makerspace for teams of students. *International Journal of Science and Mathematics Education, 17*(Suppl 1), 149-165.
- Fasso, W., & Knight, B. A. (2019). Identity development in school makerspaces: intentional design. *International Journal of Technology and Design Education, 30*(2), 275-294. <https://doi.org/10.1007/s10798-019-09501-z>
- Finkel, A., Brown, T., Wright, J., & Wienk, M. (2020). Mapping university prerequisites in Australia. In: Australian Department of Education. <https://amsi.org.au>
- Firth, J. (2020). *Review into vocational and applied learning pathways in senior secondary schooling*. Department of Education and Training. <https://engage.vic.gov.au/vcalreview>
- Goldman, A. D., & Penner, A. M. (2016). Exploring international gender differences in mathematics self-concept. *International Journal of Adolescence and Youth, 21*(4), 403-418. <https://doi.org/10.1080/02673843.2013.847850>
- Green, J., Liem, G. A. D., Martin, A. J., Colmar, S., Marsh, H. W., & McInerney, D. (2012). Academic motivation, self-concept, engagement, and performance in high school: Key processes from a longitudinal perspective. *Journal of adolescence, 35*(5), 1111-1122. <https://doi.org/10.1016/j.adolescence.2012.02.016>

- Kidman, G., & Tan, H. (2025). Interdisciplinary Thinking in STEA+M: Cognitive Integration, Makerspaces, and Knowledge Creation. In G. Kidman, H. Tan, R. Gesthuizen, & D. D. Mangao (Eds.), *Knowledge Generation in STEM and STEAM Education* (pp. 89–121). Springer Nature Switzerland. https://doi.org/10.1007/978-3-032-03247-8_4
- LeBlanc, M., Freiman, V., & Furlong, C. (2022). From STEM to STEM: Learning from Students Working in School Makerspaces. In C. Michelsen, A. Beckmann, V. Freiman, U. T. Jankvist, & A. Savard (Eds.), *Mathematics and Its Connections to the Arts and Sciences (MACAS): 15 Years of Interdisciplinary Mathematics Education* (pp. 179–203). Springer International Publishing. https://doi.org/10.1007/978-3-031-10518-0_10
- Lyons, T., & Quinn, F. (2015). Understanding Declining Science Participation in Australia: A Systemic Perspective. In E. K. Henriksen, J. Dillon, & J. Ryder (Eds.), *Understanding Student Participation and Choice in Science and Technology Education* (pp. 153–168). Springer Netherlands. https://doi.org/10.1007/978-94-007-7793-4_10
- Marchant, T., & Kennedy, S. (2025). *Year 12 Mathematics Participation Report Card: Numbers remain at record lows and female participation declines*. <https://amsi.org.au/?publications=year-12-mathematics-participation-report-card-numbers-remain-at-record-lows-and-female-participation-declines>
- Marsh, H. W. (1990). The structure of academic self-concept: The Marsh/Shavelson model. *Journal of Educational Psychology*, 82(4), 623–636. <https://doi.org/10.1037/0022-0663.82.4.623>
- Marsh, H. W., Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2005). Academic self-concept, interest, grades, and standardized test scores: Reciprocal effects models of causal ordering. *Child development*, 76(2), 397–416.
- Marsh, H. W., Pekrun, R., Parker, P. D., Murayama, K., Guo, J., Dicke, T., & Arens, A. K. (2019). The murky distinction between self-concept and self-efficacy: Beware of lurking jingle-jangle fallacies. *Journal of Educational Psychology*, 111(2), 331–353. <https://doi.org/10.1037/edu0000281>
- Molina, M. F., Schmidt, V., & Raimundi, M. J. (2017). Possible Selves in Adolescence: Development and Validation of a Scale for their Assessment. *Journal of Psychology*, 151(7), 646–668. <https://doi.org/10.1080/00223980.2017.1372347>
- Namukasa, I. K., Gecu-Parmaksiz, Z., Hughes, J., & Scucuglia, R. (2023). Technology maker practices in mathematics learning in STEM contexts: a case in Brazil and two cases in Canada. *ZDM—Mathematics Education*, 55(7), 1331–1350.
- Papert, S., & Harel, I. (1991). *Situating constructionism*. Ablex Publishing Corporation.
- Sax, L. J., Kanny, M. A., Riggers-Piehl, T. A., Whang, H., & Paulson, L. N. (2015). “But I’m not good at math”: The changing salience of mathematical self-concept in shaping women’s and men’s STEM aspirations. *Research in Higher Education*, 56(8), 813–842.
- Shapka, J. D., Domene, J. F., & Keating, D. P. (2007). Trajectories of career aspirations through adolescence and young adulthood: Early math achievement as a critical filter. *Educational Research and Evaluation*, 12(4), 347–358. <https://doi.org/10.1080/13803610600765752>
- Watt, H. M. G., Hyde, J. S., Petersen, J., Morris, Z. A., Rozek, C. S., & Harackiewicz, J. M. (2016). Mathematics—a Critical Filter for STEM-Related Career Choices? A Longitudinal Examination among Australian and U.S. Adolescents. *Sex roles*, 77(3–4), 254–271. <https://doi.org/10.1007/s11199-016-0711-1>
- Wigfield, A., & Eccles, J. S. (2000). Expectancy-Value Theory of Achievement Motivation. *Contemporary Educational Psychology*, 25(1), 68–81. <https://doi.org/10.1006/ceps.1999.1015>

Teaching Mathematical Proof in the Age of Generative AI: Toward AI-Supported Proof Inquiry

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Recent advances in generative artificial intelligence (Gen AI) have enabled students to obtain mathematical solutions and proofs almost instantly, raising important questions about the teaching of proof in school mathematics. Drawing on research on proof in mathematics education, this paper argues that proof learning should extend beyond proof production toward students' abilities to interpret, critique, compare, and explain mathematical reasoning, including AI-generated proofs. The paper proposes AI-Supported Proof Inquiry (ASPI) as a pedagogical framework for supporting inquiry-oriented engagement with proof in AI-rich learning environments.

Introduction

The rapid development of generative artificial intelligence (AI) is beginning to transform many aspects of education, including the learning and teaching of mathematics. Contemporary AI systems are increasingly capable of producing symbolic manipulations, solving mathematical problems, explaining concepts, and generating mathematical proofs in response to natural language prompts. As generative AI tools such as ChatGPT, Gemini, Claude, Deep Seek, etc., become widely accessible, students can obtain step-by-step mathematical explanations and arguments with minimal effort, raising important questions about the nature of mathematical learning in the classroom. Recent research in mathematics education has begun to examine how students interact with generative AI when solving and proving mathematical statements, highlighting both new learning possibilities and potential challenges for instruction (Yoon et al., 2024).

One area where the impact of generative AI is particularly significant is the teaching of mathematical proof. Proof has long been regarded as a central component of mathematical practice and a key element of mathematical thinking (Hanna, 2000; Hanna & de Villiers, 2008; Stylianides, 2007). Within mathematics education, reasoning and proof are widely recognised as fundamental aspects of students' mathematical learning and understanding (Hanna & de Villiers, 2008). These multiple roles of proof have been widely discussed in mathematics education research (de Villiers, 1990; Hanna & de Villiers, 2008; Stylianides, 2007).

However, the emergence of generative AI capable of producing mathematical arguments challenges some of the assumptions underlying traditional approaches to teaching proof. If students can obtain a complete proof almost instantly from an AI system, the educational value of simply producing a proof may be diminished. Studies examining the integration of generative AI in mathematics learning environments have noted that while such systems can assist students in constructing mathematical arguments, they also raise concerns about overreliance on automated solutions and the need for instructional designs that promote critical engagement with AI-generated outputs (Park & Manley, 2024).

In this paper, we argue that the increasing availability of generative AI calls for a reconsideration of proof instruction in secondary mathematics education. Rather than focusing primarily on students' independent production of proofs, instruction may place greater emphasis on interpreting, critiquing, comparing, and explaining mathematical reasoning, including AI-generated arguments. At the same time, generative AI creates opportunities for students to explore alternative proof strategies and engage more deeply with proof as reasoning,

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 202–209). MERGA.

exploration, and communication. Drawing on research on proof in mathematics education, this paper proposes AI-Supported Proof Inquiry (ASPI) as a pedagogical framework for inquiry-oriented engagement with proof in AI-rich learning environments.

The Role of Proof in School Mathematics

Mathematical proof has long been regarded as a defining feature of mathematical activity and an essential component of mathematical reasoning. Within mathematics education, proof plays a central role in helping students understand why mathematical statements are true and in developing their ability to reason logically (Stylianides, 2007; Hanna, 2000). Consequently, the teaching and learning of proof has been the focus of extensive research in mathematics education. Research has shown that proof serves several important functions in mathematical practice. In a widely cited framework, de Villiers (1990) identified multiple roles of proof in mathematics, including verification, explanation, systematisation, discovery, and communication. Verification refers to establishing the truth of a mathematical statement, while explanation concerns providing insight into why the statement holds. Proof can also contribute to the organisation of mathematical relationships, support the discovery of new mathematical insights, encourage intellectual challenge, and serve as a means of communicating mathematical reasoning.

Although these functions are all important in mathematical practice, research suggests that proof in school mathematics is often presented primarily as a means of verification (Hanna, 2000; Stylianides, 2007). Students were frequently asked to produce products that confirm the correctness of a result, while other roles of proof – such as explanation, exploration, and comparison of different arguments – may receive less emphasis in classroom instruction. Verification remains important in school mathematics, but engaging students with multiple proofs may deepen understanding and reveal different mathematical structures (Hanna & de Villiers, 2008).

In this context, the emergence of generative AI presents a potentially useful development for proof instruction by making multiple mathematical arguments and alternative proof strategies more readily accessible for classroom exploration. Because AI systems can readily produce proofs of many standard results in school mathematics, classroom activities involving proof may increasingly focus not only on constructing proofs independently, but also on analysing the validity of arguments, comparing alternative strategies, and explaining the reasoning underlying mathematical claims. In this way, generative AI may create new opportunities for students to engage with proofs in ways that more fully reflect its multiple functions in mathematical practice.

Generative AI as a Mathematical Partner in Proof Learning

Recent advances in generative artificial intelligence have introduced new possibilities for engaging with mathematical reasoning in education settings. Large language models (LLMs) are increasingly capable of producing symbolic manipulations, solving mathematical problems, and generating proofs in response to natural language prompts. As a result, students now have unprecedented access to step-by-step proofs of many results encountered in school mathematics.

From the perspective of mathematics education, the availability of AI-generated proofs raises important pedagogical questions. If students can obtain complete proofs with minimal effort through generative AI, classroom activities that focus primarily on reproducing standard proofs may no longer provide insight into students' understanding of mathematical reasoning. However, rather than viewing generative AI solely as a challenge to proof instruction, it may also be understood as a mathematical partner that can support students' engagement with

mathematical reasoning. By generating alternative arguments and proof strategies, generative AI may support students in analysing, comparing, and critiquing mathematical reasoning.

To illustrate the type of reasoning that students may encounter when interacting with generative AI, consider the following response produced by an AI system to a prompt requesting a proof.

Figure 1

A Sample Proof of a Mathematical Statement Generated by ChatGPT

Example Transcript

Prompt

Prove that $n^2 - n$ is always even for all integers n .

AI-generated response

We want to show that the expression

$$n^2 - n$$

is always even.

First factor the expression:

$$n^2 - n = n(n - 1).$$

This is the product of two consecutive integers. Since consecutive integers differ by 1, one of them must be even. Therefore the product $n(n - 1)$ must be even. Hence $n^2 - n$ is always even.

Although the argument appears convincing, it contains a step requiring further justification regarding why one of two consecutive integers must be even. Examining such examples can help students reflect on the structure and completeness of mathematical reasoning.

Interactions of this kind illustrate how generative AI may support richer engagement with proof. Rather than simply copying an AI-generated argument, students can analyse its reasoning, identify missing steps, and revise the argument to make it more complete. In addition, students may prompt the AI system to generate alternative approaches to the same problem, allowing them to compare different proof strategies and reflect on which argument offers clearer explanations of the underlying mathematics. Such interactions engage students with several functions of proof identified by de Villiers (1990), including verification, explanation, discovery, and communication.

A useful analogy may be drawn with the impact of dynamic geometry software on geometry education. Just as tools such as GeoGebra supported students' exploration of geometric relationships and conjectures (Laborde, 2001), generative AI may support proof learning by enabling students to generate, compare, and analyse alternative mathematical arguments.

At the same time, the availability of AI-generated proofs also raises important considerations for assessment. Traditional proof tasks often ask students to construct complete proofs independently. When generative AI systems can readily produce such arguments, assessment practices may need to evolve to capture students' understanding of mathematical reasoning more effectively. For example, tasks that ask students to interpret, critique, or compare proofs may provide richer insights into students' reasoning than tasks that focus solely on proof production.

At the same time, the integration of generative AI into proof learning also presents important challenges. AI-generated proofs may contain incomplete reasoning, hidden assumptions, or mathematically incorrect steps that appear superficially convincing. Students may also become over-reliant on AI-generated arguments without critically evaluating their validity. These concerns highlight the importance of designing classroom activities that support

students in analysing and critiquing mathematical reasoning rather than accepting AI-generated outputs uncritically.

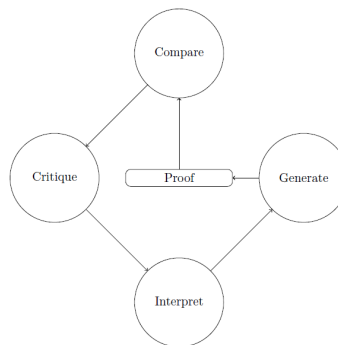
These developments suggest that generative AI may provide additional opportunities for students to engage with mathematical arguments in inquiry-oriented ways that are already valued in mathematics classrooms, such as analysing, comparing, and critiquing proofs. While such pedagogical practices are not unique to AI-supported environments, generative AI may increase students' access to multiple candidate arguments and alternative proof strategies. Further empirical research is needed to examine how these interactions influence students' proof learning in classroom settings.

AI-Supported Proof Inquiry (ASPI)

The discussion in the previous section suggests that generative AI has the potential to reshape how students engage with mathematical proof. Rather than focusing exclusively on the production of proofs, classroom activities may increasingly involve analysing, comparing, and refining mathematical arguments. To conceptualise how such interactions might be structured pedagogically, this paper proposes the framework of *AI-Supported Proof Inquiry (ASPI)*.

Figure 2

ASPI Conceptual Framework



The ASPI framework conceptualises proof learning as an inquiry process in which students engage with mathematical arguments through four complementary practices: *generate*, *interpret*, *compare*, and *critique* (Figure 2). Although the four practices are represented visually with directional arrows, the ASPI framework is not intended to describe a fixed instructional sequence. Rather, the practices of generating, interpreting, comparing, and critiquing proofs are conceptualised as interconnected and iterative forms of engagement with mathematical reasoning. Depending on the task and classroom context, students may move flexibly between these practices rather than progressing through them linearly. Generative AI serves as a tool that supports these practices by providing candidate proofs, suggesting alternative reasoning strategies, and enabling students to examine different forms of mathematical argumentation. The framework serves as both a pedagogical proposal and a conceptual model for future research on proof learning in AI-supported environments.

At the centre of the framework is *proof* itself as an object of inquiry. Students interact with proofs in multiple ways, moving between the practices of generating, interpreting, comparing, and critiquing mathematical arguments. Rather than viewing proof solely as a product to be produced, the ASPI framework emphasises proof as a dynamic process of reasoning and explanation. The four practices involve progressively deeper engagement with mathematical reasoning, from generating arguments to evaluating their validity and explanatory power.

The four components of ASPI are described below.

Generate

The first component of the framework involves generating candidate proofs or proof ideas through interaction with generative AI tools. In traditional classroom settings, students typically attempt to construct proofs independently before receiving feedback from teachers or textbooks. With generative AI, students can prompt an AI system to produce a proof or suggest a possible proof strategy for approaching a problem.

Within the ASPI framework, the purpose of generation is not merely to obtain an answer but to provide a starting point for inquiry. AI-generated proofs serve as artefacts that students can analyse and discuss. In some cases, the generated proof may be correct; in others, it may contain gaps or incomplete reasoning. Both situations provide valuable opportunities for learning. The ability to quickly generate candidate arguments also allows students to explore multiple approaches to a problem, supporting the discovery function of proof identified in mathematics education research (de Villiers, 1990).

Interpret

The second component involves interpreting the reasoning within a proof. Students examine the structure of an argument, identify the key logical steps, and explain why each step is valid. This practice emphasises understanding the meaning of a proof rather than simply verifying its correctness.

Interpretation is closely related to the explanatory role of proof discussed in the literature (Hanna, 2000). A proof that is technically correct may still fail to provide insight into why a result holds. By asking students to explain the reasoning behind each step of a proof, teachers can encourage deeper engagement with the mathematical ideas involved. Generative AI can support this process by providing explanations or alternative formulations that students can examine and discuss.

Compare

The third component of ASPI involves comparing different proofs of the same mathematical statement. In mathematics, it is common for a single result to admit multiple proofs, each highlighting different aspects of the underlying structure. For example, an algebraic identity might be proved using symbolic manipulation, geometric reasoning, or mathematical induction.

Generative AI makes it relatively easy to obtain multiple proofs of the same result. Students can prompt the system to “give another proof” or “explain the result in a different way,” producing alternative arguments that can be analysed and compared. Through such comparisons, students may gain insight into which proofs provide clearer explanations, reveal deeper structure, or generalise more readily to other contexts. This practice reflects the explanatory and systematising functions of proof described by de Villiers (1990).

Critique

This final component of the framework involves critiquing mathematical arguments. Students evaluate whether a proof is logically sound, identify gaps in reasoning, and suggest improvements to the argument. This practice emphasises proof as a process of verification and validation.

AI-generated proofs can provide particularly rich opportunities for critique. Although such systems often produce plausible reasoning, their arguments may occasionally contain subtle errors, incomplete justifications, or ambiguous explanations. When students examine these proofs critically, they develop the ability to assess mathematical arguments independently rather than relying solely on external authority. This practice aligns with the verification function of proof while also promoting mathematical judgement.

Taken together, the four practices of generating, interpreting, comparing, and critiquing proofs represent a shift in emphasis from proof production toward *proof inquiry*. Generative AI supports this shift by enabling students to interact with mathematical arguments in multiple ways and by providing a flexible environment for exploring alternative reasoning strategies. In the next section, several classroom task designs are presented to illustrate how the ASPI framework may be implemented in secondary mathematics classrooms.

Classroom Task Designs Using the ASPI Framework

To illustrate how the ASPI framework can be implemented in classroom practice, this section presents several example task designs suitable for secondary mathematics classrooms. Each task engages students with mathematical proofs through different stages of AI-Supported Proof Inquiry (ASPI). These tasks are intended to demonstrate how generative AI can support students in generating, interpreting, comparing, and critiquing mathematical arguments.

Task 1: Exploring Alternative Proofs

Consider the identity $1 + 3 + 5 + \dots + (2n - 1) = n^2$. Students are asked to prompt a generative AI system to prove this result.

Generate. Students request a proof from the AI system. The AI may produce a proof by mathematical induction showing that if the identity holds for $n = k$, it also holds for $n = k + 1$.

Interpret. Students examine the proof and explain the reasoning behind the inductive step, discussing how the expression $k^2 + (2k + 1)$ leads to $(k + 1)^2$.

Compare. Students then prompt the AI system for another proof. The AI may produce a geometric argument showing that successive odd numbers correspond to adding layers of dots that build a square of side length n . Students compare the two arguments and discuss which proof provides a clearer explanation of why the identity holds.

Through this activity, students encounter multiple proofs of the same statement and reflect on how different arguments reveal different mathematical insights.

Task 2: Comparing Proofs

Students consider the Pythagorean theorem and examine two proofs: one obtained from a textbook using similar triangles and another generated by an AI system based on geometric dissection.

Interpret. Students analyse the reasoning in each proof, identifying the key mathematical ideas involved.

Compare. Students discuss how the two proofs differ in structure and explanatory power. For example, one proof may emphasise geometric similarity while another highlights area relationships. Students then reflect on which proof helps them understand the theorem more clearly and why.

Such activities encourage students to view proofs not merely as formal procedures but as explanations that illuminate mathematical relationships.

Task 3: Critiquing AI-Generated Proofs

Students are presented with the following AI-generated proof of the statement:

“If n^2 is even, then n is even.”

Generate. A sample proof generated by ChatGPT is obtained from a prompt for a proof.

Figure 3

A Sample Proof of a Mathematical Statement

AI-generated proof

Suppose n^2 is even. Then we can write

$$n^2 = 2k$$

for some integer k . Taking square roots on both sides gives

$$n = \sqrt{2k}.$$

Since $2k$ is divisible by 2, the square root must also be even. Therefore n is even.

Interpret. Students first explain the reasoning used in the argument. They identify the key steps of the proof and describe how the conclusion is obtained from the assumption.

Critique. Students are then asked to determine whether the argument is valid. In particular, they examine the step where the square root of $2k$ is taken. Students discuss why this reasoning is problematic and identify the flaw in the argument. Students are then invited to construct a correct proof. One possible approach is to use the contrapositive:

If n is odd, then $n = 2m + 1$ for some integer m . Then

$$n^2 = (2m + 1)^2 = 4m^2 + 4m + 1 = 2(2m^2 + 2m) + 1,$$

which is odd. Therefore, if n^2 is even, then n is even.

Through this activity, students practise evaluating the validity of mathematical arguments and recognising that an apparently plausible proof may contain incorrect reasoning. Such critique tasks encourage students to engage critically with mathematical explanations, particularly when proofs are generated by AI systems.

Discussion and Implications

The examples presented in the previous section illustrate how generative AI can support richer forms of engagement with mathematical proof. Rather than viewing AI primarily as a tool that replaces students' work, the activities demonstrate how AI-generated arguments can become objects for inquiry and discussion in the classroom. Through the practices of generating, interpreting, comparing, and critiquing proofs, students interact with mathematical reasoning in ways that extend beyond the traditional emphasis on proof production.

From this perspective, the presence of generative AI invites a shift in proof pedagogy from a production-oriented approach toward a more inquiry-oriented approach. In many traditional settings, proof tasks are framed primarily as exercises in constructing correct arguments. While such activities remain important, the availability of AI-generated proof means that students can now readily access candidate arguments. This development creates opportunities for tasks that emphasise analysing, evaluating, and refining mathematical reasoning.

This shift also has implications for assessment practices. If students can easily obtain complete proofs from generative AI systems, assessments that focus solely on proof production may not adequately capture students' understanding of mathematical reasoning. Alternative forms of assessment may therefore become increasingly relevant. For example, students may be asked to interpret the reasoning within a proof, compare different proofs of the same statement, or identify and correct flaws in an argument. Such tasks provide insights into students' ability to reason about mathematical arguments rather than simply reproduce them.

More broadly, the ASPI framework highlights the potential of generative AI to function as a tool for mathematical exploration. By prompting AI systems to produce alternative proofs or explanations, students can encounter multiple ways of reasoning about the same mathematical

idea. These experiences may deepen students' understanding of the explanatory role of proof and help them appreciate the diversity of mathematical arguments.

At the same time, the integration of generative AI into proof learning raises new questions for mathematics education research. Further studies may investigate how students interact with AI-generated proofs, how such interactions influence students' understanding of mathematical reasoning, and how teachers can design tasks that effectively integrate AI into proof instruction. In this way, the ASPI framework may serve not only as a pedagogical proposal but also as a conceptual lens for examining proof learning in AI-supported environments.

Conclusion

Generative artificial intelligence is rapidly becoming part of the technological landscape in which mathematics education takes place. As AI systems become increasingly capable of producing mathematical arguments, educators face the challenge of reconsidering how proof is taught and learned in school mathematics. Rather than viewing generative AI solely as a threat to traditional proof instruction, this paper has argued that AI can serve as a productive partner in proof learning.

Drawing on research on the functions of proof in mathematics education, the paper proposed the AI-Supported Proof Inquiry (ASPI) framework as a way of conceptualising students' interactions with proofs in AI-supported learning environments. The framework emphasises four practices – generate, interpret, compare, and critique – through which students can engage with mathematical arguments as objects of inquiry.

The classroom task designs presented in this paper illustrate how generative AI can support these practices within the context of secondary mathematics. By interacting with AI-generated proofs, students can explore alternative arguments, reflect on different proof strategies, and develop the ability to evaluate the validity and explanatory power of mathematical reasoning.

References

- de Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras*, 24, 17-24.
- Hanna, G. (2000). Proof, explanation and exploration. An overview. *Educational Studies in Mathematics*, 44(1-2), 5-23. <https://doi.org/10.1023/A:1012737223465>
- Hanna, G., & de Villiers, M. (2008). ICMI Study 19: Proof and proving in mathematics education. *ZDM Mathematics Education* 40(2): 329-336. <https://doi.org/10.1007/s11858-008-0073-4>
- Park, H., & Manley, E. D. (2024). Using ChatGPT as a proof assistant in mathematics pathways course. *The Mathematical Education*, 63(2), 139-163. <https://doi.org/10.7468/mathedu.2024.63.2.139>
- Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, 6(3), 283-317. <https://doi.org/10.1023/A:1013309728825>
- Stylianides, G. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, 38(3), 289-321.
- Yoon, H., Hwang, J., Lee, K., Roh, K. H., & Kwon, O. N. (2024). Students' use of generative AI for proving mathematical statements. *ZDM Mathematics Education* 56, 1531-1551. <https://doi.org/10.1007/s11858-024-01629-0>

From Beliefs to Practice: Secondary Mathematics Teachers' Conceptions and the Teaching of Mathematical Proof in Singapore Classrooms

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This study examines secondary mathematics teachers' conceptions, beliefs, and instructional experiences related to the teaching of proof in the Singapore curriculum. Data from written proof tasks and interviews with two teachers indicate that proof is associated with reasoning and developed mathematical thinking shaped by prior experiences. Teachers also reported tensions between curricular expectations and classroom constraints, while adopting strategies to support students' justification. The findings highlight how teachers' conceptions interact with curricular contexts in shaping proof instruction.

Introduction

Mathematical proof plays a central role in the discipline of mathematics, serving not only to establish the validity of mathematical statements but also to explain why mathematical relationships hold. In mathematics education, proof has been widely recognised as an important means of developing students' mathematical reasoning and conceptual understanding (Stylianides, 2007). However, the role of proof in school mathematics differs from its role in professional mathematics. Within school contexts, proof is often shaped by curriculum expectations, students' developing mathematical knowledge, and constraints of classroom practice. In the Singapore secondary mathematics curriculum, proof is not always presented as formal deductive argument in the Euclidean sense, but often emerges through reasoning, justification, and explanation of mathematical relationships. Students are expected to articulate logical arguments, explain results, and justify procedures, even when fully formal proofs are not explicitly required (MOE, 2023, p. 3). In this study, we therefore conceptualise *mathematical proof* broadly as forms of reasoning and justification that establish the validity of mathematical claims, ranging from informal explanatory arguments to more structured deductive reasoning, while distinguishing these from purely empirical verification based on examples or patterns.

Research in mathematics education has shown that teachers' conceptions of proof vary considerably and influence how proof is interpreted and enacted in classroom practice. For example, Harel and Sowder (1998) identified different proof schemes, demonstrating that individuals may rely on external authority, empirical evidence, or deductive reasoning when justifying mathematical claims. Subsequent studies have shown that teachers, like students, may draw on multiple forms of reasoning, sometimes privileging example-based or procedural explanations over formal deductive argumentation (Stylianides, 2007; Weber, 2001). In the Singapore context, Thanabalasingam (2024) found that teachers often associate proof with advanced mathematical thinking, and may perceive formal proof as more aligned with higher-level mathematics rather than everyday classroom practice.

While these studies have documented teachers' conceptions of proof and their beliefs about its role in mathematics learning, less attention has been given to how these conceptions interact (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 210–217). MERGA.

with teachers' instructional experiences and the challenges they encounter in classroom contexts. In particular, there remains a need to better understand how teachers negotiate tensions between their beliefs about the value of proof and the practical constraints of curriculum, assessment, and student readiness.

This study builds on earlier work examining secondary mathematics teachers' conceptions and beliefs about proof in the Singapore secondary mathematics curriculum (Thanabalasingam et al., 2025). Extending this line of inquiry, the present study investigates how teachers' conceptions, beliefs, and instructional experiences collectively shape their approaches to teaching proof. By examining teachers' engagement with proof-related tasks alongside their reflections on classroom practice, this study aims to provide a more integrated account of how proof is interpreted and enacted in the secondary mathematics classroom.

The study addresses the following research questions:

1. What conceptions of mathematical proof do secondary mathematics teachers demonstrate when engaging with proof-related tasks?
2. What beliefs do teachers hold regarding the role and purpose of proof in teaching and learning in mathematics?
3. What challenges do teachers encounter when teaching proof in the secondary mathematics classroom?

By addressing these questions, this study seeks to contribute towards a deeper understanding of the interplay between teachers' conceptions of proof, their beliefs about teaching, and the contextual factors that shape proof instruction in school mathematics.

Literature Review

The Role and Functions of Mathematical Proof in Mathematics Education

Mathematical proof is widely regarded as a central component of mathematics and an important means of developing students' mathematical reasoning and conceptual understanding (Stylianides, 2007). In mathematics education, proof not only verifies mathematical statements but also supports explanation and conceptual understanding. Research has identified several functions of proof in mathematical activity, including verification, explanation, and systematisation (de Villiers, 1990; Hanna & de Villiers, 2012). Despite its recognised importance, many students struggle to transition from empirical reasoning based on examples or patterns to deductive argumentation (Harel & Sowder, 1998; Weber, 2001). In response to these challenges, researchers have argued that students should be given opportunities to engage in reasoning and justification from the early stages of mathematical learning. Such experiences can help students gradually develop an understanding of proof as a meaningful mathematical activity rather than a purely formal exercise (Stylianides, 2007). Consequently, the way in which proof is introduced in classrooms plays a crucial role in shaping students' understanding of mathematical reasoning.

Teachers' Conceptions of Mathematical Proof

Teachers' conceptions of proof have been widely studied in mathematics education research (Harel & Sowder, 1998; Stylianides, 2007; Thanabalasingam, 2024), as these conceptions influence how teachers interpret and implement proof-related activities in their classrooms. Teachers' views about what constitutes a valid proof and the purposes of proof in mathematics may shape the types of explanations they provide, the tasks they select, and the expectations they set for students.

One influential framework for analysing individuals' reasoning in proof-related contexts is the Proof Scheme Framework proposed by Harel and Sowder (1998). In this framework,

proof schemes refer to the types of arguments that individuals use to justify mathematical claims, and thus reflect their underlying conceptions of what constitutes a valid proof. These schemes are generally categorised into three broad types: external conviction schemes, empirical schemes, and analytical schemes. External conviction schemes rely on authority or ritual procedures, whereas empirical schemes validate claims using examples or patterns. Analytical schemes correspond to formal deductive reasoning (Harel & Sowder, 1998). These categories provide a useful lens for examining the nature and purpose of proof, particularly in distinguishing between reasoning based on verification through examples and reasoning based on general deductive argumentation.

Research has shown that both students and teachers may exhibit a range of proof schemes when reasoning about mathematical statements. In some cases, teachers themselves rely on empirical or perceptual reasoning when explaining mathematical ideas, particularly in classroom contexts where visual or example-based explanations are emphasised. Understanding teachers' underlying proof schemes can therefore provide insight into how they interpret mathematical arguments and how they guide students' reasoning in proof-related activities.

Teachers' Beliefs and Teaching of Mathematical Proof

In addition to their conceptions of proof, teachers' beliefs about mathematics and mathematics teaching can influence how proof is taught in classrooms. Teachers' beliefs encompass their views about the nature of mathematics, the role of proof in mathematics education, and the ways in which students should engage with mathematical reasoning.

Cabassut et al. (2012) examined teachers' beliefs related to proofs and identified several dimensions that influence instructional practices. These include beliefs about the role of proof in mathematics, beliefs about teachers' own mathematical competence, and beliefs about appropriate pedagogical strategies for teaching proof. These beliefs can shape teachers' instructional priorities and the degree to which proof is emphasised in classroom activities.

Previous studies have also shown that teachers may experience various challenges when teaching proof (Stylianides, 2007; Weber, 2001; Thanabalasingam, 2024). For instance, teachers may feel that the curriculum does not provide sufficient guidance or emphasis on proof-related activities. In addition, the absence of proof-related items in high-stakes assessments may discourage teachers from allocating substantial time to proof instruction. Teachers may also encounter difficulties in supporting students' reasoning processes, particularly when students struggle to articulate logical arguments or to understand abstract mathematical concepts.

Understanding teachers' beliefs and challenges related to proof instruction is therefore important for improving the teaching and learning of proof in school mathematics. By examining how teachers' conceptions of proof interact with their beliefs about teaching, researchers can gain deeper insight into the factors that shape the enactment of proof in classroom practice. Such understanding may inform the development of teacher education programmes, professional development initiatives, and curriculum resources aimed at strengthening the teaching of mathematical reasoning and proof.

Conceptual Framework

This study adopts an integrated conceptual framework combining Harel's Proof Scheme Framework (Harel, 2007) and Cabassut et al.'s (2012) Belief System Framework. Harel's framework focuses on the types of reasoning individuals use when justifying mathematical claims, distinguishing between external conviction, empirical, and analytical proof schemes. External conviction schemes rely on authority or ritualised procedures, empirical schemes validate claims using examples or patterns, while analytical schemes correspond to deductive

reasoning. In this study, this framework is used to analyse teachers' reasoning when engaging with proof-related tasks, and thus their conceptions of what constitutes a valid proof. Cabassut et al.'s (2012) Belief System Framework complements this perspective by examining teachers' beliefs about the nature and role of proof, their own mathematical competence, and appropriate pedagogical approaches to teaching proof. In particular, it provides a lens for understanding how teachers view the purpose of proof in the curriculum and how these beliefs shape their instructional decisions.

In this study, the two frameworks are integrated to examine both the reasoning and belief dimensions of teachers' engagement with proof. The Proof Scheme Framework is used to analyse how teachers justify mathematical claims (i.e., the forms of reasoning they employ), while the Belief System Framework is used to examine teachers' beliefs about proof and its role in teaching and learning. By bringing these perspectives together, the study investigates how teachers' conceptions of proof and their beliefs about teaching interact to shape their instructional approaches in the secondary mathematics classroom.

Methodology

Research Design

A qualitative multiple-case study design was adopted to examine teachers' conceptions of proof and their instructional experiences. In this study, two in-service secondary school mathematics teachers were examined as individual cases, allowing a detailed exploration of their conceptions of proof, their beliefs about proof instruction, and the challenges they encounter in teaching proof.

Participants

Two secondary school mathematics teachers were purposively selected. Both held degrees in pure mathematics and postgraduate teaching qualifications and taught Elementary Mathematics and Additional Mathematics at upper secondary level.

Data Collection

Data were collected using a conceptions questionnaire consisting of nine proof tasks and semi-structured interviews. Teachers' written responses to these tasks were analysed to identify the types of reasoning they used when justifying mathematical statements. The conceptions questionnaire consisted of proof-related tasks drawn from secondary school topics (e.g., geometry and algebra), requiring teachers to justify mathematical statements, evaluate the validity of given arguments, and explain their reasoning. These tasks were designed to elicit the types of arguments teachers use when engaging with proof, including empirical and deductive forms of reasoning. In addition, a beliefs questionnaire was administered to capture teachers' views about the nature and role of proof in mathematics, their perceptions of students' ability to engage with proof, and their approaches to teaching proof in the classroom. The questionnaire included open-ended items that invited teachers to elaborate on their perspectives.

To explore Research Question 3, which concerns the challenges teachers encounter when teaching proof, semi-structured interviews were conducted with each participant. The interviews provided opportunities for teachers to elaborate on their questionnaire responses, describe their experiences in teaching proof, and discuss the difficulties they face when supporting students' reasoning in proof-related tasks.

Data Analysis

Teachers' responses to the conceptions questionnaire were analysed using Harel's (2007) Proof Scheme Framework, which categorises reasoning into external conviction, empirical, and analytical proof schemes. This analysis enabled the identification of teachers' underlying conceptions of proof and addressed Research Question 1.

Responses to the beliefs questionnaire were analysed using categories derived from Cabassut et al. (2012) Belief System Framework, examining teachers' beliefs about the nature of proof, its role in the curriculum, and appropriate instructional strategies for teaching proof. The analysis provided insight into how teachers' conceptions relate to their instructional approaches, addressing Research Question 2.

Interview transcripts were analysed thematically to identify recurring patterns in teachers' reported experiences and challenges in teaching proof. The thematic analysis was conducted inductively, allowing themes to emerge from the data while being interpreted in relation to the study's conceptual framework. These themes were used to address Research Question 3, and to provide a deeper understanding of the factors shaping teachers' instructional practices.

Triangulation across three data sources – questionnaire responses, task performance and interview data – was used to enhance the credibility of the findings and to provide a more comprehensive understanding of teachers' instruction of mathematical proof.

In addition to identifying teachers' conceptions and beliefs, the analysis examined how these elements were reflected in teachers' reported instructional approaches and experience when teaching proof.

Findings

The analyses of the data revealed that teachers' conceptions of mathematical proof were closely related to ways in which they approached the teaching of proof in their classrooms. Three themes emerged: (1) alignment between teachers' conceptions and instructional goals, (2) tensions between beliefs about proof and classroom realities, and (3) pedagogical adaptations in teaching proof.

Alignment Between Teachers' Conceptions of Proof and Instructional Goals

The interview data suggest that both teachers viewed proof as requiring reasoning beyond routine procedures, particularly in explaining and justifying mathematical relationships. For instance, Teacher 1 reflected that answering some of the proof-related questions in the conceptions questionnaire required "a higher level of mathematical maturity", noting that such tasks may be challenging for students with limited exposure to advanced reasoning.

Teachers also linked their own understanding of proof to their prior experiences studying mathematics at more advanced levels. Teacher 1 attributed his confidence in approaching proof tasks to his undergraduate mathematics training, explaining that his university education helped him develop the "mathematical maturity to understand how to write proofs, and also how to think of ... the pathway into proving certain phenomena in mathematics." Similarly, Teacher 2 reported that her experiences studying mathematics in junior college (pre-university) and university had provided sufficient exposure to mathematical proof, although she noted that professional development opportunities specifically focused on the teaching of proof were relatively limited.

These reflections suggest that teachers' conceptions of proof were shaped by their prior learning experiences, with proof viewed primarily as a form of reasoning grounded in conceptual understanding. These views shaped teachers' instructional goals, emphasising students' ability to explain and justify mathematical relationships.

Evidence of teachers' conceptions of proof could also be observed in their written responses to proof-related tasks in the questionnaire. For example, in a geometry task, both teachers justified their solutions by transforming the problem into equivalent angle relationships and applying properties such as angle sums and cyclic quadrilaterals. This reflects a transformational proof scheme, in which claims are established through the application of known properties (Harel & Sowder, 1998).

At the same time, the written responses revealed subtle differences in how teachers articulated their reasoning. For instance, one teacher concluded that $\angle ABC + \angle ADC = 183^\circ \neq 180^\circ$ and therefore that the quadrilateral was not cyclic, without explicitly stating that the underlying geometric principle regarding opposite angles in cyclic quadrilaterals. In another instance, a teacher used the term *converse* when the intended logical reasoning corresponded more closely to a *contrapositive*. These examples suggest that while teachers were generally able to construct valid deductive arguments, the articulation of the underlying mathematical reasoning was sometimes implicit or imprecise.

Tensions Between Teachers' Beliefs About Proof and Classroom Realities

Although both teachers expressed positive views about the importance of proof in mathematics, the interview responses revealed several tensions between these beliefs and the realities of classroom practice. One challenge concerned the limited presence of proof-related tasks in the curriculum materials used in secondary school mathematics. Teacher 2 observed that while textbooks contain many challenging problems and investigative activities, “E-Math seldom have proving problems,” noting that explicit proving tasks appear more frequently in Additional Mathematics topics such as trigonometric identities. This observation suggests that opportunities for students to engage with proof-related reasoning may be uneven across different strands of the curriculum.

Teachers also highlighted practical constraints that affect the extent to which proof-related activities can be implemented in classroom teaching. Teacher 1 acknowledged that although exploratory activities designed to support reasoning are available in textbooks, these were not always used in practice, explaining that he “did not use much of those exploratory activities ... because of time shortage.” Such comments point to the influence of curriculum pacing and time constraints on teachers’ instructional decisions.

Student motivation was another factor shaping teachers’ experiences of teaching proof. Teacher 2 explained that because the proof of formulas is often not explicitly required in the E-Math syllabus, many students are not keen to learn how such results are derived. She noted that students tend to focus on applying formulas rather than understanding the reasoning behind them.

Taken together, these responses illustrate how teachers’ beliefs about the importance of proof can come into tension with the curricular structure, time constraints, and students’ expectations of mathematics learning. While teachers recognised the value of proof in fostering mathematical understanding, these contextual factors shaped how proof-related reasoning was enacted in classroom practice.

Pedagogical Adaptations in Teaching Proofs

In response to the challenges associated with teaching proof in secondary mathematics classrooms, the interview data suggest that teachers adopt various pedagogical strategies to support students’ engagement with mathematical reasoning. A recurring concern raised by teachers involved helping students make their reasoning explicit when constructing proofs. For example, Teacher 1 described difficulties in assessing whether students had correctly understood geometrical relationships when writing proofs. He explained that students often wrote brief statements such as “alternate angles” when justifying a step in a geometric argument, without clearly specifying which pair of angles they were referring to in diagrams that contained multiple pairs of parallel lines. As he explained, the challenge was “trying to get my students to write ... to make their thinking visibly clear.”

Teachers also described using visual or illustrative approaches to support students’ understanding of mathematical relationships before introducing more formal proofs. Teacher 2 mentioned using visual demonstrations (for instance, using dynamic pictorial proofs of the Pythagorean Theorem) when introducing certain results, explaining that such approaches could

help students grasp the underlying relationships even though these demonstrations did not constitute formal mathematical proofs. At the same time, she emphasised that understanding the reasoning behind mathematical results remained important, commenting that it would be “ridiculous for students to apply the property but not knowing how it is derived.”

These responses suggest that teachers attempt to balance the development of conceptual understanding with the formal requirements of mathematical proof. In particular, teachers appear to use explanatory discussion, visual representations, and guided reasoning as ways of helping students move toward clearer articulation of mathematical arguments. These practices support students in developing the ability to express and justify mathematical reasoning.

Discussion

The findings of this study provide insights into how teachers’ conceptions of proof interact with classroom practice in the secondary mathematics curriculum. In particular, teachers associated proof with developed mathematical reasoning shaped by their prior learning experiences, aligning with Harel’s (2007) notion of proof schemes, where individuals’ justification of mathematical claims reflects their cognitive development and prior exposure to mathematical reasoning.

At the same time, the findings highlight tensions between teachers’ conceptions of proof and the structural constraints of the curriculum. Opportunities for engaging students in proof-related reasoning appear uneven across topics, with explicit proving tasks more prevalent in areas such as trigonometric identities than in other parts of the curriculum. This reflects broader observations that the role of proof in school mathematics is shaped by curricular priorities and assessment structures (Stylianides, 2007).

The results also suggest that teachers respond to these tensions through pedagogical adaptations aimed at supporting students’ reasoning. These include encouraging clearer articulation of arguments and using visual or explanatory approaches to bridge students’ intuitive understanding and formal reasoning. Such practices indicate that proof instruction in school mathematics often involves supporting students’ progression from empirical reasoning towards more structured deductive thinking.

Taken together, the findings highlight the interplay between teachers’ conceptions of proof, their beliefs about students’ learning, and the curricular contexts in which proof is taught, contributing to a more nuanced understanding of how proof is enacted in the secondary mathematics classroom.

Conclusion and implications

By integrating Harel’s proof scheme framework with Cabassut et al.’s belief system framework, the study provides a lens for understanding how teachers’ views about proof interact with the practical realities of classroom teaching. The findings suggest that teachers’ conceptions of proof influence the instructional goals they prioritise in proof-related lessons, particularly in emphasising explanation and reasoning as important aspects of mathematical understanding. At the same time, the results reveal tensions between teachers’ pedagogical beliefs about the value of proof and the constraints of curriculum pacing, examination demands, and students’ readiness for abstract reasoning.

In navigating these tensions, teachers reported a range of pedagogical adaptations aimed at supporting students’ engagement with proof-related ideas. These adaptations included the use of examples, exploratory reasoning, and explanatory discussion as ways of bridging students’ intuitive understanding and more formal deductive reasoning. These practices suggest that proof instruction often involves a progression from empirical reasoning toward analytical proof.

The findings of this study have implications for teacher education and professional development. Within the scope of this study, which examined teachers' engagement with proof in specific mathematical topics (e.g., trigonometric identities), the results suggest that programmes designed to support the teaching of proofs may benefit from addressing not only teachers' mathematical knowledge of proof but also the pedagogical strategies needed to guide students' development of mathematical reasoning. In particular, opportunities for proof-related reasoning may be more readily embedded in some topics than others, and professional development programmes may therefore need to consider how proofs can be meaningfully integrated across different areas of the curriculum.

Supporting teachers in reflecting on their own conceptions of proof and their instructional practices may contribute to strengthening the role of reasoning and justification within school mathematics. However, given the small sample size and topic-specific nature of the study, these implications should be interpreted as exploratory rather than generalisable. Future research may examine how instructional approaches influence students' development of proof-related reasoning across a wider range of mathematical topics.

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References

- Cabassut, R., Conner, A., İşçimen, F. A., Furinghetti, F., Jahnke, H. N., & Morselli, F. (2012). Conceptions of proof in research and teaching. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education* (pp. 169–190). Springer.
- Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Pearson.
- de Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras*, 24, 17–24.
- Hanna, G., & de Villiers, M. (2012). Aspects of proof in mathematics education. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education* (pp. [chapter pages vary by edition]). Springer.
- Harel, G. (2007). Students' proof schemes revisited. In P. Boero (Ed.), *Theorems in school* (pp. 65–78). Sense Publishers.
- Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education III* (pp. 234–283). American Mathematical Society.
- Ministry of Education. (2023). *Mathematics syllabus (4052)*. <https://www.moe.gov.sg>
- Stylianides, A. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, 38(3), 289–321. <https://doi.org/10.2307/30034869>
- Thanabalasingam, N. (2024). *Proof and proving in the Singapore secondary school mathematics curriculum* [Doctoral dissertation, Nanyang Technological University].
- Thanabalasingam, N., Ho, W. K., & Kaur, B. (2025). An exploration into secondary mathematics teachers' instruction of mathematical proof in Singapore: Understanding their conceptions, beliefs, and challenges related to teaching proofs. In S. M. Patahuddin, L. Gaunt, D. Harris, & K. Tripet (Eds.), *Unlocking minds in mathematics education: Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 421–428). MERGA.
- Weber, K. (2001). Student difficulty in constructing proofs. *Educational Studies in Mathematics*, 48(1), 101–119. <https://doi.org/10.1023/A:1015535614355>

Teachers' Awareness of Students' Difficulties with Algebra Word Problems

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This paper reports findings from a diagnostic questionnaire completed by eight junior high school mathematics teachers in Ghana before participating in a Community of Practice. Analysis of the questionnaire explored how the teachers' responses revealed aspects of their pedagogical content knowledge for teaching algebra word problems, particularly their awareness of task complexity, student reasoning, and potential misconceptions. Findings suggest that while the teachers demonstrated awareness of common student difficulties, their pedagogical content knowledge was uneven, especially in relation to explanations, representations, and curriculum-task use.

Introduction

Algebra word problems continue to present substantial challenges for students in school mathematics. Unlike routine symbolic exercises, they require students to interpret language, identify quantitative relationships, represent those relationships algebraically, and reason with the resulting expressions or equations. The demands of algebra word problems therefore extend across linguistic, conceptual, and representational domains, making them an important site for examining student thinking and understanding (Kieran, 2020; Verschaffel et al., 2020). For this reason, they also provide a valuable context for examining the teaching practices needed to support students' learning.

Difficulties with algebra word problems have been widely documented. Students often struggle with the meaning of variables, the structure of multiplicative relationships, and the translation of verbal statements into algebraic form. Reversal errors are especially common when students attempt to represent comparison statements symbolically, and such errors often reflect deeper difficulties in coordinating language with algebraic structure rather than mere carelessness (Clement, 1982; Kieran et al., 2016). These challenges matter because success in algebra depends not only on symbolic fluency but also on the ability to reason about relationships, generality, and representation.

In this context, teachers' pedagogical content knowledge (PCK) is particularly important. Effective teaching of algebra word problems requires more than knowing the correct answer or procedure. It also requires teachers to anticipate likely student difficulties, interpret incorrect responses, explain relationships clearly, and select tasks or representations that support understanding (Shulman, 1987; Ball et al., 2008; Chick & Beswick, 2018). Teachers' awareness of students' difficulties is therefore an important aspect of PCK, especially in topics such as algebra where misconceptions can be conceptually persistent and instructionally consequential.

The Ghanaian context provides a strong rationale for examining these issues. Concerns about students' achievement in mathematics have persisted in national reports and related research, with algebra and solving worded problems continuing to present difficulties for many learners (Abreh et al., 2018; Osei & Agyei, 2023; West African Examinations Council [WAEC], 2023). Yet, while there is substantial discussion of students' performance, much less attention has been given to what teachers themselves know about students' difficulties with

algebra word problems and to what their responses reveal about their PCK before any targeted professional learning begins.

This paper addresses that gap through findings from a diagnostic questionnaire completed by eight junior high school mathematics teachers in Ghana prior to their participation in a Community of Practice (CoP) focused on teaching algebra word problems. The questionnaire generated two connected sets of findings. First, teachers ranked a set of algebra items according to how difficult they believed those items would be for their students and explained the reasons for their judgements. Second, teachers responded to hypothetical student answers and algebra tasks designed to reveal aspects of their PCK. Bringing these two strands together makes it possible to examine not only whether teachers were aware of likely student difficulties, but also whether that awareness was accompanied by pedagogically robust ways of responding to those difficulties. This paper is guided by two research questions:

1. What does the diagnostic questionnaire reveal about teachers' awareness of students' difficulties with algebra word problems?
2. What does the diagnostic questionnaire reveal about the pedagogical content knowledge teachers draw on in responding to those difficulties?

Theoretical Framing

This paper is framed by the concept of PCK, understood as the specialised knowledge that enables teachers to transform disciplinary content into forms that are accessible and meaningful for learners (Shulman, 1987). In mathematics education, this includes knowledge of content, knowledge of students' thinking, and the capacity to make pedagogically sound instructional decisions (Ball et al., 2008; Chick & Beswick, 2018). In algebra word problems, this is especially important because students must coordinate language, quantitative relationships, and symbolic representation, and often struggle with variables, relational statements, and translation from words to equations (Kieran, 2020; Verschaffel et al., 2020).

The analysis was guided by an adapted framework informed by Chick and Beswick (2018) and Charalambous (2016), focusing on five aspects of PCK: algebra content knowledge, selecting and using representations, providing and evaluating explanations, analysing students' errors and misconceptions, and selecting and using curriculum tasks. These aspects were selected because they are central to effective mathematics teaching and highly relevant to the instructional demands of algebra word problems (Charalambous, 2016; National Council of Teachers of Mathematics [NCTM], 2014).

Within this framing, teacher awareness is understood as more than knowing whether students succeed or fail. It includes judging likely task difficulty, interpreting incorrect responses, identifying possible sources of misconceptions, and proposing pedagogically appropriate support. At the same time, awareness alone is not sufficient. A teacher may recognise that students are likely to misinterpret variables or reverse a multiplicative relationship yet still lack the pedagogical means to address those difficulties effectively. This framing makes it possible to examine not only what the teachers noticed about students' difficulties, but also the pedagogical resources they drew on in responding to those difficulties.

Methodology

Participants and Context

This paper draws on the diagnostic phase of a broader qualitative study that investigated how a CoP supported junior high school mathematics teachers' PCK for teaching algebra word problems in Ghana. The paper focuses exclusively on the questionnaire administered before the CoP began, because these data provided baseline evidence of what the teachers knew about students' difficulties and the pedagogical resources they drew on in responding to those

difficulties. Eight teachers were purposively selected from public junior high schools in the study area using criteria that included professional teaching qualification, willingness to participate, and availability for CoP meetings. The teachers were drawn from both average- and below-average-performing schools. This number was suitable for collaborative professional learning and detailed qualitative analysis.

Instrument

A diagnostic assessment questionnaire was used to gather initial data on teachers' PCK in relation to algebra word problems. The questionnaire focused on five aspects of PCK: algebra content knowledge, selecting and using representations, providing and evaluating explanations, analysing students' errors and misconceptions, and selecting and using curriculum tasks. The questionnaire comprised three sections. Section A collected demographic information indicating variation in the teachers' ages, qualifications, and years of teaching experience. Section B required teachers to rank seven algebra items according to the level of difficulty they believed those items would present for their students and to justify those rankings. Section C presented hypothetical student answers to algebra word problems and asked teachers to judge whether the responses were correct, explain their reasoning, identify likely student mistakes, describe possible sources of those mistakes, and propose strategies for addressing them. These hypothetical answers were informed by prior literature on common student difficulties in algebra word problems (Clement, 1982; Ladele, 2013). The questionnaire was peer reviewed, piloted with five teachers outside the final sample, and revised for clarity. A copy of the questionnaire is presented in Kasimu (2024).

Data Analysis

A hybrid analytic approach was used. First, teachers' rankings of the seven algebra items in Section B were analysed descriptively using mean rankings (see Table 1). This descriptive analysis was used alongside the qualitative explanations teachers gave for their rankings. Second, teachers' responses to the hypothetical student answers to algebra word problems were analysed deductively using an adapted PCK framework informed by Chick and Beswick (2018). The analysis focused on the teachers' knowledge of algebra word problems, selecting and using representations, providing and evaluating explanations, analysing students' errors and misconceptions, and their judgements in selecting and using appropriate curriculum tasks, using representations, providing and evaluating explanations, and analysing students' errors and misconceptions. Responses were coded as satisfactory, unsatisfactory, or not evident. A response was coded as satisfactory when it showed mathematically and pedagogically appropriate understanding of the relevant task, unsatisfactory when it reflected misconceptions or significant errors in reasoning, and not evident when the relevant aspect was absent from the response. To strengthen consistency in coding, the categorisation of responses was reviewed by the co-authors, and any differences in coding were discussed and resolved through consensus. Third, teachers' open-ended responses about likely student errors, possible sources of those mistakes, and proposed strategies were analysed using thematic analysis (Braun & Clarke, 2019). The combination of descriptive, deductive, and inductive analysis enabled a more nuanced account of teachers' awareness of students' difficulties with algebra word problems and the PCK evident in their responses.

Findings

The diagnostic questionnaire revealed an uneven pattern in the teachers' awareness and PCK. The ranking data indicated that the teachers generally perceived algebra word problems as more difficult for students than symbolic equations, particularly when tasks required interpreting variables and translating verbal statements into algebraic expressions. However, analysis of the teachers' responses to the hypothetical student answers to algebra word problems

suggested that this awareness was not consistently supported by strong PCK. Although the teachers often identified likely student misconceptions, there was less evidence of clear explanations, effective representational strategies, or curriculum-oriented task reasoning in their responses.

Teachers' Awareness of Students' Difficulty with Algebra Word Problems

As shown in Table 1, the teachers' rankings of the seven algebra items showed a clear pattern. Symbolic items were consistently perceived as easier, whereas worded items were ranked as more difficult. The symbolic items received mean rankings of 1.00, 2.38, and 2.75, while the worded items received higher mean rankings of 4.63, 5.13, 5.75, and 6.38. These rankings suggest that the teachers recognised the greater demands involved in translating words into algebraic form compared with direct symbolic manipulation.

Table 1

Teachers' Ranking of Questions from 1 (Extremely Easy) to 7 (Extremely Difficult)

Nature of question type	Code	Ranked questions	Mean ranking
Symbolic: <i>Questions involving direct mathematical symbols and equations</i>	Q6	Solve for x: $5x = 50$	1.00
	Q4	Solve for x: $6/4 = 30/x$	2.38
	Q2	Solve for x in terms of a: $9a = 10x$	2.75
Worded: <i>Questions requiring translation of a word problem into a mathematical equation or concept</i>		Solve for x: $5x = 50$	1.00
	Q3	There are 900 people at a parade. There are 40 more men than women. There are twice as many children as there are men. How many children are there?	4.63
	Q5	Ahmad has four times as much money as Betty. After Ahmad spent GHC160 and Betty spent GHC 40, they each had equal amounts of money. How much money did Ahmad have at first?	5.13
	Q1	Write an equation using the variables S and P to represent the following statements: "There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors."	5.75
	Q7	Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel." Let C represent the number of cheesecakes and S represent the number of strudels ordered.	6.38

Among the worded items, a further distinction was also evident. Problems that combined known quantities and variables were perceived as less difficult than those expressed mainly in terms of variables. This suggests that the teachers recognised the added challenge students face when reasoning about relationships between abstract quantities without the support of concrete numerical values. This pattern aligns with earlier research showing that verbally presented algebra problems are often more difficult than symbolic equations because they require interpretation as well as manipulation (Clement, 1982; Nathan & Koedinger, 2000).

Thematic analysis of the teachers' reasons for their rankings generated two main themes: *conceptual understanding of variables and equations*, and *difficulty translating words into mathematical expressions*. Under the first theme, the teachers indicated that students often struggled to understand that variables represent quantities and that letters in algebra stand for numbers rather than fixed objects. Their comments suggested that students found abstract representations difficult, especially when these had to be interpreted within a problem context. The second theme concerned students' difficulty in converting verbal statements into algebraic expressions. The teachers referred to students' struggles with interpreting written statements, identifying relationships, and expressing them mathematically. Several suggested that students found symbolic equations easier when these were provided but had difficulty generating them independently from written scenarios. Together, these themes indicate that the teachers were aware of two central barriers in students' algebra learning: the abstraction involved in working with variables and the complexity of translating natural language into algebraic notation.

Teachers' Responses to the Hypothetical Student Answers and Algebra Tasks

The deductive analysis of the teachers' responses to the hypothetical student answers revealed an uneven pattern across the teaching practices associated with PCK. Although the teachers often identified likely student misconceptions, this was not consistently matched by strong explanations, representations, or curriculum-task strategies.

For the variable-equality word problems, four of the eight teachers provided satisfactory responses for algebra content knowledge, while the remaining four gave unsatisfactory responses. The same pattern was found for providing and evaluating explanations and for analysing students' thinking, errors, and misconceptions. This indicates that only half of the teachers were able to explain correctly why two different variables could represent equal quantities and to interpret the misconception embedded in the item appropriately. The unsatisfactory responses revealed uncertainty about the meaning of variables, with some teachers treating different letters as though they could not represent the same quantity.

For the verbal-to-symbolic problems, the pattern was similarly uneven, with some aspects revealing more limited content knowledge. For example, for the statement "*The number y is eight times the number z* ", five teachers gave satisfactory responses, while three did not. However, for the students-and-professors statement (*There are six times as many students as professors at this university*), only two teachers produced the correct equation, while six gave unsatisfactory responses. Many of these unsatisfactory responses reflected reversal errors or incorrect operations, suggesting that some of the teachers themselves were not fully secure in representing multiplicative relationships algebraically. This finding is important because it suggests that some of the misconceptions teachers identified in students' work were also reflected in their own responses.

A further weakness was evident in the quality of the teachers' explanations. Although some teachers provided mathematically sound reasoning, others offered explanations that were incomplete, circular, or conceptually weak. This suggests that even when a correct answer was given, the pedagogical capacity to explain the underlying algebraic relationship clearly was not always secure. Such variability matters because explanation is central to helping students make sense of algebraic structure rather than simply reproduce a procedure.

A particularly significant result was that none of the teachers explicitly referred to the use of representations in response to the verbal-to-symbolic tasks, and none explicitly addressed the selection and use of curriculum tasks. These components were therefore coded as not evident. This absence is important because it suggests that, although several teachers could identify likely student difficulties, they did not draw on visual or concrete representations, nor did they propose task-based strategies that might help students build conceptual understanding progressively.

Teachers' Views of Student Errors and Support Strategies

The teachers' open-ended responses about the kinds of mistakes students were likely to make provide further support for this interpretation. They pointed to recurring errors such as misinterpretation of mathematical symbols or operations, confusion about the meaning of "is", incorrect use of addition in place of multiplication, interchange of variables, and reversal errors in forming equations. These responses suggest that the teachers had a reasonably strong awareness of the kinds of incorrect formulations students might produce when attempting to translate verbal statements into algebraic expressions.

When asked about possible sources of these mistakes, teachers' responses clustered around two main themes: language comprehension and lack of understanding of word problems. Some pointed to students' inability to read and understand the question properly, including difficulties with English language comprehension. Others suggested that students feared word problems or assumed that mathematics should mainly involve numbers and direct operations rather than verbal situations requiring interpretation. These responses indicate that the teachers understood students' difficulties as involving both linguistic and conceptual factors.

In proposing strategies to address students' difficulties, the teachers most commonly referred to providing explanation, engaging students in problem-solving, and student-centred learning. Their suggestions included explaining the statement to students, helping them understand the problem, solving more examples with them, and encouraging students to read and attempt problems independently before receiving corrective feedback. These strategies were pedagogically sensible, but they were also quite general. The teachers rarely moved beyond explanation and practice to more specialised pedagogical tools such as visual representations or deliberately sequenced tasks designed to build understanding.

Overall, the findings suggest that the teachers' awareness of student difficulty was stronger than the pedagogical repertoire they drew on in responding to that difficulty. They were relatively better at identifying likely errors and misconceptions than at providing strong explanations, selecting representations, or drawing on curriculum-task strategies that could support learning more effectively. This reinforces the broader finding that the diagnostic assessment revealed a distinction between recognising difficulty and having the pedagogical means to address it in mathematically powerful ways.

Discussion

This study examined what a diagnostic questionnaire revealed about teachers' awareness of students' difficulties with algebra word problems and the PCK they might draw on in responding to those difficulties. The findings show that the teachers in this study were aware of students' misconceptions. Their rankings of the seven algebra items, their explanations for those rankings, and their responses to the hypothetical student answers all indicate that they recognised algebra word problems as more demanding than symbolic equations and were aware of several common student difficulties, including poor understanding of variables, reversal errors, and challenges in translating verbal statements into algebraic expressions. This pattern is consistent with research showing that algebra word problems are often more difficult than symbolic equations because they require students to coordinate language, relationships, and symbolic representation (Clement, 1982; Kieran, 2020).

At one level, this is an encouraging finding. It suggests that the teachers demonstrated an important aspect of PCK, namely awareness of how students are likely to experience difficulty in algebra. This aligns with Shulman's (1987) argument that effective teaching depends not only on subject-matter knowledge, but also on knowledge of how learners engage with that content. It also supports research identifying knowledge of content and students as a central aspect of teaching expertise (Ball et al., 2008). In this study, the teachers were often able to

anticipate incorrect formulations students might produce and to identify plausible sources of those errors, particularly in relation to language comprehension and the difficulty of translating words into symbolic form.

However, the findings also reveal a more critical point: awareness of student difficulty was not consistently matched by strong pedagogical responses. Although several teachers could identify likely misconceptions, fewer demonstrated secure algebra content knowledge across the focal tasks, and even fewer showed strong explanatory power. Most notably, there was no explicit evidence of representations or curriculum-task use in their responses to the verbal-to-symbolic tasks. This matters because PCK involves not only recognising student misconceptions, but also being able to explain concepts clearly, choose useful representations, and select tasks that support learning meaningfully (Chick & Beswick, 2018).

The distinction between recognising difficulty and responding pedagogically is one of the main contributions of the paper. A teacher may recognise that students are likely to confuse variables, reverse a multiplicative relationship, or misread a verbal statement, yet still lack the representational, explanatory, or task-based resources needed to address those difficulties effectively. In this sense, the diagnostic questionnaire did not simply show how teachers interpreted students' difficulties. It also exposed the limits of the pedagogical tools they were drawing on when imagining how to support learners. This finding supports broader arguments that effective mathematics teaching requires more than identifying error; it requires transforming mathematical ideas into forms that are accessible and instructionally productive for students (Ball et al., 2008; Chick & Beswick, 2018).

The absence of representations is especially significant. Algebra word problems require students to move between language, quantities, and symbolic forms, and representations can help bridge these forms of reasoning. Visual and concrete representations can support conceptual understanding by making relationships more visible and less abstract (Kieran et al., 2016). Yet none of the teachers explicitly proposed such representations in response to the verbal-to-symbolic tasks. A similar concern applies to curriculum-task selection. Although some teachers suggested giving students more examples or practice, there was little evidence of deliberate attention to task sequencing or to the use of tasks that build understanding progressively. Together, these omissions point to an uneven pattern of PCK rather than a simple lack of awareness.

The findings therefore suggest that the teachers had a meaningful starting point for professional learning, but one that needed to be extended and deepened, especially in relation to explanation, representation, and task design. The diagnostic questionnaire was valuable not only as a baseline measure, but also as an analytic tool that revealed where subsequent professional learning needed to focus. In this sense, the study shows how diagnostic assessment can make visible both teachers' existing strengths and the pedagogical gaps that may constrain their responses to students' difficulties.

Conclusion

This paper examined what a diagnostic questionnaire revealed about teachers' awareness of students' difficulties with algebra word problems in Ghana. The findings showed that the teachers generally recognised that worded algebra tasks were more difficult for students than symbolic ones and were able to identify several important student difficulties, including weak understanding of variables, reversal errors, and challenges in translating verbal statements into algebraic expressions. These results suggest that the teachers possessed some awareness of the conceptual and linguistic demands that algebra word problems place on learners.

At the same time, the diagnostic questionnaire also revealed uneven PCK. Although several teachers were able to anticipate likely student misconceptions, fewer demonstrated secure algebra content knowledge across the focal tasks, and none explicitly referred to representations

or curriculum-task selection in response to the verbal-to-symbolic tasks. The findings therefore suggest that awareness of student difficulty, while important, was not consistently matched by strong pedagogical resources for responding to that difficulty. Diagnostic assessment can make visible both what teachers already notice and what pedagogical resources they still need, providing an evidence-informed basis for shaping targeted professional learning.

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References

- Abreh, M. K., Owusu, K. A., & Amedahe, F. K. (2018). Trends in performance of WASSCE candidates in the science and mathematics in Ghana: Perceived contributing factors and the way forward. *Journal of Education, Society and Behavioural Science*, 26(1), 1–17. <https://doi.org/10.1177/0022057418800950>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407. <https://doi.org/10.1177/0022487108324554>
- Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative Research in Sport, Exercise and Health*, 11(4), 589–597. <https://doi.org/10.1080/2159676X.2019.1628806>
- Charalambous, C. Y. (2016). Investigating the knowledge needed for teaching mathematics: An exploratory validation study focusing on teaching practices. *Journal of Teacher Education*, 67(3), 220–237. <https://doi.org/10.1177/0022487116634168>
- Chick, H., & Beswick, K. (2018). Teaching teachers to teach Boris: A framework for mathematics teacher educator pedagogical content knowledge. *Journal of Mathematics Teacher Education*, 21(5), 475–499. <https://doi.org/10.1007/s10857-016-9362-y>
- Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, 13(1), 16–30. <https://doi.org/10.2307/748434>
- Kasimu, O. (2024). *Enhancing mathematics teachers' pedagogical content knowledge for teaching algebra word problems: A community of practice approach* [Unpublished doctoral thesis]. University of Tasmania.
- Kieran, C. (2020). Algebra teaching and learning. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (2nd ed.). Springer. https://doi.org/10.1007/978-3-030-15789-0_6
- Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (Eds.). (2016). *Early algebra: Research into its nature, its learning, its teaching*. Springer. <https://doi.org/10.1007/978-3-319-32258-2>
- Ladele, O. A. (2013). *The teaching and learning of word problems in beginning algebra: A Nigerian (Lagos state) study* [Doctoral dissertation, Edith Cowan University]. <https://ro.ecu.edu.au/theses/693/>
- Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education*, 31(2), 168–190. <https://doi.org/10.2307/749750>
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.
- Osei, W., & Agyei, D. D. (2023). Teaching knowledge and difficulties of in-field and out-of-field junior high school mathematics teachers in algebra. *Cogent Education*, 10(2), Article 2232240. <https://doi.org/10.1080/2331186X.2023.2232240>
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1–22. <https://doi.org/10.17763/haer.57.1.j463w79r56455411>
- Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: A survey. *ZDM Mathematics Education*, 52, 1–16. <https://doi.org/10.1007/s11858-020-01130-4>
- West African Examinations Council. (2023). *Chief examiners' report*.

Initial Teacher Education and Out-of-Field Teaching: Problem or Solution?

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Out-of-field teaching remains a persistent challenge in mathematics education. Initial Teacher Education (ITE) is often positioned as part of this problem because of perceived gaps in disciplinary preparation, yet it is also a key site for developing teacher capability. Using policy analysis and interview data, this paper explores how ITE is positioned with respect to accreditation constraints on program design and the limited differentiation between preparation for junior and senior secondary mathematics teaching. The analysis suggests that ITE operates simultaneously as both a constraint and an opportunity within the broader system shaping mathematics teacher supply and preparation.

Teacher shortages across many education systems have contributed to a growing reliance on out-of-field (OOF) teaching, where teachers are assigned to teach subjects for which they lack formal qualifications or disciplinary preparation (Hobbs, 2013; Ingersoll, 2002). In mathematics education, the implications of this practice are especially significant. Effective mathematics teaching requires specialised disciplinary knowledge alongside pedagogical content knowledge that supports students' conceptual understanding and teachers' capacity to interpret student thinking (Ball et al., 2008). When teachers are required to teach mathematics outside their area of expertise, this can affect their professional confidence, identity, and instructional decision-making (Du Plessis et al., 2015).

Recent work has increasingly moved beyond framing OOF teaching solely as a deficit in teacher preparation. Instead, it is understood as a systemic issue shaped by teacher supply, workforce distribution, school staffing practices, policy environments, and available pathways for professional learning and re-specialisation (Hobbs & Porsch, 2021; Ross & Hobbs, 2024). Within this broader landscape, ITE occupies a complex position. Universities are expected to prepare graduates with strong disciplinary expertise and pedagogical capability, yet teacher education programs operate within accreditation frameworks and institutional constraints that shape what can be included in that preparation. ITE is therefore often discussed in two contrasting ways. On the one hand, it may be viewed as part of the problem when graduates

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 226–233). MERGA.

are perceived as insufficiently prepared to teach mathematics (Campbell et al., 2019) or when teacher shortages persist despite the provision of teacher education (Dawborn-Gundlach, 2025). On the other hand, ITE remains one of the most important formal sites for developing the knowledge, identity, and professional capability required for effective mathematics teaching. This dual positioning raises an important question: *To what extent is ITE part of the mathematics out-of-field teaching problem, part of the solution, or both?*

This paper addresses that question, focusing on two interrelated issues identified in the analysis. Drawing on literature and illustrative interview data from mathematics-connected teacher educators (ITE providers), the paper examines how these issues shape ITE's capacity to respond to the realities of mathematics teacher supply and preparation.

Literature Review

Out-of-Field Teaching in Mathematics Education

Out-of-field teaching has been widely documented as a persistent feature of education systems in many countries, particularly in shortage areas such as mathematics and science (Goos et al., 2023; Ingersoll, 2002; Weldon, 2016). In Australia, workforce analyses suggest that a substantial proportion of teachers teaching mathematics – up to 40% – do not hold specialist qualifications in the subject (AITSL, 2021). The scale of the issue has become more visible in recent years as teacher shortages have intensified and schools have increasingly relied on pragmatic staffing responses (Ross & Hobbs, 2024).

Hobbs (2013) conceptualises OOF teaching as a boundary-crossing event in which teachers move beyond their disciplinary expertise and must renegotiate their professional identities. Teachers assigned to teach outside their field frequently report increased workload, reduced confidence, and challenges in developing subject-specific pedagogical strategies (Du Plessis et al., 2015). These challenges can be particularly pronounced in mathematics, where effective teaching requires specialised disciplinary knowledge and pedagogical content knowledge that supports students' conceptual understanding (Ball et al., 2008).

At the same time, OOF teaching should not be interpreted solely as a deficit in teacher capability. Rather, it reflects broader structural conditions within education systems, including teacher shortages, workforce distribution, and school organisational practices (Hobbs, 2020; Weldon, 2016). Recent project work has reinforced this wider view by examining how OOF mathematics teaching is shaped through workplace practices, policies, and professional education cultures (Ross et al., 2024). This broader framing is important because it shifts attention away from the simplistic assumption that ITE alone can solve the problem. For example, modelling undertaken by Prince and O'Connor (2018) estimated that to achieve the goal of reducing OOF teaching of mathematics to 10% in the next ten years, it would be necessary to recruit new graduates at 120% of the teacher retirement rate per year. It seems unfeasible for ITE to provide such a "solution".

Initial Teacher Education and Teacher Preparation

Initial Teacher Education (ITE) plays a central role in preparing teachers for professional practice. Traditionally, ITE programs aim to develop disciplinary knowledge, pedagogical knowledge, and practical teaching capability through university coursework and school-based experiences (Campbell et al., 2019; Hobbs & Porsch, 2021). In mathematics education, strong disciplinary preparation has been widely recognised as essential for effective teaching, particularly in supporting conceptual understanding, anticipating misconceptions, and making informed pedagogical decisions (Ball et al., 2008). In Australia, accreditation of ITE programs is governed by the Program Standards of the Australian Institute for Teaching and School Leadership (AITSL, 2025). The program standards set out mandatory content requirements for

discipline studies (providing content knowledge, CK) and discipline-specific curriculum and pedagogical studies (providing pedagogical content knowledge, PCK). For undergraduate programs, discipline studies must comprise “*at least a major study in one teaching area and preferably a second teaching area, which must comprise at least a minor study*” (AITSL, 2025, p.23). Graduate entry ITE programs require entrants to have completed equivalent discipline studies in a prior degree. Undergraduate and graduate entry programs require the same amount of discipline-specific curriculum and pedagogical studies, although slight variations exist between first/second teaching areas and undergraduate/graduate entry programs. However, the AITSL Program Standards do not differentiate the preparation required for teaching junior versus senior secondary mathematics, which adds to the complexity of the relationship between ITE and OOF teaching.

Campbell et al. (2019) argue that teacher education programs must prepare teachers not only for teaching within their disciplinary specialisation but also for the possibility of teaching outside it. This perspective reflects the reality that teachers’ professional trajectories do not always align neatly with the subject boundaries established during initial preparation. Hobbs and Porsch (2021) similarly note that OOF teaching presents specific challenges for teacher education, requiring more explicit attention to how teacher preparation connects with later professional contexts.

These studies collectively suggest that expectations placed on ITE programs may exceed what they can realistically achieve. Teacher education programs operate within accreditation requirements, program duration limits, and curriculum coverage demands across multiple learning areas. Moreover, subject allocation decisions are typically made within schools rather than universities. As a result, even graduates with strong disciplinary preparation may find themselves teaching outside their area of expertise because of workforce pressures or school staffing needs. These tensions highlight the importance of understanding ITE not as an isolated solution, but as one element within a broader network of policy, institutional practice, and professional learning (Prince & O’Connor, 2018; Weldon, 2016).

An Ecological Perspective on Teacher Preparation

To understand how ITE relates to OOF teaching, it is useful to consider teacher preparation within a broader system of influences. Bronfenbrenner’s ecological systems theory (1994) provides a framework for examining how individuals’ experiences are shaped by interactions among immediate environments (the microsystem), institutional relationships (the mesosystem), policy structures and other external settings that indirectly affect the individual (the exosystem), and broader cultural values (the macrosystem) (Bronfenbrenner, 1994). Within such a framework, teachers operate in interconnected systems that influence both their professional development and the conditions under which they work. This perspective is useful for OOF teaching because it helps explain why teacher preparation cannot be separated from the broader structures that shape teacher assignment and development. University programs, accreditation frameworks, school staffing practices, and professional learning opportunities all play a role in shaping who enters mathematics teaching, how they are prepared, and what kinds of support remain available when they encounter OOF contexts. Understanding the positioning of ITE within this wider system is therefore essential for developing more coherent responses to the challenges associated with OOF teaching.

Methodology

This paper forms part of a broader investigation into the system of professional education surrounding OOF teaching. The analysis in this paper focuses on mathematics and draws on two sources of data: a review of policy documents and literature on OOF teaching and

mathematics teacher preparation, and illustrative interview data from participants connected with mathematics teacher education.

Policy Review and Analysis

Phase 1 of our project involved a policy review and analysis. To this end, the whole research team collected documents from teacher professional associations, teacher registration bodies, government departments and agencies, unions, and other learned societies (110 documents in total). The mathematics teacher educators in the research team (i.e., the underlined authors of this paper) analysed all documents related to mathematics education. The documents were read iteratively and examined for statements that directly or indirectly referred to OOF teachers or teaching, to identify recurring themes.

Following individual analyses, the mathematics group met to discuss emerging themes and compare interpretations. We checked the reliability of our analysis and drew on our professional capital (Hargreaves & Fullan, 2012; Oates et al., 2021) to discern the meaning and implications of the findings (see Ross et al., 2025). Emerging themes were examined through the lens of ecological systems theory to assess their degree of influence on the broader ecosystem of support for OOF teachers. In positioning the themes, two issues emerged as particularly significant for mathematics teacher preparation: accreditation constraints affecting program design and the differentiation of preparation for different levels of secondary mathematics teaching.

Interview Data

Phase 2 of the broader project involved interviews with 37 stakeholders, including teacher educators, professional association representatives, and policy actors. In this paper, however, the interview data are not treated as a standalone dataset for analysis. Instead, anonymised statements are selected which address the two key issues identified in the documentary analysis. Because of our focus on ITE, these excerpts are drawn from three mathematics teacher educators and used illustratively, woven through the findings. Their words bring to life and exemplify the themes derived from the documentary analysis, highlighting how they are experienced, interpreted, and negotiated by participants, and illuminating the opportunities, tensions, and challenges embedded in ITE for mathematics. As such, the interview material is presented to illustrate broader patterns rather than as independent evidence for claims.

Findings and Discussion

The analysis highlighted two closely related issues shaping the role of ITE in responses to OOF mathematics teaching (see Table 1).

Table 1

Summary of key issues in positioning ITE as a problem and a solution

Issue	Problem	Potential solution
Accreditation constraints and program responsiveness	Rigid accreditation and registration requirements can restrict flexible responses to workforce needs and limit timely program adjustment.	More responsive program design and clearer pathways may strengthen mathematics teacher preparation while maintaining professional standards.
Differentiating preparation for junior and senior mathematics teaching	A single mathematics preparation threshold may obscure different content and pedagogical demands across junior and senior secondary school mathematics.	More differentiated preparation pathways could better align teacher education with the specific demands of junior and senior mathematics teaching contexts.

The first issue concerns how accreditation and program structures can constrain responsiveness in teacher education, while the second relates to the limited differentiation

between preparation for junior and senior secondary mathematics teaching. Operating between the meso- and exo- levels of the system, these issues highlight how institutional arrangements and broader policy settings interact to shape practice in ITE. Together, they illustrate how ITE may function as both a problem and a potential solution within the broader system that shapes mathematics teacher preparation.

Accreditation constraints and program responsiveness

Accreditation frameworks play a central role in shaping the structure of ITE programs. These frameworks establish expectations for disciplinary preparation, professional experience, and pedagogical development, thereby contributing to the professional credibility of teacher education. At the same time, accreditation requirements may limit the flexibility of teacher education programs in responding to emerging workforce needs.

Nancy highlighted the force of external registration requirements directly: *“VIT won’t register if subject requirements aren’t met ... NESA won’t ... QCT won’t.”* This quote captures the regulatory weight of teacher registration bodies across jurisdictions. It points to a central tension: accreditation and registration standards are designed to ensure professional quality, yet they can also restrict the range of possible pathways into mathematics teaching. This concern was evident across the broader data, where accreditation and registration requirements were repeatedly described as both necessary for maintaining standards and limiting for program responsiveness.

Several interviewees questioned the AITSL program standards requirement that undergraduate ITE programs include third-year level university mathematics. Isobel suggested:

Maybe what we need is a... profound understanding of secondary level mathematics... do we want them to have studied up to second year level university mathematics... or do we want them to have a really rich, deep connection about what’s going on with the secondary level mathematics, with a sense of where it’s going?

This comment highlights the problematic boundaries within the discipline of mathematics that are often experienced by both pre-service and practising teachers. In particular, research suggests that secondary school mathematics teachers find it difficult to see connections between the mathematics they learn at university and the mathematics they teach in school (Wasserman et al., 2018). However, ITE programs typically maintain this boundary by separating, rather than integrating, the teaching of these two forms of mathematical knowledge.

These constraints are not limited to external accreditation and registration requirements. Internal university governance can also restrict the capacity for rapid curricular change and the introduction of more responsive pathways. Even where teacher educators recognise a clear need to provide additional preparation for prospective teachers who may be required to teach mathematics, institutional processes may make timely adjustments difficult. In this respect, the issue is not simply whether teacher educators know what should be done, but whether program structures allow it.

Recent work on mathematics OOF upskilling in Australia points to a similar tension. There is growing recognition of the need for clearer, more coherent pathways to support mathematics teacher supply, yet existing arrangements remain fragmented and difficult to scale (Barker et al., 2024; Goos & Marchant, 2025). Although those reports focus mainly on practising teachers, their implications extend to ITE because both domains are shaped by the same wider pressures to maintain standards while responding to the shortage.

Nevertheless, accreditation frameworks also contribute positively to teacher preparation by ensuring that mathematics teachers possess recognised disciplinary and pedagogical preparation. In this sense, accreditation operates simultaneously as a regulatory constraint and as a mechanism that protects the professional standards of mathematics teaching. The challenge lies in balancing these two functions so that teacher education programs can maintain rigorous

preparation while remaining responsive to the realities of mathematics teacher supply. One possible response is to develop more flexible program structures and clearer mathematics pathways that retain standards while allowing institutions to respond more effectively to workforce needs.

Differentiating preparation for junior and senior mathematics teaching

A second issue concerns the limited differentiation between preparation for junior and senior secondary mathematics teaching. School mathematics encompasses a wide range of conceptual demands, from foundational number, algebra, and proportional reasoning in lower secondary years to advanced mathematical topics in senior secondary curricula. However, teacher education pathways frequently treat mathematics preparation as a single disciplinary requirement.

This issue is also evident in the way teacher education courses are structured. Katrina described the fragmentation that can occur within programs: *“Courses might have one unit that focuses on teaching and another on learning, another on assessment, another on planning ... they’re kind of more pedagogy first, and then it can be a bit random what content areas they focus on.”* This observation suggests that the organisation of coursework may sometimes privilege generic pedagogical categories without ensuring a sufficiently coherent relationship between disciplinary mathematics and mathematics pedagogy. Similar concerns appeared elsewhere in the data, where participants described teacher education structures that separate mathematics-specific content from pedagogical preparation in ways that can weaken the coherence of teacher preparation.

Participants also suggested that a uniform approach to mathematics preparation may restrict potential pathways into teaching, especially for graduate-entry applicants seeking a career change. Prospective teachers with partial mathematics preparation may be able to teach junior secondary mathematics effectively if they are provided with targeted pedagogical and disciplinary support. Yet rigid disciplinary thresholds may prevent such candidates from entering mathematics teaching pathways at all. This has implications for both teacher supply and for the design of ITE programs.

Interview data highlighted the importance of integrating disciplinary knowledge and pedagogical preparation in mathematics teacher education. The relationship between content and pedagogy was made explicit by Isobel, who stated: *“It’s very hard to talk seriously about pedagogy without talking about content...pedagogical content knowledge actually needs content knowledge.”* This reinforces a core point in mathematics teacher education: content and pedagogy cannot be treated as separate or sequential domains. Preparation for teaching mathematics requires attention to the interplay between what is taught and how it is taught. Across our analysis, this issue was linked to questions about what kind of mathematical preparation is required for different forms of secondary mathematics teaching, rather than assuming that one undifferentiated threshold adequately serves all school levels.

The issue is not whether standards should be lowered, but whether they should be more carefully differentiated. A single undifferentiated specialisation threshold may obscure the distinct demands of junior and senior secondary mathematics teaching. A more nuanced approach could better align teacher preparation with different mathematics teaching contexts while maintaining disciplinary integrity. This is especially important in shortage conditions where schools often need teachers for junior secondary mathematics, but current pathways may be structured around expectations more closely aligned with senior secondary specialisation. A possible way forward is to create preparation pathways that recognise the differing pedagogical and disciplinary demands of junior and senior secondary mathematics teaching.

ITE within the broader system of mathematics teacher preparation

Taken together, these issues illustrate the complex role ITE plays in the wider system that shapes mathematics teacher preparation. Program accreditation and disciplinary expectations influence who enters mathematics teaching pathways and how teacher education programs are structured. At the same time, teacher education institutions remain critical sites for developing the knowledge, professional identity, and pedagogical expertise required for effective mathematics teaching. These dynamics reflect an interplay between the mesosystem (where institutional practices and program structures are enacted) and the exosystem (encompassing policy frameworks and external settings that indirectly shape teacher education).

This suggests that it is unhelpful to attribute ITE as a source of deficiencies in teacher preparation for out-of-field teaching. Rather, it is one component within a broader system that includes policy frameworks, institutional practices, and professional learning opportunities, operating across and between these system levels. The same features that help preserve standards can also restrict flexibility, just as the structures that create barriers to responsiveness may simultaneously provide the formal recognition needed to sustain teacher quality.

Seen in this way, ITE can be both part of the problem and part of the solution. It may contribute to the persistence of OOF teaching when pathways are too rigid, too generic, or insufficiently differentiated. Yet it also remains one of the strongest sites for developing future mathematics teachers' disciplinary confidence, pedagogical capability, and professional identity. Strengthening mathematics teacher preparation, therefore, requires attention not only to teacher education programs, but also to the broader structures that shape their operation.

Conclusion

Out-of-field teaching continues to present challenges for mathematics education systems, particularly in contexts where teacher supply and disciplinary preparation are unevenly distributed. ITE occupies a central position within these dynamics because it represents the primary pathway through which new mathematics teachers enter the profession.

The analysis presented in this paper suggests that ITE should not be viewed simply as either problem or solution. Accreditation frameworks and disciplinary expectations can limit program responsiveness, yet they also maintain professional standards. Similarly, current structures may restrict pathways into mathematics teaching, while teacher education institutions remain essential sites for developing disciplinary and pedagogical expertise.

This analysis is limited to the ITE perspective and does not yet incorporate the lived experiences of teachers teaching mathematics OOF. Even so, it suggests that some of the constraints shaping ITE are located in exosystemic policy and regulatory structures, while some potential responses may emerge through more responsive mesosystem relationships within and across institutions. Understanding ITE as part of a broader system therefore provides a more nuanced perspective on how teacher preparation can respond to the challenges of OOF mathematics teaching. Future research should continue to examine how policy frameworks, institutional structures, professional learning opportunities, and teachers' lived experiences interact to shape the preparation and support of mathematics teachers.

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References

Australian Institute for Teaching and School Leadership (AITSL) (2021). *Australian Teacher Workforce Data National Teacher Workforce Characteristics Report*.

- Australian Institute for Teaching and School Leadership (AITSL) (2025). *Accreditation of initial teacher education programs in Australia. Standards and procedures.*
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407. <https://doi.org/10.1177/0022487108324554>
- Barker, M., Goos, M., & Coupland, M. (2024). *Analysis of out-of-field secondary mathematics teacher upskilling initiatives in Australia.* AMSI. <https://amsi.org.au/?publications=relieving-out-of-fieldteaching-in-australian-secondary-mathematics>
- Bronfenbrenner, U. (1994). Ecological models of human development. In *International Encyclopedia of Education* (Vol. 3, 2nd ed.). Elsevier.
- Campbell, C., Hobbs, L., & Porsch, R. (2019). Initial teacher education: Roles and possibilities for preparing capable teachers. In L. Hobbs & G. Törner (Eds.), *Examining the phenomenon of teaching out-of-field* (pp. 243–267). Springer. https://doi.org/10.1007/978-981-13-3366-8_10
- Dawborn-Gundlach, M. (2025). Addressing the problem of teacher shortage in the Australian educational context. In A. Gehrmann & P. Germer (Eds.), *Teacher shortage in international perspectives: Insights and responses* (pp. 127–144). Springer. https://doi.org/10.1007/978-3-658-45398-5_7
- Du Plessis, A. E., Carroll, A., & Gillies, R. M. (2015). Understanding the lived experiences of novice out-of-field teachers in relation to school leadership practices. *Asia-Pacific Journal of Teacher Education*, 43(1), 4-21.
- Goos, M. & Marchant, T. (2025). *Relieving out-of-field teaching in Australian secondary mathematics: Upskilling program design elements.* Australian Mathematical Sciences Institute (AMSI).
- Goos, M., Ni Riordáin, M., Faulkner, F., & Lane, C. (2023). Impact of a national professional development programme for out-of-field teachers of mathematics in Ireland. *Irish Educational Studies*, 42(3), 401-421.
- Hargreaves, A. & Fullan, M. (2012). *Professional capital: Transforming teaching in every school.* Teachers College, Columbia University.
- Hobbs, L. (2013). Teaching out-of-field as a boundary-crossing event: Factors shaping teacher identity. *International Journal of Science and Mathematics Education*, 11(2), 271–297.
- Hobbs, L. (2020). Learning to teach science out-of-field: A spatial-temporal experience. *Journal of Science Teacher Education*, 31(7), 725–745. <https://doi.org/10.1080/1046560X.2020.1718315>
- Hobbs, L., & Porsch, R. (2021). Teaching out-of-field: Challenges for teacher education. *European Journal of Teacher Education*, 44(5), 601–610. <https://doi.org/10.1080/02619768.2021.1985280>
- Ingersoll, R. M. (2002). Out-of-field teaching and the limits of teacher policy. *Educational Evaluation and Policy Analysis*, 24(1), 21–42. <https://doi.org/10.3102/01623737024001021>
- Oates, G., Muir, T., Murphy, C., Reaburn, R. & Maher, N. (2021). What influences mathematics teacher educators’ decisions in course design: Activity theory and professional capital as an investigative approach. In M. Goos & K. Beswick (Eds.), *The learning and development of mathematics teacher educators* (pp. 345–366). Springer Nature. https://doi.org/10.1007/978-3-030-62408-8_18
- Prince, G., & O’Connor, M. (2018). *Crunching the numbers on out-of-field teaching.* AMSI Occasional Paper 1. <https://schools.amsi.org.au/2019/01/14/crunching-the-numbers-out-of-feild-teaching/>
- Ross, E., Goos, M., Caldis, S., Cirkony, C., Delaney, S., Dutton, J., Hobbs, L., Oates, G., & Speldewinde, C. (2024). Exploring the culture of out-of-field professional education for mathematics teachers. In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education: Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (p. 566). MERGA.
- Ross, E., Goos, M., Oates, G., Hobbs, L., Speldewinde, C., Caldis, S., Cirkony, C., Delaney, S., & Dutton, J., (2025). “Anyone can teach maths”: Workplace perspectives on out-of-field teaching of mathematics. In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia, Canberra* (pp. 389–396). MERGA.
- Ross, E., & Hobbs, L. (2024). Stretching the boundaries: The rise and rise of out-of-field teaching in Australia. *Curriculum Perspectives*, 44, 579–585. <https://doi.org/10.1007/s41297-024-00282-7>
- Wasserman, N., Weber, K., Villanueva, M., & Mejia-Ramos, J. P. (2018). Mathematics teachers’ views about the limited utility of real analysis: A transport model hypothesis. *Journal of Mathematical Behavior*, 50(1), 74–89. <https://doi.org/10.1016/j.jmathb.2018.01.004>
- Weldon, P. R. (2016). *Out-of-field teaching in Australian secondary schools.* Policy Insights (Issue 6). ACER. <https://research.acer.edu.au/policyinsights/6/>.

Selling Success: A Critical Examination of Mathematics Tutoring Advertisements in Australia

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Private mathematics tutoring has expanded rapidly in Australia, yet little research has examined how agencies advertise their services or how promotional claims align with school curriculum and pedagogy. Using Critical Discourse Analysis (CDA), this study analyses promotional texts from five Australian mathematics tutoring agencies. Cross-case analysis identified discursive themes through which advertisements interact with students, parents, and tutors to legitimise tutoring participation. The paper shows how these discourses shape understandings of mathematics learning and success.

Introduction

The tutoring industry has expanded rapidly across many education systems worldwide (Bray & Kwo, 2014) and has become particularly prominent in Australia over the past decade (Li et al., 2025). Recent estimates suggest that more than 40% of Australian students now participate in some form of private tutoring (Apex Tuition Australia, n.d.). Within Australia's multi-billion-dollar tutoring market (ATA, 2022), mathematics tutoring holds the largest market share. This growth is often linked to stakeholders' heightened emphasis on mathematics achievement, driven by the belief that mathematical performance underpins broader academic success and future opportunities beyond school (Li et al., 2025).

Despite this growth, little attention has been directed towards examining how tutoring services are advertised. Specifically, whether their promotional claims align with or contradict how mathematics is taught in formal schooling. Existing research has documented tensions between teachers and tutors: teachers may criticize tutor's reliance on drilling and sole focus on exam preparation, while tutors may position school teaching as insufficient for producing the results students and parents seek (Guo et al., 2020; Zheng et al., 2020). Yet it remains unclear how tutoring advertisements may reinforce or intensify these tensions. In this study, stakeholders are understood as occupying distinct but overlapping instructional roles. Teachers are considered to be curriculum-aligned educators advocating holistic mathematical proficiency development. Tutors are positioned as specialised instructors oriented towards exam performance. Whereas parents and students are seen as evaluators of educational effectiveness whose decision shape tutoring participation. These roles are not neutral, as they actively shape how tutoring is valued, justified, and enacted within the local educational context.

As such, to address this gap and explore how tutoring services' advertisements align with or contradict formal schooling's skill-focused approach in teaching mathematics (ACARA, n.d.), this paper employs CDA to answer the question: How do Australian mathematics tutoring agencies construct tutoring services, stakeholders, and relationships to formal schooling through advertisements? Through systematic analysis of five tutoring agencies' advertisements, the linguistic patterns, recurring themes, advertisements' potential influence on stakeholders in making educational decision, and the broader discursive functions of tutoring advertisements were investigated. From this, we aim to illuminate how particular discourses are utilised to construct and legitimise tutoring services and explore the extent to which these discourses align with the philosophies of teaching and learning mathematics in formal schools in Australia.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 234–241). MERGA.

Literature Review

Private tutoring, often conceptualised as ‘shadow education’ (Bray, 2009), refers to fee-paying supplementary learning activities taking place outside school hours, that mirrors formal schooling in content and structure. In this study, tutoring is treated as a form of shadow education, but the term ‘tutoring’ is adopted to reflect its everyday usage in the Australian context. This conceptualisation situates tutoring not as an isolated practice, but as one that evolves in direct relation to formal schooling and its assessment structures.

Although some forms of tutoring emphasise enrichment, confidence building, or remediation for learners who struggle to keep pace with school-based instruction (Bray, 2009), a large proportion of tutoring services are explicitly framed around examination preparation and target performance gains (Bray & Kwo, 2014). The main purpose of tutoring, in many students’ and parents’ views, is to boost grades in exams and to secure access to desirable career pathways (Guo et al., 2020). This purpose is evident across many education systems that have witnessed exponential growth in tutoring (Guill et al., 2022), including Australia (Dhall, 2024).

The Australian education system is strongly meritocratic, with university entrance largely dependent on the Australian Tertiary Admission Rank (ATAR), which represents a student’s academic standing relative to peers across states, based on performance in senior secondary subjects. High-stakes examinations, that contribute to ATAR calculations, therefore carry substantial weight in shaping students’ post-school options (Zheng et al., 2020). These mechanisms contribute significantly to the growth of tutoring, as families seek any perceived advantage in an increasingly competitive educational environment (Bray & Kwo, 2014; Zhang et al., 2021). However, several important questions remain insufficiently addressed in the literature: (1) To what extent is tutoring effective in increasing academic performance and longer-term learning outcomes? (2) What impact does tutoring have on formal schooling? (3) How are parents and students persuaded to participate in tutoring, and what messages and representations lead them to place trust in tutoring providers?

While there’s lack of data in Australia to answer the first question, international research provides a useful basis for informed postulation. Studies conducted in a range of jurisdictions indicate that tutoring appears to have heterogeneous impacts on students’ achievement depending on prior attainment, socio-economic status, gender, geographic location, tutor qualifications and experience, and the intensity and duration of participation (Ömeroğulları et al., 2020; Zhang & Liu, 2022). In other words, tutoring does not consistently guarantee improved academic outcomes many students and parents expect, in some cases, it may offer negligible or negative returns relative to financial and emotional costs (Zhang & Liu, 2022). One possible explanation is that tutor’s and teachers’ practices often serve different purposes (Sriprakash et al., 2016), shaping how each group conceptualises their role and the kinds of learning outcomes they prioritise (Zhang et al., 2021). These differences make collaboration between schools and tutors inherently challenging.

In Australian mathematics curriculum policy documents, teachers are required to develop students’ overall mathematical proficiency, including conceptual understanding, procedural fluency, problem solving, and reasoning, rather than a narrow focus on procedural fluency (ACARA, n.d.; Li, 2024). Accordingly, school mathematics pedagogy increasingly advocates for inquiry-based learning, problem-solving tasks, formative assessment, and the holistic development of students’ mathematical skills that foster confident, critical and reflective learners of mathematics (Li, 2024). However, the extent to which these aims align with how mathematics is taught by tutoring agencies remains unclear. Given that tutoring participation is often driven by parents and students – whose expectations may centre solely on achieving competitive results in a meritocratic education system (Sriprakash et al., 2016) – tutoring agencies may adopt practices that focus on enhancing students’ procedural fluency, instead of

well-balanced development of mathematical proficiency (Li, 2024). This sits in tension with government-endorsed educational goals and pedagogies embedded in curriculum documents (ACARA, n.d.). Research suggests that in contexts where university entrance and future opportunities are strongly influenced by comparative examination results, tutoring agencies frequently present themselves as efficient, targeted, and results-driven solutions to these pressures (Guo et al., 2020). In doing so, they may focus on rapid content coverage, examination techniques, “shortcut” strategies, and performance guarantees, in ways that contradicts the educational philosophies and classroom practices in formal schooling (Li et al., 2025).

Tensions between school-based teaching and learning, and marketised, output-focused tutoring has been documented in several countries (Guo et al., 2020; Ömeroğulları et al., 2020). Yet, despite growing recognition of these tensions, limited attention has been given to the discursive discourses through which tutoring services are described, legitimised, and made appealing to students and parents, particularly in Australia. Existing research on tutoring has largely focused on patterns of participation, equity implications, or outcome measures, rather than on the language and discourse used to promote tutoring as desirable and necessary. At the same time, broader work on the marketisation of education has shown how schools and universities increasingly adopt commercial marketing strategies, deploying advertisements that emphasise competitiveness, choice, and individualised returns on investment (Sriprakash et al., 2016). However, systematic analyses of how such marketised discourses operate within the tutoring sector, and how they interact with students, parents, and teachers, remain limited.

Given the rapid expansion of the tutoring market and its growing influence on students’ educational trajectories, there is a need to interrogate how tutoring is advertised. Specifically, by understanding how tutoring services are promoted, parents and students can gain an understanding of difference in philosophies of teaching mathematics between schools and tutoring agencies and thus make more informed educational decisions on tutoring participation. Examining the promotional materials of tutoring agencies also allows researchers and teachers to understand how tutoring is framed as an educational solution through the discursive constructions of parents, teachers, and student roles.

Research Design and Methodology

A CDA approach was applied to analyse advertisements of selected mathematics tutoring agencies. By focusing on the linguistic patterns, recurring themes, and discursive functions evident in these promotional texts, the study aims to illuminate how particular discourses are employed to promote tutoring services and to assess the extent to which these discourses align with, or potentially undermine, the way in which mathematics is taught in schools.

Sampling, Selection Criteria, and Corpus Building Criteria

Given the scope of this project, a purposive sampling strategy was adopted. The selection criteria used to identify five tutoring agencies for inclusion in the CDA were:

1. **Mathematics Focuses:** Each selected agency was required to specialise in tutoring mathematics for secondary students. While agencies could offer additional subjects, mathematics had to be the primary or clearly dominant area of provision.
2. **Representative Tutoring Models:** The agencies were chosen to exemplify common organisational and delivery models within the Australian tutoring sector. This included, for example, a large, franchised tutoring centre, a provider offering hybrid (online and in-person) modes of attendance, and a specialist mathematics coaching college, thereby capturing variation in how tutoring is structured and delivered.
3. **Accessible Advertising Content:** Agencies were required to have readily accessible advertising materials from which advertisement/promotional texts could be extracted. Agencies were required to have advertisements/promotional texts and associated promotional pages, with

sufficient written content to support detailed discourse analysis, that are listed publicly on a website or equivalent online platform.

4. **Coverage Across Australia:** The selected agencies needed to operate in major regions or cities in Australia, rather than being confined to a single suburb. This criterion was intended to enhance the relevance of the analysis beyond a narrowly localised context.

With the above criteria being used, five agencies that met all 4 criteria were chosen. To maintain the anonymity of individual providers and to keep the focus on discursive patterns, tutoring agencies are referred to by pseudonyms (Agency A, B, C, D & E). A short justification of each agency's alignment with criteria 2 and 4 is presented in Table 1.

Table 1:

Justification of 5 Agencies' Inclusion

Agency	A	B	C	D	E
Criteria 2 Alignment	Small-group classes; Senior maths specialist	Online 1-to-1 tutoring; interactive platform; Australian Curriculum mapped	Exam-focused small-group classes; Senior maths specialist	After-school learning centre franchise; small-group face-to-face tutoring	Large international franchise; centre-based and online small-group/individual tutoring
Criteria 4 Alignment	Metropolitan Sydney and Melbourne	All Australian states and territories	Greater Sydney area	Major cities across Australia	All Australian states and territories

After selecting agencies, the promotional text from their websites was extracted to form the advertisement corpus for CDA. To define what promotional text was included in the CDA, 'promotional' was defined operationally rather than in strictly legal or formal terms. Any text or visual material that communicated the values, legitimacy, or superiority of a tutoring service, with an explicit or implicit aim of influencing enrolment decisions, was considered as promotional text and thus eligible for inclusion in the critical discourse analysis. Conversely, purely administrative content (e.g., timetables, enrolment forms, terms and conditions) and individual testimonial statements in advertisements were excluded, as they do not function as promotional discourse authored by the agency itself.

Critical Discourse Analysis and Comparison across Agencies

Guided by Fairclough's (1992) three-dimensional framework for Critical Discourse Analysis, this study positioned each agency's tutoring advertisements simultaneously as a text, a discursive practice, and a social practice.

At the textual level, each agency's promotional texts (corpus) were analysed to reveal its linguistic features. By examining the frequency and use of vocabulary, grammatical structures, modality, evaluative language, and rhetorical organisation, the analysis sought to identify how students, parents, schools, and mathematics tutoring were linguistically constructed, and how particular representations of teaching, learning, and success were normalised or marginalised within the corpus. At the level of discursive practice, the focus shifted to how the promotional texts were distributed and consumed. The analysis considered what assumptions about intended audiences were embedded in the language and imagery; how different audiences were positioned; and how the advertisements drew intertextually on wider educational, marketing, and policy discourses. At the level of social practice, the study explores how the discourses evident in the advertisements share similarities in contributing to broader social structures and relations within the Australian tutoring industry, which reveals implications for public's understanding of tutoring's promotion and interaction with formal schooling.

The three-dimensional CDA was conducted for all five agencies, after which the findings were brought together in a cross-agency comparison to examine whether a recurring pattern of tutoring portrayal existed. This comparative stage identified recurring themes among agencies, patterns in promotional text corpus, and contrasts in the discourses used to promote tutoring services across agencies. Attention was also given to how promotional texts may influence interactions between students, parents, and teachers. By examining these convergences and divergences, the analysis aimed to identify dominant and competing discourses about mathematics tutoring, and to illuminate their implications for how tutoring is understood in relation to formal schooling, educational equity, and students' learning trajectories.

Findings and Discussion

Textual and Discursive Practice Levels

The CDA of advertising and promotional texts from each agency revealed several recurring discursive patterns. At the textual level, agencies drew upon a relatively limited but strategically deployed set of linguistic and rhetorical resources to promote and legitimise their services. These recurring patterns were subsequently organised into four overarching themes that capture the dominant ways tutoring is represented across the dataset. The first theme concerns tutoring goal constructions, which articulate the intended purposes of tutoring, such as examination excellence, confidence development, or academic remediation. The second theme involves personalisation claims, whereby agencies emphasise that instruction is tailored to the needs of individual students. The third theme consists of curriculum alignment statements, which position tutoring as closely connected to, or supportive of, formal school curricula. The fourth theme relates to the mobilisation of quantitative evidence, in which numerical indicators are used to signal effectiveness, credibility, or success. Taken together, these themes contribute to the construction of a distinctive discursive identity for each agency. Depending on how particular themes were emphasised, agencies were represented variously as elite examination academies, technologically enabled personalised learning services, nurturing confidence-builders, or professional providers of academic remediation. A cross-case synthesis of these themes and the corresponding dominant identities is presented in Table 2.

At the level of discursive practice, the analytical focus shifts from what is said in the advertisements to how these texts relate their intended audiences and producers. Across all five agencies, students are consistently represented as learners who are unable to realise their full academic potential through school alone and who therefore require ongoing external support, specialised expertise, and structured guidance. This positioning is evident in recurring promotional claims such as:

“Our proven methodology has helped ... to catch up and keep up in ... maths.”

“... to teach children basic mathematical concepts ... so they can catch up at school.”

“... with insights into exam techniques and an ability to explain concepts clearly.”

“... so they can focus on their studies and reach their full potential.”

Collectively, these statements portray students as learners at risk of falling behind unless supplementary instruction is provided. This deficit- and risk-oriented framing is particularly pronounced in texts that foreground high-stakes examination performance, where success is portrayed as contingent upon specialised preparation beyond the classroom. Within this discourse, tutoring is implicitly positioned as a necessary intervention besides formal schooling.

Parents, in turn, are discursively positioned as responsible decision-makers and active managers of their children's academic trajectories. Promotional texts frequently address parents directly, suggesting that “good” or “attentive” parents closely monitor school performance, interpret assessment outcomes, and strategically select supplementary educational services. This positioning is reinforced through personalisation claims and curriculum-alignment

statements, which reassure parents that tutoring will be tailored to their child's individual needs while remaining closely connected to school requirements. In doing so, the discourse invites parents to view enrolment in tutoring not merely as an optional enhancement, but as a rational example of good parenting, and at times a necessary response to competitive schooling.

Table 2:

Summary of the findings of each agency's advertisement/promotional text

Category	Agency A	Agency B	Agency C	Agency D	Agency E
Tutoring goal construction	Exam excellence, marks	Grades and confidence	High stake exam performance	Confidence, enjoyment, steady progress	Catch-up with personalised teaching
Personalisation claim	Moderate ("small classes")	Very strong (adaptive programs)	Weak (no individualised programs)	Strong ("learning plans tailored to styles")	Strong ("individualised program")
Curriculum alliance statement	Claims relevance; no alignment details	Content matches curriculum; no skills reference	Little or no explicit curriculum link	Alignment claimed; no supporting detail	Explicit reference to state curriculum topics
Use of ATAR statistics	63% of text reference ATAR	47% of text reference ATAR	78% of text reference ATAR	0% of text reference ATAR	0% of text reference ATAR
Dominant discursive identity	High-performance exam academy	Personalised tech-enable tutoring	Elite results-driven institution	Confidence-focused support provider	Teacher-led remediation service

Social Practice Level and Implications of the Promotional Texts

At the level of social practice, the promotional texts reveal several recurring discursive patterns that shape stakeholder's perceptions on tutoring and influence them to engage in participating in tutoring. These shared discourses do more than advertise services; they actively construct social norms surrounding tutoring participation and influence students', parents', and teachers' daily interactions in formal schools.

A dominant pattern across all five agencies is the discursive construction of mathematics tutoring as an essential, rather than optional, component of contemporary schooling. Despite substantial variation in business models, target markets, and declared pedagogical philosophies, each agency positioned tutoring as necessary. This shared framing normalises tutoring as part of the expected educational trajectory, transforming what was once considered supplementary support into an assumed requirement for academic success. Collectively, the promotional texts across five agencies promoted a form of educational dependency, encouraging parents and students to enrol in mathematics tutoring as a precautionary or default response rather than through careful consideration of their actual learning needs. A second recurring pattern is the pervasive claim of personalised learning, presented as a distinguishing feature across agencies. All agencies asserted that their programs respond to the individual learner (e.g., "... the content ... is structured according to your skill level and individual needs"; "Individual plans are tailored to meet the learning needs..."). However, these claims were rarely accompanied by detailed pedagogical explanations. Personalisation therefore functioned largely as a marketable discourse rather than a clearly articulated instructional approach, reflecting parental expectations on tailored support for effective mathematics learning. A third pattern concerns the instrumentalisation of emotional outcomes. While some agencies foregrounded enjoyment and confidence-building, others mobilised exam-related anxiety or urgency to justify

participation. These emotional discourses framed mathematics tutoring as both a psychological remedy and a performance-enhancing intervention, appealing simultaneously to parental hopes and concerns (e.g., "... the right help to support your ATAR goals"; "... will determine how your child is really doing in class."). In doing so, tutoring was positioned not merely as an academic service but also as a form of emotional and motivational management within increasingly competitive schooling environments. Another recurring pattern was the widespread invocation of curriculum alignment (e.g., "... covers all curriculum topics ..."). However, these references tended to be symbolic rather than substantive. None of the agencies examined offered explicit evidence of how their instructional content mapped onto syllabus outcomes or how their pedagogical approaches fostered students mathematical proficiency as outlined in curriculum documents. Curriculum alignment therefore operated primarily as a discursive legitimacy device, reassuring parents that tutoring remains connected to formal schooling even when such alignment is not transparently demonstrated. Finally, across agencies there is a consistent emphasis on quantifiable outcomes, including ATAR statistics, parental satisfaction percentages, and diagnostic assessment scores. These numerical indicators reinforced a broader neoliberal logic in which learning is framed as measurable, optimisable, and comparable. Notably, agencies that emphasise personalised learning often provided the least transparent evidence of instructional impact, highlighting tensions between marketing narratives and pedagogical substantiation.

Taken together, these recurring patterns demonstrate that tutoring advertisements do more than promote commercial services; they reproduce and legitimise broader social expectations surrounding academic competition, individual optimisation, and the perceived necessity of supplementary education. In this sense, the findings illuminate several important implications for key stakeholders. For students and parents, the analysis suggests that tutoring advertisements may overstate claims of individualised learning, curriculum alignment, and academic effectiveness, while offering limited transparency about pedagogy. This highlights the need for more critical engagement with promotional claims so that participation decisions are based on informed educational judgement rather than marketing appeal alone. For teachers whose pedagogical aims and educational philosophies may differ from those promoted by tutoring agencies, these discourses may further complicate relationships with students and families by reinforcing the perception that formal schooling is insufficient on its own. This raises important questions about how teachers might communicate the purposes of school mathematics more clearly and, where appropriate, engage more constructively with tutoring as part of students' wider learning ecology. For policymakers and researchers, the findings point to a broader systemic issue: if tutoring industries gain influence by appealing to perceived gaps between curriculum ideals and competitive educational realities, then greater attention is needed not only to tutoring advertisements, but also to the wider policy and admission structures that sustain demand for such services.

Conclusion and Future Research Implications

This study examined how five tutoring agencies discursively construct their services, stakeholders, and relationships to formal schooling through promotional texts. Using Fairclough's (1992) three-dimensional framework, the analysis showed that agencies draw on a shared set of discourses that normalise mathematics tutoring as an essential rather than supplementary component of schooling. Across providers, tutoring is framed as necessary for coping with school demands and succeeding in high-stakes examinations. Within this discourse, students are positioned as learners at risk without external support, parents as responsible managers of academic trajectories, and tutors as expert or nurturing agents capable of delivering improved outcomes. Such constructions risk narrowing understandings of successful mathematics learning to measurable outcomes such as grades and examination performance and may contribute to tensions between tutoring providers and schoolteachers by implying that

formal schooling alone is insufficient. While based on a small purposive sample and not intended to represent the entire sector, the study highlights the importance of examining how tutoring is marketed and how such discourses shape educational expectations and decision-making. Future research could extend this work through larger samples, multimodal promotional materials, and direct engagement with stakeholder perspectives.

References

- Apex Tuition Australia. (n.d.). *Why get private tutoring? Exploring the benefits of one-on-one instruction*. Retrieved March 16, 2025, from <https://www.apextuitionaustralia.com/our-blog/why-get-private-tutoringexploring-the-benefits-of-one-on-one-instruction>
- Australian Curriculum, Assessment and Reporting Authority (ACARA). (n.d.). *Essential Mathematics* (Version 8.4). The Australian Curriculum. <https://v8.australiancurriculum.edu.au/senior-secondary-curriculum/mathematics/essential-mathematics/>
- Australian Tutoring Association [ATA]. (2022). *Within Tuition*. Retrieved from <https://ata.edu.au/wp-content/uploads/2022/10/ATA-Newsletter-September-2022.pdf>
- Bray, T. M. (2009). *Confronting the shadow education system: What government policies for what private tutoring?* United Nations Educational, Scientific and Cultural Organization; International Institute for Educational Planning.
- Bray, T. M., & Kwo, O. W. Y. (2014). *Regulating private tutoring for public good: Policy options for supplementary education in Asia*. UNESCO and Comparative Education Research Centre, HKU.
- Dhall, M. (2024). A journey along the frontier: My attempts to bring accountability to shadow education in Australia and beyond. *ECNU Review of Education*, 7(1), 111-130.
- Fairclough, N. (1992). Discourse and text: Linguistic and intertextual analysis within discourse analysis. *Discourse & Society*, 3(2), 193-217.
- Guill, K., Lüdtke, O., & Schwanenber, J. (2022). The effects of private tutoring on students' academic performance: A within-student approach. *European Journal of Psychology of Education*, 37(2), 385–405. <https://doi.org/10.1007/s10212-021-00581-x>
- Guo, Y., Chen, Q., Zhai, S., & Pei, C. (2020). Does private tutoring improve student learning in China? Evidence from the China Education Panel Survey. *Asia & the Pacific Policy Studies*, 7(3), 322–343. <https://doi.org/gqm6r3>
- Li, Z. (2024). The Assessment of Mathematical Proficiency in Written Exams: A Perspective from New South Wales (NSW). In J. Visnovska, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education*. (Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia) pp. 335-342. Gold Coast: MERGA.
- Li, Z., Reid O'Connor, B., Zunica, B. (2025). Private Tutoring and Mathematics Education: A Review of the Current Research Landscape and Future Directions. In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education*. (Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia) pp. 253–260. Canberra: MERGA.
- Ömeroğulları, M., Guill, K., & Köller, O. (2020). Effectiveness of private tutoring during secondary schooling in Germany: Do the duration of private tutoring and tutor qualification affect school achievement? *Learning and Instruction*, 66, 101306. <https://doi.org/gqmvs5>
- Sriprakash, A., Proctor, H., & Hu, B. (2016). Visible pedagogic work: Parenting, private tutoring and educational advantage in Australia. *Discourse: Studies in the Cultural Politics of Education*, 37(3), 426–441.
- Zhang, E., & Liu, Y. (2022). Effects of private tutoring intervention on students' academic achievement: A systematic review based on a three-level meta-analysis model and robust variance estimation method. *International Journal of Educational Research*, 112, 101949. <https://doi.org/gqkstc>
- Zhang, Y., Dang, Y., He, Y., Ma, X., & Wang, L. (2021). Is private supplementary tutoring effective? A longitudinally detailed analysis of private tutoring quality in China. *Asia Pacific Education Review*, 22, 239–259. <https://doi.org/ppvb>
- Zheng, X., Wang, C., Shen, Z., & Fang, X. (2020). Associations of private tutoring with Chinese students' academic achievement, emotional well-being, and parent-child relationship. *Children and Youth Services Review*, 112, 104934. <https://doi.org/g8n9xg>

Teaching Experience and Teacher Competence for Multiplicative Thinking in the Early Years

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Early development of multiplicative concepts in students has been associated with success in mathematics in later years of schooling. This paper reports on a study that employed an embedded mixed methods approach to investigate the association between primary school teachers' teaching experience and pedagogical content knowledge for multiplicative thinking in the early years. Our focus was on teachers' capacity to support student transition from additive to multiplicative thinking. The study used a questionnaire with 62 primary school teachers in Australia at different levels of teaching experience from Pre-service teachers to Novice, Experienced, and Expert teachers. Longer teaching experience was important but not a condition for sound pedagogical content knowledge.

Introduction

The importance of multiplicative thinking in supporting students' learning of key topics and success in further mathematics is widely and clearly stated in mathematics education literature. It forms the basis for understanding proportions, patterns, fractions, measurement, rates, percentages, statistical thinking, the development of algebraic thinking and understanding the complex issues in society (Askew et al., 2019). However, there is evidence of low student performance in this area (Siemon, 2013). Fennema and Franke (1992) emphasise that teachers' pedagogical content knowledge (PCK) is critical in determining students' learning attainment. In relation to multiplicative thinking, Siemon and Breed (2006) suggested that the transition from additive thinking to multiplicative thinking, in contrast to the implication that most curricula provide, is a major conceptual leap for most students who continue to rely on repeated addition strategies for problems that require efficient multiplicative strategies. It requires expert pedagogical understanding by their teacher to shift their spontaneous conceptions to the conceptually correct "scientific conceptions". The study reported here interrogated the capacity of teachers (Pre-service, Novice, Experienced, and Expert) to support this transition.

Literature Review

Student transition from additive to multiplicative thinking

Research in mathematics education is clear that additive thinking forms the basis for multiplicative thinking (Askew et al., 2019; Malola et al., 2020). For instance, Askew et al., (2019) acknowledged that although multiplicative thinking can be developed through repeated addition in primary schools, the teaching of multiplicative thinking focusing on proportional reasoning supports students' learning of higher-level multiplicative thinking problems and success in mathematics subject. Siemon et al. (2006) suggested that the learner transition from additive thinking to multiplicative thinking, in contrast to the implication that most curricula provide, is a major conceptual leap and for most students requires expert pedagogical understanding by their teacher.

Accordingly, the focus of this research study is to examine how teachers support students' transition from additive to multiplicative thinking (the transitional teaching stage). Teachers therefore need to possess appropriate understanding of the conceptual field and diverse

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 242–249). MERGA.

pedagogies to successfully move learners from additive to multiplicative thinking. Further to this argument, Jacob and Willis (2001) emphasised that teachers need to understand the difference between additive and multiplicative thinking to effectively support students' development and understanding of multiplicative thinking in their further schooling. Teachers' PCK of multiplicative thinking becomes important because the teachers are key to students' development of this concept.

Teaching experience and competence

The current decade is seeing a growing interest among mathematics educators inquiring into the association between teaching experience (number of years one has been teaching) and teacher competence (how teaching practices influence student learning outcomes). For instance, In Australia, Gore et al. (2023) worked with 512 primary school teachers to investigate the relationship between the number of years spent teaching (teaching experience) and teaching quality. These teachers had a wide range of teaching experience (< 1 to 24 years), and the lessons were conducted over a period of 4 years (2014–2015, then 2019–2021). They found no significant differences in teaching quality across the range of teaching experience.

In Germany, König et al. (2021) investigated the connection between teachers' mathematical competence and students' achievement. This large-scale study involved 89 teachers and 3,496 students in junior secondary school in the federal states of Hamburg and Thuringia. The researchers found a positive relationship between teachers' mathematical competency and students' achievement on national examinations. On the other hand, Irvine (2019) reviewed several studies on the relationship between teaching experience and teacher effectiveness. He concluded that this relationship “as measured by student learning gains is complex nuanced, and non-linear” (page 1). While several studies have examined the association between the total number of years of teaching and teacher competence, the studies are general with limited attention to the teaching of complex mathematical concepts such as multiplicative thinking in the early primary (K-4) school years. This study intended to contribute knowledge to the teaching profession and empirical evidence to mathematics education community on the relationship between teaching experience and teacher competence in developing foundational mathematical competencies such as multiplicative thinking in students. This has implications to school practices in allocating teaching classes to teachers.

Mathematical pedagogical content knowledge

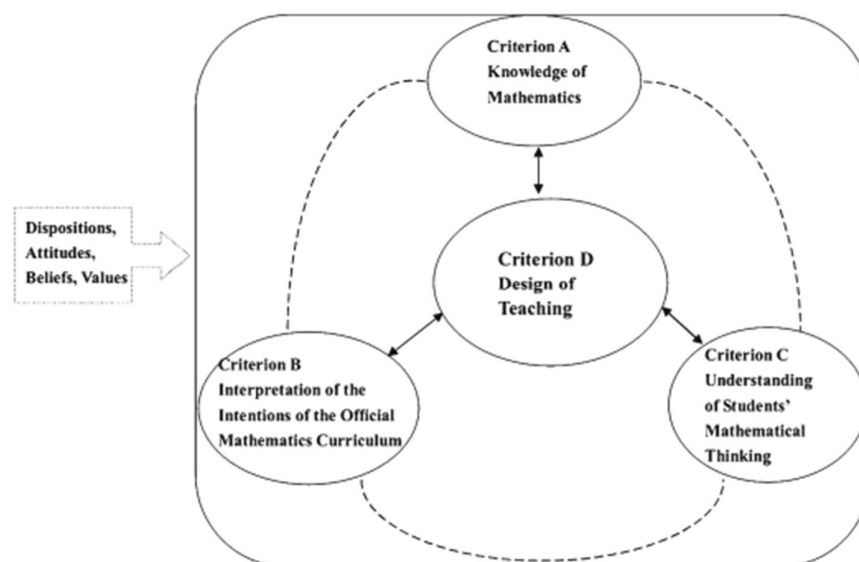
Informed by Shulman's (1986) seminal work on PCK, various scholars (Baker & Chick, 2006; Ball et al., 2008; Fennema & Franke, 1992; Zhang & Stephens, 2013) have developed models for mathematical knowledge for teaching (MKT). These models have provided useful lenses through which teachers' knowledge for teaching mathematics can be examined. Ball et al. (2008) and all the above authors underscore the claim that MKT is necessary to effectively teach complex concepts such as multiplicative thinking which was the focus of our research. This study used the model of Teacher Capacity (Zhang & Stephens, 2013) as the lens through which teachers' pedagogical content knowledge for developing foundational concepts of multiplicative thinking was understood.

The teacher capacity model

In developing a construct of teacher capacity, Zhang and Stephens (2013) worked with 120 teachers from Australia and China. Figure 1 presents the four elements of teacher capacity model to help investigate teachers' capacity for teaching mathematics: knowledge of mathematics, knowledge of curriculum, knowledge of students' thinking, and design of instruction (p. 489). Collectively, these elements influence teachers' PCK for Mathematics

Figure 5

Zhang and Stephens Model of Teacher Capacity



For this study, these four elements were understood in terms of teacher capacity for multiplicative thinking.

Summary of components of the Teacher Capacity model

- **Mathematical knowledge:** Mathematical language, multiplicative ideas, the effectiveness of solution strategies, student correct and incorrect responses, variety of teaching/solution strategies, connection to key topics, knowledge of possible student future challenges with the use of inefficient strategies.
- **Knowledge of students:** Knowledge of where students “should be” supporting individual students, knowing when to move students forward, knowledge of potential challenges and successes, identifying and correcting errors.
- **Curriculum knowledge:** Identifying multiplicative thinking in curriculum, knowing where multiplicative thinking begins and aspects of multiplicative thinking suitable for each year level, knowing where multiplicative thinking knowledge is applicable in curriculum, and knowing how multiplicative thinking is connected to other key topics in mathematics.
- **Design of instruction:** Lesson design, addressing misconceptions, adherence to curriculum standards, variety of resources solution strategies, and choice of teaching.

Research Design

The following research question guided this study: What is the association between teaching experience and teachers’ pedagogical competence for supporting students’ development of multiplicative thinking in the lower primary (K-4) school years? This study employed an embedded mixed methods approach and used the Zhang & Stephens (2013) model of Teacher Capacity as the framework for teacher questionnaire design and data analysis where Mathematical Knowledge, Knowledge of Students’ Thinking, Curriculum Knowledge, and Design of Instruction are all seen as essential components of teachers’ PCK for mathematics.

Participants

The participants in this study were 62 primary school preservice and in-service teachers in Australia. For this study, Preservice teacher participants were Master of Teaching (MTeach) students from one institution of higher learning in the State of Victoria. These students already held a first degree in other fields of study at the time they enrolled for the MTeach degree. Table 2 shows the distribution of teacher participants according to teaching experience.

Table 2

Distribution of participants according to teaching experience

Experience	Frequency	Percent	Valid percent	Cumulative percent
Preservice	11	17.7	17.7	17.7
Novice	5	8.1	8.1	25.8
Experienced	13	21.0	21.0	46.8
Expert	33	53.2	53.2	100.0
Total	62	100	100.0	

The study initially aimed at obtaining 25 participants from each group of teachers. These groups based on a classification used by DEECD (2004) are: Graduate, Novice (1-3 years), Experienced (4-7 years), and Expert (> 8 years). However, this was not achieved due to challenges encountered in the data collection phase of this study in a Covid-19 environment. Nevertheless, the performed statistical procedures controlled the possible limitations to study results and conclusions associated with the uneven sample sizes.

Data collection

A questionnaire comprised of four extended responses items that underwent expert validation and piloting was used to collect both quantitative and qualitative data. A task suitable for Year 3 & 4 students was used and informed the development of questionnaire items that interrogated teachers' capacity for supporting students' transition from additive to multiplicative thinking.

The task was "A new theme park has opened in Geelong, and you are going there for your birthday party. You and five friends are going to the party. Tickets cost \$43 each. How much will it cost altogether?" Figure 3.2 is a response from Student C.

Figure 3.2

Solution Strategy Used by Student C.

The image shows a handwritten student response on a grid. The grid contains the following numbers and symbols:

	1	4	3	
+	1	4	3	
+	1	4	3	
+	1	4	3	
+	1	4	3	
+	1	4	3	
+	1	4	3	
	2	5	8	

Student C

This problem was used to develop four questionnaire sub-items addressing the four components of the Model of Teacher Capacity (Zhang & Stephens, 2016). Table 3 is a matrix of questionnaire items (sample of first four items out of seven items) and components of Teacher Capacity model which they address.

Table 3*Questionnaire item versus element(s) of Teacher Capacity Model.*

Items	MK	CK	KS	DI
Item 1.1. What Year level is this question suitable for?		✓		
Item 1.2. Can you identify other additive strategies for solving the above problem which are more efficient than used by student C?	✓		✓	✓
Item 1.3. What are the present and future challenges that students are likely to encounter if they continue to rely on additive strategies to solve this problem?	✓		✓	
Item 1.4. How will you support students moving forward from relying on additive strategies to using more efficient (multiplicative) strategies? Please provide as many as possible.	✓			✓

Data analysis

The teachers were assigned pseudonyms (Teacher 1 to Teacher 62). Their responses to the questionnaire were scored using a rubric and were recorded on a spreadsheet ready for analysis. Again, the rubric underwent multiple stages of expert evaluation and validation before its final use in this study. The total possible score for this questionnaire was 22 points spread across the four elements of the Teacher Capacity model. The data were analysed quantitatively using SPSS and also qualitatively providing further understanding of quantitative analysis.

Results and Discussion

Table 4*Estimated Marginal Mean (EMM) scores for the four groups of teachers.*

Experience	Mean	Standard error	95% confidence interval	
			Lower bound	Upper bound
Preservice	15.8	1.0	13.7	17.8
Novice	14.4	1.3	11.8	16.9
Experienced	16.1	1.0	14.1	18.2
Expert	17.1	.57	16.0	18.3

Table 4 shows scores for groups of teachers ordered based on the EMMs from the highest to lowest. The Expert teachers scored the highest (17.1), followed by Experienced teachers (16.1), then Preservice (15.8), then Novice teachers (14.4) who scored the lowest. A thematic analysis of the teachers' responses to the questionnaire was conducted based on the quality and depth of responses originating from teachers at different levels of teaching experience and helped make connections to the quantitative data.

In relation to Item 1, teachers from preservice to expert demonstrated high PCK by including skip counting, partitioning, doubling, halving, area model, using known multiplication facts, times tables, multiplying by place value, and grouping in their responses

as efficient multiplicative strategies for solving the problem. On the other hand, limited PCK was evident when respondents wrote “multiplication” (Teacher 37, Experienced) and “extending multiplicative strategies” (Teacher 56, Preservice). In relation to Item 1.3, teachers demonstrated high PCK by including responses such as using arrays, teaching times tables, splitting larger numbers, teaching ratios, place value, skip counting, and doubling and halving. However, limited PCK was evident when teachers wrote responses, for example, “showing them tricks” (Teacher 51, Novice) and “using concrete pictorial abstracts” (Teacher 33, Expert). Likewise, for Item 1.4, teachers demonstrated high PCK by including responses such as “place value” (Teacher 37, Experienced), “basic money concepts” (Teacher 8, Expert), “skip counting” (Teacher 55, Preservice), “partitioning” (Teacher 52, Preservice), “arrays” (Teacher 33, Expert), and “doubling and halving” (Teacher 4, Expert). On the other hand, low PCK was evident when teachers wrote responses such as “crossing the decade when adding” (Teacher 44, Experienced), and “self-efficacy around multiplication” (Teacher 51, Novice).

Two exceptional Expert teachers: What did they know and show differently?

The discussion in this section focuses on Item 1.2 on the teacher questionnaire. Two Expert teachers, Teacher 4 and Teacher 8, whose excerpts of responses to the survey items at the transitional teaching stage are presented in Figures 2 and 3, all scored twenty-two points, representing scores of 100%. Teacher 4 was a leading teacher, while Teacher 8 was an assistant principal, and they both worked in different States and territories of Australia.

These two teachers mentioned that additive strategies will not be helpful in the learning and solving of problems involving fractions. They both highlight that those students who rely on additive strategies will encounter challenges working with decimals, for example, 1.33×2.41 . Teacher 8 went further to mention that with continued reliance on additive strategies, students’ learning of the measurement topic will be impacted. This demonstrates that these two teachers understand the limitation of additive strategies and therefore the need to support students with more efficient multiplicative strategies. They also understand the connection between multiplicative thinking and other topics as impacting students’ success in mathematics.

Figure 2

An Exceptional Expert Teacher (Teacher 4)

1.1. Can you suggest other additive and/or multiplicative strategies for solving the above problem which are more efficient than used by student C?

40x6+3x6=258 OR doubling 86+86+86=80+80+80+6+6=258 or algorithm

1.2. What are the present and future challenges that students are likely to encounter, if they continue to rely on additive strategies to solve this problem?

When the size of number gets bigger, students will experience work of overload. Repeated addition will not work for decimals or fractions such as 1.33×2.41 .

1.3. How will you support students moving forward from relying on additive strategies to using more efficient(multiplicative)strategies? Please provide as many as possible.

Ensure students can recognise the structure of equal groups, develop an understanding of many to one counting, encourage using the doubling and halving strategies.

1.4. Can you suggest any prior mathematical knowledge and skills that students may need to solve the above problem? List as many as possible.

Understanding of place value, repeated addition, doubling, partitioning, multiplication facts

Figure 3

An Exceptional Expert Teacher (Teacher 8)

1.1. Can you suggest other additive and/or multiplicative strategies for solving the above problem which are more efficient than used by student C?

1. Partitioning to multiply 40 and 3 utilising place value to multiply by 6. 2. Multiplying 43 by 10, halving it and adding 43. 3. Using the area model to multiply 43×6 4. Providing models with arrays.

1.2. What are the present and future challenges that students are likely to encounter, if they continue to rely on additive strategies to solve this problem?

In K-6 additive strategies restrict mental computation, fluency, and flexibility. It restricts thinking in looking at fractions as division, decimals, limit abilities in measurement. So many challenges but mostly using inefficient strategies impacts negatively on a student's self-perception as a mathematician.

1.3. How will you support students moving forward from relying on additive strategies to using more efficient(multiplicative)strategies? Please provide as many as possible.

Share models and different representations. Build a strong foundation of multiplicative place value. Let children investigate with concepts like commutativity and inverse operations and show them the how these concepts work. Implement a relational pedagogy and utilise different pedagogies in teaching math's; explicit teaching, investigation, problem solving. Manage the cognitive load of students.

1.4. Can you suggest any prior mathematical knowledge and skills that students may need to solve the above problem? List as many as possible.

Unpacking word problems. Place value, addition but I'd prefer to see multiplication, mathematical language, basic money concepts.

It would possible also to point to high MKT amongst several Preservice teachers in the study.

Conclusion

This study showed that teaching experience tends to support effective teaching of key ideas associated with multiplicative thinking. The complexity of transitioning students from additive to multiplicative thinking and its pedagogical challenges are well emphasised in mathematics education literature (Clark & Kamii, 1996; Malola et al., 2020). While high level responses were demonstrated by Pre-service, Novice and Experienced teachers, Expert teachers tended to demonstrate higher pedagogical competence in supporting students through this process by clearly stating limitations with additive strategies suggesting the need for more efficient multiplicative strategies. Expert teachers also demonstrated understanding of the connection between multiplicative thinking and the learning of key topics in mathematics, a process that is well supported by developing efficient multiplicative strategies in students. Their capacity to support students' transition from additive to multiplicative thinking can be attributed to enhanced professional growth and development opportunities, engagement with peers and curriculum resources, and several opportunities for self-reflection on the lessons taught.

While there is a growing body of research demonstrating a weak association between teaching experience and pedagogical competence, this study has indicated that both MKT and experience are important for effective teaching of foundational mathematics concepts such as multiplicative thinking. This cautions us against discrediting the role of teaching experience. School leaders should consider allocating more experienced teachers to teach mathematics in the early years to effectively support students' development of foundational concepts (e.g., multiplicative thinking). The relatively small sample sizes used in this study point to the need to further research among the four groups of teachers on PCK for developing students' multiplicative thinking, especially in identifying which if any components of the Teacher Capacity model are improved as a result of teaching experience.

References

- Askew, M., Mathews, C., Takane, T., Venkat, H., Ramsingh, V., & Roberts, N. (2019). Multiplicative reasoning: An intervention's impact on Foundation Phase learners' understanding. *South African Journal of Childhood Education*, 9(1), 1–10. <https://journals.co.za/doi/abs/10.4102/sajce.v9i1.622>
- Baker, M., & Chick, H. (2006). Pedagogical content knowledge for teaching primary mathematics: A case study of two teachers. *Proceedings of the 29th Conference of the Mathematics Education Research Group of Australasia*, 1, 60–67.

- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education* 59(5), 389–407. <https://doi.org/10.1177/0022487108324554>
- Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in Grades 1–5. *Journal for Research in Mathematics Education*, 27(1), 41–51. <https://doi.org/10.5951/jresmetheduc.27.1.0041>
- Department of Education and Early Childhood Development. (2004). *Victorian Government Schools Agreement*. Victorian Government.
- Fennema, E., & Franke, M. L. (1992). Teachers’ knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 147–164). Macmillan.
- Gore, J., Rosser, B., Jaremus, F., Miller, A., & Harris, J. (2023). Fresh evidence on the relationship between years of experience and teaching quality. *The Australian Educational Researcher*. <https://doi.org/10.1007/s13384-023-00612-0>
- Irvine, J. (2019). Relationship between Teaching Experience and Teacher Effectiveness: Implications for Policy Decisions. *Journal of Instructional Pedagogies*, 22, 1-19
- König, J., Blömeke, S., Jentsch, A., Schlesinger, L., née Nehls, C. F., Musekamp, F., & Kaiser, G. (2021). The links between pedagogical competence, instructional quality, and mathematics achievement in the lower secondary classroom. *Educational Studies in Mathematics*, 107, 189–212. <https://link.springer.com/article/10.1007/s10649-020-10021-0>
- Malola, M., Symons, D., & Stephens, M. (2020). Supporting students’ transition from additive to multiplicative thinking: A complex pedagogical challenge. *Australian Primary Mathematics Classroom*, 25(2), 31–36.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.
- Siemon, D. (2013). Launching mathematical futures: The key role of multiplicative thinking. *Mathematics: Launching Futures*, 36–52.
- Siemon, D., Breed, M., Dole, S., Izard, J., & Virgona, J. (2006). Scaffolding numeracy in the middle years: Project findings, materials and resources: *Final report*. RMIT University.
- Zhang, Q., & Stephens, M. (2013). Utilising a construct of teacher capacity to examine national curriculum reform in mathematics. *Mathematics Education Research Journal*, 25(4), 481–502. <https://link.springer.com/article/10.1007/s13394-013-0072-9>

Primary Teacher Confidence as a Mediating Factor in Content-Specific Mathematics Resource Selection

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This study examines how Queensland primary school teachers select mathematics curriculum resources for specific mathematical content. Through semi-structured interviews with eight primary teachers, we explored resource choices across primary mathematics curriculum. Teacher confidence emerged as the most significant mediating factor, shaped by three patterns: prior success with familiar resources, avoidance of resources teachers felt unable to use authentically, and conservative choices among teachers working alone at a year level. These findings highlight confidence as a key affective factor in primary mathematics teachers' resource practice.

Introduction

Curriculum resource selection is a well-established field of inquiry in mathematics education research. Mathematics teachers do not simply implement curriculum materials as written, they interpret, adapt, and are in turn shaped by the resources they use and how they use them (Adler, 2000; Remillard, 2005). Research has examined the qualities of specific resource types across mathematical content areas, including physical manipulatives, digital tools, and textbooks (Pepin et al., 2013). Yet less attention has been paid to the affective factors that shape why teachers choose particular resources for particular mathematical concepts. When those decisions are driven by familiarity rather than conceptual fit, students miss opportunities for the rich mathematical experiences that conceptual understanding requires, given that the cognitive demand of the tasks and resources teachers select is directly linked to students' capacity for mathematical thinking and reasoning (Sullivan et al., 2013b).

Research has established which resources best support particular mathematical concepts. For example, place value instruction benefits from structured physical representations, where meta-analytic evidence confirms significant learning advantages over abstract instruction alone, geometric reasoning develops through dynamic spatial tools (Lowrie et al., 2017) and probability concepts become more accessible through physical randomisation devices and data collection activities (Jones et al., 2007). However, knowing which representation best supports a given concept is insufficient if teachers lack the confidence to enact unfamiliar approaches in their classrooms.

Teacher confidence and self-efficacy in mathematics education have been studied primarily in relation to subject matter knowledge (Goos, 2013) and pedagogical approaches (Sullivan et al., 2013b). While self-concept reflects people's beliefs and attitudes which often arises from their different experiences (often social ones), self-efficacy and self-confidence are closely linked with both generally reflecting how one feels about themselves (Sander & Sanders, 2006). Teacher self-efficacy (Bandura, 1997) is a generalised belief in one's capability to execute the actions required to produce a specific outcome, and has been extensively studied in mathematics education, primarily through instruments measuring teachers' beliefs about their capacity to teach mathematics effectively (Bates et al., 2011; Enochs et al., 2000). This body of research treats self-efficacy as a relatively stable, domain-level judgement about general mathematics teaching capability. In this study the term confidence is used and is conceptualised more broadly as 'assurance'. Specifically, confidence is investigated in respect of resources, that is how assured a teacher feels using a particular resource for a particular mathematical concept.

Teachers' confidence with some resources and uncertainty with others, even in the same subject, is something general self-efficacy measures do not capture (Pajares, 1996).

A teacher may report high general mathematics teaching self-efficacy while experiencing genuine uncertainty about whether they can use a virtual manipulative authentically to develop students' probabilistic reasoning. It is this resource-specific confidence and its consequences for the representational experiences available to students that this study investigates. Considerably less research has examined confidence as it specifically relates to content-specific resource knowledge, that is, teachers' confidence in selecting and using particular resources for particular mathematical concepts. Where this gap has been acknowledged, Gueudet and Trouche (2009) demonstrate that teachers' documentary work with resources is deeply shaped by their prior resource experiences, forming stable schemes of utilisation that guide future selection. When those schemes are based on a limited set of familiar resources, teachers may be constrained in their capacity to provide the diverse and conceptually rich representational experiences that primary mathematics learning requires.

This study examines content-specific resource selection among Queensland primary teachers implementing the Australian Curriculum: Mathematics (ACARA, 2023), with particular attention to how teacher confidence mediates those decisions. Two research questions guide the inquiry:

1. How do mathematical content demands and curriculum expectations shape primary teachers' resource selection across different mathematical areas?
2. How does teacher confidence mediate content-specific resource decisions, and with what implications for primary mathematics instruction?

In addressing these questions, the study moves beyond descriptive accounts of which curriculum resources teachers use toward an explanatory account of the affective factors that constrain or enable their choices highlighting the role of confidence in shaping primary mathematics teachers' everyday resource practice.

Theoretical Framework

We draw on two interconnected frameworks from mathematics education research. The first is Remillard's (2005, 2018) conceptualisation of the teacher-curriculum relationship, which considers resource use as an active, design-oriented practice rather than passive implementation. Remillard's later construct of pedagogical design capacity (PDC) captures teachers' ability to perceive and mobilise the affordances of curriculum resources in the service of instructional goals (Remillard, 2018). Crucially, PDC is not solely a cognitive capacity; it is shaped by teachers' orientations, their beliefs, dispositions, and affective responses to materials (Remillard & Kim, 2020). We extend this framework by theorising teacher confidence as an affective dimension of PDC that operates with particular force in content-specific resource decisions. A teacher may possess the curriculum knowledge to identify an appropriate resource yet lack the confidence to enact it, a gap that PDC alone does not fully theorise.

The second framework is Gueudet and Trouche's (2009) documentational approach, which argues that, through repeated resource use, teachers develop documents, stable combinations of resources, and schemes of utilisation that guide future selection and enactment. In the primary mathematics context of this study, these schemes tend toward efficiency over flexibility: they reproduce what has worked before, and unfamiliar materials do not easily displace them. This tendency is consequential because the range and quality of mathematical representations students encounter shapes not only their conceptual understanding but also their affective dispositions toward mathematics, beliefs, attitudes, and emotions that, once formed, are resistant to change (Grootenboer & Marshman, 2016). Teacher confidence, operating through schemes of utilisation, thus functions as a structural constraint on the representational diet available to students. Together, these two frameworks illuminate how confidence becomes

embedded in teachers' resource practices and why disrupting that embedding requires deliberate professional intervention rather than simply exposure to new materials.

Methodology

This qualitative study employed semi-structured interviews consistent with established practice in mathematics education research (Bobis et al., 2021; Sullivan et al., 2013a). Eight primary teachers were recruited through purposive sampling (Patton, 2015), prioritising diversity in teaching experience (1–31 years), year level (Prep–Year 5), and school context (suburban state, rural-urban state, and independent schools). This sample size is consistent with qualitative interview research in mathematics education where the aim is analytical depth rather than statistical generalisation (Creswell & Poth, 2018). Participants included Peter (first year, Year 5), Amy (fourth year, Year 4), Marilyn (31 years, Year 5), John (seven years, Year 3), Joanna (eight years, Year 5), Clare (seven years, Year 5), Nick (fifth year, Prep), and Andrea (fourth year, Year 3).

Interviews of 45–60 minutes were conducted between October 2024 and January 2025. The protocol asked teachers to describe resources used for recently taught mathematical topics, their selection rationale, integration strategies, and challenges. Teacher confidence was not initially a focus but emerged inductively from participants' accounts, a finding that arose from the data rather than being assumed by the research design. All interviews were video recorded with participant consent and fully transcribed. Analysis followed Braun and Clarke's (2006) reflexive thematic analysis, with initial inductive coding identifying resource types and selection factors, and second-cycle coding organising themes by mathematical content area.

The two theoretical frameworks shaped analysis in distinct ways. Remillard's (2005, 2018) theorisation of the teacher-curriculum relationship directed attention to teachers' orientations toward materials, their affective responses and confidence, not merely their cognitive knowledge of resources, and guided coding of moments where affective stance shaped selection. Gueudet and Trouche's (2009) documentary approach informed the second-cycle coding, which examined whether teachers' accounts revealed stable schemes of utilisation: recurrent patterns of resource selection tied to prior experience. Together, these frameworks allowed the analysis to move from descriptive coding of resource choices to an explanatory account of the affective and structural factors shaping those choices. Pseudonyms protect confidentiality throughout.

Findings

Content Demands and Resource Selection Across Mathematical Strands

Across the six strands of the Australian Curriculum: Mathematics (ACARA, 2023), teachers' resource selections were broadly appropriate to content demands. For Number and Algebra, teachers used physical manipulatives such as MAB blocks and colour-coded connecting blocks alongside digital tools including Polypad for virtual representations of place value and division. For Space and Measurement, spatial and interactive tools predominated - angle creation tools on interactive televisions, Scratch programming for coordinate concepts, and iPads for photographing and manipulating geometric transformations. For Statistics and Probability, randomisation devices made abstract chance concepts observable: online spinners, dice-rolling games, paddle pop stick draws, and structured tally-to-graph data collection activities. This alignment was largely tacit: teachers could not consistently articulate why a resource suited a concept, but their choices reflected accumulated professional knowledge consistent with the research literature on effective representational practice (Clements & Sarama, 2011; Jones et al., 2007; Lowrie et al., 2017). What varied substantially was not the appropriateness of their selections but the confidence with which those selections were made and whether teachers felt equipped to consider alternatives.

The most analytically instructive example from this strand came from Andrea, who was the most deliberate in her sequencing: she began with hands-on materials, transitioned to digital tools, and used the shift itself as a diagnostic — students who struggled to manage the transition were identified as needing to return to physical materials. This represents a high-confidence enactment of the concrete-representational-abstract progression, providing a useful contrast with the confidence-constrained practices examined in the section that follows.

Teacher Confidence as a Mediating Factor in Resource Selection

Across all three content areas, teacher confidence emerged as the study's most analytically significant finding: a mediating factor that shaped not simply which resources were selected, but whether teachers felt able to experiment with unfamiliar alternatives. This finding extends mathematics education research on teacher beliefs and resource use (Remillard & Kim, 2020) by demonstrating that confidence operates with content specificity. Teachers could feel confident using connecting blocks for place value while feeling deeply uncertain about resources for probability or measurement. Three patterns structured the data.

Pattern 1: Confidence built on prior success with familiar resources

For many participants, confidence was a product of longevity and the repeated observation of a resource's efficacy. This familiarity created a professional safety zone, a tested repertoire teachers could draw on without deliberation:

[I feel] more confident when I've used it before... the ones that are kind of well known... I can see it real clearly and I'm confident that they're going to feel better using that. (Peter)

Because I've been teaching for so long... I'm all good. (Marilyn)

[Basic manipulatives are] just something I've learned over time that this is what's gonna help kids. (Joanna)

[I] often [return] to the usual that you know that works. (Joanna)

The contrast between these positions is analytically instructive. Peter's confidence is resource-specific and grounded in enacted success with a particular tool for a particular concept. Marilyn's is an experience-based general confidence that, on closer inspection, may reflect a settled repertoire rather than broad flexibility. Joanna describes familiarity with basic manipulatives as accumulated professional wisdom, 'learned over time', while also acknowledging that time pressure drives her back to the known.

In each case, the outcome is a narrower representational exposure for students than the research literature would support (Mainali, 2020): familiar materials are reproduced and alternatives are not seriously considered. Familiarity-driven selection, in this study, operates not as pedagogical wisdom but as a structural constraint on students' representational experiences. This operates as a structural constraint in which schemes of resource utilisation, once formed, actively resist disruption (Gueudet & Trouche, 2009). The structural conditions sustaining this pattern centre on time. As Joanna acknowledged:

The curriculum is so jam packed that you really only get to do a surface level... you don't really get to delve into things. (Joanna)

It is worth noting that Joanna's comment reflects a general curriculum-management anxiety rather than a resource-specific judgement. However, this broader pressure affects resource confidence directly: the conditions needed to develop familiarity with new materials are precisely what a packed curriculum rules out. Time pressure does not merely limit planning; it actively eliminates representational diversity, narrowing the mathematics students encounter (Bobis et al., 2021; Pepin et al., 2013). The cost-benefit calculation of trialling an unfamiliar resource, with limited preparation time, in a packed curriculum, with real consequences if it goes wrong, consistently favours what is already known to work.

Pattern 2: Avoidance of resources teachers felt unable to use authentically

A distinct and more critical form of confidence constraint emerged when teachers identified gaps in their own content-specific resource knowledge. The concern was not simply unfamiliarity with a tool but a deeper worry: that using a resource without truly understanding it would produce hollow or misleading instruction. Andrea articulated this most directly:

My confidence is definitely increasing. I am... a bit of a way to go still... I might possibly not even know how to use some of these things to teach... in the most authentic way. (Andrea)

If I'm kind of floating through, then they're [the students] probably just going to float through too. (Andrea)

The metaphor of “floating through” is precise: Andrea was not describing ignorance of content but a felt mismatch between her grasp of a resource and the mathematical depth she believed the concept deserved. Without genuine resource confidence, instruction risks becoming procedural in spite of itself, a dynamic directly linked to the quality of mathematical discourse teachers generate with students (Goos, 2013).

Nick, in his early career at Prep level, noted that “early on it is a bit tricky to find out what is needed”, illustrating how content-specific resource knowledge develops unevenly and with effort. Peter, already resourceful and reflective, articulated an aspiration that implied a felt gap in his own content-specific resource knowledge:

[I] would like to probably have a broader range of knowledge around what's available.

His use of ChatGPT to generate extension tasks in unfamiliar contexts reflected both resourcefulness and an honest awareness of his own knowledge limits. AI tools are increasingly being recruited to fill gaps in content-specific pedagogical knowledge. Amy introduced a time-based dimension. Her uncertainty was not about a resource currently in use but about pedagogical choices made in previous years and their consequences:

I think I failed those students by not doing the rote learning enough [referring to multiplication facts]... I really needed to spend more time on that... It's those kinds of things that I still have a lot of uncertainty around—what kind of time do you allocate to [rote facts]. (Amy)

Amy's reflection reveals the longitudinal weight of resource decisions: the awareness of how early pedagogical choices play out across students' subsequent learning carries a form of professional guilt that shapes present-day uncertainty (Ma, 2020). This affective dimension of resource confidence has received little attention in the existing literature.

Pattern 3: Conservative resource choices among teachers working alone at a year level

A third pattern concerned the amplifying effect of professional isolation on the stakes of resource decisions. Where teachers were the sole practitioner at a year level, selection carried a qualitatively different weight—no colleague to debrief with, no one to validate an experiment, no shared practice to fall back on. John described this directly:

Being the only year 3 teacher here... [it is] a big weight to get right or as perfect as you can be. (John)

It is a big weight... you can be a little bit more reserved with what you're picking and choosing. (John)

The word “reserved” is significant. John was not describing paralysis but a deliberate, self-protective conservatism, a calculated reduction in risk because the consequences of a lesson that did not work fell on him alone. Professional isolation amplifies accountability risk in a specific way: the absence of collegial debriefing raises the personal cost of failure, producing a conservatism that narrows the range of resources students encounter (Sullivan et al., 2013a).

Clare described a related but distinct affective experience, the moment when a resource choice goes wrong in real time, and the teacher internalises the consequences:

There'll be times where you'll be like, Oh my God, this is not working... you start... taking it on yourself sometimes. (Clare)

This “taking it on yourself” is not simply professional discomfort. It describes an internalisation of failure that shapes future risk-taking. A teacher who has experienced what Clare describes will think twice before trialling something unfamiliar again, not because they lack knowledge, but because the emotional cost of a public failure with a new resource is remembered and avoided. This is the affective mechanism through which confidence constraints self-perpetuate and utilisation schemes become more rigid over time.

An instructive contrast emerged in integrated STEM units. When designing inquiry-based projects, several teachers reported increased confidence and creative agency. Integrated STEM units appear to function as low-stakes rehearsal spaces for resource experimentation, a mechanism not previously identified in studies of teacher confidence in resource selection. John, ordinarily reliant on textbooks, described finding “freedom” and his “personality coming out” in STEM contexts. Clare drew on a school “Recycle shed” for open-ended design challenges. Andrea designed activities where students built structures using specified geometric shapes. Resources became tools for inquiry and mathematical modelling rather than vehicles for demonstration. Integrated STEM units are typically inquiry-based, cross-disciplinary contexts where teachers are not accountable to discrete content descriptors on a lesson-by-lesson basis, which reduces the specific accountability stakes around any single resource choice. Where that pressure is lower, there is more room to try something new, a finding with direct implications for how professional learning contexts might be structured to support resource experimentation.

Discussion

The findings contribute to mathematics education research in two interconnected ways. First, they provide empirical evidence that teacher confidence, as an affective dimension of pedagogical design capacity (Remillard, 2018) functions as a mediating variable in content-specific mathematics curriculum resource selection. Resource decision-making is not solely a function of curriculum alignment (ACARA, 2023) or knowledge of available materials. It is shaped by the affective confidence that emerges, or fails to emerge, from prior successful implementation of resources for specific mathematical concepts.

Second, the findings illuminate a problematic feedback loop: teachers tend to favour resources they have used successfully, which limits exposure to alternative approaches, which maintains confidence gaps around unfamiliar materials. This loop is consistent with Gueudet and Trouche’s (2009) account of scheme formation in teachers’ documentary work, but our findings add an affective specificity the documentational approach does not foreground. For mathematics education, the loop has representational consequences: students may consistently encounter a narrower range of mathematical representations than their learning requires, given that the ability to move between and connect multiple representations is a defining condition for conceptual understanding in mathematics (Duval, 2006; Kilpatrick et al., 2001).

The three confidence patterns identified, familiarity-based safety zones, fear of unauthentic teaching, and the amplifying effect of professional isolation are analytically distinct and suggest different professional learning responses. Marilyn’s settled repertoire calls for structured exposure to a wider range of content-specific materials. Joanna’s time-driven conservatism calls for protected curriculum space and structures that make experimentation feasible. Andrea’s and Nick’s concerns about authenticity call for deeper content-specific pedagogical development, not simply access to more resources. Clare’s experience of internalising public failure and John’s professional isolation both point to the importance of collegial collaboration structures around resource use, whether teachers can share, debrief, and trial together matters as much as individual knowledge.

Joanna’s return to “the usual that you know that works” under time pressure is a rational, self-protective response to accountability demands, not pedagogical conservatism in any simple

sense (Sullivan et al., 2013a). Understanding this mechanism is important for professional learning design. Providing access to new resources is insufficient unless accompanied by structured, low-stakes opportunities to develop confidence through guided implementation.

Integrated contexts appear to reduce teachers' confidence-related inhibitions around resource use (English, 2016). If integrated STEM contexts genuinely produce greater resource confidence, they may function not only as curriculum delivery modes but as opportunities for primary mathematics teachers' professional development, a possibility that warrants direct empirical investigation.

The study's limitations include its sample size and geographic concentration within Queensland. Findings reflect the reported practices of eight teachers rather than observed instruction. The study also did not examine the origins of teachers' initial resource schemes, whether shaped by initial teacher education, professional learning workshops, or early career experience. Future research employing classroom observation alongside interviews, tracing how resource confidence develops across career stages and teacher education programs, and examining the specific role of professional learning would extend these findings.

Conclusion

This study demonstrates that teacher confidence is a significant and under-examined mediating factor in content-specific mathematics resource selection. Teachers do not simply identify the best resource for a mathematical concept and deploy it. They navigate between what they know to be effective and what they feel confident enacting, and the latter exerts substantial weight in decision making. Where confidence is rooted in a narrow set of familiar resources, students may be systematically disadvantaged in the representational richness and mathematical quality of their learning experiences. Research consistently demonstrates that encountering and connecting mathematical ideas across multiple representations is a defining feature of conceptual understanding and mathematical proficiency (Kilpatrick et al., 2001).

For school leaders, the findings suggest that effective support for primary mathematics teaching must go beyond resource provision. Structured, collaborative resource review processes, drawing on models such as mathematics lesson study (Takahashi & McDougal, 2016), could create professional communities in which teachers share, trial, and build confidence with content-specific materials across year levels. Targeted professional learning that explicitly connects mathematical concepts to appropriate representational tools, and provides supported implementation opportunities, may interrupt the familiarity-dependency loop identified in this study. Further research directions include longitudinal studies tracking how content-specific resource confidence develops across career stages, and classroom-based studies directly connecting teacher resource confidence to students' representational fluency and conceptual understanding in primary mathematics.

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References

- Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, 3(3), 205–224. <https://doi.org/10.1023/A:1009903206236>
- Australian Curriculum, Assessment and Reporting Authority [ACARA] (2023). *Australian Curriculum: Mathematics*. ACARA. <https://www.australiancurriculum.edu.au/curriculum-information/understand-this-learning-area/mathematics>
- Bandura, A. (1997). *Self-efficacy: The exercise of control*. Freeman.

- Bates, A. B., Latham, N., & Kim, J. (2011). Linking preservice teachers' mathematics self-efficacy and mathematics teaching efficacy to their mathematical performance. *School Science and Mathematics, 111*(7), 325–333. <https://doi.org/10.1111/j.1949-8594.2011.00095.x>
- Bobis, J., Russo, J., Downton, A., Feng, M., Livy, S., McCormick, M., & Sullivan, P. (2021). Instructional moves that increase chances of engaging all students in learning mathematics. *Mathematics, 9*(6), 582.
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101.
- Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education, 14*(2), 133–148. <https://doi.org/10.1007/s10857-011-9173-0>
- Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches* (4th ed.). SAGE.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*, 103–131. <https://doi.org/10.1007/s10649-006-0400-z>
- English, L. D. (2016). STEM education K–12: Perspectives on integration. *International Journal of STEM Education, 3*(1), 1–8. <https://doi.org/10.1186/s40594-016-0036-1>
- Enochs, L. G., Smith, P. L., & Huinker, D. (2000). Establishing factorial validity of the mathematics teaching efficacy beliefs instrument. *School Science and Mathematics, 100*(4), 194–202.
- Goos, M. (2013). Knowledge for teaching secondary school mathematics: What counts? *International Journal of Mathematical Education in Science and Technology, 44*(7), 972–983. <https://doi.org/10.1080/0020739X.2013.826387>
- Grootenboer, P., & Marshman, M. (2016). *Mathematics, affect and learning: Middle school students' beliefs and attitudes about mathematics education*. Springer. <https://doi.org/10.1007/978-981-287-679-9>
- Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218. <https://doi.org/10.1007/s10649-008-9159-8>
- Jones, G. A., Langrall, C. W., & Mooney, E. S. (2007). Research in probability: Responding to classroom realities. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 909–955). NCTM.
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up. Helping children learn mathematics*. Washington, DC: National Academy Press.
- Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students' mathematics performance. *British Journal of Educational Psychology, 87*(2), 170–186.
- Ma, L. (2020). *Knowing and teaching elementary mathematics*. <https://doi.org/10.4324/9781003009443>.
- Mainali, B. (2020). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science and Technology, 9*, 1–21. <https://doi.org/10.46328/ijemst.1111>.
- Pajares, F. (1996). Self-efficacy beliefs in academic settings. *Review of Educational Research, 66*(4), 543–578. <https://doi.org/10.3102/00346543066004543>
- Patton, M. Q. (2015). *Qualitative research and evaluation methods* (4th ed.). SAGE.
- Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teachers' work and interactions: A collective perspective on resources, their use and transformation. *ZDM Mathematics Education, 45*(7), 929–943. <https://doi.org/10.1007/s11858-013-0534-2>
- Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246. <https://doi.org/10.3102/00346543075002211>
- Remillard, J. T. (2018). Examining teachers' interactions with curriculum resource to uncover pedagogical design capacity. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues* (pp. 69–88). Springer.
- Remillard, J. T., & Kim, O.-K. (2020). *Elementary mathematics curriculum materials: Designs for student learning and teacher enactment*. Springer.
- Sander, P., & Sanders, L. (2006). Understanding academic confidence. *Psychology Teaching Review, 12*(1), 29–42.
- Sullivan, P., Clarke, D. J., Clarke, D. M., Farrell, L., & Gerrard, J. (2013a). Processes and priorities in planning mathematics teaching. *Mathematics Education Research Journal, 25*(4), 457–480.
- Sullivan, P., Clarke, D., & Clarke, B. (2013b). *Teaching with tasks for effective mathematics learning*. Springer.
- Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: Maximising the impact of lesson study. *ZDM Mathematics Education, 48*(4), 513–526. <https://doi.org/10.1007/s11858-015-0752-x>

A Mathematical Resilience Intervention for Upper Primary Students: A Design to Address Mathematics Anxiety

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This paper is part of a broader mixed-methods study exploring how a literature-informed mathematical resilience intervention impacts mathematical resilience and mathematics anxiety in Year 5 and 6 Australian students. This paper details the creation of a five session, whole-class mathematical resilience intervention for upper primary students. The intervention was developed through a synthesis of mathematical resilience literature, focusing on building students' knowledge and skills related to the four constructs of mathematical resilience described by Johnston-Wilder and Lee (2024): growth mindset, struggle, value and support.

Mathematics anxiety negatively impacts mathematics performance, learning behaviours and academic and vocational choices (Luttenberger et al., 2018). It is an international concern; results from the 2022 Program for International Student Assessment (PISA) show a negative relation between mathematics anxiety and student mathematics achievement across all countries (OECD, 2023). Consequently, there has been renewed interest into how to decrease mathematics anxiety and one way may be to increase mathematical resilience. This paper describes a research-informed mathematical resilience intervention for Year 5 and 6 students.

Literature Review

Mathematics Anxiety

Mathematics anxiety has been defined as “feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems” (Richardson and Suinn, 1972, p.551). Research suggests that mathematics anxiety is a distinct form of anxiety, separate from test anxiety and general anxiety (Devine et al., 2012; Williams et al., 2025). It is typically assessed via self-report questionnaires, where participants rate their anxiety in various mathematics scenarios. PISA 2022 results found 35% of Australian 15-year-old students reported that they get very nervous doing mathematics problems and 61% feel anxious about failing in mathematics, with female students reporting significantly higher mathematics anxiety than male students (De Bortoli & Underwood, 2025). This gender discrepancy is reflected in the literature (Devine et al., 2012; Gokhool et al., 2022; Williams et al., 2025). While some studies have found no gender differences in mathematics anxiety in primary and secondary students (Erturan & Jansen, 2015; Mitchell & George, 2022), no studies report mathematics anxiety to be higher in males.

Evidence suggests that mathematics anxiety develops in primary school and increases with age (Dowker et al., 2016). In Hembree's (1990) seminal meta-analysis, mathematics anxiety was found to peak in Grades 9-10, but studies suggest that mathematics anxiety can appear as early as Year 1, with young children reporting high levels of mathematics anxiety (Erturan & Jansen, 2015; Ramirez et al., 2016).

Many factors impact on the development of mathematics anxiety, including environmental factors (such as parents, teachers, socio-economic status and poor mathematics experiences),

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 258–265). MERGA.

personal factors (such as general anxiety, genetic predispositions, gender and appraisal), and reciprocal variables, such as mathematics self-concept and self-efficacy, prior mathematics knowledge and motivation (Luttenberger et al., 2018; Ramirez et al., 2018). Mathematical resilience is a powerful construct that could potentially address several factors that shape mathematics anxiety.

Mathematical Resilience

Johnston-Wilder and Lee (2010) coined the term ‘mathematical resilience’ as “a positive approach to mathematics that allows people to overcome any affective barriers presented when learning mathematics” (p. 2). They advocate that it is an antidote to mathematics anxiety, supported by their early work with adult apprentices (Johnston-Wilder et al., 2014). They describe four aspects of mathematical resilience: having a growth mindset, recognising the personal and societal value of mathematics, understanding that mathematics involves struggle and knowing how to find support (Johnston-Wilder & Lee, 2024).

Evidence that supports the link between mathematics anxiety and mathematical resilience is still emerging and relatively recent, but mathematical resilience is increasingly viewed as both a preventative and interventional approach to reducing mathematics anxiety. Studies with secondary and tertiary students have consistently found a negative relationship between mathematics anxiety and mathematical resilience, with varying effect sizes (Gokhool et al., 2022; Hadji-Sonni, 2023; Lovelace, 2022; Trigueros et al., 2020).

Research on this relationship in primary school children is limited. A systematic review conducted by Akkan and Horzum (2024) examined thirty-one studies on increasing mathematical resilience published from 2000 to 2021 and found that 38.7% of studies used high school students and none of the studies in their review had primary school aged participants.

Intervention studies that have attempted to decrease mathematics anxiety by increasing mathematical resilience in school-aged students show promising results, however many are limited by small sample sizes and generally focus on high school students. Kirkland (2020) examined how a one-on-one expressive writing and mathematical resilience intervention affected mathematics anxiety in three UK primary students. While the intervention reduced mathematics anxiety, mathematical resilience was not measured. A case-study in a Brazilian high school conducted a mathematical resilience intervention with 32 high school students, utilizing the three tools discussed later in this paper, and found that mathematics anxiety appeared to decrease, however, due to the small sample size, they were unable to obtain a p-value lower than 0.05 (Para & Johnston-Wilder, 2023). Their qualitative results, however, showed increases in mathematically resilient thinking. Hadji-Sonni (2023) conducted a study with 125 French high school learners from Year 6 to 10 and found that the mathematical resilience intervention, also using these tools, reduced mathematics anxiety and increased mathematical resilience. A study with seventeen Year 12 UK students found that mathematical resilience increased and mathematics anxiety decreased after the intervention (Johnston-Wilder et al., 2015). With the exception of Hadji-Sonni (2023), these studies are limited by small sample sizes, restricting generalisability.

One aspect of being mathematically resilient involves having a growth mindset (Johnston-Wilder & Lee, 2024). In the PISA 2022 results, growth mindsets were related to lower levels of mathematics anxiety (OECD, 2023). Although studies are limited, interventions aimed at building growth mindsets in mathematics have also demonstrated positive effects on mathematics anxiety. For instance, Clark (2021) conducted mathematics growth mindset lessons with fourteen fourth grade children using an online course and found that an increase in growth mindset scores was linked with a decrease in mathematics anxiety scores.

Building Mathematical Resilience

This paper focuses on building mathematical resilience by targeting all four factors of mathematical resilience and using mathematical resilience tools in dedicated instructional sessions, but this is only one possible approach. Alternative approaches involve teachers providing students with challenging mathematical tasks to normalise uncertainty and support productive engagement with challenge (Buckley & Sullivan, 2023). These approaches also address key aspects of resilience, particularly Struggle and Support.

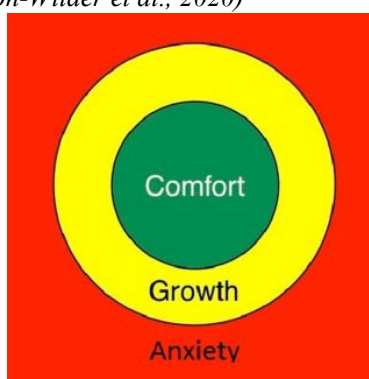
One aspect of mathematical resilience includes having a growth mindset, which can be developed through teaching students about what a growth mindset is and how it is beneficial, conveying information about the neuroplasticity of the brain and supporting learners to try mathematics problems that challenge them (Johnston-Wilder & Lee, 2024).

Mindset interventions largely have three objectives, according to Jiang and colleagues' (2024) meta-analysis. The first is to explain the scientific basis behind how human brains change and learn (Jiang et al., 2024). This is supported by a meta-analysis of ten studies by Sarasin and colleagues (2018), that found that teaching students about neuroplasticity had an overall positive effect on motivation, achievement and brain activity. The second objective is to give credibility to mindset theory, often through sharing stories of people overcoming challenges with growth mindsets (Jiang et al., 2024; Yeager & Dweck, 2020). The third objective is to enable students to internalise the message, often through writing activities, role play or experiential learning (Jiang et al., 2024).

Mathematical resilience tools have been used successfully with adults and high school students, however, the effectiveness of these tools for increasing mathematical resilience in primary school students has not yet been studied. The Growth Zone Model (GZM; see Figure 1), adapted by Johnston Wilder and colleagues (2020) from Vygotsky's (1978) Zone of Proximal Development, aims to help students recognise the difference between productive nervousness and debilitating anxiety.

Figure 1

Growth zone model (GZM; Johnston-Wilder et al., 2020)



Note. From “A toolkit for teachers and learners, parents, carers and support staff: Improving mathematical safeguarding and building resilience to increase effectiveness of teaching and learning mathematics,” by S. Johnston-Wilder, C. Lee, S. Pardoe, and S. Richards, 2020, *Creative Education*, 11, Figure 2 (10.4236/ce.2020.118104). CC BY 4.0.

The green circle of the GZM is the comfort zone, where learners feel confident and safe. The yellow ring, the growth zone, is where learners experience challenge, make mistakes and may require support. The red zone is the anxiety zone, where learners feel threatened and their cognition decreases.

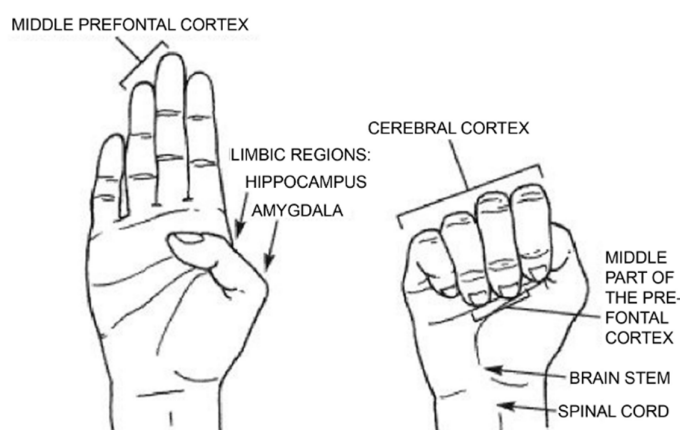
Using the GZM, along with the Hand Model of the Brain (Siegel, 2011; Figure 2) and the relaxation response (Benson, 2000), Johnston-Wilder and colleagues (2021) found the tools helped an adult learner focus on their emotional responses and manage their mathematics

anxiety. Similarly, a one-on-one intervention with thirteen Year 7 students in England found that the GZM increased students' emotional awareness and supported students to reengage with their learning (Baker, 2021).

The Hand Model of the Brain (HMB) describes how the brain reacts when threatened, shutting off higher-order thinking skills. Johnston-Wilder and colleagues (2020) describe how it can be used when explaining about mathematics anxiety and how the brain works. The closed fist represents a regulated brain ready to learn, while the open hand represents a dysregulated brain and corresponds to the anxiety zone in the GZM. Johnston-Wilder and colleagues (2020) argue that teaching the HMB explains how the brain reacts to stress and reframes stress responses as the brain trying to protect the individual. It can also be used by learners to show what state they are in using the hand signals and gives a name to mathematics anxiety that is simpler- learners can say "I've flipped".

Figure 2

Hand Model of the Brain (Siegel, 2010)



Note. From *Mindsight: Transform your brain with the new science of kindness* (p.15), D. Siegel, 2010, Oneworld Publications. Copyright 2010 by Siegel.

The relaxation response (Benson, 2000) is a mindfulness strategy that learners can use to calm their body when they feel they have 'flipped', engaging their parasympathetic nervous system (Johnston-Wilder et al., 2020). Recommended techniques include 7-11 breathing, 3-3-3 grounding (naming three things they can hear, see and feel) and going for a walk (Johnston-Wilder et al., 2020). Focused deep breathing exercises have been found to decrease mathematics anxiety in adult and child samples (Brunyé et al., 2013; Khng, 2017).

Rationale

While studies have found a negative relationship between mathematics anxiety and mathematical resilience, few studies have actually tested how one variable impacts on the other. Additionally, most research on mathematical resilience has been conducted with secondary students and adult populations, with few studies investigating its application in primary school contexts (Akkan & Horzum, 2024). This lack of mathematical resilience research at the primary level is concerning, given that primary school seems to be when mathematics anxiety develops, and where preventative interventions could be most effective (Barroso et al., 2021; Namkung et al., 2019). If mathematical resilience interventions were found to be effective in decreasing primary school students' mathematics anxiety, they could be used in schools to support struggling students and combat the negative effects of mathematics anxiety.

To the best of the authors' knowledge, no mathematical resilience programs exist in primary schools currently. This paper describes the creation of a five-session mathematical resilience intervention for Year 5/6 students, based on the interventions published in the literature

conducted with adolescent samples and other recommendations. This intervention is part of a larger mixed-methods research project which seeks to investigate the relationship between mathematical resilience and mathematics anxiety across year level and gender, before and after the intervention. Results will follow later in the year.

The Intervention

The intervention goals are based on the constructs of mathematical resilience: Growth, Struggle, Value and Support (Johnston-Wilder & Lee, 2024). Table 1 shows an overview of the learning goals for each session of the intervention and the mathematical resilience focus.

Across the five sessions, students will:

- Develop a growth mindset about mathematics
- Understand that mathematics involves struggle and making mistakes are part of the process of learning mathematics
- Learn strategies they can use when they get stuck
- Understand the personal and societal value of mathematics

Session 1 introduces group expectations, emphasising a respectful environment in which students feel safe to make mistakes and seek challenges, an important condition for developing mathematical resilience (Johnston-Wilder & Lee, 2024). Students learn about neuroplasticity and how the brain learns, which has been recommended when teaching for a growth mindset (Jiang et al., 2024; Johnston-Wilder & Lee, 2024; Sarrasin et al., 2018). Students watch a video illustrating how repeated effort strengthens neural connections and then explain the concept to a partner and create their own analogy. They are introduced to the Hand Model of the Brain and play a game identifying different brain parts and functions. Finally, the definition of a growth mindset is explicitly introduced and students label fixed or growth mindset thoughts before writing about a time they demonstrated a growth mindset.

Table 1

Learning Goals and Focus of the Mathematical Resilience Intervention Sessions

Learning Goals	Mathematical Resilience Focus
1 I can explain how our brain learns. I can point to parts of the brain using the Hand Model of the Brain. I can differentiate between growth and fixed mindset thoughts and describe a time I used a growth mindset.	Growth Mindset
2 I can describe and explain which zone of the GZM others are in. I can explain why it is important to struggle in maths. I can write about how I can use a growth mindset in my maths learning.	Growth Mindset Struggle
3 I can write about when I have been in each zone of the GZM. I can describe relaxation strategies I can use to regulate myself when anxious.	Struggle Support
4 I can list ways to get 'unstuck'.	Support
5 I can identify the value that maths has for me and in society. I can apply mathematical resilience tools and strategies that I have learnt to overcome struggles when problem-solving maths questions.	Value Support Struggle

Session 2 introduces the GZM to students and has students performing short skits in small groups, where the rest of the class tries to figure out which zone of the GZM the 'main character' is in. Role play can be used to enable students to internalise the messages of mindset theory (Jiang et al., 2024), specifically that we want to be working in the growth zone, and that struggle is part of learning. The skits were created to be relatable and recognisable scenarios, e.g. struggling with completing homework. The session finishes with a free-write answering two

questions: Why is it important to struggle in maths? and How will I use a growth mindset in maths? Mathematically resilient learners recognise that mathematics challenges involve struggle, and can regulate the associated negative emotions (Johnston-Wilder & Lee, 2024).

Session 3 has students relating the GZM to themselves by writing and sharing about when they have been in each zone, to consolidate their understanding of the model. Students are introduced to three relaxation strategies: Hand breathing, 3-3-3 grounding and muscle relaxation. These techniques were selected for their suitability in primary classrooms, with hand breathing chosen as an alternative to the 7-11 breathing recommended by Johnston-Wilder and colleagues (2020), deemed too difficult for this age group. Hand breathing has students tracing their hand, breathing in when tracing up a finger and breathing out when tracing down for five slow breaths. Research on progressive muscle relaxation has found it reduces stress and anxiety among children (Ayed et al., 2021; Köse Kabakcıoğlu & Ayaz-Alkaya, 2024). The session finishes with a video and discussion about Hellen Keller, providing credibility to mindset theory by showing someone who overcame challenges (Jiang et al., 2024; Yeager & Dweck, 2020).

Session 4 focuses on finding support. When students are in the Growth zone of the GZM, they should be getting stuck and making mistakes, but they need to be taught strategies for how to get 'unstuck'. Students brainstorm ways they can get unstuck and then are shown different strategies. They then make their own poster with at least 5 strategies.

In Session 5, students brainstorm all the ways we use mathematics, for themselves and in society, which focuses on the mathematical resilience construct of Value. When children do not understand the practical uses of mathematics, this can be linked to decreased motivation and negative mathematics attitudes (Ashby, 2009). Students then apply what they have learnt throughout the sessions while tackling a challenging mathematics problem.

Conclusion

This mathematical resilience intervention will be conducted in four Year 5 classes and three Year 6 classes, and work samples and observational notes will be collected. Before and after the intervention, students will fill out a mathematical resilience survey and mathematics anxiety survey. After the intervention, students will also complete a qualitative written survey of thoughts and opinions on the intervention. These data will allow further exploration of the extent to which mathematical resilience interventions can enhance mathematical resilience and reduce mathematics anxiety in upper primary students, as well as whether the impact of such interventions varies by gender or year level. Additionally, qualitative data will be used to identify what aspects of the intervention students found most and least helpful to build mathematical resilience and why, offering valuable insight for refining future interventions.

More broadly, this work contributes a practical approach to supporting students who struggle with low mathematical resilience, by explicitly targeting the four aspects of mathematical resilience. Although the effectiveness of the intervention will be explored in subsequent research, it offers a research-informed and classroom-based approach that has the potential to improve students' mathematics experiences and attitudes towards mathematics and guides efforts to foster resilience and potentially reduce anxiety in the primary classroom.

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References

Akkan, S. N., & Horzum, T. (2024). Illuminating the landscape of mathematical resilience: A systematic review. *Journal of Pedagogical Research*, 1. <https://doi.org/10.33902/JPR.202420093>

- Ashby, B. (2009). Exploring children's attitudes towards mathematics. In M. Joubert (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* 29(1) March 2009.
- Ayed, M. M. A., Farrag, J. M., Mahmoud, S. A., Abu Salem, E. M. M., Ali Hegazy, A. E.-S., Mahmoud, T. M., & Khalifa, S. M. (2021). Effect of mothers' implementing progressive muscle relaxation technique on anxiety and sleep quality among children with COVID-19. *Port Said Scientific Journal of Nursing*.
- Baker, J. (2021). "You see it differently once you calm down": Developing an intervention to support learners to address their mathematics anxiety [University of Warwick]. <http://wrap.warwick.ac.uk/166960>
- Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A meta-analysis of the relation between math anxiety and math achievement. *Psychological Bulletin*, 147(2), 134–168. <https://doi.org/http://dx.doi.org/10.1037/bul0000307>
- Benson, H. (2000). *Relaxation response*. HarperCollins.
- Brunyé, T. T., Mahoney, C. R., Giles, G. E., Rapp, D. N., Taylor, H. A., & Kanarek, R. B. (2013). Learning to relax: Evaluating four brief interventions for overcoming the negative emotions accompanying math anxiety. *Learning and Individual Differences*, 27, 1–7. <https://doi.org/10.1016/j.lindif.2013.06.008>
- Buckley, S., & Sullivan, P. (2023). Reframing anxiety and uncertainty in the mathematics classroom. *Mathematics Education Research Journal*, 35(1), 157–170. doi:10.1007/s13394-021-00393-8
- Bucur, S. M., Crişan, I. M., Cocos, D. I., Bud, E. S., & Galea, C. (2025). Observational study on progressive muscle relaxation and breathing control for reducing dental anxiety in children. *Medicina*, 61(5), 876.
- Clark, J. E. (2021). *Case study of growth mindset and the impact on math anxiety* [Ed.D., Brenau University]. <https://www.proquest.com/docview/2572612113/abstract/3774871460FF4E3FPQ/1>
- De Bortoli, L., & Underwood, C. (2025). *PISA 2022. A closer look at mathematics in Australia*. Australian Council for Educational Research. <https://doi.org/10.37517/978-1-74286-786-1>
- Devine, A., Fawcett, K., Szűcs, D., & Dowker, A. (2012). Gender differences in mathematics anxiety and the relation to mathematics performance while controlling for test anxiety. *Behavioral and Brain Functions*, 8(1), 33. <https://doi.org/10.1186/1744-9081-8-33>
- Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology*, 7. <https://doi.org/10.3389/fpsyg.2016.00508>
- Eidlin-Levy, H., Avraham, E., Fares, L., & Rubinsten, O. (2023). Math anxiety affects career choices during development. *International Journal of STEM Education*, 10(1), 49.
- Erturan, S., & Jansen, B. (2015). An investigation of boys' and girls' emotional experience of math, their math performance, and the relation between these variables. *European Journal of Psychology of Education*, 30(4), 421–435. <https://doi.org/10.1007/s10212-015-0248-7>
- Gokhool, F., Lawson, D., & Hodds, M. (2022). Investigating the relationship between mathematics anxiety, mathematical resilience and mathematics support engagement: An analysis of demographic and cohort factors. *MSOR Connections*, 20(2), 82–93. <https://doi.org/10.21100/msor.v20i2.1321>
- Gonzalez-DeHass, A. R., Furner, J. M., Vásquez-Colina, M. D., & Morris, J. D. (2024). Undergraduate students' math anxiety: The role of mindset, achievement goals, and parents. *International Journal of Science and Mathematics Education*, 22(5), 1037–1056. <https://doi.org/10.1007/s10763-023-10416-4>
- Hadji-Sonni, A. (2023). *Developing mathematical resilience while addressing mathematics anxiety in disadvantaged areas in France* [Phd, University of Warwick].
- Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21(1), 33–46. <https://doi.org/10.2307/749455>
- Jiang, X., Mueller, C. E., & Paley, N. (2024). A systematic review of growth mindset interventions targeting youth social-emotional outcomes. *School Psychology Review*, 53(3), 251–272.
- Johnston-Wilder, S., Baker, J. K., McCracken, A., & Msimanga, A. (2020). A toolkit for teachers and learners, parents, carers and support staff: Improving mathematical safeguarding and building resilience to increase effectiveness of teaching and learning mathematics. *Creative Education*, 11(8).
- Johnston-Wilder, S., Brindley, J., & Dent, P. (2014). *A survey of Mathematics Anxiety and Mathematical Resilience among existing apprentices*. The Gatsby Charitable Foundation.
- Johnston-Wilder, S., & Lee, C. (2010). *Developing Mathematical Resilience*. In: BERA Annual Conference 2010, 1-4 Sep 2010, University of Warwick.
- Johnston-Wilder, S., & Lee, C. (2024). *The Mathematical Resilience Book* (1st ed.). Routledge. <https://doi.org/10.4324/9781003334354>
- Johnston-Wilder, S., Lee, C., Brindley, J., & Garton, E. (2015, November 18). Developing mathematical resilience in school students who have experiences repeated failure. *Research and Innovation*. 8th International Conference of Education.
- Johnston-Wilder, S., Lee, C., & Mackrell, K. (2021). Addressing mathematics anxiety through developing resilience: Building on self-determination theory. *Creative Education*, 12(09), 2098–2115.

- Khng, K. H. (2017). A better state-of-mind: Deep breathing reduces state anxiety and enhances test performance through regulating test cognitions in children. *Cognition and Emotion*, 31(7), 1502–1510.
- Kirkland, H. R. (2020). *An exploration of maths anxiety and interventions in the primary classroom* [Thesis, University of Leicester]. <https://doi.org/10.25392/leicester.data.12659741.v1>
- Köse Kabakcıoğlu, N., & Ayaz-Alkaya, S. (2024). The effect of progressive muscle relaxation on stress, anxiety, and depression in adolescents: A quasi-experimental design. *Journal of Pediatric Nursing*, 78, 89–96. <https://doi.org/10.1016/j.pedn.2024.06.014>
- Lovelace, D. R. (2022). *Does mathematical resilience address mathematics anxiety? A measurement validation study for high school students* [Ed.D., California State University, Sacramento]. <https://www.proquest.com/docview/2696119123/abstract/6387B1C9FBA941E1PQ/1>
- Luttenberger, S., Wimmer, S., & Paechter, M. (2018). Spotlight on math anxiety. *Psychology Research and Behavior Management*, Volume 11, 311–322. <https://doi.org/10.2147/PRBM.S141421>
- Macnamara, B. N., & Burgoyne, A. P. (2023). Do growth mindset interventions impact students' academic achievement? A systematic review and meta-analysis with recommendations for best practices. *Psychological Bulletin*, 149(3–4), 133–173. (2023-14088-001). <https://doi.org/10.1037/bul0000352>
- Mitchell, L., & George, L. (2022). Exploring mathematics anxiety among primary school students: Prevalence, mathematics performance and gender. *International Electronic Journal of Mathematics Education*, 17(3), em0692. <https://doi.org/10.29333/iejme/12073>
- Namkung, J. M., Peng, P., & Lin, X. (2019). The relation between mathematics anxiety and mathematics performance among school-aged students: A meta-analysis. *Review of Educational Research*, 89(3), 459–496.
- OECD. (2023). *PISA 2022 results (Volume I): The state of learning and equity in education*, PISA, OECD Publishing, Paris, <https://doi.org/10.1787/53f23881-en>.
- Para, T., & Johnston-Wilder, S. (2023). Addressing mathematics anxiety: A case study in a high school in Brazil. *Creative Education*, 14(2), Article 2. <https://doi.org/10.4236/ce.2023.142025>
- Ramirez, G., Chang, H., Maloney, E. A., Levine, S. C., & Beilock, S. L. (2016). On the relationship between math anxiety and math achievement in early elementary school: The role of problem solving strategies. *Journal of Experimental Child Psychology*, 141, 83–100. <https://doi.org/10.1016/j.jecp.2015.07.014>
- Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, 53(3), 145–164.
- Richardson, F. C., & Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale: Psychometric data. *Journal of Counseling Psychology*, 19(6), 551–554. <https://doi.org/10.1037/h0033456>
- Sarrasin, J. B., Nenciovici, L., Foisy, L.-M. B., Allaire-Duquette, G., Riopel, M., & Masson, S. (2018). Effects of teaching the concept of neuroplasticity to induce a growth mindset on motivation, achievement, and brain activity: A meta-analysis. *Trends in Neuroscience and Education*, 12, 22–31.
- Siegel, D. J. (2010). *Mindsight: Transform your brain with the new science of kindness*. Oneworld Publications.
- Trigueros, R., Aguilar-Parra, J. M., Mercader, I., Fernández-Campoy, J. M., & Carrión, J. (2020). Set the controls for the heart of the maths. The protective factor of resilience in the face of mathematical anxiety. *Mathematics*, 8(10), 1660. <https://doi.org/10.3390/math8101660>
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
- Williams, K., White, S. L. J., & English, L. D. (2025). Profiles of general, test, and mathematics anxiety in 9- and 12-year-olds: Relations to gender and mathematics achievement. *Mathematics Education Research Journal*, 37(1), 161–186. <https://doi.org/10.1007/s13394-024-00485-1>
- Yeager, D. S., & Dweck, C. S. (2020). What can be learned from growth mindset controversies? *American Psychologist*, 75(9), 1269–1284. (2020-99903-019). <https://doi.org/10.1037/amp0000794>

Tick or Fix: Enacting Daily Reviews in Primary Mathematics Classrooms and Implications for Inclusive Teaching

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Daily Reviews are widely used in primary mathematics classrooms, yet little is known about how teachers enact these routines or the opportunities they create for inclusive teaching. We observed Daily Reviews in two primary classrooms using a descriptive research design to explore (1) how the routine was enacted and (2) how these enactments reflected inclusive practices. Although both teachers used a structured, slide-driven review built around a write–reveal cycle, their enactments differed in instructional variation and student participation. Implications for future implementation suggesting that routines may benefit from practices that make students’ thinking more visible.

Daily Review in mathematics has become increasingly encouraged across Australian schools and promoted by state and territory education departments, as well as by national bodies such as the Australian Education Research Organisation (AERO, 2024). It involves fast-paced mathematical questions, completed on mini whiteboards for the first 15-20 minutes of a lesson. Teachers design their own questions or source prepared slide decks (e.g., Ochre Education, 2026). The Daily Review aligns with broader instructional models that prioritise retrieval, rehearsal, and explicit teaching (Victorian Department of Education, 2026), influenced by Rosenshine’s (2012) *Principles of Effective Instruction*. These principles reflect a belief that frequent rehearsal strengthens recall and supports well-connected mathematical knowledge. AERO similarly recommends reviewing prior content at the beginning of lessons to activate knowledge, assess students’ understanding and plan support. While cognitive load theory (Sweller et al., 2011) supports explicit models of instruction (Kirschner et al., 2006), researchers caution against its mechanical approach, particularly when it may limit opportunities for reasoning or student agency. As Attard (2025) argues, these practices must not come “at the expense of flexible and responsive teaching, student engagement, and authentic mathematical problem solving and inquiry” (p. 4).

Inclusive teaching in mathematics involves using strategies such as scaffolding, differentiation, reflection and feedback to ensure all students can participate and make progress (Darragh et al., 2025; Ingram et al., 2020). Inclusive teaching practices as referred to in our study include the pedagogical moves, participation structures, and interactional routines through which teachers support students to participate in mathematics. A Daily Review that incorporates these elements has the potential to support inclusive teaching practices. Yet little is known about how teachers enact these routines in practice, or whether the structure supports inclusive teaching. Given this potential and widespread uptake of Daily Reviews, our study addresses a gap in the literature by analysing how primary school mathematics teachers enact Daily Reviews and situate these enactments within the broader discussions of inclusive teaching. As part of a larger study, we observed six primary school teachers (Foundation to Year 6) implementing Daily Review. For this paper, we focus on two teachers who implemented the Daily Review but enacted it differently in practice. Our guiding research questions were:

How is the Daily Review enacted in two primary classrooms?

In what ways do these enactments reflect inclusive teaching practices?

Review of literature

This study is theoretically informed by cognitive science perspectives on memory and instruction (Rosenshine 2012; Sweller et al., 2011) and by inclusive pedagogy, which emphasises participation, access, and responsiveness to learner diversity (Darragh et al., 2025).

The Daily Review

The following review considers the principles of instruction, benefits and limitations related to the implementation of the Daily Review, and practices that reflect inclusive pedagogies.

Daily Review is positioned as a pedagogical routine that supports long term retention, reduces cognitive load, and prepares students for the acquisition of new content. By prompting students to deliberately recall previously taught material, this routine supports the consolidation of knowledge in long term memory, which in turn reduces the cognitive load experienced during new learning (Sweller et al., 2011). This is based on the premise that when working memory and cognitive load increase beyond what students can manage, unnecessary demands are placed on the learner, hindering effective engagement with the task (Sweller et al., 2019). Existing literature frames Daily Review as part of Rosenshine's research informed ten principles of instruction including beginning a lesson with a short review of previous learning; presenting new material in small steps with student practice after each step, asking a large number of questions and checking the responses of all students; checking for student understanding and providing scaffolds for difficult tasks (Rosenshine et al., 2012, p. 12).

Some describe the Daily Review as process driven, focusing on what teachers do and what students learn (e.g., Hiebert & Stigler, 2023). This process-driven approach prioritises response production and correctness over exploration of why answers are correct or incorrect. While practices such as *teacher-directed unison responding* (Merlo, 2024) ensures high student participation, the level of participation does not necessarily recognise diversity and inclusion in relation to students' mathematical thinking and sense-making.

While Daily Reviews have been promoted as an important part of mathematics lessons, concerns have been raised about their implementation (e.g., Swan, 2023). Some teachers, for example, create overly long PowerPoint presentations, student participation is dominated by chanting slide content and answers without genuine understanding. If instruction is delivered too quickly, it reduces students' opportunity to read, and process slide content at their own pace. Similarly, the dominant Initiation-Response-Feedback (IRF) pattern (Sinclair & Coulthard, 1975) is teacher-dominated and has been found to reduce the opportunities to engage students or to shape students' thinking (Myhill, 2006).

Inclusive Teaching Practices

Teachers play a central role in creating inclusive teaching environments, drawing on their expertise to plan, teach, and adapt learning in ways that attend to students' mathematical needs (Attard, 2025). Inclusive teaching practices go beyond behaviour participation and should involve designing and delivering learning experiences that respond to the diverse strengths, backgrounds, and needs of all students (Darragh et al., 2025). Additionally, inclusive practices provide students with multiple pathways for understanding and engaging with mathematical concepts. Research shows that the highest learning gains occur when students engage in high-level thinking, reasoning, and problem solving (Stein & Lane, 1996). Students' mathematical connections are further strengthened when they reflect on prior learning, use concrete materials, construct their own knowledge, and discuss and learn from their mathematical mistakes (e.g., Ingram et al., 2020). Contemporary mathematics practices promote dialogic teaching, with a focus on openness for dialogue, inviting student contribution, fostering dialogic participation, and encouraging students to take risks (Hennessy et al., 2016).

Consistent with this, Alexander (2008) argues that students require opportunities to participate in mathematical talk, to justify, and refine their thinking.

Student thinking can also be explored through an equity lens considering aspects of access, achievement, identity, and power (Gutierrez, 2012). Students should have access to high-quality tasks and develop their identity as learners and doers of mathematics, and as participants in the classroom community (Sherin & van Es, 2026). Gutierrez's (2012) power dimension is particularly relevant to considering who are the sources of mathematical knowledge within the classroom, and how students are positioned to learn that mathematical knowledge.

Policy documents also emphasise inclusive teaching practices. Equity is a key principle of the National Council of Teachers of Mathematics (NCTM, 2000), while the Australian Curriculum: Mathematics (ACARA, 2025) advocates responsive teaching that builds on students' mathematical thinking rather than rigid adherence to scripted instruction. It is also important to emphasise that no single pedagogical approach can meet the needs of every mathematics learner (MERGA, 2025). Effective mathematics teaching relies on a range of intentional and high-quality practices that can be tailored to specific contexts, learners and purposes (MERGA, 2025)

Methodology

A descriptive research design (Kervin et al., 2016) enabled detailed analysis of two case-study teachers' Daily Review enactments and their inclusive teaching opportunities.

Background and context

The study took place in an Australian non-government K–12 International Baccalaureate (IB) school whose philosophy emphasises critical thinking, inquiry-based learning, and developing independent thinkers. The school was implementing a year-long mathematics improvement initiative, which included all classes adopting Daily Review as part of their regular mathematics lessons. Teachers engaged in professional learning on explicit instruction and completed a one-hour Daily Review module. While adoption of explicit instruction and Daily Review routines may seem at odds with the IB philosophy, teachers were encouraged to adapt the routines to their context.

Data collection and analysis

Data were collected through field notes and video-recorded observations of six teachers (Foundation-Year 6) conducting a Daily Review (15-20 minutes) at the beginning of a mathematics lesson. From the six observed teachers, two were purposively selected as case-study teachers to provide an analysis of contrasting Daily Review enactments. Tessa and Kimba (pseudonyms) taught Year 2 and Year 4 respectively; both had 15 years of teaching experience.

Data analysis occurred in two cycles. First-cycle, descriptive coding identified the structural components of each teacher's Daily Review (e.g., pacing, task types, questioning patterns). Second-cycle pattern coding examined how these routines created or limited opportunities for inclusive participation. Codes included *strategy sharing*, *misconception visibility*, *peer reliance*, and *teacher positioning*. Comparative case analysis enabled identification of similarities and differences across the two classrooms, and their implications for inclusion.

Results

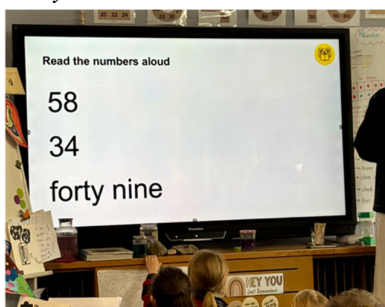
This section presents the results of the Daily Review observations for each teacher. Tables have been used to document the mathematical focus, pedagogical routines, student participation, and instructional variations for each Daily Review.

Daily review: Tessa (Grade 2)

Tessa's enactment of the Daily Review was highly structured and teacher-directed, with Ochre-generated slides (e.g., see Figure 1), sequencing short response items at a steady pace. Students were seated on the floor in front of an Interactive Whiteboard (IWB) where the slides were projected. The teacher sat to the side of the IWB and remained there throughout the session.

Figure 1.

Scene depicting representation of a Daily Review slide



Classroom observations showed that students responded either with choral responses (all students respond aloud together, at the same time, to a teacher's prompt) or worked individually and silently, writing answers on mini-whiteboards and holding them under their chins for rapid teacher checking. Students were then instructed to 'tick or fix' their answers. Some students corrected their responses and then erased their work, ready to move onto the next question. Although the teacher occasionally acknowledged that alternative methods may have been used, these acknowledgements did not lead to invitations for students to share or justify their reasoning. Interaction was minimal, with limited opportunities for mathematical talk or comparison of approaches. The Daily Review lasted for 15 minutes.

Table 1 provides an overview of the structure of Tessa's Daily Review, including mathematical focus, pedagogical routine, instructional variations, and student participation patterns. The write-reveal cycle refers to the repeated instructional sequence whereby students write independently, then 'hover' (place board face down or hold to chest), 'chin it' (hold boards under chin or in air), after which the teacher scans responses and reveals the answer on IWB. Students then 'tick or fix' their answers before they 'bin it' (rub out their answer).

Table 1.

Structure of Tessa's Daily Review Lesson

Mathematical focus (slides)	Pedagogical routine	Instructional variations	Student participation pattern
Skip counting (2s, 3s, 10s) using number chart	Slide-based instruction; teacher read prompt and progressed slides		Choral whole-class responses
Place value tasks (expanded form; number	Slide-based write-reveal cycle	Placed 3 students close to her who required	Individual whiteboard responses; answers held under

Mathematical focus (slides)	Pedagogical routine	Instructional variations	Student participation pattern
of the day; nearest 10; before/after)		additional assistance with interpreting and writing	chin; self-correction following reveal
Additive reasoning representations (number bonds; bar models; number lines; partitioning)	Slide-based write-reveal cycle	As above; prompt to compare with neighbour; acknowledgement that different strategies may have been used	Individual whiteboard responses; brief peer comparison when prompted

The table shows that the instructional routine remained consistent across tasks, with variation limited to mathematical content and occasional prompts for brief peer comparison.

Field notes showed that the format enabled efficient whole-class monitoring of answers but afforded restricted insight into students' thinking processes. Because only final answers were visible, the teacher had limited access to students' reasoning or misconceptions, making it challenging to provide targeted support to students or identify when follow-up was needed to address misconceptions, struggle, or peer reliance. When answers were revealed, there were visible displays of enthusiasm (e.g., "Yes!") from some students, indicating they also valued correct answers and opportunities to check. The structure and positioning of students as well as the teacher allowed for potential non-thinking behaviours (e.g., such as faking or copying) to remain unnoticed. The teacher remained largely in one position, supporting a small group seated nearby, and intermittently recorded observational notes. When asked, Tessa explained that she sometimes adapted the format to differentiate for a small group of students who worked through an 'extended' Daily Review together rather than the whole-class administered version. She reported positively on the review process, and that the routine builds retention through rehearsal, had strengthened her instructional confidence, and was positively received by students.

Daily review: Kimba (Grade 4)

Kimba's Daily Review followed a similar slide-based, write-reveal structure to Tessa's, but differed markedly in interaction pattern. Students were frequently asked to explain and demonstrate their strategies and encouraged to ask questions. Alternative strategies were accepted and discussed, and choral responding was minimal. Kimba remained standing throughout and moved constantly around the room. While the lesson was fast paced, she welcomed students' contributions. The Daily Review ran for 20 minutes. Using the same format as Table 1, Table 2 provides an overview of Kimba's Daily Review structure.

Table 2.

Structure of Kimba's Daily Review Lesson

Mathematical focus (slides)	Pedagogical routine	Instructional variations	Student participation pattern
Continuing patterns; numbers in descending order	Slide-based instruction; write-reveal cycle	Asked students to describe rule for pattern; invited explanations before revealing answers; selected students to justify reasoning	Individual whiteboard responses; boards held under chin; self-correction
Equations	Slide-based write-reveal cycle	Asked 'How do you know?'; elicited multiple solution strategies; revoiced and compared students' methods	Individual whiteboard responses

Mathematical focus (slides)	Pedagogical routine	Instructional variations	Student participation pattern
Vertical algorithms (addition/subtraction)	Slide-based write-reveal cycle	Probed place value reasoning; asked students to explain regrouping; addressed misconceptions openly (working from left to right)	Individual whiteboard responses showing working out; students modelling of strategies on class IWB
2x multiplication facts; dividing by 2	Write-reveal cycle	Asked students to connect doubling and halving; prompted generalisations (ending in even number); brief discussion of strategy efficiency	Some choral responses (skip counting); individual recording on whiteboards; some partner checking
Area of a rectangle	Write-reveal-cycle	Asked students to explain meaning of area	Individual whiteboard responses with drawings
Draw a hexagon	Write-reveal-cycle	Addressed misconception of hexagon having 8 sides	Individual whiteboard drawings

Table 2 shows several instructional variations that mainly involved eliciting students to explain their thinking. Choral responding was minimal and enacted as whole class skip counting. Video analysis showed that while the lesson was fast-paced, Kimba encouraged extended exchanges and there was wide student participation in these exchanges. There were at least two opportunities where errors or misconceptions were revealed, and these were addressed as teachable moments (unplanned teaching opportunities). Students were observed using post it notes to record PowerPoint slide numbers they found challenging. When asked, Kimba explained that these would be addressed through ‘targeted mini lessons’ prior to students re-encountering the content again, thereby catering for diversity.

Discussion

While both teachers implemented a structured, slide-driven Daily Review built around a write-reveal cycle, the enactment differed substantially in instructional variations and student participation. Tessa’s lesson prioritised pace, accuracy, and whole-class synchrony, while providing limited opportunities for students to explain and make visible their mathematical reasoning, both of which are recognised indicators of inclusive teaching practices (Ingram et al., 2020). In contrast, Kimba demonstrated inclusive practices through embedded frequent invitations for strategy sharing, comparison, and public unpacking of misconceptions, allowing for students to discuss and learn from their mathematical mistakes (e.g., Ingram et al., 2020),

Although both lessons were highly routinised and teacher-directed, Kimba’s enactment incorporated more dialogic and adaptive moves, particularly through inviting students’ contributions and encouraging students to take risks (Hennessy et al., 2016). This was evidenced by inviting students to demonstrate on the IWB how they would solve the algorithms. Students were active participants in the classroom community (Sherin & van Es, 2026) and positioned to add to the teacher, as sources of mathematical knowledge (Gutierrez, 2012).

Although students were instructed to ‘tick or fix’ their responses, observations showed that this rarely involved reflection on why an answer was incorrect. Instead, students typically erased and replaced their response once the answer was revealed. This suggests that the routine functioned more as answer verification than as a process of diagnosing and addressing misunderstandings. Kimba appeared more aware of this limitation and created inclusive opportunities for students to explain strategies and publicly discuss and examine misconceptions.

Participation largely consisted of students responding in ways that were prompted by the slides or teacher, such as writing answers, holding boards under their chins, and choral responses. Tessa's Daily Review was dominated by unison choral responding (Merlo, 2024). While this ensured widespread participation, it did not necessarily support inclusive teaching, as the 'power' (Gutierrez, 2012) remained with the teacher and students had limited opportunities to contribute their thinking. As noted by Alexander (2008), students need opportunities to engage in meaningful talk; therefore, highly procedural Daily Review routines may restrict valuable interaction with peers and the teacher.

The rapid pacing and tightly sequenced slides meant that students progressed through tasks simultaneously. While that might support fluency rehearsal (Rosenshein, 2012), it risks sweeping students along without ensuring conceptual understanding, particularly when answers are quickly revealed and the next questions appears immediately. Kimba attempted to mitigate this through questioning and discussion of strategies; however, the fast pace continued to limit opportunities for individual reflection and the sharing of diverse mathematical thinking, both of which are important aspects of inclusive teaching practices.

Conclusions and Implications

In this paper we report on two classrooms and observations of two single Daily Review lessons at one point in time. We did not observe the full range of mathematics instruction enacted by these teachers, and the routines described here represent only one component of their broader mathematical teaching practice. It is also important to note that the findings should not be interpreted as a critique of the teachers or the Daily Review routine itself. Both teachers implemented the mandated structure thoughtfully and with clear commitment to supporting students' learning. Daily review routines can provide valuable opportunities for rehearsal, consolidation, and whole-class participation. However, the structure of the routine, particularly the rapid write-reveal cycle and emphasis on answer verification may limit teachers' access to students' reasoning and constrain opportunities for responsive and inclusive teaching. While such routines may appear inclusive because all students participate simultaneously, participation alone does not necessarily reveal individual understanding, enable instruction to be adjusted (Sherin & van Es, 2026), or position students as contributors to mathematical knowledge (Gutierrez, 2012)

The findings reveal that Daily Reviews are not enacted uniformly, with enactment choices influencing opportunities for inclusive practices. Rather than evaluating the effectiveness of Daily Reviews, this small-scale study contributes nuanced insight into how the practice is interpreted and enacted within classroom contexts. By making visible the pedagogical variations within ostensibly similar routines, the study provides an important foundation for future research examining the relationship between Daily Review enactment, student participation, and inclusive mathematics teaching practices. Complementary instructional approaches, such as number talks, promote inclusive practices by encouraging students to articulate strategies, compare methods, and reason collectively. Integrating such dialogic practices alongside Daily Review routines may provide teachers with richer insights into student thinking and strengthen opportunities for responsive and inclusive mathematics teaching.

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References

Alexander, R. J. (2008). *Towards dialogic teaching: Rethinking classroom talk* (4th ed.). Dialogos.

- Attard, C. (2025). Unlocking minds: Exploring the pendulum of mathematics pedagogy. In S. M. Patahuddin, L. Gaunt, D. Harris, & K. Trippet (Eds.), *Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 3-10). MERGA.
- Australian Curriculum Assessment and Reporting Authority (ACARA). (2025). *Australian Curriculum: Mathematics v.9*. <https://www.australiancurriculum.edu.au/curriculum-information/understand-this-learning-area/mathematics>
- Australian Education Research Organisation (AERO). (2024). *Revisit and review. Regularly revisit and review learning*. <https://www.edresearch.edu.au/sites/default/files/2024-11/AERO-practice-guide-revisit-and-review-aa.pdf>
- Darragh, L., MacArthur, J., Ell, F., & Morton, M. (2025). Weaving a web to catch them all: Inclusive pedagogies in mathematics. *Mathematics Education Research Journal*, 38, 459-483. <https://doi.org/10.1007/s13394-025-00535-2>
- Gutiérrez, R. (2012). Context matters: How Should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in Discourse for Mathematics Education: Theories, Practices, and Policies* (pp. 17-33). Springer Netherlands. https://doi.org/10.1007/978-94-007-2813-4_2
- Hennesy, S., Rojas-Drummond, S., Higham, R., Torreblanca, O., Barrera, M. J., Marquez, A. M., García Carrión, R., Maine, F., & Ríos, R. M. (2016). Developing a coding scheme for analysing classroom dialogue across educational contexts. *Learning, Culture and Social Interaction*, 9, 16-44.
- Hiebert, J., & Stigler, J. W. (2023). Theorizing teaching: Current status and open issues. In A. K. Praetorius & C. Y. Charalambous (Eds.), *Creating practical theories of teaching* (pp. 23-56). Springer. <https://doi.org/10.1007/978-3-031-25613-4>
- Ingram, N., Holmes, M., Linsell, C., Livy, S., McCormick, M., & Sullivan, P. (2020). Exploring an innovative approach to teaching mathematics through the use of challenging tasks: a New Zealand perspective. *Mathematics Education Research Journal*, 32(3), 497-522. <https://doi.org/10.1007/s13394-019-00266-1>
- Kervin, L., Vialle, W., Herrington, J., & Okely, T. (2016). *Research for educators (2nd ed.)*. Cengage Learning Australia.
- Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist*, 41(2), 75-86.
- Mathematics Education Research Group of Australasia (MERGA). (2025). *MERGA position statement on effective mathematics teaching*. <https://merga.net.au/positionstatement/>
- Merlo, S. (2024). *The science of maths and how to apply it*. Centre for Independent Studies.
- Myhill, D. (2006). Talk, talk, talk: Teaching and learning in whole class discourse. *Research Papers in Education*, 21(1), 19-41.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. NCTM.
- Ochre Education. (2026). *Ochre learning*. <https://ochre.org.au/ochre-learning>
- Rosenshine, B. (2012). Principles of instruction. Research-based strategies that all teachers should know. *American Educator*, Spring, 12-39.
- Sherin, M. G., & van Es, E. A. (2026). Noticing Student Thinking to Create Equity. *Mathematics Teacher: Learning and Teaching PK-12*, 119(1), 6-15. <https://doi.org/10.5951/mtlt.2024.0151>
- Sinclair, J. & Coulthard, M. (1975). *Towards an Analysis of Discourse*. Oxford University Press.
- Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation: An International Journal of Theory and Practice*, 2(1), 50-80.
- Swan, P. (2023). *The Daily Review*. <https://drpaulswan.com.au/the-daily-review/>
- Sweller, J., Ayres, P., & Kalyuga, S. (2011). *Cognitive Load Theory*. Springer.
- Sweller, J., van Merriënboer, J. J. G., & Paas, F. (2019). Cognitive Architecture and Instructional Design: 20 Years Later. *Educational Psychology Review*, 31(2), 261-292. <https://doi.org/10.1007/s10648-019-09465-5>
- Victorian Department of Education. (2026). *Mathematics retrieval resources guidance*. <https://arc.educationapps.vic.gov.au/learning/sites/lesson-plans-guidance/2225/Mathematics-Retrieval-Resources-guidance>

Finding Common Ground: Perspectives on Spatial Reasoning and Patterning in Mathematics Education

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Patterning and spatial skills are central to mathematics achievement and are precursors to generalising in mathematics. This paper draws together three complementary lines of research on the role of pattern and structure, number sense, and STEM in early learning to consider the interrelations between patterning and spatial skills, and their influence on mathematical achievement. An overview of the ‘Pattern and Structure’ project, studies on patterning and number sense, and the Early Years STEM Australia (ELSA) project highlight commonalities in approaches and findings. An integrated approach aims to develop an improved understanding on how and why patterning, spatial skills and number sense are linked and are ‘transferred’ to mathematical achievement.

There is an increasing interest in identifying key skills that underpin early mathematics learning (Resnick et al., 2026). One line of work focuses on the role of patterning and structure, highlighting how the ability to detect, represent and generalise regularities contributes to the development of early algebraic and numerical concepts (Mulligan & Mitchelmore, 2009; Rittle-Johnson et al., 2019). A second, largely parallel line of research has emphasised spatial thinking skills, finding that skills such as mental rotation and perspective taking are robust predictors of mathematics performance from preschool onwards (e.g., Gilligan et al., 2020; Mix, 2019).

Although these bodies of work have developed with different emphases and methodological traditions, more recent research suggests patterning and spatial skills reflect interdependent foundations of mathematical reasoning (Lowrie & Logan, 2023; Mulligan et al., 2020a; Rittle-Johnson et al., 2019). Patterning tasks often require children to utilise spatial processes (e.g., structuring, transformation, and visualisation) with their developing understanding of numerical or algebraic relations. At the same time, spatial skills may support children’s capacity to perceive and abstract structure within patterns. Emerging studies suggest that these domains mutually reinforce one another, providing complementary pathways into the development of mathematical generalisation, number sense, and problem solving (Junker et al., 2025; Lowrie & Logan, 2023; Mulligan et al., 2020a; Rittle-Johnson et al., 2019). Indeed, developmental progressions (trajectories) and learning pathways on patterning, have involved spatial skills (Baroody et al., 2021; Mulligan et al., 2020a).

This convergence signals a need for a more integrated theoretical treatment of patterning and spatial skills in mathematics learning. Accordingly, the aim of this paper is to synthesise these complementary lines of research and suggest a more integrated framework of how patterning and spatial skills jointly contribute to early mathematical learning. We provide a summary of our studies focused on the interrelated roles of patterning and spatial skills in: mathematical structure, number sense, and STEM learning.

The role of patterning in early mathematical development

There is strong evidence that patterning is central to mathematics achievement and is a precursor to generalising in mathematics (e.g., Acosta et al., 2026; Larkin et al., 2024; Mulligan & Mitchelmore, 2009; Zippert et al., 2020). Repeating patterning ability remains a unique predictor of success, even after accounting for a range of demographic factors such as receptive mathematics language, geometry, counting and cardinality, gender, age, ethnicity, basic number skills, or prior mathematics knowledge and spatial skills (Gilligan et al., 2020; Hawes et al., (2022), Rittle-Johnson et al., 2019; Wijns et al., 2019). Repeating patterning skills at the end of PreK have been found to predict mathematics achievement in first and fifth grade (Rittle-Johnson et al., 2013), and other studies have found pattern understanding a predictor of early reading and arithmetic skills (e.g., Burgoyne et al., 2019).

There are large individual differences among four-year olds' repeating pattern knowledge, with 77% of the four-year-olds able to copy a simple pattern, approximately 50% able to extend patterns, 33% capable of translating (abstracting) a repeating pattern, and only 10% being able to identify the 'unit of repeat' (Rittle-Johnson et al., 2019). A large scale, naturalistic Australia-wide study confirms existing laboratory-based research, whilst also indicating that some preschool children were more capable at creating three and four element patterns than previously thought (Larkin et al., 2024). In a suite of studies with four-to-eight year-olds Mulligan and colleagues found that the recognition of the 'unit of repeat' was more advanced than simple repetition, and that the unit of repeat was critical to abstraction and generalisation (mathematical structure), and multiplicative reasoning (e.g., Mulligan & Mitchelmore, 2009; Papic et al., 2011). In a mixed-method study of three-to-seven year-olds Lüken and Sauzet (2021) found a clear developmental progression in patterning skills: simple repetitions as alternate items, to those with a more advanced composite structure of the 'unit of repeat', and to the extension of growing patterns. These findings were consistent with Papic et al. (2011).

Recent studies on patterning have extended to the investigation of growing and spatial patterns (e.g., Wijns et al., 2021) and how these may be related to concepts such as counting, multiplication, number sense, transformations and dimensionality (Junker et al., 2025; Mulligan, 2025; Rittle-Johnson et al., 2019). Lüken (2025) investigated pre-schooler's self-initiated patterns, with most comprising transformations, constructed as both repeating and growing patterns. However, the role of different types and modes of patterning has not been sufficient to establish causal relations between patterning and spatial skills, and numerical competencies (Mulligan, 2025; Resnick & Lowrie, 2023).

The role of spatial reasoning in early mathematical development

The relation between spatial skills and mathematics achievement across development is consistent, predictive, and strengthens over time (Resnick et al., 2019). Higher levels of spatial skills are correlated with higher performance in mathematics (Mix et al., 2019). This relation begins in preschool (Verdine et al., 2014) and is increasingly associated with mathematics achievement over the intermediary through adult years (Hawes et al., 2022). For example, spatial skills at age three accounts for 27% of the variance in mathematics performance at age four, even after controlling for SES, gender, fine-motor skills, vocabulary, and executive function (Verdine et al., 2017).

Preschool children demonstrate a broad range of spatial skills, even before formal schooling (for review, see Davis & Spatial Reasoning Study Group, 2015). By age three, many children can mentally represent and manipulate simple spatial configurations, identify shapes, navigate familiar environments using landmarks, and complete basic spatial visualisation tasks such as assembling puzzles or block constructions. Across the preschool years, spatial thinking becomes more flexible and efficient, which reflects both increasing experience with spatially-rich activities and a growing capacity for relational reasoning.

Patterning and spatial skills in developing mathematical structure

Awareness of Mathematical Pattern and Structure (AMPS) has been observed, described, and measured as a general construct that underpins structural thinking and its ties to emergent generalisation in early years' mathematics (Mulligan & Mitchelmore, 2009). AMPS comprises two interdependent components: one cognitive, a knowledge of structure, and one meta-cognitive, a tendency to seek and analyse patterns.

Over the past decades the Pattern and Structure Project conducted a series of related studies on AMPS with four-to-six year-olds (including task-based interviews, Rasch-modelled scales of AMPS, longitudinal interventions, and developmental design studies). A Pattern and Structure assessment interview (PASA) and program (PASMAP) was developed and evaluated, comprising an interrelated network of mathematical concepts and relationships, implemented through a pedagogical framework: modelling, visualising, representing, generalising and sustaining.

Students demonstrating highly-advanced AMPS spontaneously searched for mathematical patterns, noticed and represented common structural features, and formed emergent generalisations. In contrast, children showing pre-structural AMPS used idiosyncratic ideas and did not recognise structural features. The majority of children were able to spontaneously construct simple repeating patterns, while others applied a 'unit of repeat' and constructed growing patterns. 'Spatial structuring' and visualisation skills were found integral to AMPS and related mathematical knowledge. For example, structuring 'equal groups' required spatial skills such as co-linearity of arrays and grids in two and three dimensions, and transformation and orientation skills were used in extending patterns systematically.

Secondary qualitative analyses examined how different pattern types, including growing patterns, accounted for wide variation in children's AMPS, illustrating how spatial and patterning skills are interrelated (Mulligan, 2025). These analyses described how multiplicative thinking, transformation, navigation (directionality) and perspective-taking skills influenced the child's perception of both repetitions and growing patterns, and how the use of dynamic spatial visualisation determined whether the structure of a pattern could be noticed and generalised.

A Spatial Reasoning Mathematics Program (SRMP) extended the Pattern and Structure studies, developed through a 2-year intervention, and a two-phase design study in Grades 3 and 4 (Mulligan et al., 2020b). The SRMP involved complex repetitive and growing patterns including spirals; area/perimeter/shape relations; directionality, and perspective taking. In summary, the intervention study found significant gains by the experimental group on two interview-based assessments (comprising rotation, reflection, equipartitioning, directionality, perspective-taking tasks) and marked growth in patterning and spatial structuring skills, for example, visualising and structuring arrays and grids and extending rotational growing patterns. The SRMP also enabled the integrated structural development of related numerical and measurement concepts. Further studies aim to describe how and why the SRMP concepts are inextricably linked to improved number sense, mathematical achievement, and problem solving.

Patterning and spatial skills in number sense learning

Although there has been an unprecedented number of studies on early patterning in recent decades (e.g., Björklund & Pramling, 2014; Mulligan, 2025; Pitta-Pantazi et al., 2025; Wijns et al., 2019), there remains limited understanding of how patterning is associated with specific aspects of number sense, particularly for students with high or low mathematical ability. Junker (2024) theorised that AMPS is essential for the development of early number sense and formulated a new study to investigate the interrelations between pattern and number sense.

A cross-sectional mixed-method doctoral study of 75 Norwegian 6-year-olds analysed counting, repeating and sequencing patterning, number-sense performance, and strategy use through a digital assessment, and task-based interviews (Junker et al., 2025). Fifty-two embedded case studies differentiated high (HNS) and low number sense (LNS) groups. A digital pre-assessment of number sense included 69 items: eight interdependent skills drawn from Andrews and Sayers' (2015) Foundational Number Sense framework: *Numerical Identification*, *Systematic Counting*, *Number* and *Quantity*, *Estimation*, *Quantity Discrimination*, *Arithmetic Competence*, *Number Patterns*, and *Subitising*. Number patterns included *Sequencing patterning* focused on systematic counting tasks and repeated patterning involving duplication, extension, transfer, and unit isolation.

For students with LNS, success in extending repeating patterns was linked to quantity discrimination, while success in sequencing $N \pm 1$ and $N - 2$ patterns was linked to number and quantity and additive concepts—areas in which these students struggled. For HNS students, success in extending repeating patterns was associated with arithmetic, and success in sequencing $N \pm 1$ patterns was associated with estimation. In-depth multimodal analysis of the LNS and HNS students' strategies found a statistically significant group difference for extending repeating patterns. Both groups used recursive strategies but only students with high number sense used relational strategies. LNS students used more variations of recursive patterning strategies, such as omitting, adding or mirroring the order of pattern elements. In contrast to HNS students, LNS students attended to superficial pattern features, which likely hindered identification of a pattern's underlying structure. For example, LNS students constructed patterns based on colour and shape rather than attending to the underlying structure that the colours and the shapes represented. By contrast, those with a high number sense exhibited (de)composition, a strategy indicative of relational understanding. Notably, repeating and sequencing patterning activities requiring unit identification significantly distinguished the student groups and predicted success.

Follow-up case studies of HNS and LNS students were selected on the basis of assessment data and their use of finger and gestural strategies. The study primarily aimed to video students' spontaneous hand and finger movements and analyse these in relation to number sense. *Systematic Counting* utilised traditional tasks: verbal counting, forward/backward sequences 0–20, counting on/back from 7 and 14, and counting by twos from 0, 10, and 20. Manipulatives were available, though rarely used. For the LNS case, counting occurred as rote verbal sequences with no evidence of decomposing quantities or their relations, unlike the HNS case. Video analysis revealed two themes: (1) decomposition of individual quantities, which may or may not indicate structural awareness, and (2) more advanced decomposition of whole or partial counting sequences into unequal or equal parts (composite units), where unequal parts may be associative and equal parts multiplicative (e.g., Mulligan & Mitchelmore, 2009).

The findings demonstrated that LNS students can demonstrate unitary and skip counting, cardinality, ordering, number-line skills, and basic facts to 10 without displaying structural awareness, or using decomposition or additive structure. The study has important implications for pedagogy and curricula that should emphasise the relationships between embodied counting and structural awareness. Another key insight was the students' use of dynamic embodied spatial skills, including use of finger patterning, shape, symmetry, and directional finger movements to track the count and visualise decomposed groups.

Patterning and spatial reasoning in early STEM learning

Patterning and spatial skills are critical in STEM (science, technology, engineering, and mathematics) learning (Resnick & Lowrie, 2023). Thus, the Early Learning STEM Australia (ELSA) programs emphasise patterning and spatial skill development as core components of the programs, which span preschool to Year 2. For example, the ELSA F-2 program includes

topics such as *sorting objects based on spatial and numerical attributes; aligning numbers to spatial patterns; giving and following directions using landmarks* (Foundation); *measuring and estimating distance with informal units; decoding representations using maps; sorting and ordering visual data* (Year 1); *graphing using and X- and Y- axis; Fraction Families – Wholes, halves, quarters and eights, and transformations and rotations* (Year 2). Notably, this program begins by developing the requisite spatial skills, then spatial patterning, building towards numerical patterning understanding.

A randomised control trial (RCT) of the ELSA preschool program (Resnick & Lowrie, 2023), found that engagement with a play-based spatial program (which included playing patterning and spatial games) led to better overall spatial reasoning and transferred to better numeracy understanding for students compared with a business-as-usual control. The results underscore the importance of embedding spatial learning within strong pedagogy and authentic learning contexts, supported by the use of engaging digital games that can be played independently by the students with their learning automatically reported back to teachers, via a digital dashboard, at individual and cohort levels.

A study involving a nationally representative sample of 697 preschool children and 1011 kindergarten children (n=1708) found that patterning and spatial skills are separate but correlated, with each providing unique variance in predicting numeracy (Resnick et al., under review). Specific pathways remained stable over the transition from play-based (preschool) to more formal (kindergarten) learning environments, highlighting that the relations between patterning, spatial skills, and numeracy appear consistent. The interdependence of spatial skills and patterning is also evident in work adopting a strength-based perspective on children's spatial competencies. Harris et al. (2025) demonstrated that preschool children's spatial language and patterning skills are strongly related and that their expression is largely context dependent. In game-based assessment environments, children from diverse socio-economic and geographic backgrounds demonstrated robust spatial language and patterning skills that were not always apparent in formal test-based measures. This finding reinforces the idea that spatial patterning and spatial language develop together through engagement with rich, embodied, and contextualised experiences—experiences that are central to authentic STEM learning.

Further research directions and implications for pedagogy and curricula

Taken together these studies have not yet established *how* and *why* patterning, spatial skills, and number sense are interrelated, or which particular spatial skills are critical to improved and broader mathematics achievement. Hence our approach is to adopt a complementary perspective to leverage existing research and find 'common ground' in order to collaboratively develop a more coherent body of research on the role of spatial reasoning in mathematics learning. Based on our joint research findings, some development and integration of our theoretical and pedagogical frameworks may be proposed. Further research may design and implement interdisciplinary, 'mixed methods' studies, some at scale, that focus on more complex multi-dimensional patterning and spatial skills (beyond simple repetitions and rotation). This approach aims to find strong associations between patterning spatial reasoning, and number sense and advance understanding of the mechanisms of 'transfer' to mathematics achievement.

A collaborative, complementary approach may provide more convincing and cohesive evidence for impactful reform in mathematics pedagogy and curricula that currently limits the place of pattern and structural development, and spatial reasoning as core aspects of number sense. Our broad aim is to positively impact teacher practice: pedagogy, curricula, and assessment, that articulates and prioritises spatial reasoning, and embeds spatially-oriented programs within learning. More conceptually-advanced and explicit descriptors of spatial/mathematical interrelations are needed for us to achieve this aim.

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References

- Acosta, Y., Alsina, A., Ayala-Altamirano, C., & Rodrigues-Silva, J. (2026). Pattern recognition and generalization in 4-year-olds: The impact of task contexts and teaching resources on the development of algebraic thinking. *Journal of Mathematical Behavior*, 82, 101315
- Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal*, 43(4), 257–267. <https://doi.org/10.1007/s10643-014-0653-6>
- Baroody, A. J., Yilmaz, N., Clements, D. H., & Sarama, J. (2021). Evaluating a basic assumption of learning trajectories: The case of early patterning learning. *Journal of Mathematics Education*, 13, 8–32.
- Burgoyne, K., Malone, S., Lervag, A., & Hulme, C. (2019). Pattern understanding is a predictor of early reading and arithmetic skills. *Early Childhood Research Quarterly*, 49, 69–80. <https://doi.org/10.1016/j.ecresq.2019.06.006>
- Björklund, C., & Pramling, N. (2014). Pattern discernment and pseudo-conceptual development in early childhood mathematics education. *International Journal of Early Years Education*, 22(1), 89–104. <https://doi.org/10.1080/09669760.2013.809657>
- Davis, B., & the Spatial Reasoning Study Group. (2015). *Spatial reasoning in the early years: Principles, assertions, and speculations*. Routledge.
- Gilligan, K. A., Thomas, M. S., & Farran, E. K. (2020). First demonstration of effective spatial training for near transfer to spatial performance and far transfer to a range of mathematics skills at 8 years. *Developmental Science*, 23(4), e12909. <https://doi.org/10.1111/desc.12909>
- Hawes, Z., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology*, 58(1), 112–137. <https://doi.org/10.1037/dev0001281>
- Harris, D., Resnick, I., Logan, T., & Lowrie, T. (2025). Pathways from spatial skills to mathematics: The roles of gender and fluid reasoning. *Developmental Science*, 28, e13602. <https://doi.org/10.1111/desc.13602>
- Junker, A. (2024). *Patterning in grade 1 students with low and high number sense proficiency*. <https://ntnuopen.ntnu.no/ntnu-xmlui/handle/11250/3164680>
- Junker, A., Nortvedt, G. A., & Farsani, D. (2025). Patterning strategies in grade 1 students with low and high number sense proficiency. *Educational Studies in Mathematics*, 118(1), 29–51. <https://doi.org/10.1007/s10649-024-10341-5>
- Larkin, K., Resnick, I., & Lowrie, T. (2024). Preschool children's repeating patterning skills: Evidence of their capability from a large scale, naturalistic, Australia wide study. *Mathematical Thinking and Learning*, 26(2), 127–142. <https://doi.org/10.1080/10986065.2022.2056320>
- Lowrie, T., & Logan, T. (2023). Spatial visualization supports students' math: Mechanisms for spatial transfer. *Journal of Intelligence*, 11(6), Article 127. <https://doi.org/10.3390/jintelligence11060127>
- Lüken, M. M. (2025). Young children's self-initiated pattern-making during free play. *Mathematical Thinking and Learning*, 27(2), 221–241. <https://doi.org/10.1080/10986065.2023.2276798>
- Lüken, M. M., & Sauzet, O. (2021). Patterning strategies in early childhood: A mixed methods study examining 3- to 5-year-old children's patterning competencies. *Mathematical Thinking and Learning*, 23(1), 28–48. <https://doi.org/10.1080/10986065.2020.1719452>
- Mix, K. S. (2019). Why are spatial skills and mathematics related? *Child Development Perspectives*. <https://doi.org/10.1111/cdep.12323>
- Mulligan, J. T. (2025). The role of spatial reasoning in growing and spatial-repeating patterns in first and second graders' structural development of mathematics. *Education Sciences*, 15(11), 1479. <https://doi.org/10.3390/educsci15111479>

- Mulligan, J. T., & Mitchelmore, M. C. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, 21(2), 33–49. <https://doi.org/10.1007/bf03217544>
- Mulligan, J., Oslington, G., & English, L. (2020a). Supporting early mathematical development through a ‘pattern and structure’ intervention program. *ZDM–Mathematics Education*, 52(4), 663–676. <https://doi.org/10.1007/s11858-020-01147-9>
- Mulligan, J., Woolcott, G., Mitchelmore, M., Busatto, S., Lai, J., & Davis, B. (2020b). Evaluating the impact of a spatial reasoning mathematics program (SRMP) intervention in the primary school. *Mathematics Education Research Journal*, 32(2), 285–305. <https://doi.org/10.1007/s13394-020-00324-z>
- Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers’ mathematical patterning. *Journal for Research in Mathematics Education*, 42(3), 237–268. <https://doi.org/10.5951/jresmetheduc.42.3.0237>
- Pitta-Pantazi, D., Demosthenous, E., Schindler, M., Lilienthal, A. J., & Constantinos, C. (2025). Structure sense in students’ quantity comparison and repeating pattern extension tasks: An eye-tracking study with first graders. *Educational Studies in Mathematics*, 118(3), 339–357. <https://doi.org/10.1007/s10649-023-10290-5>
- Resnick, I., Newcombe, N.S., & Jordan, N.C. (2019). The relation between spatial reasoning and mathematics achievement in children with mathematics learning difficulties. In P. Rasanen (Ed.), *International handbook of mathematical learning difficulties*. Springer.
- Resnick, I., & Lowrie, T. (2023). Spatial reasoning supports preschool numeracy: Findings from a large-scale nationally representative randomized control trial. *Journal for Research in Mathematics Education*, 54(5), 295–316. <https://doi.org/10.5951/jresmetheduc-2022-0051>
- Resnick, I., Harris, D., & Bruce, C. (2026). Considering the relations between geometry and spatial reasoning. In T. Lowrie & A. Gutiérrez (Eds.) *Advances in geometry education*. Springer.
- Resnick, I., Larkin, K., Logan, T., Levido, A., & Lowrie, T. (under review). The role of spatial skills and spatial language on numeracy in the early years of schooling.
- Rittle-Johnson, B., Fyfe, E., McLean, L., & McEldoon, K. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development*, 14(3), 376–396. <https://doi.org/10.1080/15248372.2012.689897>
- Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*, 46, 166–178. <https://doi.org/10.1016/j.ecresq.2018.03.006>
- Verdine, B., Golinkoff, R., Hirsh-Pasek, K., & Newcombe, N. (2017). Links between spatial and mathematical skills across the preschool years. *Monographs of the Society for Research in Child Development*, 82(1), 730. <https://doi.org/10.1111/mono.12285>
- Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2014). Deconstructing building blocks: Preschoolers’ spatial assembly performance relates to early mathematical skills. *Child Development*, 85(3), 1062–1076. <https://doi.org/10.1111/cdev.12165>
- Wijns, N., Torbeyns, J., De Smedt, B., & Verschaffel, L. (2019). Young children’s patterning competencies and mathematical development: A review. In K. Robinson, H. Osana, & D. Kotsopoulos (Eds.), *Mathematical learning and cognition in infancy and early childhood: Integrating interdisciplinary research into practice* (pp. 139–161). Springer. https://doi.org/10.1007/978-3-030-12895-1_9
- Wijns, N., Verschaffel, L., De Smedt, B., & Torbeyns, J. (2021). Associations between repeating patterning, growing patterning, and numerical ability: A longitudinal panel study in 4- to 6-year olds. *Child Development*, 92(4), 1354–1368. <https://doi.org/10.1111/cdev.13490>
- Zippert, E., Douglas, A., & Rittle-Johnson, B. (2020). Finding patterns in objects and numbers: Repeating patterning in pre-K predicts kindergarten mathematics knowledge. *Journal of Experimental Child Psychology*, 200. <https://doi.org/10.1016/j.jecp.2020.104965>

Pedagogic Identity and Maths Talk in the Early Years: A Study of Two Teachers

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The introduction of pedagogic strategies to support student talk in mathematics classrooms has had varying success. This study explored the pedagogic identity of two teachers within a professional development project aimed at developing student talk in early years mathematics. Interview data with the two teachers were analysed deductively according to Lefstein's four dimensions of dialogic teaching and related to pedagogic identity. Findings revealed contrasting ideologies that influenced the teachers' adoption of talk strategies and suggest that, to sustain changes in student talk, teacher pedagogic identities should be recognised and accommodated.

Studies have shown that, when students' awareness of talk in learning is raised and they engage in content specific reasoned discussions, then the quality of group activity and learning outcomes are greatly improved (e.g., Howe et al., 2019) leading to a consensus that type of classroom talk impacts on student learning (Bouton & Asterhan, 2023). However, professional development (PD) to develop productive dialogue is not always successful (Moser et al., 2022; Vrikki et al., 2019). Within mathematics classrooms, many factors, including competing goals, student background, and the uncertainties of both teachers and students, impact on changes to classroom practice (O'Connor & Michaels, 2019). Maass (2011) has further indicated that teacher beliefs can limit potential changes. What is less clear is how such beliefs and factors can be further understood, particularly in early years mathematics classrooms. Understanding and accommodating these beliefs and factors may help to adapt PD programmes and support teachers in developing use of productive talk in their classrooms.

In this research report, we present data from our larger project, *Talking Maths*, that involved the design and implementation of a professional development (PD) programme to promote young students' talk and learning in mathematics. We determine how the perspectives of two teachers influenced their capacity to promote and sustain talk in early years mathematics classrooms. Specifically, we refer to pedagogic identity as the beliefs, perspectives and pedagogical experiences of the teachers (Hong et al., 2024; Murphy, 2015) and ask the question: To what extent can pedagogic identity function as a theoretical lens for understanding the tensions in sustaining productive talk in mathematics classrooms?

Literature Review

The relevance of talk on student learning in the classroom has been promoted by academics such as Alexander (2008) who defined dialogic teaching as teaching that focuses on talk that engages student interest, stimulates thinking, and advances understanding. Authenticity, response, and cognitive demand are seen to be inherent qualities of dialogue. These qualities of dialogue contrast with the often prevailing Initiation-Response-Feedback (IRF) model (Sinclair & Coulthard, 1975), where the teacher initiates a question, the student provides a response, and the teacher follows with feedback.

Supporting students' talk in the mathematics classroom, both whole class and in collaborative groups, requires scaffolding. There has been much interest in how experienced

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 281–288). MERGA.

teachers manage such scaffolding while maintaining students' ownership of their own thinking (e.g., Klemp, 2020; Murphy et al., 2025). Other literature has provided support on the use of prompts and ground rules for small group work (Mercer & Sams, 2006) and the use of talk moves in supporting teachers to take up productive talk in whole class teaching (O'Connor & Michaels, 2019).

The availability of literature presenting support structures for teachers, for example Talk Moves (Anderson et al., 2011) and Number Talks (Parrish, 2010), has provided evidence that interventions based on practical strategies can impact on teachers' practice and dialogue in mathematics classrooms. Nevertheless, there remains a scarcity of productive talk in mathematics (Moser et al., 2022). PD has not always been successful, with potential factors such as time constraints, teacher familiarisation of dialogic principles, and the need to provide tools to support teachers in supporting and sustaining student talk both whole class and in group work. What has not always been considered is the teacher's capacity in extending talk in their classroom to move beyond sharing strategies towards cumulative student contributions (Ni et al., 2021). A further consideration relates to a teacher's view of mathematics and their role in relation to their authority as a teacher of mathematics (Campbell & Teo, 2022). Such findings suggest that "doing" PD (Heyd-Metzuyanim et al., 2018) is not always sufficient in countering misalignment between PD intentions and teachers' practices. These findings point towards the impact of teachers' capacity, views of mathematics, and their perceived role in teaching mathematics on alignment or misalignment with the PD intentions. Our premise is that anticipating and accounting for these elements in relation to pedagogic identity can help in understanding and addressing potential misalignment.

Theoretical Perspectives

While teacher instruction is recognised as social, this social notion has contrasting ideologies (Walkerline, 1990). Much of the literature underpinning PD on talk in the classroom draws on theories related to dialogic practice (e.g., Alexander, 2008). Based on Vygotsky's (1978) sociocultural perspective, collaborative and joint reasoning are determined through the intermental plane of cognition and the genesis of intramental functioning. In this way, classroom talk happens on a social plane and acts as the conduit to the individual plane of a student. Such an ideology relates to non-positivist viewpoints where mathematics is seen as generative meaning making. Talk in the classroom is not about acquiring a set of mathematical structures but is about meaning making through negotiation of ideas and recognises that "discursive contributions that may pull others forward into their increasing participation in mathematical speaking/thinking" (Lerman, 2001, p.89). This non-positivist viewpoint is intrinsic to dialogic practice. In contrast, a positivist viewpoint does not recognise the social nature of knowledge and learning in the same way. A positivist structuralist ideology suggests mathematical knowledge is static and stable. Whilst teaching is still seen as a social practice, the teacher tells the students what they need to know.

An important element of these contrasting ideologies is the teachers' own perspectives, that is their identity (beliefs and perceived confidence) and experiences towards mathematics pedagogy (Murphy, 2015). As such, teaching practices are seen as "enactments of an individual's combined teaching identity and experiences" (Hong et al., 2024, p. 308). In this way, a teacher's pedagogic identity is interpreted as their sense of self, that is their knowledge, beliefs and interests as much as their professional experiences and development. Furthermore, a teacher's pedagogic identity may be swayed by perceptions of policy directives and the expectations of themselves as teachers. In addition, practices may "thicken" as teacher experiences and PD accumulate (Cribbs & Utley, 2024). In these regards, identity is defined at a specific time and determines how teachers understand their own practice within a given context (Hong et al., 2024). It is within a specific context, parallel early years Australian

classrooms, that we investigated how two teachers evaluated the implementation of the PD to support student talk in mathematics.

Research method

In this research report, we focus on two teachers in the same school that were involved in the project for one school year. The two teachers, Helen and Edith (pseudonyms) taught grade one and grade two students (six- to seven-year-olds) at a school situated in a low socio-economic area in a regional Australian city. Helen had been teaching in the primary sector for ten years but had recently transferred to teaching this younger age range. Edith had twenty years teaching experience in various roles with six years teaching this age range. Helen taught full-time and Edith taught part-time at the school.

The PD was delivered across five workshops. Workshop one focused on classroom discourse norms and drew on pedagogical strategies to promote students' confidence to speak and willingness to engage and listen to others through short partner work and whole class repeating, reporting, and revoicing. Workshop two introduced talk prompts for group work directed by the teacher as part of whole class teaching. Workshop three extended the use of the talk prompts to independent small group work and short challenging tasks. Workshop four consolidated talk within collaborative group work as part of the intended curriculum content. Workshop five focused on evaluation across the PD.

A range of data were collected as part of the larger project. Data used in this research report were gathered through paired semi-structured interviews in each of the five workshops. The agenda for the interview in workshop one focused on teaching experiences, including any previous PD, and reasons for joining the project. Further questions related to their perceived proficiency in teaching mathematics and their approach to teaching. Interviews in workshops two to five focused on the two teachers' self-reported experiences when introducing the phased talk strategies in their classrooms. Interviews were managed as conversations between the interviewer and the two teachers. Whilst the interviewer followed a semi-structured protocol, the teachers extended their conversations as a pair, allowing them to make sense of their teaching practice and of their experiences in implementing phases of the PD. In that way, the data present as teacher narratives and the interviewer lived the story with the teachers (Clandinin & Caine, 2008).

Teacher narratives were analysed deductively. Themes for the analysis of workshop one interview relate to key elements from identity literature: previous experiences (Hong et al., 2024); reported levels of confidence (Cribbs & Utley, 2022); and teachers' understanding of their own practice (Hong et al., 2024). Analysis of workshops two to five focused on the successes and tensions that the teachers experienced and we relate these to Lefstein's (2010) dimensions. Lefstein posited that aspects of school practice may constrain the way that dialogue is conducted and are not always commensurate with idealistic models of dialogic teaching. Based on this premise, Lefstein presented four dimensions that recognise the inherent tensions between teacher-student and student-student dialogue in classrooms: 1) Metacommunicative – “the nature of the conversation itself” and “dialogue as interactional structure” (p.173); 2) Interpersonal – non-cognitive, emotional relations perspective and how “personal animosity can obstruct or derail dialogue's progress” (p.173); 3) Ideational – cognitive activity as a means of learning between two or more people's thoughts or perspectives or understandings; 4) Aesthetic – setting the scene for engagement in relation to the topic and activity as well as the space and organisation with the classroom. Accepting rather than ignoring these tensions and constraints are useful in understanding the nature of dialogue. In the discussion, we determine each teachers' pedagogic identity and relate to their experiences in managing the tensions and constraints of the four dimensions.

Results

Workshop One Interview: Pedagogic identities

Previous experiences

The last PD that Edith had attended in mathematics was six years ago. She felt that she had mostly forgotten the training and that it no longer impacted on her teaching. Helen attended PD in teaching early numeracy through structured inquiry two years ago and she continued to work with the approach. Both teachers referred to a lack of consistency and guidance in teaching mathematics. They felt that they were finding their own way and that the “landscape of teaching math... has chopped and changed so much” (Helen) and that “it feels quite a jumble of what I’m putting into the teaching” (Edith).

Reported levels of confidence

Edith indicated that mathematics was not her favourite subject to teach, admitting that “sometimes I feel like a fool.” She felt she had missed a lot of the “first steps” in her own mathematics and how that had “carried through” into her own teaching. She further suggested that “I feel like my explanations with maths are really not that great.” Helen also expressed some reticence stating that “I don’t think I’m bad at teaching maths...I’m just not sure yet.” She saw her recent move to early years mathematics as a shift to a pedagogy that is “far more concrete, far more explicit in many ways... And you kinda just go... I think I’m doing this right.”

Understanding of their own practice

Edith indicated that she liked “prescribed things, ‘cause math is not something I’m particularly strong with...” She further indicated that she was keen to see what the children could do as individuals “I’m gonna give you a task. I’ll see if you can do it. Here’s me explaining it to you... and here’s the job that I want to see [what you can do].” As such, Edith had used little shared talk with her students. Helen referred to her promotion of inquiry through collaboration, participation and differentiation saying, “You’re not doing the talking they are, they’re sharing strategies, they’re building on each other’s strategies.” These perspectives in relation to sense of self were further reflected in the teachers’ reasons for joining the project. Helen indicated that she wanted “to build quality practice in teaching maths” and to “develop skills to empower students and for them to feel confidence in mathematics.” Edith indicated that she wanted “just to” improve her mathematics instruction by gaining confidence and “to explain to my children and making sure I’m using language that they understand.”

Workshops Two to Five Interviews: Dialogic Dimensions

Metacommunicative: Dialogue as Interactional Structure

Both teachers reflected on the introduction of talk prompts to be used by the students in small groups. Edith felt that the way she had presented them (“I just wrote mine out very plain”) was insufficient as the students did not use them directly but that they may have reminded them to ask questions in their group. Helen had been more proactive. “I’d have to go around and kind of say ... well, I don’t agree with that. Well, what do you need to do?” This reminded students to hold up the talk prompt and become more confident in using the language. Further to this, Helen monitored students’ use of questions “Have you used the prompts? Are you sharing the knowledge? Are you teaching this person?” She also modelled the language in whole class contexts. “So we’d have solutions on the board... and we’d talk about, well, how did you come up with this?... What do we notice here?” Helen endeavoured to break a habit of taking turns, prompting students with “You are doing it together... Well, that was his answer. And I will say, well, no, because you’re a group so this is a group answer.”

Interpersonal: Dialogue as relation

Edith noticed that the students enjoyed “the opportunity to work with somebody to do maths ... to talk and share their ideas.” She further noted that they “were listening to each other, and there was a real willingness to engage.” Helen also referred to students’ sharing of knowledge, confidence to speak, and listening to each other. Helen further indicated how “Every partner gets to share and that students cannot hide. You’re accountable to your partner, and you’re accountable to the class and to me.” In workshop three, Edith and Helen discussed how the groups were composed. Helen noticed that students often chose partners they felt comfortable to challenge “in a safe pairing” and where they could “have a go without worrying about being wrong.” Edith also commented that “they’re in a safe environment with children with similar abilities.” However, both Edith and Helen noticed that some students did not choose partners based on trust but on support, where a more able student would do the mathematics for them. Edith commented that the stronger mathematician would be “taking the lion’s share” and that this pattern had not been broken.

Ideational: Dialogue as cognitive activity

Helen reported how the group work supported students’ learning.

When we come back to share ... a lot of the children who didn’t come up with the answer are still able to explain what the other person has done.... [they] obviously felt confident enough to kind of say, well, hang on, are these the right numbers?

Edith, however, felt less certain that her students were able to consider other perspectives, and that they would “Default to something that they know... They couldn’t look at it any other way...” Both teachers indicated a disparity between the students’ skill in the mathematics and their ability to explain the mathematics. Edith suggested students may have been hindered by their language level “only the more articulate students could do this.” Helen observed how going beyond an explanation to justify their mathematics was problematic. “If they explain it and ... people ask questions, they don’t know how to go that bit further.” Helen further observed that some students “just do not have the social skills to actually understand that they are in a group and it’s not all about them.” She commented that “grade ones are really only just starting to learn how to actually explain what they’ve done.”

Aesthetic: Setting the scene for engagement

Both teachers discussed how their choice of questions and activities could create or limit opportunities for talk. Edith referred to finding “the right level of questions” commenting “it’s hard to get that meatiness of not too much and not too little.” Helen also noted that it was important to get the right stimulus. “When something’s easy, kids will say, I just know. And they do ... they know the concept really well. And so, when you say, how do you know? It’s not a challenge for them, so they don’t have to think about their thinking.” Edith further commented on the nature of the mathematics. “Maths does require that system, ... to be methodical about it.” Students “get frustrated when those answers are not there in their minds straight away, ... like they don’t necessarily want to have to work for those answers.” Edith further considered how her students were reliant on working with closed questions. “That’s what they’re expecting.” She then related to her aims in teaching, indicating that “I’m always trying to drive towards what I think is correct. So, I think I’m in that mindset.” She then added “Do I need to sort of not do that?” Helen’s perspective suggested a contrast. “I love wrong answers because then we get to talk about them. We can say, who agrees? Who doesn’t agree? Great. Tell me more.” She indicated that her students seemed resilient in managing this “I’ve never had a kid upset about that ... I thought it was right, but we come to a shared understanding. And the kids are all like, yeah, we solved it. It was great.”

Both teachers commented on time pressures in managing the monitoring of the talk in groups. Edith referred to getting “very stuck in that housekeeping of just making sure everyone actually looks like they’re okay” and admitted that she needed “to get down and really listen to these children.” Helen also noted that it was “tricky to get around all the groups, ... you certainly didn’t have time to get around all the groups. You just kind of had to trust, in some ways, they were actually holding each other accountable.”

Discussion

In the discussion, we consider each teachers’ pedagogic identity and relate these to their different experiences in implementing the PD and sustaining productive talk in the two parallel classrooms. Three elements of the two teachers’ pedagogic identity - experiences and context; level of confidence; and understanding of their own practice - are critiqued in relation to Lefstein’s dimensions - Metacommunicative, Interpersonal, Ideational, and Aesthetic. This critique is used to determine the extent to which the pedagogic identity helps to understand how the tensions and constraints were experienced by the two teachers.

In relation to the Aesthetic dimension and opportunities for student engagement, both teachers suggested some lack of confidence in teaching mathematics, Edith in relation to her own learning experiences and Helen due to lack of consistency in pedagogical approaches. Helen’s more recent PD had focused on structured inquiry (e.g., Stein & Smith, 2011), suggesting she was more familiar with the authentic nature of dialogic teaching (Alexander, 2008). Further to this, Helen referred to the constructive use of wrong answers with her students suggesting an epistemological non-positivist viewpoint of meaning making, recognising the potential to create openings for dialogue in her classroom. She appeared to aim for discursive contributions in the way that Lerman (2001) had suggested by pulling her students into the classroom discourse. This viewpoint is further reflected in her aim to empower students in their learning. As such, Helen experienced fewer tensions in realising the Aesthetic dimension. In contrast, Edith’s interview responses suggested her practice was more aligned with the IRF model (Sinclair & Coulthard, 1975) as defined in the introduction. She further expressed a view that mathematics requires systems, suggesting an epistemological positivist viewpoint where mathematics is static and stable (Walkerdine, 1990). She focused on closed questions and was concerned for her students’ frustration in not getting to the correct answer. As such, Edith experienced greater tensions in realising the Aesthetic dimension. Nevertheless, she felt that her teaching had become less formal, suggesting a change in her practice. She also questioned her mindset in asking whether she needed to always “drive towards what was correct”.

In relation to the Metacognitive dimension, Helen seemed confident in scaffolding the use of the talk strategies. She modelled and reinforced the use of the talk prompts and felt that the students had moved to their own spontaneous questioning in their group work. Edith, however, seemed to have been less proactive in modelling the use of prompts. Nevertheless, she felt the prompts had acted as a physical reminder and that the students did engage in paired work. It is possible that Edith’s lack of confidence and limited familiarity in dialogic practices hindered her scaffolding. A further possibility is that Edith was concerned with time constraints as she was employed part-time, hence, she may have had less opportunity to scaffold the use of the prompts. Nevertheless, Edith was finding her own ways to adopt the strategies into her teaching within the limitations of her experiences and her employment.

In relation to the Ideational dimension, Edith’s positivist perspective led to an interpretation of cognitive activity as being pertinent to the students’ own knowledge, rather than a student appreciating another student’s perspective. Furthermore, Edith noted that strong mathematicians often dominated the mathematics talk. On the other hand, Helen’s non-positivist perspective related to an interpretation of cognitive activity as an act of taking on another student’s mathematical thinking. Such an act would then enable a student to question

their own thinking in determining meaning making. Her aim was for students to become accountable to each other rather than acting individually or allowing one student to dominate.

In relation to the Interpersonal domain, both teachers experienced similar successes. They both reported that their students enjoyed sharing their mathematics and were willing to engage in talk in pairs or small groups. However, they both experienced similar contextual tensions relating to the young students' language and social abilities. The issue of time restrictions was also common to both. Such contextual tensions were common to the pedagogic identity of both teachers in the study.

Concluding remarks

The findings suggest that curriculum context, time pressures, and the social and language abilities of young students limited opportunities for both teachers to move beyond reporting and revoicing to encouraging students to justify their own thinking. However, further to these common constraints, the findings suggest that the two teachers' contrasting ideologies, evident through the interview conversations in the workshops, played a substantial role in each teachers' implementation of the PD programme. Helen's non-positivist ideology and her aim to empower students to be accountable contrasted with Edith's positivist ideology in working towards method and correct solutions. These contrasting ideologies were essential to the pedagogic identities of the two teachers. Nevertheless, Edith suggested her teaching had become less formal and that her students enjoyed sharing their mathematics. Involvement in the PD may have "thickened" her practice (Cribbs & Utley, 2024) which, in turn, may have influenced aspects of her pedagogic identity.

In conclusion, we suggest that, in this study, pedagogic identity functioned as a theoretical framework in understanding the tensions in promoting and sustaining productive talk in the two teachers' early years mathematics classrooms beyond the common constraints. Further to previous studies that focused on contextual constraints and teacher capacity (e.g., Campbell & Teo, 2022; Moser et al., 2022; Ni et al., 2021), we propose that recognising the alignment or misalignment of teachers' pedagogic identities with PD intentions helps to understand the tensions experienced by different teachers. Recognising pedagogic identities suggests a shift from "doing" PD to accommodating differing pedagogic identities with the potential to help teachers adapt to new practices. Beliefs may be hard to change (Maass, 2011) but, by incorporating new practices, teachers' pedagogic identity can further develop.

We note that the findings in this research report are drawn from teacher self-reporting. Whilst such reporting can reveal teacher perceptions, they do not confirm classroom practice. Analysis of the classroom observation data is desirable in substantiating any claims. Investigations are underway, involving the larger project's classroom video data, to further determine the claims of the study presented in this research report.

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References

- Alexander, R. (2008). Culture, dialogue and learning: Notes on an emerging pedagogy. In N. Mercer & S. Hodgkinson (Eds.) *Exploring talk in school: Inspired by the work of Douglas Barnes* (pp. 91-114). Sage. <https://doi.org/10.4135/9781446279526.n6>
- Anderson, N., Chapin, S., & O'Connor, C. (2011). *Classroom discussions in math: A facilitator's guide to support professional learning of discussion and the common core*. Scholastic Inc.

- Bouton, E., & Asterhan, C. (2023). In pursuit of a more unified method to measuring classroom dialogue: The dialogue elements to compound constructs approach. *Learning, Culture and Social Interaction*, 40, 100717. [10.1016/j.lcsi.2023.100717](https://doi.org/10.1016/j.lcsi.2023.100717)
- Campbell, T., & Teo, S. (2022). Professional noticing of coordinated mathematical thinking. *British Educational Research Journal*, 48, 488-509. <http://dx.doi.org/10.1002/berj.3778>
- Clandinin, D. J., & Caine, V. (2008). *Narrative inquiry. The Sage encyclopedia of qualitative research*. Sage.
- Cribbs, J., & Utley, J. (2024). Exploring K-12 mathematics teachers' identity and beliefs about mathematics and teaching. *Mathematics Teacher Education and Development*, 26(1), Article 5.
- Klemp, T. (2020). Early mathematics – teacher communication supporting the pupil's agency. *Education 3-13*, 48(7), 833-846. <https://doi.org/10.1080/03004279.2019.1663893>
- Heyd-Metzuyanim, E., Munter, C., & Greeno, J. (2018). Conflicting frames: A case of misalignment between professional development efforts and a teacher's practice in a high school mathematics classroom. *Educational Studies in Mathematics*, 97, 21-37. <http://dx.doi.org/10.1007/s10649-017-9777-0>
- Hong, J., Cross Francis, D., & Schutz, P. A. (2024). Reconceptualizing teacher identity development. *Educational Psychologist*, 59(3), 159-176. <https://doi.org/10.1080/00461520.2023.2292713>
- Howe, C., Hennessy, S., Mercer, N., Vrikki, M., & Wheatley, L. (2019). Teacher-student dialogue during classroom teaching: Does it really impact upon student outcomes? *Journal of the Learning Sciences*, 28(4-5), 462-512. <https://doi.org/10.1080/10508406.2019.1573730>
- Lefstein, A. (2010). More helpful as problem than solution: Some implications of situation dialogue in classrooms. In K. Littleton & C. Howe (Eds.) *Educational dialogues* (pp. 182-203). Routledge. <https://doi.org/10.4324/9780203863510>
- Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics*, 46, 87- 113. <http://dx.doi.org/10.1023/A:1014031004832>
- Maass, K. (2011). How can teachers' beliefs affect their professional development? *ZDM-Mathematics Education* 43, 573–586. <https://doi.org/10.1007/s11858-011-0319-4>
- Mercer, N. & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education*, 20(6), 507-528. <https://doi.org/10.2167/le678.0>
- Moser, M., Zimmermann, M., Pauli, C., Reusser, K., & Wischgoll, A. (2022). Student's vocal participation trajectories in whole-class discussions during teacher professional development. *Learning, Culture and Social Interaction*, 34, 100633. <https://doi.org/10.1016/j.lcsi.2022.100633>
- Murphy, C. (2015). Changing teachers' practices through exploratory talk in mathematics: As discursive pedagogical perspective. *Australian Journal of Teacher Education*, 40(5), 61-84. <http://ro.ecu.edu.au/ajte/vol40/iss5/4>
- Murphy, C., Muir, T., & Thomas, D. (2025). Scaffolding collaboration in early years mathematics: A practice-based case study in teaching multiplicative grouping. *Early Childhood Education Journal*, 53, 3207-3217. <https://doi.org/10.1007/s10643-025-01928-5>
- Ni, Y., Shi, L., Cheung, A., Chen, G., Ng, O-L., & Cai, J. (2021). Implementation and efficacy of a teacher intervention in dialogic mathematics classroom discourse in Hong Kong primary schools. *International Journal of Educational Research*, 107, 101758. <https://doi.org/10.1016/j.ijer.2021.101758>
- O'Connor, C., & Michaels, S., (2019). Supporting teachers in taking up productive talk moves: The long road to professional learning at scale. *International Journal of Educational Research*, 97, 166-175.
- Parrish, S. (2010). *Number Talks: Helping children build mental math and computation strategies, Grades K-5*. Heinemann Educational Books.
- Sinclair, J. M., & Coulthard, R. M. (1975). *Towards an analysis of discourse: English used by teachers and pupils*. Oxford University Press.
- Stein, M. K., & Smith, M. (2011). *Five practices for orchestrating productive mathematics discussions*. National Council of Teachers of Mathematics.
- Vrikki, M., Wheatley, L., Howe, C., Hennessy, S., & Mercer, N. (2019). Dialogic practices in primary school classrooms. *Language and Education*, 33(1), 85–100. <https://doi.org/10.1080/09500782.2018.1509988>
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
- Walkerdine, V. (1990). *The Mastery of Reason: Cognitive development and the production of rationality*. Routledge.

Primary and Secondary Teacher Confidence in Teaching Mathematics Concepts

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This study examined differences in mathematics confidence between primary and secondary teachers. Survey data was collected from 225 Victorian teachers in the Momentum Schools program. Primary teachers reported highest confidence in foundational number concepts, while secondary teachers showed greater confidence in measurement and data concepts. Areas of low confidence, fractions for primary teachers, foundational concepts for secondary teachers, align with student achievement challenges. These findings will inform differentiated learning design and highlight the importance of role-specific pedagogical content knowledge in mathematics teaching reform.

Introduction

Mathematics education across Australia is experiencing a significant shift toward explicit, evidence-based practices. This approach has gained momentum nationally, with the Australian Education Research Organisation (AERO) advocating for explicit instruction as central to effective mathematics teaching, particularly in foundational years (AERO, 2023a). Several state and territory systems have focussed on explicit teaching approaches in mathematics, aligning with suggestions from Grattan Institute (2025).

In Victoria, the implementation of the Victorian Curriculum 2.0 and the Victorian Teaching and Learning Model 2.0 (VTLM 2.0), prioritises explicit, sequential teaching approaches grounded in research on how students learn (Department of Education Victoria, 2024). The Victorian Curriculum 2.0, has streamlined mathematics content and clearer sequencing of mathematical concepts and proficiencies. The VTLM 2.0, released in 2022 for mandatory implementation from 2025-2027, positions explicit teaching as the core pedagogical approach. For successful implementation, strengthening teacher confidence in new pedagogical approaches will be essential (Gresham, 2008).

This shift toward explicit teaching reflects broader national curriculum reforms in response to Australian student performance in international mathematics assessments. While Year 4 Trends in International Mathematics and Science Study (TIMSS) results have improved from 495 in 1995 to 525 in 2023, Year 8 results have remained relatively stable, fluctuating between 496 and 517 over the same period (Australian Council for Educational Research [ACER], 2026a). More concerning are Programme for International Student Assessment (PISA) results for 15-year-olds, which declined by 37 points between 2003 and 2022, with mathematics achievement declining more sharply than reading and science (ACER, 2026b). These declining trends, coupled with concerns about teacher preparation identified in national reviews (Masters, 2016), have prompted significant reforms across both initial teacher education and school pedagogical practices in Australia.

Teacher's Beliefs and Perceptions Regarding Mathematics Concepts

Teachers' beliefs about mathematics teaching fundamentally shape instructional practices. Understanding what teachers need to know to teach mathematics effectively begins with Shulman's (1987) foundational concept of pedagogical content knowledge (PCK). Shulman argued that effective instruction requires more than subject matter knowledge alone; teachers must transform content into forms accessible to learners. Building on this foundation, Ball et al., (2008) developed the Mathematical Knowledge for Teaching (MKT) framework, (2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 289–296). MERGA.

distinguishing common content knowledge (mathematical knowledge expected of educated adults), specialised content knowledge (knowledge unique to teaching, such as analysing errors or choosing representations), and knowledge of content and teaching (making mathematics comprehensible to learners). Research confirms that teachers' beliefs interact with their knowledge base to shape instructional practices. Studies demonstrate that beliefs and practices are interconnected, with alignment between them contributing to effective instruction (Aljaberi & Gheith, 2018; Muir, 2008).

Self-efficacy, defined by Bandura (1997) as beliefs about one's capability to organise and execute actions required for specific outcomes, provides a critical theoretical lens for understanding teacher confidence in mathematics instruction. In mathematics teaching contexts, self-efficacy is both task-specific and domain-specific (Hackett & Betz, 1989), meaning teachers' confidence varies across different mathematical concepts and pedagogical situations. Bandura identified four sources of self-efficacy: mastery experiences, vicarious experiences, verbal persuasion, and emotional states. More recently, McDaid (2024) developed the Building Teacher Self-Efficacy Framework specifically for Australian preservice teachers, identifying six interconnected factors: collaborative relationships, resources, teacher feedback, teaching practice, motivation and self-regulation, and reflection. Importantly, research demonstrates that self-efficacy is malleable, targeted professional development can strengthen mathematics teaching confidence (Gresham, 2008), particularly when addressing both content knowledge and pedagogical approaches. This is especially relevant as Victorian teachers implement explicit teaching frameworks through the VTLM 2.0, where confidence in new pedagogical approaches becomes essential for successful curriculum reform.

Central to the implementation of curriculum and pedagogical approaches, is understanding teacher confidence and perceptions regarding mathematics concepts. Research has established that teachers' mathematical knowledge, encompassing both subject matter knowledge and pedagogical content knowledge, positively impacts student mathematics achievement (Ball et al., 2008; Hill et al., 2005). However, possessing mathematical knowledge alone is insufficient; teachers must also have confidence in their ability to teach these concepts effectively.

The Victorian Curriculum: Mathematics 2.0 has refined content descriptions with clearer sequencing. These changes necessitate that teachers possess both strong mathematical content knowledge and confidence in implementing explicit teaching frameworks, which emphasise breaking down content into manageable chunks, providing clear explanations through modelling and demonstration, and using gradual release of responsibility approaches (AERO, 2023b).

The distinction between primary and secondary teachers' perceptions of mathematics concepts is particularly significant in this reform context. Primary teachers typically hold generalist qualifications and are expected to teach mathematics alongside other learning areas, while secondary mathematics teachers generally possess specialist mathematical training. This structural difference in teacher preparation may lead to variations in confidence levels across different mathematical concepts. Understanding these differences is essential for targeted professional learning and support as schools implement explicit teaching approaches, whether through Victoria's VTLM 2.0 and Lesson Plans, or similar initiatives in other jurisdictions.

This research examines teacher confidence in teaching various mathematics concepts prior to engaging in professional learning on evidence-based explicit teaching practices. By investigating the rank ordering of teacher confidence for specific mathematical concepts (such as addition and subtraction algorithms) and analysing the differences between primary and secondary teacher responses, this study aims to identify where professional learning support is most needed. The findings will have direct implications for the ongoing rollout of evidence-based pedagogical frameworks in Victoria and contribute to the broader national conversation about supporting teachers through major pedagogical transitions. As systems across Australia

commit to explicit teaching approaches, understanding teacher confidence becomes critical to ensuring reforms translate into improved classroom practice and student outcomes. Specifically, this study addresses the following research question: *What are the significant differences in confidence levels between primary and secondary teachers for specific mathematics concepts?*

Research Design

The data presented in this paper originates from a broader Australian study examining teachers' knowledge and confidence in teaching literacy and mathematical concepts. The larger study employed a quantitative and qualitative survey design to investigate teachers' self-reported understanding and pedagogical confidence across various domains within schools participating in La Trobe's Momentum Schools program. This program aims to provide participating schools with guidance and support to transition to explicit teaching instruction, in line with VTLM2.0. Specifically, this paper focuses on the quantitative survey components related to primary and secondary teacher confidence in teaching mathematics concepts, examining patterns and differences across teaching levels. The baseline data findings presented in this paper are intended to inform the design of targeted interventions and support structures for mathematics teaching with participating schools.

Research Setting

Staff from 64 schools across Victoria participated in this study. All participating schools were enrolled in La Trobe's Momentum Schools program.

Participants

Staff from the participating schools responded to an initial survey to indicate their knowledge of and confidence teaching mathematical concepts. The survey had 225 respondents, of which 183 (81%) worked at government schools and 42 (19%) worked at independent schools. Of the respondents, 146 identified as primary teachers and 54 identified as secondary teachers. The remaining 25 respondents did not specify their teaching level.

Participants represented a range of teaching experience levels. Of the 223 teachers that answered the item on years of registration, 61 teachers reported having 0-5 years of experience, 54 teachers had 6-10 years of experience, 40 teachers had 11-15 years of experience, 25 teachers had 16-20 years of experience, and 43 teachers had more than 20 years of experience as registered teachers.

Data Collection

All teachers were invited to complete the survey via a letter from the research team that was distributed to schools together with a Participant Information Statement and Consent form. Survey data were collected and managed using Research Electronic Data Capture (REDCap) electronic tools. The survey was distributed via an email containing a hyperlink to REDCap. REDCap is a secure, web-based application which has been designed to support data capture for research studies and has automated export procedures for data downloads to common statistical packages, such as SPSS (Harris et al., 2009).

The survey items were adapted from the SOLAR Lab Teacher Survey (Stark et al., 2016), designed to explore teachers' beliefs, confidence and experience with explicit and systematic reading instruction. Additional survey items were developed by the research team to explore teachers' confidence teaching mathematical concepts. The confidence items asked participants to rate their confidence to teach a mathematical concept on a scale from 0 to 100, with 0 representing very low confidence and 100 representing very high confidence. Background items collected demographic information, such as teacher specialisation, years of registration

and year levels taught. Multiple choice items were used to collect information on where teachers felt they obtained the majority of their pedagogical knowledge.

Data Analysis

The survey data was downloaded to SPSS for descriptive statistical analysis and hypothesis testing. Normality testing was conducted using Shapiro-Wilk and Kolmogorov-Smirnov tests for the secondary and primary samples respectively.

Results

Teachers reported obtaining knowledge of instructional practice from a variety of sources (Table 1). Professional development activities were the most frequently cited source, selected by 90% of primary and 83% of secondary teachers. Classroom teaching experience was similarly prominent (84% primary, 80% secondary). In contrast, fewer teachers attributed their knowledge to university study, with only 38% of primary and 46% of secondary teachers identifying pre-service teacher training as a source.

Table 1

Main source of pedagogical knowledge – multiple selections permitted

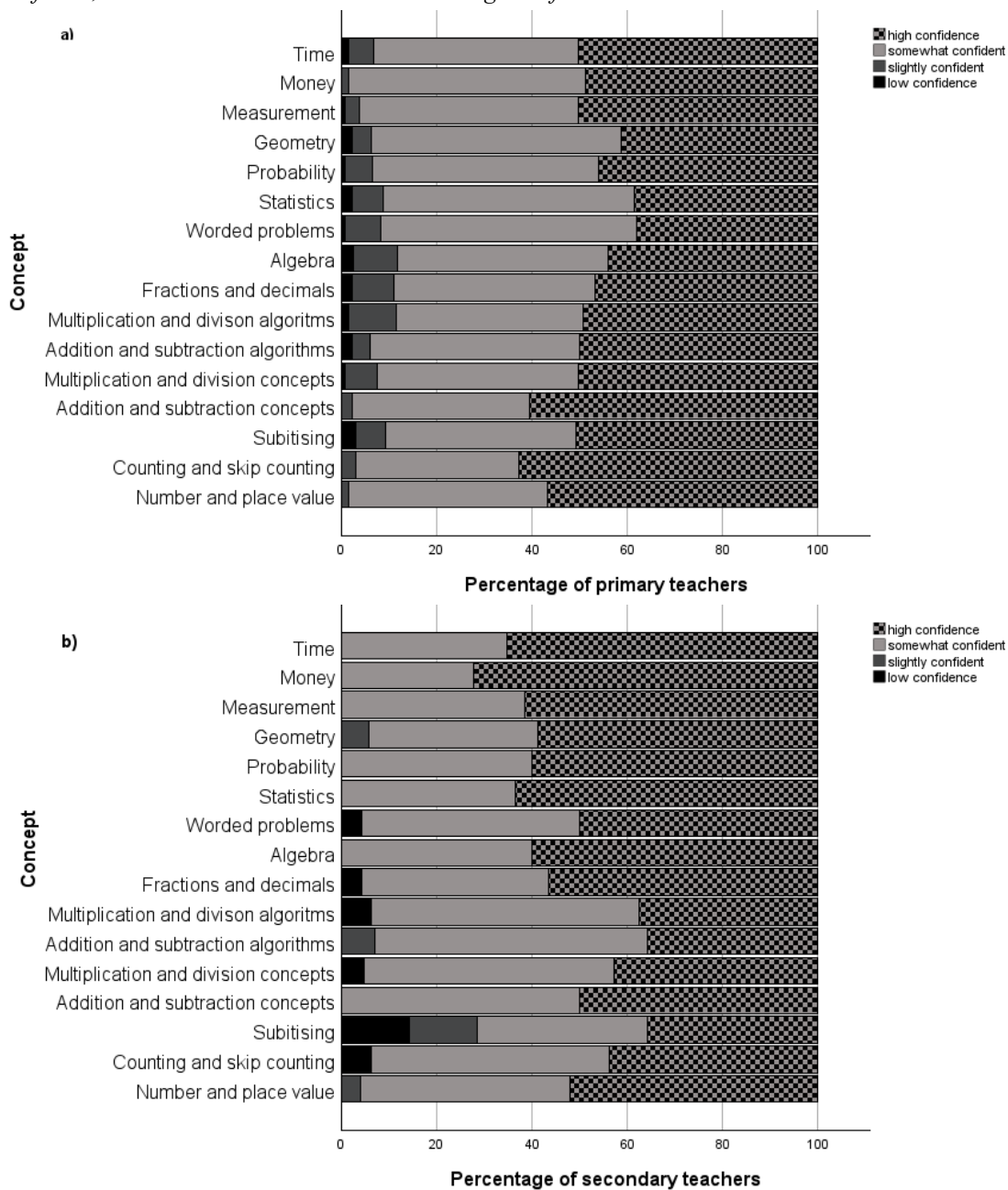
Source	N primary	% primary	N secondary	% secondary
Professional development	131	90%	45	83%
Classroom teaching experience	122	84%	43	80%
Colleagues	106	73%	32	59%
Research and reading	92	63%	32	59%
University preservice teacher training	55	38%	25	46%
University postgraduate study	31	21%	20	37%
Their own school experience	11	8%	6	11%

Teachers were asked to rank their confidence teaching a mathematics concept from 0 (low confidence) to 100 (high confidence). When all teachers were surveyed, the lowest average confidence was for the teaching of fractions and decimals ($M=70.3$, $SD=20.7$), followed by early algebra such as missing numbers and simple written algorithms ($M=70.5$, $SD=19.6$) and geometry ($M=70.8$, $SD=17.0$). The highest average confidence was for teaching addition and subtraction concepts ($M=77.6$, $SD=13.8$), counting and skip counting ($M=76.9$, $SD=14.8$) and money ($M=76.1$, $SD=13.9$).

However, confidence levels varied between the primary and secondary teachers surveyed. Figure 1 shows the self-rated teacher confidence from primary (Figure 1a) and secondary (Figure 1b) teachers. 63% of primary teachers were highly confident teaching counting and skip counting compared to 44% of secondary teachers (Figure 1). Conversely, 60% of secondary teachers were highly confident teaching early algebra concepts compared to 44% of primary teachers. The secondary teachers surveyed were from a range of specialisations, covering mathematics, science, humanities, English and health.

Figure 1

Primary and secondary teacher confidence teaching a mathematical concept. Rankings < 25 were considered low confidence, ranking 25 to <50 were considered slightly confident, 50 to <75 somewhat confident, and 75 and above were considered high confidence.



Through independent samples t-tests, statistically significant differences in the mean confidence were detected for:

- Counting and skip counting, with primary (M= 78.1, SD=14.0) and secondary teachers (M=68.8, SD=18.7). The primary respondents demonstrated significantly more confidence, $t(148)=2.427$ and $p = .016$.
- Subitising, with primary (M= 72.0, SD=18.7) and secondary teachers (M=58.1, SD= 26.4). The primary respondents demonstrated significantly more confidence, $t(140)=2.527$ and $p = .013$.
- Early algebra, with primary (M=69.1, SD=19.8) and secondary teachers (M=79.0,

SD=18.4). The secondary respondents demonstrated significantly more confidence, $t(138) = -2.089$ and $p = .039$.

When asked which concepts were not applicable to their role, 69% and 72% of secondary teachers deemed counting and skip counting and subitising as not relevant, respectively. 11% of primary teachers deemed early algebra as being not relevant to their role.

Discussion

This paper addressed the question: *what are the significant differences in confidence levels between primary and secondary teachers for specific mathematics concepts?* The findings reveal distinct confidence profiles that reflect both the different pedagogical approaches of teaching levels and teachers' perceptions of role-relevant content. Primary teachers demonstrated highest confidence in foundational number concepts, with over 60% reporting high confidence teaching counting, skip counting, addition and subtraction concepts. Conversely, their lowest confidence areas included fractions, decimals, multiplication, division algorithms, and early algebra, with more than 10% expressing low or slight confidence in these domains. Secondary teachers' confidence patterns differed remarkably, with over 60% reporting high confidence in the teaching of money, time, measurement, probability and statistics concepts (Figure 1), yet 29% indicating low confidence in teaching subitising. Statistical analysis confirmed significant differences between primary and secondary teacher groups.

These confidence patterns appear partially explained by teachers' perceptions of role relevance. While 72% of secondary teachers rated counting and skip counting and subitising as irrelevant to their role, only 11% of primary teachers considered early algebra irrelevant despite it being among their lowest confidence areas. This discrepancy is particularly noteworthy given that early algebra concepts form critical bridges between primary and secondary mathematics learning. The finding suggests that primary teachers recognise the curricular importance of algebraic thinking even when lacking confidence to teach it effectively, pointing to a gap between perceived responsibility and self-efficacy that professional learning interventions must address.

The confidence patterns emerging from these data align with Bandura's (1997) self-efficacy theory and Ball et al.'s (2008) Mathematical Knowledge for Teaching framework. Primary teachers' high confidence in foundational concepts likely reflects frequent mastery experiences, while lower confidence in fractions and early algebra suggests limited pedagogical content knowledge in these challenging areas. Secondary teachers' dismissal of foundational concepts as 'not applicable' may reflect insufficient understanding of how these underpin advanced mathematical thinking.

The implementation of VTLM 2.0 requires differentiated professional learning addressing these distinct profiles. Primary teachers need targeted support for fractions, multiplicative thinking, and early algebra concepts, where both teacher confidence and student achievement show persistent challenges. Secondary teachers require professional learning connecting foundational number concepts to their practice, demonstrating how gaps in counting strategies, subitising, and number sense impact student performance in advanced mathematics.

Only 38% of primary and 46% of secondary teachers identified initial teacher education as a main source of pedagogical knowledge. While requiring further investigation, this finding suggests potential misalignment between pre-service preparation and classroom realities, highlighting the critical role of ongoing professional learning in developing effective mathematics teaching practices.

Conclusion

In conclusion, this study provides critical baseline data on mathematics teacher confidence across primary and secondary contexts within Victorian schools implementing the VTLM2.0 framework. The findings reveal distinct confidence profiles between teacher groups, with primary teachers demonstrating greater confidence in foundational concepts while secondary teachers show higher confidence in measurement and data-related concepts. The data from this study highlight the need for differentiated role-specific professional learning that addressed both the pedagogical content knowledge with a self-efficacy focus. As Victorian schools transition towards changes in teaching practices, understanding these confidence patterns enables more targeted and effective support for teachers at different year levels, and with different experience.

While these findings offer important insights, several limitations should be acknowledged. The number of secondary teachers was relatively small compared to the number of primary teachers, so caution should be exercised when generalising findings. Also, this study captures baseline confidence measures only and does not examine how professional learning interventions impact teacher confidence over time. Additionally, the reliance on self-reported confidence data means findings may not directly correspond to teachers' actual pedagogical content knowledge or classroom practice effectiveness. The sample was also limited to teachers in the Momentum Schools program; findings may not generalise to teachers in non-participating schools. Future research should track confidence changes following targeted interventions and explore relationships between teacher confidence, pedagogical practice, and student outcomes.

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References

- Aljaberi, N. M., & Gheith, E. (2018). In-service mathematics teachers' beliefs about teaching, learning and nature of mathematics and their mathematics teaching practices. *Journal of Education and Learning*, 7(5), 156-173. <https://doi.org/10.5539/jel.v7n5p156>
- Australian Council for Educational Research. (2026a). *TIMSS 2023 Australia*. ACER. <https://www.acer.org/au/timss>
- Australian Council for Educational Research. (2026b). *PISA 2022 Australia*. ACER. <https://www.acer.org/au/pisa>
- Australian Education Research Organisation. (2023a). *How students learn best: An evidence-based approach*. Australian Government. https://www.edresearch.edu.au/sites/default/files/2023-11/how-students-learn-best-aa_0.pdf
- Australian Education Research Organisation. (2023b). *Teach explicitly: Practice guide*. Australian Government. <https://www.edresearch.edu.au/guides-resources/practice-guides/teach-explicitly>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Bandura, A. (1997). *Self-efficacy: The exercise of control*. W. H. Freeman.
- Department of Education Victoria. (2024). *Victorian Teaching and Learning Model 2.0*. Victorian Government. <https://www2.education.vic.gov.au/pal/victorian-teaching-learning-model>
- Grattan Institute. (2025). *The maths guarantee: A guide for principals*. Grattan Institute. <https://grattan.edu.au/wp-content/uploads/2025/05/The-Maths-Guarantee-a-guide-for-principals-Grattan-Institute-WEB.pdf>
- Gresham, G. (2008). Mathematics anxiety and mathematics teacher efficacy in elementary pre-service teachers. *Teaching Education*, 19(3), 171-184. <https://doi.org/10.1080/10476210802250133>

- Hackett, G., & Betz, N. E. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal for Research in Mathematics Education*, 20(3), 261-273. <https://doi.org/10.2307/749515>
- Harris, P. A., Taylor, R., Thielke, R., Payne, J., Gonzalez, N., & Conde, J. G. (2009). Research electronic data capture (REDCap)—A metadata-driven methodology and workflow process for providing translational research informatics support. *Journal of Biomedical Informatics*, 42(2), 377–381. <https://doi.org/10.1016/j.jbi.2008.08.010>
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers’ mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406. <https://doi.org/10.3102/00028312042002371>
- Masters, G. N. (2016). *Policy insights: Issue 5. Five challenges in Australian school education*. ACER. <https://research.acer.edu.au/policyinsights/5/>
- McDaid, K. (2024). Factors that influence primary preservice teachers’ self-efficacy while teaching mathematics during professional practice. In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education: Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 375-382). MERGA. <https://files.eric.ed.gov/fulltext/ED661018.pdf>
- Muir, T. (2008). Principles of practice and teacher actions: Influences on effective teaching of numeracy. *Mathematics Education Research Journal*, 20(3), 78-101. <https://doi.org/10.1007/BF03217531>
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-23.
- Stark, H. L., Snow, P.C., Eadie, P.A., & Goldfeld, S.R. (2016). Language and reading instruction in early years’ classrooms: the knowledge and self-rated ability of Australian teachers. *Annals of Dyslexia*, 66, 28-54. <https://link.springer.com/article/10.1007/s11881-015-0112-0>

Incremental Professional Development in Algebra: Considering Teachers' Opinions on Instructional Nudges

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This study investigates a model of incremental teacher professional development (PD) based on small “instructional nudges”—concise, optional teaching suggestions shared asynchronously. Forty secondary teachers received 18 nudges during an academic year and then participated in heatmap interviews wherein they expressed opinions about each nudge and reported their frequencies of implementation. They tried an average of eleven nudges and regularly continued approximately three. The most and least positively viewed nudges are reported on, and results suggest that optional, small-scale instructional suggestions may support modest but sustainable instructional change.

Teacher professional development (PD) is a common mechanism for promoting improvement in mathematics instruction. Many PD efforts require intensive, transformational instructional changes based on innovative or ambitious visions for mathematics pedagogy, but those transformations have not yet worked at scale, with conventional, teacher-centred forms of instruction still pervasive (Arbaugh, 2025; Clarke et al., 2006; Every et al., 2025; Wilkie, 2019). Thus, scholars have begun investigating alternative models of PD such as incremental PD with support that may be brief and optional (Jansen et al., 2024), or include substantial elements of choice for teachers (Lee et al., 2024; Litke et al., 2025; Partridge, 2025), or possibly involve only slight adjustments to their instruction (Cortina & Višňovská, 2023; Otten et al., 2022; Star, 2016). These incremental efforts, modest in their aims, may be able to sustain and spread to a greater degree than the past attempts at transformational PD (Otten et al., 2025).

One particular type of incremental PD is nudge-based PD (Otten et al., 2025) in which small teaching suggestions (“instructional nudges”) are encapsulated on a single page, with a brief description, instructional notes, and an illustrative example or classroom-based comic strip. These instructional nudges are shared with teachers asynchronously online and the teachers are free to choose any (or none) of the nudges and may implement them as they see fit. Figure 1 contains two examples of instructional nudges and other examples are contained in the following sections. It should be noted that nudges are concise and may seem simple but a great deal of work goes into their design (e.g., drawing from research but making the instructional moves modest and clear, choosing those with high probabilities of immediate success, providing a title/subtitle that “hooks” teachers and increases the likelihood of teachers sharing the nudge with others; see www.PracticeDrivenPD.com, for more detail). We hypothesize that nudge-based PD has the potential to spread instructional change to many teachers, particularly those who may not be reached by transformational PD efforts (Otten et al., 2025). As Star (2016) argued, there is value in making small changes across many teachers, complementing longstanding efforts to make large changes with a small group of teachers.

But nudge-based PD will only work if the instructional nudges offered to teachers are, in their view, worth trying. It is also imperative that the nudges be viewed positively by teachers after they have tried them, so they continue implementing and possibly spreading them. Thus,

Figure 1

Examples of Two Instructional Nudges (Excerpted from One-Page “Nudge Sheets”)

PUT A BOW ON IT

"In case you missed it, here's the recap."

TRY IT

Before the very end of the class or an instructional segment, wrap it up by making a concluding statement about the lesson topic.

WHY TRY IT?

This instructional nudge:

- gives students an opportunity to reflect on the goals of the lesson to provide closure
- highlights important mathematical ideas from the lesson

YOU SPEND TIME INSTRUCTING YOUR STUDENTS ABOUT THE MATH CONTENT.

STUDENTS WORK ON PROBLEMS

EITHER INDIVIDUALLY OR IN GROUPS

SHARE THE MAIN IDEAS OF THE LESSON

TEACHER NOTES

Use this instructional nudge after you have given students time at the end of the class to work on problems.

This is meant to be no more than five minutes of you (the teacher) summarizing the lesson goals.

The summary could include important definitions, formulas, concepts, or procedures.

CAUTION

This is not meant to be a time to go over the solutions to problems.

LEAVE A TRACE

Leave ideas in place so students keep them in mind

TRY IT

As you are going over problems, rather than clearing the board, keep up important ideas for students to refer back to.

WHY TRY IT?

This instructional nudge:

- keeps ideas accessible for students
- supports students in making connections between strategies and representations
- promotes students' understanding of math ideas

WHEN WORKING PROBLEMS ON THE BOARD

PUT MULTIPLE PROBLEMS ON THE BOARD AT THE SAME TIME

KEEP DEFINITIONS, GENERALIZATIONS, OR FORMULAS UP AS YOU MOVE FORWARD

TEACHER NOTES

Keep important definitions, generalizations, or formulas up for the next set of problems.

You do not need to keep up all the work, just the important ideas.

Keep the work up as long as it is relevant to the problems you are working on.

CAUTION

Do not compromise the size and visibility just to fit more problems on the board.

we seek to understand what factors shape teachers' opinions of nudges. Drawing upon teacher interview data from a multi-year PD project, we address the following research questions:

- RQ1: When offered 18 instructional nudges via email over the course of an academic year, how many nudges do secondary mathematics teachers choose to implement and how many do they continue implementing relatively frequently?
- RQ2: Which nudges are regarded by secondary mathematics teachers the most positively and which are regarded negatively?
- RQ3: For those nudges that arise in RQ2, what reasons did the secondary mathematics teachers provide for their opinions?

Our theoretical framing is built upon the nudge-based model of incremental PD (Otten et al., 2025), with key conceptual facets being high levels of teacher choice, low levels of teacher time commitment, close proximity of proposed changes to pre-existing practice, and the considered potential of uptake of instructional suggestions, continuation of instructional changes, and spread to other teachers. These broader ideas connect to practicality theory (Doyle & Ponder, 1977) and PD scale (Coburn, 2003) but for the present analysis the focus here is on teacher choice and uptake of instructional suggestions.

Method

This study comes from a multi-year project investigating nudge-based incremental PD for algebra teachers in the U.S. The project involved one year of baseline data collection in each teacher's classroom (3 observations per year), with teachers coming from diverse school districts across 14 states, and then the creation of 18 instructional nudges. During the subsequent academic year, all 18 nudges were made available to the teachers online and were also sent via email (links to the nudge sheets) in groups of 9 nudges in the first month of the academic year and then the other 9 in the second month (teachers received different groups of nudges but all eventually had access to all 18 nudges by the second month). The present analysis focuses on “heatmap” interviews with 40 teachers, who varied from second-year teachers to those with 20+ years of experience. Within the interview, they were shown the titles of all 18 nudges on a slide and asked to place them spatially (Fig. 2). Their horizontal placement indicated the extent to which they hated or loved the nudge and vertical placement indicated how many times they tried the nudge in practice. There were also regions to place nudges they had not yet tried but intended to and nudges they did not intend to try. During the interview we asked various questions to elicit the teachers' perspectives on the nudges and we video-recorded their movements and transcribed their verbal remarks.

Figure 2

Example of the Heatmap Layout Used to Investigate Teachers' Opinions about Instructional Nudges



For RQ1, any nudges teachers placed above the horizontal line (i.e., in the positive “# of Tries” area) were considered to have been tried, and any nudges placed within one-nudge-box-

height of the top of their used graphical space was considered to have been many times (e.g., *Getting Unstuck*, *Pivot*, *Leave a Trace*, and *Put a Bow On It* in Fig. 2). For RQ2, the nudge placed rightmost by an individual teacher was considered “most positive” (e.g., *Put a Bow On It* in Fig. 2) and any other nudges that overlapped horizontally with that nudge were also considered to be in the group of “most positive” (e.g., *Leave a Trace* in Fig. 2). Non-positive opinions were rarer and so we included any nudges placed on the midline (e.g., *Pairing*) and any to the left (e.g., *Here’s Where I’m At* and *Rate and Review*) as well as those placed in the “Do Not Plan on Trying It” section as being “least positive”. Two coders confirmed these categorisations of the nudges and tallies were made of all the most positive and the non-positive opinions of nudges.

For RQ3, two coders reviewed the interview transcripts for any mention of the most positive and non-positive nudges and conducted open coding of the reasons teachers provided for their opinions. A single nudge could have multiple reasons expressed. After open coding, the two coders met to combine related themes and clarify subtle distinctions, yielding a set of refined codes (Tables 3 and 4 in Results). The co-authors conferred on the results and interpretations.

Results

Number of Instructional Nudges Attempted and Continued (RQ1)

Aggregating across the 40 secondary mathematics teachers, all reported trying at least some of the instructional nudges. In fact, all teachers tried at least three nudges during the course of the academic year and one teacher reported trying all 18. The mean number of nudges tried was 11.0 (median=11.5, mode=13, standard deviation=3.8). As a reminder, within this nudge-based approach to PD, the teachers were free to choose which nudges they implemented and how they enacted them with their own students. Our interest is not in “fidelity” but rather the uptake of the instructional suggestions that were provide without training or required follow-up.

With regard to continuing to implement the nudges, as measured by their placement of the nudge toward the top of the number-of-tries dimension on the heatmap, all teachers had at least one nudge that they reported using regularly. The mean number of such nudges was 2.9 (median=3, mode=3, standard deviation=1.2). In a few cases, these frequently-implemented nudges were teaching practices that were similar to something the teacher had already been doing, but in these cases, the teachers expressed that they still valued the nudges positively as a reinforcement of their own practice or as something that can give them a refined idea of how to implement the teaching strategy.

Positively- or Negatively-Viewed Instructional Nudges (RQ2)

Table 1 displays the five nudges rated most positively by at least 10 teachers. *Put a Bow On It* was rated most positively by 18 teachers (45%); it is the suggestion to teachers to provide a brief summary of the mathematical idea of the lesson before students depart (Shimizu, 2006), inspired by the fact that baseline data showed nearly all lessons ending abruptly without any mathematical conclusion. *Leave a Trace* was rated most positively by 14 teachers (35%); it is the suggestion to place worked examples or problem solutions on the front board in a side-by-side fashion rather than displaying only one worked example or one problem at a time (Rittle-Johnson & Star, 2011). *Leave a Trace* can also involve leaving key information visible while students work. Other relatively popular nudges were *Confidence Meter* (asking students for their confidence level in addition to their answer on a mathematical problem), *Pivot* (shifting the activity structure at least every 10–15 minutes), and *Getting Unstuck* (providing students with options for what to do if they get stuck during work time).

Table 1*Five Instructional Nudges (out of 18) Rated Most Positively by the Most Teachers (n=40)*

Nudge Title	Nudge Hook	No. of Teachers Rating it Most Positively
Put a Bow On It	In case you missed it, here's the recap.	18
Leave a Trace	Leave ideas in place so students keep them in mind.	14
Confidence Meter	Drive instruction based on your students' confidence.	12
Pivot	Change up your format to keep students engaged.	12
Getting Unstuck	When you're outnumbered, expand students' options.	11

Table 2 displays the three nudges viewed non-positively by pluralities of teachers. *Rate and Review*, viewed negatively by 12 teachers (30%), is an activity wherein students are provided a written mathematical solution and asked to rate it out of five stars and provide a written review, explaining their rating (Stewart et al., 2025). *Here's Where I'm At*, viewed negatively by 11 teachers (28%), is the suggestion to have students pause in the middle of work time, move across the room to discuss their progress with a peer, and then continue working afterward. *Pairing*, viewed negatively by 9 teachers (23%), is the suggestion to supplement a problem set by asking students to identify a pair of problems they see as similar (or different) and explain why. It should be noted that several teachers misremembered the nudge (which they had perhaps viewed briefly several months earlier) as asking the students to pair up with a peer rather than asking them to find pairs of problems to compare/contrast; they often expressed negative opinions of this peer-based alternative version of Pairing. We have subsequently revised the nudge to *Pairing Problems* and are conducting further field tests with it.

Table 2*Three Instructional Nudges (out of 18) Rated Negatively by the Most Teachers (n=40)*

Nudge Title	Nudge Hook	No. of Teachers Rating it Negatively
Rate and Review	You rate movies. You rate restaurants. You can also rate math solutions!	12
Here's Where I'm At	Work, move, meet.	11
Pairing	Sometimes two problems are better than one.	9

Rationales Teachers Expressed About Instructional Nudges (RQ3)

With regard to positive opinions of instructional nudges (Table 3), the reasons expressed by teachers often focused on their students (23 instances) but almost as frequently focused on their own work and preference as a teacher (17 instances). The Student-Focused category included teachers' reports of benefits of the nudges on students' learning (12 instances) ("if you don't leave what you've done [on the board]...it doesn't click as quickly, so *Leave a Trace* is really good"). In other cases, teachers viewed the learning as going beyond mathematics ("You need to be able to get unstuck...it's really what you do with regular life when you have problems and you're stuck"). Other reasons (5 instances) focused on classroom management and supporting students' interpersonal dynamics ("I need to pivot and do something else to keep them engaged"). There were also 6 instances where the teacher simply stated that students responded well when they implemented the nudge.

Table 3

Reasons Teachers Provided for Why They Liked the Popular Instructional Nudges

Nudge	Student-Focused Benefits			Teacher-Focused Benefits		
	Learning	Management	Positive Response	Improved Instruction	Gather Feedback	Positive Response
Put a Bow On It	1	0	1	1	2	5
Leave a Trace	6	1	3	2	0	0
Confidence Meter	1	1	1	0	2	3
Pivot	0	3	1	1	0	0
Getting Unstuck	4	0	0	1	0	0
TOTALS	12	5	6	4	4	8

Table 3 also enumerates the Teacher-Focused reasons for positive opinions of the nudges. In 4 instances, teachers described how the nudge entailed effective instruction or an opportunity for them to improve upon what they had been doing before (“[*Put a Bow On It*] is just a really good way to pull a summary together and, like, remind the students at the end of class, ‘Remember, this is what we learned today’”). In another 4 instances, teachers expressed the specific benefit of two nudges (*Put a Bow On It* and *Confidence Meter*), being that they allowed them to gather feedback about how the students were doing in the lesson. The most frequent Teacher-Focused reason for their positive opinions was simply that the teacher liked the nudge or that it had gone well when they tried it (“I do love doing [*Getting Unstuck*]”; “I do [*Pivot*] very often, I like it”).

Teachers’ negative opinions (Table 4) were largely driven by their views on how students reacted to the nudge; they expressed 23 reasons in the Student-Focused Concern category and only 12 reasons in the Teacher-Focused Concern category (4 reasons were too vague to be categorised). *Pairing* garnered negative opinions primarily because of concerns about how students would respond to the nudge, be it learning concerns (“Other [students] are like, ‘Yeah, whatever, it’s just two problems,’ so they’re indifferent”) and management/engagement concerns (“They’re really lazy, and so I found [*Pairing*] to be more work than our kids were willing to try”). *Here’s Where I’m At* and *Rate and Review* were quite different from one another in the reasons for being disliked. *Here’s Where I’m At* was disliked primarily (5 instances) due to concerns about classroom management (“It turns out sometimes to be more of a managing thing than I want”; “[Students] moving around the room can be a lot for us”), but 3 teachers also expressed that they did not like *Here’s Where I’m At* because they already had a particular way they organized their student interactions during work time (e.g., teacher-selected groups without the standing and mixing that the nudge suggests). In contrast, it was student learning concerns (7 instances) that prevailed for *Rate and Review* (“I’ve really seen kids struggle and get discouraged with the writing”). Additionally, 3 teachers expressed they were unsure of the purpose or how to implement the nudge (“I don’t know if I did it incorrectly...they couldn’t explain to me why they gave [their rating]”).

Table 4*Reasons Teachers Provided for Disliking (or Not Trying) Unpopular Instructional Nudges*

Nudge	Student-Focused Concerns		Teacher-Focused Concerns			
	Learning	Management	Already Do Similar	Do Not My Style	Unclear Purpose	Not Enough Time
Pairing	4	3	0	1	2	0
Here's Where I'm At	2	5	3	1	0	0
Rate and Review	7	2	1	0	3	1
TOTALS	13	10	4	2	5	1

Discussion

This nudge-based approach to instructional change is based on the observation that intensive, transformational PD has not succeeded in a widescale sense because it is difficult to make large, ambitious changes away from conventional instruction and it is unlikely for such changes to easily spread (Otten et al., 2025). But nudge-based PD is also unlikely to succeed if teachers are not able or are unwilling to try instructional suggestions based only on brief one-page descriptions. It was not obvious to us beforehand that teachers would actually implement small instructional nudges without being required to do so and without receiving any training or direct support from professional developers. Although there may have been implicit pressure because they were enrolled in the ongoing project, we explicitly informed teachers many times that the nudges were voluntary and that we knew they were busy and did not mind if they chose to forego the nudges. Yet, results for RQ1 show that all teachers were willing to try some of the nudges and were able to do so based on the brief instructional nudge content. Moreover, all teachers had at least one nudge, and typically three nudges, that they were happy to continue implementing fairly regularly. This result does not generalise to all teachers, because these were individuals who were willing to sign up for a university-based research project, but it is at least an encouraging result for initial uptake of instructional nudges, and perhaps such teachers can be the initiators of further spread since the nudges are packaged in easily-shareable forms. (Analysis of the data about their spreading/sharing behaviours is beyond the scope of this brief report, but we have indications of some spontaneous sharing of the teachers' favourite nudges.)

As teachers are considering whether to try or continue instructional nudges, results show that they may focus primarily on how the nudges affect or will be received by students, which corroborates findings by Partridge (2025) at the elementary level. Yet, there were also some reasons for nudge preferences that were teacher focused, with teachers opting against instructional suggestions in areas where they already have a preferred practice or against suggestions that do not resonate with their personal style, but opting for modest suggestions they felt were slight improvements to practice. In designing nudges, it can be challenging to find the distinction between nudges that are similar to what they are already doing and viewed as an enhancement versus nudges viewed as unnecessary because teachers already have a similar means for achieving the same ends. Our data suggests these distinctions are not to be found for nudges in general but, rather, the distinction depends on individual teachers. Thus, it may be essential in the nudge-based incremental PD approach to provide a variety of options to teachers and allow them to choose which nudges they want to implement and how. After all, in this study, even the most popular nudge was only rated most highly by less than half of the participants, but all teachers had some nudges they rated very highly. This suggests potential in this PD approach, and we are interested in how the possible criteria for success in this nudge-based approach may compare, or more likely *contrast*, with the known criteria for transformational PD (e.g., Desimone, 2011; Feiman-Nemser, 2001).

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References

- Arbaugh, F. (2025). Commentary: Personal transformations and the possibilities of incremental progress in mathematics teacher professional development. *Education Sciences*, 15(1), 30.
- Clarke, D., Emanuelsson, J., Jablonka, E., & Mok, I. A. C. (Eds.) (2006). *Making connections: Comparing mathematics classrooms around the world*. Sense Publishers.
- Coburn, C. E. (2003). Rethinking scale: Moving beyond numbers to deep and lasting change. *Educational Researcher*, 32(6), 3–12.
- Cortina, J. L., & Višňovská, J. (2023). Designing instructional resources to support teaching. In T. Lamberg & D. Moss (Eds.), *Proceedings of the 45th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 15–26). Univ. Nevada.
- Desimone, L. M. (2011). A primer on effective professional development. *Phi Delta Kappan*, 92(6), 68–71.
- Doyle, W., & Ponder, G. A. (1977). The practicality ethic in teacher decision-making. *Interchange*, 8(3), 1–12.
- Every, E., Ball, L., & van Driel, J. (2026). Mathematics teachers' perceptions of teaching strategies when adopting a STEM approach. *Mathematics Education Research Journal*, 38, 1–33.
- Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record*, 103(6), 1013–1055.
- Jansen, A., Botello, M., & Silla, E. M. (2024). Rough Draft Math as an evolving practice: Incremental changes in mathematics teachers' thinking and instruction. *Education Sciences*, 14(11), 1266.
- Lee, H. S., Thrasher, E., Mojica, G. F., Graham, B. M., Lee, J. T., & Kuhlman, A. (2024). Examining teachers' professional learning in an online asynchronous system: Personalized supports for growth and engagement in learning to teach statistics and data science. *Education Sciences*, 14(11), 1236.
- Litke, E., Wilson, J., & Hill, H. C. (2025). Equity-focused, rubric-based coaching: An incremental improvement approach to supporting teachers to shift toward more equitable mathematics instruction. *Education Sciences*, 15(4), 444. <https://doi.org/10.3390/educsci15040444>
- Munter, C., Stein, M. K., & Smith, M. S. (2015). Is there a common pedagogical core? Examining instructional practices of competing models of mathematics teaching. *NCSM Journal of Mathematics Education Leadership*, 16(2), 3–13.
- Otten, S., de Araujo, Z., & Candela, A. G. (2025). The benefits of modesty: Considering incremental professional development for mathematics teachers. *Education Sciences*, 15(4), 473.
- Otten, S., de Araujo, Z., Candela, A. G., Vahle, C., Stewart, M. E. N., Wonsavage, F. P., & Baah, F. (2022). Incremental change as an alternative to ambitious professional development. In A. Lischka & J. F. Strayer (Eds.), *Proceedings of the 44th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1463–1468). PME-NA.
- Partridge, E. (2025). *Investigating instructional nudges as a model for elementary mathematics professional development* [dissertation]. University of Missouri, Columbia, MO, USA.
- Rittle-Johnson, B., & Star, J. R. (2011). The power of comparisons in learning and instruction: Learning outcomes supported by different types of comparisons. *Psychology of Learning and Motivation: Cognition in Education*, 55, 199–226.
- Shimizu, Y. (2006). How do you conclude today's lesson? The form and functions of 'matome' in mathematics lessons. In D. Clarke, J. Emanuelsson, E. Jablonka, & I. A. C. Mok (Eds.), *Making connections: Comparing mathematics classrooms around the world* (pp. 127–145). Sense Publishers.
- Star, J. R. (2016). Improve math teaching with incremental improvements. *Phi Delta Kappan*, 97(7), 58–62.
- Stewart, M. N., Brown, N., Candela, A., Otten, S., & de Araujo, Z. (2025). Rate and review. *Mathematics Teacher: Learning and Teaching PK-12*, 118(1), 45–57.
- Wilkie, K. J. (2019). The challenge of changing teaching: Investigating the interplay of external and internal influences during professional learning with secondary mathematics teachers. *Journal of Mathematics Teacher Education*, 22(1), 95–124.

Beyond Boundaries: How Mathematics Education Research Frames Generative AI Use Through ZPA–ZFM

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This paper investigates how generative artificial intelligence (GenAI) use is explicitly promoted in mathematics education research and how these promoted actions are constrained by reported challenges and ethical concerns. Using the Zone of Promoted Action (ZPA) as an analytic lens, we systematically examine empirical studies to identify promoted practices, sources of promotion, and their structural boundaries. Findings indicate that GenAI is most commonly promoted for instructional planning, feedback, and assessment support, while concerns related to reliability, training, curriculum alignment, and ethics delimit the scope of these promoted uses.

Introduction

Generative artificial intelligence (GenAI) tools have rapidly entered educational discourse and practice, with mathematics education emerging as a prominent area of exploration. Recent advances in GenAI have enabled automated generation of explanations, feedback, lesson plans, assessment tasks, and worked examples, raising both opportunities and concerns for the teaching and learning of mathematics (e.g., Canonigo, 2024; Davis, 2025; Henkel et al., 2025; Kim et al., 2025; Taani & Alabidi, 2024). In response, mathematics education researchers have begun to examine how these tools might support instructional practice, student learning, and teacher professional development (Biton & Segal, 2025; Kim et al., 2025; Pepin et al., 2025; Walkington, 2025).

To date, much of the empirical research on GenAI in mathematics education has focused on adoption, perceptions, and impact. Studies commonly investigate teachers' attitudes towards GenAI, their willingness to integrate such tools into practice, or perceived benefits and risks associated with their use (Busuttil & Calleja, 2025; Wahba et al., 2024; Zhuang & Zhang, 2025). Others examine student engagement, learning outcomes, or performance when GenAI-supported tools are introduced into instructional settings (Fardian et al., 2025; Ramprakash et al., 2024; Segal & Biton, 2024; Torres-Peña et al., 2024). While this body of work has contributed valuable insights into how GenAI is taken up and experienced, it often positions GenAI use as an individual choice or pedagogical intervention, foregrounding questions of effectiveness, acceptance, or readiness.

Existing GenAI-in-mathematics-education studies report uses and outcomes but rarely interrogate how the literature itself constructs and legitimises “appropriate” use through designs, pedagogical framings, PD models, and AI-mediated prompts. Crucially, challenges—accuracy, transparency, privacy, integrity, curriculum alignment—are treated as afterthoughts rather than structural conditions shaping what gets promoted (e.g., Gurl et al., 2025; Karaman & Goksu, 2024). This study is novel in shifting the unit of analysis from “reported practice” to the field’s promotional logics, and in jointly analysing promotion and constraint as co-configuring legitimate GenAI practice. Methodologically, we conduct a PRISMA-guided systematic review (Page et al., 2021), develop and iteratively refine a coding scheme distinguishing promoted actions and reported constraints, and map co-occurrence patterns to surface configuration logics. We ask: *What forms of GenAI use are explicitly promoted in*

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 305–312). MERGA.

mathematics education research, and how are these constrained or related by reported challenges, limitations, and ethical concerns?

Conceptual Framework: ZPA and Structural Boundaries

This study adopts the Zone of Promoted Action (ZPA) as its primary conceptual lens for examining how GenAI use is positioned within mathematics education research. Building on Goos et al. (2003), ZPA refers to the deliberate shaping of learning environments to create opportunities for particular forms of action. In classroom contexts, such shaping occurs through pedagogical decisions that prompt exploration, encourage mathematical reasoning, and extend student activity. ZPA thus captures how teaching involves not only enabling action, but also structuring the space within which action is possible.

In technology-rich mathematics learning environments, pedagogical shaping is increasingly mediated by digital tools. From this perspective, features such as prompt design, automated feedback, and AI-generated scaffolds can be understood as digital forms of ZPA, as they foreground particular approaches, representations, or problem-solving pathways when learners engage with GenAI (Biton et al., 2025; Slobodsky & Durcheva, 2025). ZPA therefore captures how particular actions are *invited and foregrounded* through design features and pedagogical moves.

Goos (2005) further conceptualises ZPA as the actions, strategies, and practices promoted by experts, mentors, and professional programs. For pre-service teachers, ZPA may be shaped through coursework, professional development, or supervision that *models, exemplifies, and invites uptake of* specific pedagogical approaches. This highlights that ZPA is not solely produced through individual teaching decisions but is socially and institutionally constituted through *endorsed professional discourses and exemplars* that *foreground* what is valued as effective practice.

Extending this notion, the present study examines ZPA as it is evidenced within empirical studies of mathematics education. ZPA is operationalised as the practices, strategies, or uses of GenAI that are explicitly reported as being encouraged, modelled, or required within the study contexts, such as professional development activities, teacher education programs, classroom interventions, or AI-mediated learning environments. In this sense, ZPA refers to promoted actions within the empirical settings described by the studies, rather than to individual teacher choice or inferred practice.

To maintain analytic precision, the ZPA is defined narrowly to include only practices explicitly promoted in empirical studies—such as stated recommendations, guided or modelled uses, or GenAI presented as an intended component of instruction, assessment, or professional learning. No inferences are made beyond what is reported. The identified ZPA reflects promotion occurring across the system, including PD initiatives, teacher-educator guidance, institutional expectations, and built-in AI supports within GenAI tools.

This study's central contribution is to analyse promoted actions (ZPA) *together with* reported challenges, limitations, and ethical concerns as *boundary-setting conditions* that shape promotion. Analytically, this study distinguishes *promotive moves* (what is encouraged or modelled: ZPA) from *boundary conditions* (what is permitted/feasible: Zone of Free Movement (ZFM)), while recognising they co-occur in study descriptions. It treats these *boundary conditions* as structural, boundary-setting conditions that determine how far—and in what ways—GenAI use is promoted in mathematics education research. Thus, promoted actions and boundary conditions are analytically inseparable in reporting—for example, *promoting* GenAI for planning (ZPA) while *restricting* assessment use due to integrity concerns (ZFM).

Methodology

This study employed a systematic literature review to examine how GenAI use is explicitly promoted in mathematics education research and how these promoted actions are constrained by reported challenges, limitations, and ethical concerns. The review followed the Preferred Reporting Items for Systematic Review and Meta-Analysis (PRISMA 2020) guidelines (Page et al., 2021) to ensure transparency and rigour in study identification, screening, and reporting. Searches were conducted across ERIC, Scopus, and Web of Science using Boolean combinations of GenAI-related and mathematics-education keywords. The search was limited to peer-reviewed journal articles published between 2018 and 2025, and reference lists of included studies were also checked for additional sources. Studies were included if they were written in English, explicitly focused on mathematics education, examined GenAI as a central analytic object, and employed empirical qualitative, quantitative, or mixed-methods designs involving students, in-service or pre-service teachers, or teacher educators. Conceptual papers, broader STEM studies without a clear mathematics focus, and studies mentioning AI only peripherally were excluded. A total of 344 records were imported into Covidence; after removing duplicates, 328 titles and abstracts were screened followed by full-text screening of 91 potentially relevant studies. The screening of each abstract and full-text were conducted by two reviewers. Disagreements between reviewers were resolved through discussion or independently by the third reviewer, resulting in a final corpus of 43 empirical studies.

Data were extracted using a structured coding template informed by a ZPD–ZFM–ZPA¹³ framework (Goos et al., 2003; Goos, 2005), with this paper focusing specifically on ZPA and ZFM. ZPA was operationalised as practices, behaviours, or uses of GenAI that were explicitly encouraged or promoted within the study, including through professional development, teacher educator guidance, institutional or curricular directives, or AI-embedded supports. This study analytically distinguishes ZPA—promotive moves that encourage or model particular actions—from ZFM—boundary conditions that define what is permitted or feasible—while recognising that, in empirical reports, these often co-occur and are difficult to disentangle because promoted actions are typically presented alongside their enabling and constraining conditions. To maintain analytic rigour, only information explicitly stated in the studies was coded. No inferences were made beyond the text. When a promoted action, challenge, limitation, or ethical concern was not explicitly reported, it was coded as N/A. Each coded item was accompanied by a reference to its location in the article (e.g., section heading and paragraph or page number).

After completing data extraction in Covidence, we exported the outputs to an Excel matrix for subsequent coding. An initial attempt to analyse the exported fields using freely available institutional and one of the most popular GenAI tools produced inconsistent and sometimes inaccurate classifications; therefore, we adopted manual coding. Two researchers independently coded all data extraction, compared code applications, and resolved discrepancies through discussion to ensure interpretive consistency and analytic trustworthiness. Each code was operationalised using binary indicators (1 = present; 0 = absent). Coding focused on four domains: promoted GenAI use, mediations of promoted GenAI use, reported challenges/limitations, and reported ethical concerns. In total, 40 codes were developed, organised into four major themes and 18 sub-themes (listed in the Supplementary Materials). We also coded contextual descriptors (i.e., year of publication, country, population, mathematics content/strand).

³ ZPD (zone of proximal development) refers to the range of learning possible with support; ZFM (zone of free movement) refers to the contextual constraints and possibilities shaping action; and ZPA (zone of promoted action) refers to the actions and practices encouraged within that setting. Together, these constructs frame how learning and participation are enabled, constrained, and directed.

To examine how promoted forms of GenAI use relate to reported challenges and ethical concerns, we conducted a co-occurrence analysis of the binary coding matrix. Pairwise associations were estimated using Jaccard similarity to capture shared presences across the 43 studies. A weighted, undirected network ($\text{Jaccard} \geq 0.20$) was then constructed to represent the strongest associations and was complemented by a Jaccard heatmap to show the broader similarity structure (Podani & Schmera, 2011). All computations and visualisations were performed in Python using standard scientific libraries.

Findings and Discussion

Research Demographic Background

The publication-year distribution reveals a recent and concentrated evidence base: 97.6% of studies were published in 2024–2025, with only one study from 2023 (2.3%). The reviewed literature focuses mainly on GenAI use by teachers (60.5%), students (34.9%), or both teachers and students (4.7%) in mathematics education, largely within school and teacher-education settings. This recency and concentration suggest a field still in active formation, where conceptual frameworks, research designs, and pedagogical claims remain fluid.

Geographical coverage was uneven. The largest share of studies originated from North America (20.9%), followed by the Middle East (16.3%), Europe (11.6%), and Southeast Asia (11.6%). Representation was markedly lower for East Asia (7.0%) and Sub-Saharan Africa (2.3%), and nearly one-third of studies (30.2%) did not specify a national context. When reclassified using a Global North–Global South framing, 44% were situated in the Global North and 26% in the Global South. This imbalance mirrors long-standing structural asymmetries in educational technology research and constrains both contextual generalisability and theorisation sensitive to sociocultural and infrastructural variation.

Participant profiles were similarly concentrated. Pre-service teachers ($n = 21$; 48.8%) and students ($n = 15$; 34.9%) accounted for 83.7% of the sample, whereas in-service teachers and teacher educators were each examined in only two studies (4.7%). This distribution reflects a dominance of early-stage exploratory work situated in controlled learning environments, rather than sustained investigations of classroom enactment or professional practice. Content coverage followed a parallel pattern: mathematics was often addressed generically (23.1%), with fewer studies engaging specific domains such as Algebra (17.3%), Number Sense (17.3%), or Geometry/Measurement (15.4%). Advanced mathematics appeared infrequently (11.5%), while Calculus and Statistics/Probability were least represented (7.7% each). This pattern suggests that current evidence is drawn largely from foundational content areas, limiting how far claims about GenAI can yet extended across the breadth of school mathematics.

Promoted GenAI Practices: Constraints and Ethical Concerns

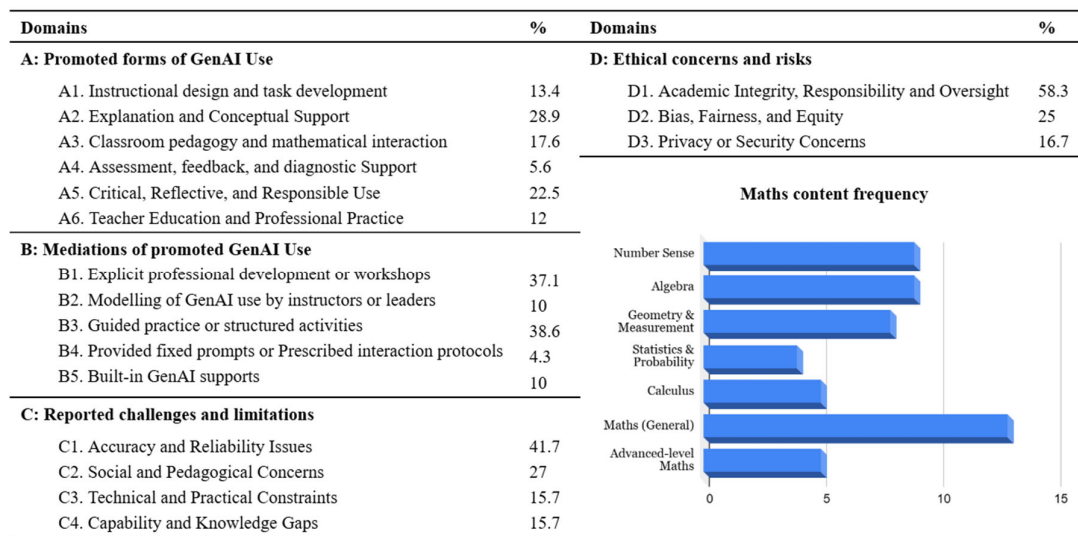
Next, we address the research question by synthesising what mathematics education research explicitly promotes educators and learners to do with GenAI, and how these advocated uses are shaped by reported challenges, limitations, and ethical concerns. We first characterise the dominant forms of promoted GenAI use across the included studies. We then examine how these promotions are mediated by constraints—such as reliability, pedagogical fit, access and workload—and by ethical considerations including academic integrity, responsibility and oversight, privacy, and fairness. Rather than treating promotion and constraint as separate themes, we interpret their co-occurrence as indicative of the conditions under which GenAI is positioned as educationally legitimate, feasible, and defensible. The synthesis therefore clarifies not only what is being advocated, but also under what boundaries and with what cautions these advocated practices are framed.

Across the four analytic domains (illustrated in Figure 1), GenAI use in mathematics education is situated within a cautiously delimited zone of promoted action. Promotion

concentrates most strongly on explanation and conceptual support (A2; 28.9%), followed by critical, reflective, and responsible use (A5; 22.5%) and classroom pedagogy and mathematical interaction (A3; 17.6%), with comparatively limited attention to assessment and feedback (A4; 5.6%). These promoted uses are typically enacted through guided practice or structured activities (B3; 38.6%) and explicit professional development (B1; 37.1%), while system-level scaffolds—such as fixed prompts or built-in AI supports—remain marginal. This action space is shaped by constraints primarily related to accuracy and reliability (C1; 41.7%), followed by social and pedagogical concerns (C2; 27.0%), technical constraints (C3; 15.7%), and capability and knowledge gaps (C4; 15.7%). Ethical considerations add another layer of limitation, with strong emphasis on academic integrity and oversight (D1; 58.3%), and less frequent attention to bias and equity (D2; 25.0%) or privacy and security (D3; 16.7%), reflecting the continued influence of governance and institutional risk management.

Figure 1

The ZPA and ZFM analytic domains and mathematics content focus



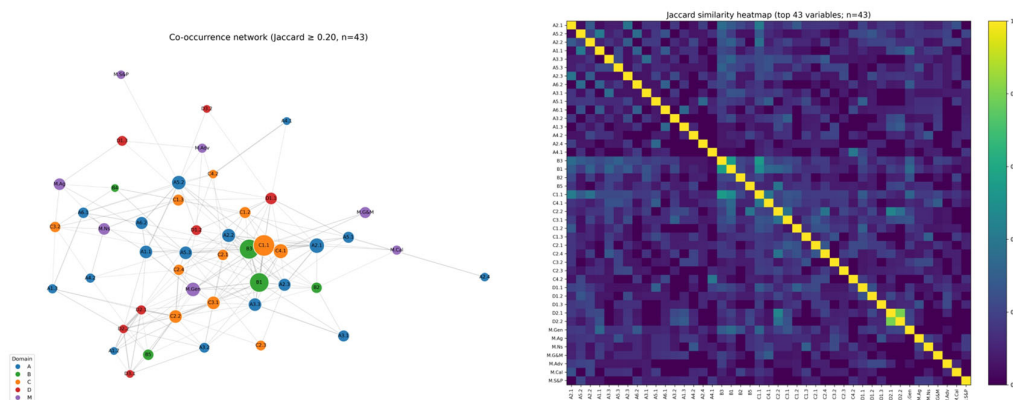
The co-occurrence network ($Jaccard \geq .20$, $n = 43$) accompanied by the heatmap (see Figure 2) further clarifies how these patterns operate in combination. The field is organised around a practice-centred explanatory core, where the A2 uses—explanations, worked solutions, alternative strategies, and sense-making—form dense ties with B1 (professional development) and B3 (guided practice). These connections sit alongside two recurring constraints: C1.1 (GenAI output unreliability) and C4.1 (AI literacy limitations). Together, they indicate that explanatory uses are framed not as direct affordances but as practices requiring structured human mediation and ongoing verification. A3.3 (interactive questioning) aligns with this core, whereas A4 (assessment and diagnosis) and most D-domain concerns remain peripheral. Mathematics-content codes are also weakly integrated: only Maths (General) shows moderate connectivity, while specific strands such as Algebra, Geometry, Number, Advanced, and Statistics sit at the edges. Overall, the evidence base privileges pedagogical enactment over disciplinary specificity, and teacher- or program-led scaffolding over systemic or ethical analyses.

Taken together, these results provide a coherent picture of how GenAI is currently positioned within mathematics education research: promoted in carefully bounded ways, mediated through structured support, and accompanied by recurring concerns about accuracy, capability, and ethical oversight. The co-occurrence patterns, in particular, highlight the centrality of explanation-focused uses and the foundational role of professional development

and guided practice in shaping how these uses are enacted. At the same time, several areas—including assessment, domain-specific mathematical content, and system-level ethical considerations—remain relatively underexplored. These patterns point toward important directions for future inquiry, which we elaborate in the following discussion.

Figure 2

Promoted uses and mediating constraints co-occur



Note. Code labels are defined in the Supplementary Materials; the full code list provides complete descriptions of each code.

1. Moving beyond scaffolded use towards classroom enactment

The co-occurrence network shows that promoted uses are tightly coupled with professional development (B1) and guided practice (B3). While this indicates responsible, well-scaffolded experimentation, it also reveals a field that is heavily anchored in controlled learning contexts rather than authentic classroom settings. There is a need for design-based, classroom-embedded, and longitudinal studies that trace how teachers' judgement, students' reasoning, and patterns of feedback evolve as GenAI becomes entangled in everyday teaching. Such studies would clarify not only what GenAI can do, but how teachers actually take it up within real pedagogical and institutional constraints.

2. Bridging reliability concerns with capability building

Across the literature, C1.1 (unreliability) and C4.1 (AI literacy gaps) consistently co-occur with explanatory uses (A2). This pairing suggests an opportunity for research focused on mechanisms of verification, correction, and sense-making. Rather than treating inaccuracy as an inherent limitation, future studies could investigate instructional routines—such as cross-checking, re-prompting, or pair-reasoning—that strengthen students' and teachers' capacity to evaluate GenAI outputs (Park & Choo, 2025). This direction aligns with broader trends in mathematics education emphasising critical engagement with tools rather than simple adoption (Cullen & Hertel, 2023).

3. Deepening the mathematical substance of GenAI research

The predominance of generic mathematics codes and the marginality of Algebra, Geometry, Number, and Statistics indicate that current studies rarely examine how GenAI interacts with the structure of mathematical ideas. This gap matters, because explanation quality, conceptual accuracy, and error-patterns differ substantially across domains. Future work would benefit from content-specific task design, enabling closer analysis of how GenAI handles symbolic manipulation, geometric reasoning, or stochastic representations, and how teachers and students evaluate these outputs. Such work would also support stronger claims about the disciplinary

integrity of GenAI-mediated reasoning.4. Integrating ethical and equity considerations into pedagogical inquiry

Ethical concerns, especially academic integrity (D1), appear in the corpus but remain structurally peripheral in the network, with equity (D2) and privacy (D3) even less connected. Given the differential access to GenAI, linguistic inequalities, and the risks associated with data exposure, future work should integrate system-level ethics directly into pedagogical research designs. Studies in multilingual and under-resourced contexts are particularly needed to understand how GenAI behaviour, institutional policy, and infrastructural realities shape pedagogical possibilities.

5. Strengthening methodological transparency and reporting

Finally, the patterns observed in the network highlight the need for stronger methodological transparency. Future empirical work should report the prompting strategies, model versions, verification procedures, and AI-literacy baselines used in studies, as these factors directly shape the validity and replicability of findings. Pairing co-occurrence or descriptive analyses with mechanism-focused designs (e.g., mediation analysis, process tracing, or microgenetic methods) would help identify why certain GenAI uses are effective and how mediations mitigate or exacerbate risks.

In conclusion, the review highlights a field in transition—experimenting with GenAI’s potential while negotiating its constraints. Future research must move beyond early trials to examine how GenAI interacts with mathematical thinking, classroom practice, and the diverse realities of learners and teachers. Although policy is not this paper’s primary focus, the findings may also inform mathematics education policy by identifying conditions for the thoughtful, responsible, pedagogically meaningful integration of emerging technologies.

Acknowledgement

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References

- Biton, Y., & Segal, R. (2025). Learning to craft and critically evaluate prompts: The role of Generative AI (ChatGPT) in enhancing pre-service mathematics teachers’ TPACK and problem-posing skills. *International Journal of Education in Mathematics, Science and Technology*, 13(1), 202-223. <https://doi.org/10.46328/ijemst.4654>
- Biton, Y., Segal, R., & Alush, K. (2025). How utilizing Generative AI when addressing pedagogical and mathematical events contributes to mathematics teacher educators’ TPACK (Technological Pedagogical Content Knowledge). *International Journal of Education in Mathematics, Science and Technology*, 13(4), 895-913. <https://files.eric.ed.gov/fulltext/EJ1475751.pdf>
- Busuttil, L., & Calleja, J. (2025). Teachers’ beliefs and practices about the potential of ChatGPT in teaching Mathematics in secondary schools. *Digital Experiences in Mathematics Education*, 11(1), 140-166. <https://doi.org/10.1007/s40751-024-00168-3>
- Canonigo, A. M. (2024). Levering AI to enhance students’ conceptual understanding and confidence in mathematics. *Journal of computer assisted learning*, 40(6), 3215-3229. <https://doi.org/10.1111/jcal.13065>
- Cullen, C. J., & Hertel, J. T. (2023). Improving our criticism of technology in mathematics education. *Mathematics Teacher: Learning and Teaching PK-12*, 116(5), 314-321. <https://doi.org/10.5951/mtlt.2023.0004>
- Davis, J. D. (2025). Evolving techniques and emerging schemes: Prospective teachers’ transformation of ChatGPT. *International Journal of Science and Mathematics Education*, 23, 2181–2198. <https://doi.org/10.1007/s10763-024-10533-8>
- Fardian, D., Suryadi, D., Prabawanto, S., & Jupri, A. (2025). Integrating Chat-GPT in the classroom: A study on linear algebra learning in higher education. *International Journal of Information and Education Technology*, 15(4). <https://doi.org/10.18178/ijiet.2025.15.4.2279>

- Gurl, T. J., Markinson, M. P., & Artzt, A. F. (2025). Using ChatGPT as a lesson planning assistant with preservice secondary mathematics teachers. *Digital Experiences in Mathematics Education*, 11(1), 114–139. <https://doi.org/10.1007/s40751-024-00162-9>
- Goos, M. (2005). A sociocultural analysis of the development of pre-service and beginning teachers' pedagogical identities as users of technology. *Journal of Mathematics Teacher Education*, 8(1), 35–59. <https://doi.org/10.1007/s10857-005-0457-0>
- Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology-mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior*, 22(1), 73–89. [https://doi.org/10.1016/S0732-3123\(03\)00005-1](https://doi.org/10.1016/S0732-3123(03)00005-1)
- Henkel, O., Horne-Robinson, H., Dyshel, M., Thompson, G., Abboud, R., Ch, N. A. N., ... & Vanacore, K. (2025). Learning to love LLMs for answer interpretation: Chain-of-thought prompting and the AMMORE dataset. *Journal of Learning Analytics*, 12(1), 50–64. <https://doi.org/10.18608/jla.2025.8621>
- Kim, Y. R., Park, M. S., & Joung, E. (2025). Exploring the integration of artificial intelligence in math education: Preservice teachers' experiences and reflections on problem-posing activities with ChatGPT. *School Science and Mathematics*. <https://doi.org/10.1111/ssm.18336>
- Karaman, M. R., & Goksu, I. (2024). Are lesson plans created by ChatGPT more effective? An experimental study. *International Journal of Technology in Education*, 7(1), 107–127. <https://doi.org/10.46328/ijte.607>
- Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., ... & Moher, D. (2021). The PRISMA 2020 statement: An updated guideline for reporting systematic reviews. *bmj*, 372. <https://doi.org/10.1136/bmj.n71>
- Park, J., & Choo, S. (2025). Generative AI prompt engineering for educators: Practical strategies. *Journal of Special Education Technology*, 40(3), 411–417. <https://doi.org/10.1177/01626434241298954>
- Pepin, B., Buchholtz, N., & Salinas-Hernández, U (2025). A scoping survey of ChatGPT in mathematics education. *Digital Experiences Mathematics Education* 11, 9–41. <https://doi.org/10.1007/s40751-025-00172-1>
- Podani, J., & Schmera, D. (2011). A new conceptual and methodological framework for exploring and explaining pattern in presence – absence data. *Oikos*, 120(11), 1625–1638. <https://doi.org/https://doi.org/10.1111/j.1600-0706.2011.19451.x>
- Ramprakash, B., Nithyakala, G., Bhumika, K., & Avanthika, S. (2024). Comparing traditional instructional methods to ChatGPT: A comprehensive analysis. *Journal of Engineering Education Transformations*, (37), 612–620. <https://journaleet.in/index.php/jeet/article/view/2477/2297>
- Segal, R., & Biton, Y. (2024). The Contribution That utilizing Generative AI for problem posing makes to pre-service high school mathematics teachers' TPACK. *International Journal of Education in Mathematics, Science and Technology*, 12(6), 1559–1582. <https://files.eric.ed.gov/fulltext/EJ1465780.pdf>
- Slobodsky, P., & Durcheva, M. (2025). Using ChatGPT errors to encourage student exploration and self-learning in mathematics. *International Journal of Mathematical Education in Science and Technology*, 1–25. <https://doi.org/10.1080/0020739X.2025.2505200>
- Taani, O., & Alabidi, S. (2024). ChatGPT in education: Benefits and challenges of ChatGPT for mathematics and science teaching practices. *International Journal of Mathematical Education in Science and Technology*, 1748–1777. <https://doi.org/10.1080/0020739X.2024.2357341>
- Torres-Peña, R. C., Peña-González, D., Chacuto-López, E., Ariza, E. A., & Vergara, D. (2024). Updating calculus teaching with AI: A classroom experience. *Education Sciences*, 14(9), 1019. <https://doi.org/10.3390/educsci14091019>
- Wahba, F., Ajlouni, A. O., & Abumosa, M. A. (2024). The impact of ChatGPT-based learning statistics on undergraduates' statistical reasoning and attitudes toward statistics. *Eurasia Journal of Mathematics, Science and Technology Education*, 20(7), em2468. <https://doi.org/10.29333/ejmste/14726>
- Walkington, C. (2025). The implications of generative artificial intelligence for mathematics education. *School Science and Mathematics*. <https://doi.org/10.1111/ssm.18356>
- Zhuang, Y., & Zhang, S. (2025). Pre-service mathematics teachers' perceptions of using GenAI for practicing teacher questioning: A semester-long study. *EURASIA Journal of Mathematics, Science and Technology Education*, 21(9), em2689. <https://doi.org/10.29333/ejmste/16764>
- Note:* Due to the MERGA page limit, we do not include the full list of the reviewed 43 empirical studies in the paper. The complete list is available in the Supplementary Materials.

From Text Direction to the Number Line: How Cultural Reading Systems and Education Shape Numerical-Spatial Mapping

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Numerical magnitude is often represented along a spatial continuum known as the mental number line, with magnitudes increasing from left to right. Cross-cultural studies indicate that reading direction influences the orientation of these spatial numerical mappings, while developmental research suggests that formal schooling plays a crucial role in promoting more precise number line representations. This paper reviews how reading direction and education shape spatial-numerical associations, discusses implications for future research and highlights the need to understand the connections between the mental number lines, number sense and learning mathematics.

Introduction

The concept of the mental number line (MNL) can be traced back to Francis Galton's work in 1880, which first documented the spatial nature of numerical thinking by showing that individuals often visualise numbers arranged in space (Dehaene, 2011). Building on this early insight, Dehaene (2011) defined the MNL as the internal representation of numerical quantities, which is "metaphorically thought of as being oriented in space" (p. 81). It is commonly conceived as a left-to-right spatial continuum on which smaller numbers occupy the left side and with numbers increasing towards the right.

The phenomenon of left/small to right/large representation of numbers is called the Spatial-Numerical-Associations of Response Codes (SNARC) effect (Dehaene, 2011). However, the spatial-numerical mapping is not universal, and may be influenced by reading direction and education. Additionally, little is known about the implications of a MNL for number sense, how the development of the MNL could be enhanced by teaching, and then the implications for learners with an impaired MNL. Hence, the present paper aims to review how the MNL is studied and how cultural reading systems and education shape how we map numbers onto space. Finally, implications for future research and for mathematics teaching are discussed.

Studies about the mental number line

In the literature, the term MNL is sometimes used interchangeably with "number line", yet the latter may also refer to a concrete, physical instructional representation used in mathematics teaching. Schneider and colleagues (2009) explicitly distinguished the internal number line, referring to mental magnitude representation, from the external number line, referring to its physical visual display. Despite this distinction, both forms share key structural properties, comparatively representing numerical magnitude, typically as a one-dimensional scale (Schneider et al., 2009).

Origin of MNL and its connection with number sense and mathematics learning

The origins of the MNL remain a subject of debate. Some evidence has shown that the MNL is biologically rooted, as studies with babies and animals indicate early sensitivity to numerical magnitude. In contrast, Nunez (2011) assumes that there is no innate MNL, with the MNL acquired only through cultural practices and educational exposure. Supporting this view, he (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 313–320). MERGA.

described research involving the Mundurucu group, a remote and isolated Indigenous community with limited exposure to formal education or measuring tools. These people reported difficulty in representing even small numbers such as one, two and three on a line segment. Nunez therefore concluded that a linear number line described by Siegler and Opfer (2003) was a product of culture and formal education.

Closely related to the mental number line is the concept of number sense. Dehaene (2011) argues that humans are born with an intuitive ability to perceive and compare numerical quantities. Infants can precisely recognise small quantities (typically up to three or four) and estimate larger quantities approximately. Evidence of number sense is also observed in adults' subitizing ability, which allows individuals to recognise small quantities without counting (Dehaene, 2011). Humans possess an innate approximate number system. How this is represented is not known, though it may be represented on a mental number line.

There is also a connection between the MNL, number sense and mathematics learning. Griffin et al. (1994) designed the RightStart training program. Before the training, almost all kindergarten participants from low-income families in Canada and the US failed the 6-year-old level of number knowledge tests, and even many of them failed at items for the 4-year-old level. They could not tell which number is larger: 6 or 8, or which of 2 or 6 is closer to 5. These difficulties indicate the absence of general number knowledge inherent in a mental comparative representation, such as on a number line. The RightStart program was designed to focus on the intuitive representation of quantities on a mental number line. After the forty 20-minute sessions of training with the RightStart program, taking around 4 to 5 months, the great majority of the children passed the tests, and they even outperformed the control group who received a traditional curriculum. Their gains were also consolidated in the next school year. The success provides evidence to support the importance of the MNL and intuition of numbers in mathematics learning.

Where the brain stores the mental number line

In addition to the origin of MNL and its link with number sense and mathematics learning, studies of the MNL have investigated if there is a specific region of the brain for MNL representation. Many of these have used clinical studies, such as imaging and patient observation. Other studies have focused on understanding characteristics of MNL representations.

Evidence supports the parietal cortex playing a crucial role in mental representations of numbers (Dehaene, 2011). Abdullaev and Melnichuk (1996), using implanted cortical electrodes, recorded activity of neurones of the parietal cortex when doing arithmetic and language tasks. They found neurones in both left and right parietal cortex became active only when numerals were shown and most neurones fired more strongly during counting and calculation than during reading the same numbers as words. This finding indicates the parietal cortex is involved in number processing operations. A later meta-analysis review based on neuroimaging and neuropsychological evidence by Dehaene and colleagues (2003) found that the horizontal segment of the intraparietal sulcus appears to be the most likely site for representing numerical magnitudes.

Further evidence arising from clinical observation by a German neurologist, Gerstmann (1940 discussed in Dehaene, 2011) described a patient who suffered a lesion of the left inferior parietal region, who showed a deficit in quantitative representations of numbers. This suggests that this area of the brain plays a key role in processing numerical magnitudes and the damage to the inferior parietal area may be responsible for his loss of number sense (Dehaene, 2011).

Finally, the way the brain represent numbers appears to follow principles that are very similar to the structure of the hand's layout (Dehaene, 2011). For example, the above patient confused his index finger with his middle finger when prompted. Dehaene (2011) thus assumed

a possible connection between numbers and finger use indicating that body maps, spatial maps and the mental number line may all stem from a single structural principle that shapes connectivity in the inferior parietal cortex. It is unclear at this stage where the brain stores a MNL representation, though it is plausible that it is in the parts of the brain that are known to be used for other representations of quantity.

Studies about characteristics of MNL representations

The MNL is commonly investigated using three different approaches: number line estimation tasks, the distance effect and the SNARC effect. These are discussed in turn.

Number line estimation tasks

The number line estimation task is popular for investigating the mental number line as it explicitly requires mapping numbers onto space (Siegler & Opfer, 2003). The tasks include number-to-position tasks and position-to-number tasks. In number-to-position tasks, participants are asked to place a given number on a number line; while in position-to-number tasks, participants are required to identify the number corresponding to a given position.

The number line estimation task has been used to test three different ways participants might use to represent numbers: the logarithmic-ruler representation, the linear-ruler representation or variable representations, depending on the age of the participant and the numerical context in which the task was given. “Logarithmic representation exaggerates the distance between magnitudes of numbers at the low end and minimizes the distance at the middle and high ends” (Siegler & Booth, 2004, p. 429). For example, the represented distance between 1 and 75 would be bigger than that between 75 and 1000. In linear-ruler representations, numbers are spaced evenly with linearly increasing magnitudes (Siegler & Booth, 2004).

Distance effect

In tasks that require participants to judge which of two numbers is larger, a distance effect (first documented by Moyer and Landauer, 1967) has been noted. When the distance between two numbers is large, for example 2 and 9, participants answer quickly and accurately; however, when two numbers are closer, for example 5 and 6, response times are slower. Additionally, for equal distances between pairs of numbers such as 1 and 2, and 8 and 9, with both having the same distance of one unit, the response time for the pair 8 and 9 is slower than the pair 1 and 2. The pattern also occurs in comparing two-digit numerals, that the response time increases continuously when the numbers are increasingly close (Dehaene et al., 1990). For example, 71 and 65 takes more time to compare than 79 and 65. The authors concluded the strong distance effect supports analogue magnitude processing, which means two-digit numbers were encoded as whole-magnitude numbers, rather than comparing the digits (tens and units) separately, and transformed into mental number line representations.

SNARC effect

SNARC effect, another indicator of a MNL, refers to the mental representation of numbers, typically in a left-to-right orientation (Dehaene, 2011). This effect was observed in a study where participants were instructed to classify two numbers as smaller or larger. They held two response keys, one on the left and another one on the right. The experiment found that the subjects responded faster with fewer errors when they pressed the left key for small numbers and the right key for larger numbers (Dehaene, 2011). Similarly, in other experiments, participants were required to determine if one-digit numbers were odd or even, on a “parity judgment” task (Dehaene et al., 1993). Half of the participants were asked to press the right response key for an odd number and the left response key for an even number, then they went through with the reverse responses to keys. The other half of participants did the same but with reverse order. Regardless of instruction, they also found the same effect, with the left-hand

responses faster than right hand responses for small odd or even numbers, while the right-hand responses were faster for large odd or even numbers.

Cultural Reading Direction and Spatial–Numerical Mapping

The generation of the MNL has two possible origins: an innate, representation of biological origin or one that is at least influenced by teaching approaches and educational experience.

Cross-Cultural Evidence

One of the strongest challenges to the idea of a universal, biologically fixed MNL comes from cross-cultural research examining reading direction. Studies comparing left-to-right and right-to-left reading populations consistently show differences in SNARC direction (Dehaene et al., 1993; Shaki & Fischer, 2008, 2012; Shaki et al., 2009; Wood et al., 2008; Zebian, 2005). For example, in populations whose writing systems proceed from left to right such as English, French, and Canadian readers, the classic SNARC pattern is consistently observed, with numerical magnitude increasing from left to right (Dehaene et al., 1993; Fischer, 2008; Shaki et al., 2009; Zebian, 2005). These findings initially appeared to support the notion of a left-to-right spatial representation of numerical magnitude for learners in general.

However, evidence from right-to-left reading cultures complicates this interpretation. Studies involving Palestinian, Iranian, Hebrew, and Arabic readers have reported either reversed SNARC effects or attenuated patterns consistent with their habitual scanning direction (Dehaene et al., 1993; Shaki et al., 2009; Zebian, 2005). In these populations, smaller numbers are often associated with the right side of space and larger numbers with the left. Such findings indicate that reading direction systematically influences the orientation of spatial–numerical mapping rather than reflecting a universally fixed spatial structure.

Moreover, reading orientation is not limited to horizontal formats. In cultures where texts are traditionally arranged vertically such as Chinese, Taiwanese, and Japanese contexts, spatial–numerical associations can also follow vertical patterns. For example, Japanese participants have been shown to associate smaller numbers with lower spatial positions and larger numbers with upper positions (Ito & Hatta, 2004). Intriguingly, this is opposite to their reading direction. In contrast, Hung et al. (2008) found that Taiwanese participants demonstrated spatial–numerical mappings consistent with the vertical orientation of traditional Chinese texts, specifically, small numbers on the top and large numbers on the bottom.

To show consistent spatial mapping, reading systems need to be the same direction for both words and numbers. For example, Israeli students read words from right to left while reading numbers from left to right. In a study by Shaki et al. (2009), participants did not exhibit a clear SNARC effect, possibly due to conflicting spatial experiences. This finding suggests that congruency between reading habits for words and numbers may be necessary for stable spatial–numerical mapping.

Together, these findings suggest that reading orientation, whether left-to-right, right-to-left, or top-to-bottom plays a significant role in shaping how numerical magnitude is spatially organized.

Bilingual and Bidirectional Readers

Research on bilingual and bidirectional readers provides important insight into how reading experience shapes spatial–numerical mapping. Individuals who are fluent in languages with opposing reading directions often display context-dependent SNARC effects (Shaki & Fischer, 2008). In other words, the orientation of spatial–numerical associations can shift depending on the linguistic context activated during the task. For example, the authors asked Russian–Hebrew bilingual university students, whose mother-tongue was Russian and spoke Hebrew for at least seven years, to read a short paragraph in either Russian (left-to-right) or Hebrew (right-to-left)

before completing a parity judgment task. After reading Russian, participants exhibited the typical left-to-right SNARC effect; but after reading Hebrew, the effect was substantially weaker. A similar study with Russian-Hebrew university bilinguals by Fischer and colleagues (2009) found that when the participants classified number words (presented in either Russian or Hebrew) and Arabic digits, the SNARC effect emerged in the Russian context but disappeared in the Hebrew context. Notably, even brief and recent exposure to a single word in a particular script was sufficient to influence spatial–numerical mapping. These findings show that spatial–numerical associations remain flexible and can shift according to the active reading system.

At the same time, language proficiency appears to play a crucial role. When second-language competence is limited, spatial–numerical mapping tends to align with the dominant language. For instance, Zebian (2005) reported that both Arabic–English bilingual adults and children showed a reversed SNARC effect when their English proficiency was low. Similarly, Iranian participants who had recently migrated to France demonstrated reversed SNARC patterns consistent with right-to-left reading habits, whereas Iranian individuals with greater familiarity with left-to-right systems showed weaker reversed effects (Dehaene et al., 1993). Thus, the strength and direction of spatial–numerical mapping are influenced not only by exposure to multiple reading systems but also by the level of language proficiency and contextual activation. The evidence leads to a further question: in individuals who have not yet acquired literacy or who have had limited formal education, does a directional MNL emerge at all?

Illiterate and emerging literacy populations

Research on illiterate populations offers further important insight into the role of literacy in shaping the MNL. Zebian (2005) examined an illiterate Lebanese group who could recognise and read numbers but were unable to read words. Although these individuals were immersed in a right-to-left reading culture, they did not exhibit a reversed SNARC effect; rather, they showed no reliable spatial–numerical association. This finding suggests active engagement with written language may be necessary for spatial–numerical mapping. Given that young children, like illiterate adults, can often count objects and recite numbers before acquiring reading skills, do preschool children also show absent or unstable spatial–numerical associations?

Research (Göbel et al., 2018; Patro et al., 2016) suggests that directional tendencies can emerge before formal literacy instruction begins. For example, Opfer and Thompson (2006) found that 98% of children aged 2.5 to 5.5 years counted objects from left to right. They also found in addition and subtraction tasks, children in the MNL group (mean age = 11.9 years) more consistently added from left to right and subtracted from right to left compared to children in the no MNL group (mean age = 6.36 years). Moreover, children who displayed stable directional patterns performed more accurately when estimating larger quantities beyond the subitising range, even after controlling for age. In Ashkenazi and Mark-Zigdon's (2021) study, Hebrew-speaking preschoolers aged 3.5 to 5 years showed more accurate number-line estimations when the line was oriented from right to left rather than left to right. These findings support the hypothesis that spatial–numerical associations begin to develop early and may be based on involvement in active engagement or passive observation of spatial oriented events, such as reading books, drawing a line, or organising toys on a shelf (Patro et al., 2016).

Among spatial oriented events, shared reading books seems to directly shape young children's counting direction. Even after a short, shared reading session of only two to three minutes, children's preferred counting direction shifted toward the observed orientation (Göbel et al., 2018). Importantly, this effect occurred even when explicit counting activities were not included. Exposure to directional motion in videos did not change their counting preferences.

This shifting is only temporary after the intermediate exposure as their counting direction returned to culturally dominant patterns in a follow-up test, four weeks later.

Studies on finger counting and the SNARC effect in children and adults show different results in participants who read from left to right. In one study (Fischer, 2008), for subjects who prefer counting fingers starting from their left hands, a typical SNARC effect was found and no reliable SNARC effect in the right-starter group. However, other studies have provided contradictory results (see for example, Prete & Tommasi, 2020; Sato & Lalain, 2008). Such investigations remain limited in cultures demonstrating clear reversed SNARC effects (Pitt & Casasanto, 2014). Thus, the precise contribution finger-counting habits have to the formation of the MNL remains unclear (Pitt & Casasanto, 2014).

Overall, findings from preschool children who have not received formal schooling such as reading and mathematics instruction indicate that early spatial–numerical mapping arises from multiple interactions. However, research shows that a proportion of preschoolers (Ashkenazi & Mark-Zigdon, 2021; Opfer & Thompson, 2006) and illiterate adults (Zebian, 2005) display no, weak or inconsistent spatial–numerical associations. These patterns suggest that while directional tendencies may emerge early, formal education plays a crucial role in consolidating and refining them into a stable mental number line.

Education and spatial-numerical mapping

Formal schooling plays a crucial role in shaping how numerical magnitude is spatially organized. In left-to-right reading cultures, educational materials consistently present numbers as increasing from left to right. Tools such as number lines, rulers, graphs, number charts and written arithmetic layouts reinforce this directional structure. Among these, the number line is one of the most explicit instructional devices for representing magnitude. In primary mathematics classrooms, numbers are arranged sequentially along a horizontal line, allowing teachers to visually illustrate concepts of magnitude, numerical distance, and arithmetic operations such as addition and subtraction through spatial movement (Göbel et al., 2011). The illustration of external number lines supports the development of the MNL by providing a spatial model that children can map onto their internal representations (Schneider et al., 2009).

Research demonstrates that instructional use of number lines strengthens children's number sense, numerical understanding and arithmetic performance (Griffin et al., 1994; Ramani & Siegler, 2008; Siegler & Ramani, 2008). Targeted interventions including linear board games (Ramani & Siegler, 2008; Siegler & Ramani, 2008) and structured number lines have been shown to improve numeracy estimation accuracy (Diezmann et al., 2010), which relates strongly to mathematics achievement (Siegler & Booth, 2004). Studies have found numeracy estimation patterns shifted developmentally from predominantly logarithmic representations to a mixture of logarithmic and linear patterns, and eventually to largely linear representations across kindergarten, first-grade and second-grade children (approximately six to eight years old) (Siegler & Booth, 2004) which is a similar estimation pattern to that found by Siegler and Opfer (2003) but with older groups (grade 2, grade 4, grade 6 and adults). Specifically, second graders (mean age: 7.9 years) followed a logarithmic pattern while sixth graders (mean age: 11.8 years) and adults followed a linear pattern. The results indicate numeracy estimation accuracy improvement with age and schooling (Siegler & Opfer, 2003), which is consistent with the strength of the SNARC effect in a meta-analysis by Wood and colleagues (2008) who found that the strength of SNARC effect increases with age. As children gain mathematical experience, their estimation as well as spatial representations of number become more precise, systematic, and aligned with culturally standardised formats. This enhances the precision of the mental number line.

Implications

While there has been a growing body of research improving understanding of how reading systems shape numerical-spatial mapping in adults and typical developing children, little is known how students who might need extra support, such as those with mathematics learning disability, and bilingual children with different reading systems, can be assisted to develop a stable linear MNL. It is unknown whether students with mathematical learning disabilities shape their numerical mapping similarly or differently to typically developing children in a monolingual culture or whether their orientation of the MNL is consistent with their reading systems. Future research is needed to deepen the understanding of the development of the MNL for these students. There follows important implications for mathematics teaching. For example, the impact of explicit teaching on the use of external number lines may make it easier for children to represent numbers on a structure that is similar to their internal number line, thus supporting children to develop their MNL (Schneider et al., 2009). In addition, external number lines represent abstract mathematics ideas that help students better understand and operate with different types of numbers (Teppo & Heuvel-Panhuizen, 2014). It is therefore essential to undertake further research to investigate the impact of external representations on MNL development and the subsequent impact on mathematics education for students with learning disabilities and other learning support needs.

Conclusion

In this paper, we have canvassed research literature that has explored how humans store representations of quantity on a mental number line. Much is still to be understood in this field, not least where in the brain this is recorded, and how the representation is developed. It would appear that cultural context and education, along with age-related growth and development are important. Reading directions and education play an important role in shaping how learners map numerical information onto space. Teaching is likely to guide students in how to map the spatial-numerical association aligned with their reading system whether left-to-right, right-to-left or top-to-bottom reading direction. This helps construct the MNL, which supports magnitude understanding, arithmetic and mathematics learning more broadly. It is therefore critical to understand the impact on learners who may require extra support from teachers.

References

- Abdullaev, Y. G., & Melnichuk, K. V. (1996). Counting and arithmetic functions of neurons in the human parietal cortex. *NeuroImage (Orlando, Fla.)*, 3(3), S216–S216.
- Ashkenazi, S., & Mark-Zigdon, N. (2021). Directionality of number space associations in Hebrew-speaking children: Evidence from number line estimation. In M. Herzog, E. Gürsoy, & A. Fritz (Eds.), *Diversity Dimensions in Mathematics and Language Learning* (Vol. 24, pp. 163–182). Walter de Gruyter GmbH.
- Dehaene, S. (2011). *The number sense: How the mind creates mathematics*. OUP USA.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology. General*, 122(3), 371–396.
- Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology. Human Perception and Performance*, 16(3), 626–641. <https://doi.org/10.1037/0096-1523.16.3.626>
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20(3–6), 487–506.
- Diezmann, C. M., Lowrie, T., & Sugars, L. A. (2010). Primary students' success on the structured number line. *Australian Primary Mathematics Classroom*, 15(4), 24–28.
- Fischer, M. H. (2008). Finger counting habits modulate spatial-numerical associations. *Cortex*, 44(4), 386–392. <https://doi.org/10.1016/j.cortex.2007.08.004>
- Fischer, M. H., Shaki, S., & Cruise, A. (2009). It takes just one word to quash a SNARC. *Experimental Psychology*, 56(5), 361–366. <https://doi.org/10.1027/1618-3169.56.5.361>
- Göbel, S. M., McCrink, K., Fischer, M. H., & Shaki, S. (2018). Observation of directional storybook reading influences young children's counting direction. *Journal of Experimental Child Psychology*, 166, 49–66.

- Göbel, S. M., Shaki, S., & Fischer, M. H. (2011). The cultural number line: A review of cultural and linguistic influences on the development of number processing. *Journal of Cross-Cultural Psychology*, 42(4), 543-565. <https://doi.org/10.1177/0022022111406251>
- Griffin, S. A., Case, R., & Siegler, R. S. (1994). RightStart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In K. McGilly (Ed.) (1996), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 25–49). The MIT Press. <https://doi.org/10.7551/mitpress/1861.003.0007>
- Hung, Y.-h., Hung, D. L., Tzeng, O. J. L., & Wu, D. H. (2008). Flexible spatial mapping of different notations of numbers in Chinese readers. *Cognition*, 106(3), 1441-1450.
- Ito, Y., & Hatta, T. (2004). Spatial structure of quantitative representation of numbers: Evidence from the SNARC effect. *Memory & Cognition*, 32(4), 662-673. <https://doi.org/10.3758/BF03195857>
- Moyer, R. S., & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature*, 215(5109), 1519–1520. <https://doi.org/10.1038/2151519a0>
- Nunez, R. E. (2011). No innate number line in the human brain. *Journal of Cross-Cultural Psychology*, 42(4), 651–668. <https://doi.org/10.1177/0022022111406097>
- Opfer, J. E., & Thompson, C. A. (2006). Even early representations of numerical magnitude are spatially organized: Evidence for a directional magnitude bias in pre-reading preschoolers. *Proceedings of the Annual Meeting of the Cognitive Science Society*, 28(28). Retrieved from <https://escholarship.org/uc/item/4477g2wg>
- Patro, K., Nuerk, H.-C., & Cress, U. (2016). Mental number line in the preliterate brain: The role of early directional experiences. *Child Development Perspectives*, 10(3), 172–177.
- Pitt, B., & Casasanto, D. (2014). Experiential origins of the mental number line. *Proceedings of the Annual Meeting of the Cognitive Science Society*, 36(36).
- Prete, G., & Tommasi, L. (2020). Exploring the interactions among SNARC effect, finger counting direction and embodied cognition. *PeerJ (San Francisco, CA)*, 8(3), Article e9155.
- Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, 79(2), 375-394.
- Sato, M., & Lalain, M. (2008). On the relationship between handedness and hand-digit mapping in finger counting. *Cortex*, 44(4), 393-399. <https://doi.org/10.1016/j.cortex.2007.08.005>
- Schneider, M., Grabner, R. H., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in grades 5 and 6. *Journal of Educational Psychology*, 101(2), 359-372. <https://doi.org/10.1037/a0013840>
- Shaki, S., & Fischer, M. H. (2008). Reading space into numbers – a cross-linguistic comparison of the SNARC effect. *Cognition*, 108(2), 590-599. <https://doi.org/10.1016/j.cognition.2008.04.001>
- Shaki, S., & Fischer, M. H. (2012). Multiple spatial mappings in numerical cognition. *Journal of Experimental Psychology: Human Perception and Performance*, 38(3), 804-809.
- Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. *Psychonomic Bulletin & Review*, 16(2), 328-331.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, 75(2), 428-444. <https://doi.org/10.1111/j.1467-8624.2004.00684.x>
- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, 14(3), 237-243.
- Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, 11(5), 655-661.
- Teppo, A., & Heuvel-Panhuizen, M. (2014). Visual representations as objects of analysis: the number line as an example. *ZDM*, 46(1), 45–58. <https://doi.org/10.1007/s11858-013-0518-2>
- Wood, G., Willmes, K., Nuerk, H.-C., & Fischer, M. H. (2008). On the cognitive link between space and number: a meta-analysis of the SNARC effect. *Psychology Science*, 50(4), 489.
- Zebian, S. (2005). Linkages between number concepts, spatial thinking, and directionality of writing: The SNARC effect and the reverse SNARC effect in English and Arabic monoliterates, biliterates, and illiterate Arabic speakers. *Journal of Cognition and Culture*, 5(1-2), 165-190.

From Uncertainty to Enacted Agency: A Mother's Developing Agency in Supporting Mathematics Learning at Home

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This instrumental case study examines how a Queensland mother developed agency in supporting her Year 1 child's mathematics learning at home during COVID-19 school closures (2020–2021). Findings suggest one possible trajectory from constrained agency, through emerging agency shaped by relational recognition of what others could contribute (Edwards, 2010), to enacted pedagogical judgement that the relational process made possible. This trajectory indicates that parental agency in mathematics education is not acquired through knowledge alone but developed through relational conditions, with implications for how schools design support for families.

When schools closed during the COVID-19 pandemic, parents were unexpectedly required to take on sustained responsibility for their children's mathematics learning at home. How parents developed the capacity to do this and what relational conditions made it possible is not yet well understood.

Introduction

During COVID-19 lockdowns, educational institutions across the globe transitioned to emergency remote education, an unplanned, crisis-driven shift from face-to-face teaching and learning (Bond, 2020). In Australia, schools rapidly implemented remote teaching and learning approaches to maintain continuity of learning (Brown et al., 2020; Kalogeropoulos et al., 2021). In Queensland, Australia, school-aged children experienced several interruptions and learning@home (Queensland Department of Education, 2020) during 2020 and 2021. As a result, home became the central site of learning for many students. This shift raised urgent questions about parents' capacity for decision making and sense of agency when supporting their children's learning in unfamiliar and uncertain circumstances, particularly in mathematics, the subject that parents often find most difficult to support (Muir, 2018; Wadham et al., 2019).

Research has documented how parents experienced and navigated the challenge of supporting their children's mathematics learning at home during COVID-19 school closures (e.g., Darragh & Franke, 2022; Harper et al., 2021; Murphy et al., 2023). However, there is limited insight into how parents' agency in supporting their children's mathematics learning developed over time, or what relational conditions enabled that development.

Drawing on data from a larger qualitative doctoral research project conducted in Brisbane, Queensland, the study addresses the research question: How did a mother develop agency in supporting her child's mathematics learning at home during COVID-19 school closures?

Literature Review and Theoretical Framework

Although parental engagement is encouraged in children's learning, mathematics is widely considered the most challenging subject for parents to support at home (Muir, 2018; Wadham et al., 2019). Compared to reading, families engage less frequently in their children's mathematics learning at home (Muir, 2018; Sheldon et al., 2010). Reasons for this include unfamiliar pedagogical approaches that differ from how parents themselves learned

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 321–328). MERGA.

mathematics (Bofferding et al., 2016; Jay et al., 2018; Palfy et al., 2020), mathematics anxiety or negative prior learning experiences (Jay et al., 2018), and challenges in recognising and enacting the mathematical potential of everyday home contexts (Metzger et al., 2019). The difficulty parents face in supporting mathematics learning at home became particularly visible during COVID-19 school closures, when parents were required to take on sustained responsibility for their children's mathematics learning in an emergency context for which most felt unprepared (Cahoon et al., 2021; Garbe et al., 2020).

A common response to these challenges has been to focus on building parental mathematical knowledge and competence, with evidence showing improved parental engagement and increased children's mathematics achievement (Knapp et al., 2017). Intervention-based initiatives such as Muir's (2018) take-home numeracy bags and maths club for parents, Daniel et al.'s (2022) take-home game kit, and Berkowitz et al.'s (2015) structured family mathematics interactions have shown positive impacts on supporting parental engagement and students' mathematics learning. However, less attention has been given to how parents develop the capacity to sustain mathematics engagement over time, particularly through the relational and contextual conditions that shape whether parents are able to act on whatever knowledge or resources they have access to.

Central to addressing this gap is a distinction between two different orientations to parental engagement. Goodall and Montgomery (2014) propose a parental engagement continuum that distinguishes between school-oriented involvement, where parents respond to school-defined activities, and learning-centred engagement, where parents exercise agency and make purposeful choices about their children's learning. Later, Goodall (2022) extends this by arguing that parental engagement is not a set of discrete activities but a relational phenomenon, shaped by ongoing interactions among parents, children, school staff, and the wider community, a process to be sustained over time rather than accomplished. However, neither framework fully addresses how parents develop this capacity to move towards learning-centred engagement, particularly in contexts of disruption and uncertainty.

Edwards' (2010) concept of relational agency addresses this directly. Grounded in both sociocultural and activity theory, Edwards (2010) conceptualises agency as relational: it emerges through interaction with others and the resources available in a complex activity system, rather than as a personal trait that individuals possess internally. The concept involves responding to a complex problem that cannot be solved alone, recognising the resources and motives that others contribute to that problem, and developing the capacity to align one's own responses to the expanded understandings that emerge from working with them. In this study, relational agency is not used to position the parent as a professional teacher, or to evaluate the quality of her instructional practices in supporting her child's mathematics learning at home. Rather, it provides a lens for understanding how the parent's agency was achieved through interaction with relational supports over time in a context of uncertainty, responsibility, and lack of formal expertise.

Research has documented what parental engagement in mathematics education at home during COVID-19 looks like. For example, in the United States, Harper et al. (2021) drew on social media data and parent surveys to show that parents reimagined the home as a site of mathematics learning by combining school-provided materials with curated digital resources and everyday activities, highlighting both the emergence of networked supports and parents' continued reliance on teachers as key relational partners. In New Zealand, Darragh and Franke (2022) found significant variability in parents' capacity to support mathematics learning at home, often depending on the support and resources provided by schools and teachers rather than parents alone, suggesting that parental agency is shaped by broader relational conditions. In Australia, Murphy et al. (2023) identified three types of parental engagement, namely monitors, facilitators, and enhancers, based on the roles parents assumed and the forms of

capital that they activated during school closures. Monitors completed school-set tasks with minimal personal involvement; facilitators worked alongside their children, adapting tasks to their circumstances; and enhancers drew on their confidence in mathematics to critique set tasks and source alternative learning experiences. These three categories suggest a progression along Goodall and Montgomery's (2014) parental engagement continuum from school-oriented involvement towards learning-centred engagement. What distinguished enhancers was not simply greater mathematical knowledge, but their capacity to recognise the resources and motives that others could contribute, which aligns with Edwards' (2010) relational agency.

Drawing on this body of work, the study conceptualises parental agency as having two interconnected dimensions. The first concerns what parental agency involves: the capacity to make purposeful choices and judgements about how to support children's mathematics learning, rather than simply complying with school-directed activities (Goodall & Montgomery, 2014). The second concerns how such agency is achieved: not as a fixed individual capacity acquired through knowledge alone, but through a growing ability to recognise what others contribute to the problem of supporting mathematics learning and to align one's own responses to the understandings that emerge from those interactions (Edwards, 2010).

Methodology

Research Design

This paper draws on data from a larger qualitative doctoral research project examining variation in parents' experiences of supporting their children's mathematics learning at home during COVID-19 school closures in Brisbane, Queensland, Australia. Fifteen parents of Preparatory (Prep) to Year 3 students were purposefully recruited using pre-interview criteria to represent variation in family contexts and home learning circumstances. For this paper, an instrumental case study design (Stake, 2003) was adopted, with one mother selected from this larger sample whose retrospective account provided sufficient detail over time to examine the processes through which parental agency develops.

Participant and Context

The participant, referred to throughout by the pseudonym Belinda, is a mother of three children. During the COVID-19 school closures, her eldest child was in Year 1 at a Brisbane Catholic primary school. She was a flight attendant on maternity leave at the time and also cared for a toddler and a newborn. Her husband was a Fly-In-Fly-Out (FIFO) worker on a "two-on, two-off" roster. Belinda's mother stayed with the family during the lockdown period, providing additional support. The family had access to digital devices and internet connections. The school provided structured learning tasks via an online platform, school-recommended YouTube educators, and scheduled regular online check-ins with students.

Data Collection and Analysis

A 45-minute face-to-face semi-structured interview was conducted with Belinda in May 2025. The interview invited Belinda to reflect on her experiences of supporting her child's mathematics learning at home during COVID-19 school closures, including challenges, strategies, sources of support, and changes over time. Although retrospective accounts over a five-year period carry inherent limitations in recall accuracy, they also provide valuable insight into how participants narrate their own development over time, which is central to this study's focus. To mitigate this, Belinda was encouraged to draw on digital artefacts such as email communications with her child's school and teacher, and photos of home learning setups.

Interview data were transcribed verbatim using an institutionally approved online transcription service and analysed iteratively using NVivo. Relational agency (Edwards, 2010)

informed the analysis as an interpretive lens, guiding attention towards how Belinda positioned herself and others in relation to the problem of supporting mathematics learning at home. Coding focused on passages in which Belinda described her role, her decision making, and the relational supports she drew upon, with attention to how these descriptions shifted across the timeline of her account. Coding and emerging interpretations were discussed with the supervisory team to support analytical rigour. The three phases reported here were not pre-determined but emerged inductively from qualitatively distinct shifts in Belinda's self-positioning and her recognition of relational others across her account.

Findings

Belinda's retrospective account is interpreted here as suggesting one possible trajectory, organised into three phases.

Phase 1 Constrained Agency: Replicating School

In the early stage of learning from home, Belinda described feelings of uncertainty and self-doubt about her capacity to support her child's mathematics learning alongside a fear of failing him. She expressed:

I hope he passes grade one...he's got me as his teacher. I was just worried that would I know how to teach him, would I be a good teacher for him... I'm not a teacher, so... the ways that I show him is that the correct way that they'd be taught in class.

Here Belinda framed mathematics teaching as requiring the expertise of a professional teacher. She articulated concerns about legitimacy ("I'm not a teacher"), pedagogical uncertainty about whether her methods aligned with school approaches, and anxiety about her child's school progress, a form of self-positioning documented in research on parents navigating home schooling during COVID-19 (Hoskins & Wainwright, 2025). Belinda also connected this anxiety explicitly to her own past experiences of learning mathematics: "Learning was hard for me when I was at school... it's my own learning insecurities coming out for my own child." Although relational resources such as family support, teacher communication, and access to the school's online platform were available, Belinda had not yet recognised the challenge of supporting mathematics learning at home as one that could be worked on through the resources and motives that others around her could contribute.

Phase 2 Emerging Agency: Recognising Relational Support

As learning from home continued, Belinda described a shift towards taking action despite her ongoing uncertainty, drawing on relational resources to make mathematics learning work for her child. She explained:

I tried to make maths learning fun. So, there were like colouring-ins... I laminated Post-it notes, counting 2, 4, 6, 8, 10, and I put them around, and then he had to go and like a hide and seek and find the numbers... We did lots of making shapes out of paper. We watched YouTube videos... I tried to do songs.

The activities of skip-counting and geometry construction, the use of school-recommended YouTube videos, and arithmetic songs were not random substitutes for schoolwork. Rather, they represented Belinda's own interpretations of the Year 1 mathematics curriculum, translated into forms that she considered workable and engaging at home. The shift from passive compliance with school-provided work to active, curriculum-aligned experimentation signalled the beginning of emerging agency expressed through pedagogical initiative.

Relational supports were central to sustaining this shift and Belinda's engagement with them reflected Edwards' (2010) relational agency: recognising the motives and resources that others could contribute to a complex problem, and working with these to expand what the problem is understood to require. Three instances of this are particularly significant. First,

Belinda described her mother as a “neutraliser” who intervened during moments of frustration in mathematics sessions:

When she could hear I was getting really frustrated, she would put out the candle like she was the neutraliser, OK, now it's a break time, we'll move on to the next topic, so she helped to keep us on track.

Belinda's mother contributed not mathematical knowledge but an emotional motive: to sustain the conditions under which learning could continue. By recognising and working with this, Belinda implicitly expanded her understanding of what supporting mathematics at home required. It was not only a pedagogical challenge but an emotional labour challenge that required relational management. Second, Belinda extended her network beyond the household, contacting a teacher friend who provided supplementary mathematics worksheets. What Belinda recognised here was a different kind of motive and resource: professional, curriculum-aligned knowledge, which expanded what mathematics engagement at home could include. Third, Belinda sought ongoing guidance from her child's classroom teacher, asking whether her instructional approaches were aligned with school methods: “I would have communication with his teacher... I would be asking her for guidance on, this is how he did this, this is how I showed him. Is that correct? She always replied to that.” By proactively seeking guidance, rather than waiting for the school or teacher to direct her, Belinda showed initiative and positioned the classroom teacher as a resource rather than a distant authority.

Phase 3 Enacted Agency: Acting with Pedagogical Judgement

The transition to enacted agency was marked not by teacher feedback or formal assessment, but by an unexpected moment at home. When her husband called during the lockdown period, Belinda overheard her child spontaneously recount their mathematics activities and demonstrate his mathematical understanding:

Little things like when Scott (pseudonym) would ring and he'd ask him, how was your day? What did you do? And I could hear him answer him with things that we had done... he had counted, odds or evens for Scott over the phone. I guess that made me feel a bit of relief. He got it.

This moment marked an important shift in how Belinda interpreted her child's learning. Her child was not responding to a direct mathematics question but answering “how was your day?” His spontaneous explanation of odd and even numbers indicated that the concept had been genuinely internalised, not simply completed as a task. Her response of “he got it” signalled the first moment in which she interpreted her child's learning as evidence that her own support was effective. This marked a critical shift, from seeking external verification to trusting her own judgement of the situation. Further validation came from hearing her child describe the home learning experience positively to his teacher:

I found it quite amazing hearing him talk so happily to his teacher about the at home learning experience... and I was worried, was I doing a good job? He has only positive memories of it...He just thought it was terrific.

Her earlier fear of having failed her child was directly contradicted by her child's positive recollection. These moments collectively shifted how Belinda evaluated her own capability from measuring herself against a professional teaching standard she felt she could not meet, to recognising her child's learning as a meaningful indicator of her own effectiveness.

By the end of the school closure, Belinda's account reflected enacted agency, where she made independent pedagogical decisions based on her own judgement of her child's learning needs without requiring school or teacher validation. The shift was captured in her reaction to the announcement that schools would reopen: “I was actually sad when Anastasia [Premier] announced that the kids were going back to school cos I thought, no, no, no, I've got it now. Like I'm not nervous and worried and I know how I can do this.” The phrase “I've got it now” is the direct inversion of Phase 1's central anxiety about whether she was doing it right. What

had changed was not her mathematical knowledge, but her capacity to act on her own judgement about what her child needed.

This enacted agency was evident in Belinda's pedagogical judgement. Rather than treating school-provided tasks as fixed scripts, she adapted them into contextualised learning experiences. For example, the school's "number of the day" task became an opportunity for hands-on mathematical reasoning. As Belinda explained, "he had to go around and find 20 of something... it could have been 20 pieces of Lego... then he had to draw it out in groups of five to make 20." This task engaged core Year 1 mathematics content such as grouping, early place-value ideas, and reasoning about numbers through counting and representation in a physical, contextualised form that Belinda judged appropriate for her child. More importantly, Belinda made this adaptation herself. The school provided the task structure, but she decided how to enact it. The Lego pieces functioned as a material resource through which an abstract number concept was made tangible. Belinda's capacity to make this kind of contextualised pedagogical decision was built on the expanded understanding of her child's learning that had developed through the relational interactions of Phase 2.

Phase 3 did not represent a departure from relational support but its consolidation. What changed was not that Belinda no longer needed others, but that she no longer needed school or teacher validation. This distinction was most clearly evidenced in her decision, immediately after schools reopened, to arrange weekly mathematics tutoring. Rather than feeling solely and anxiously responsible, she now understood herself as part of a purposefully constructed support network. She explained, "If I didn't have the time or I was unable to help him, I knew that she was. If he missed it in class, then I knew that she was able to fill in those gaps."

Five years after the school closures, the mathematically rich home environment Belinda has constructed and maintained reflects a further dimension of this transformed understanding:

On the stairs at home, I have the three times table and the four times table, so every time we walk up and down the stairs... we have to say three times one is three, three times two... I have the even numbers and the odd numbers on different sides of the hallway. We have maths tables as placemats. It might be accidental learning... it's all around our house, even to today.

The COVID experience transformed how Belinda understood her ongoing role in supporting her child's mathematics learning: not as a temporary emergency substitute for school, but as a sustained and purposeful dimension of family life.

Discussion

Belinda's trajectory was not triggered by formal parent training or increased mathematical content knowledge. Rather, her capacity to act with pedagogical judgement emerged through her growing ability, particularly through Phase 2, to recognise what others could contribute to the problem of supporting mathematics learning at home, and to align her own responses to the expanded understandings that emerged (Edwards, 2010).

Building on Murphy et al.'s (2023) typology, this study reveals that monitors, facilitators, and enhancers can be understood not only as co-existing patterns across a sample but as phases through which a single parent developed over time. Through the lens of relational agency (Edwards, 2010), the key turning points were not moments of knowledge acquisition but moments of relational recognition: her mother's emotional support enabled persistence; her teacher friend expanded her sense of what mathematics engagement could include; the classroom teacher's feedback provided pedagogical verification; and her child's spontaneous explanation of odd and even numbers confirmed evidence of learning. In doing so, this study responds to Murphy et al.'s (2023) call for further research to develop their typology, offering a relational account of how movement between engagement modes may be explained.

These findings are consistent with Darragh and Franke's (2022) argument that the success of mathematics home learning often depended on the support and resources available to families

rather than on parents alone. Belinda's case illustrates this structurally, and extends it: in Phase 1, the relational supports were present but she had not yet recognised them as resources for the problem of supporting mathematics learning. The shift that enabled her agency was not the arrival of new resources but a change in how she understood the problem as one to be worked on through others rather than solved alone. It suggests that what constrains parental agency in mathematics education may not always be a deficit of knowledge or resources, but the absence of relational conditions that enable parents to recognise and work with what is already available.

The implications for practice are therefore relational as well as instructional. Many initiatives designed to strengthen parental engagement in children's mathematics learning focus on building parents' mathematical content knowledge and pedagogical competence, with the aim of improving students' mathematics achievement (e.g., Knapp et al., 2017; Muir, 2018). If parental agency in supporting mathematics learning develops through relational conditions, then schools may need to do more than simply provide mathematical resources for families. They may also need to cultivate the relational conditions that enable parents to recognise and mobilise support, including accessible communication with teachers, peer networks beyond the school, and ongoing dialogue about mathematics learning. When such conditions are present, parents who do not see themselves as mathematically confident can still provide rich and sustained opportunities for mathematics learning at home. These findings suggest that the relational network surrounding a parent may matter as much as the mathematical knowledge they bring to the task.

This study draws on a single instrumental case and does not represent the experiences of all parents supporting mathematics learning at home during COVID-19 school closures. Instead, the value of the instrumental case study is analytical depth. It reveals a developmental process that studies focusing on patterns across a larger sample cannot easily capture. The findings rely on a retrospective account situated within the specific context of pandemic schooling in Brisbane, Queensland. It should be noted that Belinda's relational context was relatively well-resourced, including an extended family network, digital access, and a responsive classroom teacher. The developmental process identified here may operate differently where such conditions are absent.

Acknowledgments

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References

- Berkowitz, T., Schaeffer, M.W., Maloney, E. A., Peterson, L., Gregor, C., Levine, S. C., & Beilock, S. L. (2015). Math at home adds up to achievement in school. *Science*, 350(6257), 196–198. <https://doi.org/10.1126/science.aac7427>
- Bofferding, L., Kastberg, S., & Hoffman, A. (2016). Family mathematics nights: An opportunity to improve preservice teachers' understanding of parents' roles and expectations. *School Science and Mathematics*, 116(1), 17–28. <https://doi.org/10.1111/ssm.12109>
- Bond, M. (2020). Schools and emergency remote education during the COVID-19 pandemic: A living rapid systematic review. *Asian Journal of Distance Education*, 15(2), 191–247.
- Brown, N., Te Riele, K., Shelley, B., & Woodroffe, J. (2020). *Learning at home during COVID-19: Effects on vulnerable young Australians* (Independent Rapid Response Report). Peter Underwood Centre for Educational Attainment, University of Tasmania.
- Cahoon, A., McGill, S., & Simms, V. (2021). Understanding home education in the context of COVID-19 lockdown. *Irish Educational Studies*, 40(2), 443–455.
- Daniel, L., Doyle, T., & Kaeschagen, C. (2022). Supporting parents to co-produce differentiated learning opportunities in mathematics. *Education* 3–13, 50(5), 612–626.
- Darragh, L., & Franke, N. (2022). Lessons from lockdown: Parent perspectives on home-learning mathematics during COVID-19 lockdown. *International Journal of Science and Mathematics Education*, 20(7), 1521–1542. <https://doi.org/10.1007/s10763-021-10222-w>

- Edwards, A. (2010). Relational agency: Working with other practitioners. In *Being an expert professional practitioner: The relational turn in expertise* (pp. 61–79). Springer. https://doi.org/10.1007/978-90-481-3969-9_4
- Garbe, A., Ögurlu, U., Logan, N., & Cook, P. (2020). Parents' experiences with remote education during COVID-19 school closures. *American Journal of Qualitative Research*, 4(3), 45–65. <https://doi.org/10.29333/ajqr/8471>
- Goodall, J., & Montgomery, C. (2014). Parental involvement to parental engagement: A continuum. *Educational Review*, 66(4), 399–410. <https://doi.org/10.1080/00131911.2013.781576>
- Goodall, J. (2022). A framework for family engagement: Going beyond the Epstein Framework. *Wales Journal of Education*, 24(2). <https://doi.org/10.16922/wje.24.2.5>
- Harper, F. K., Rosenberg, J. M., Comperry, S., Howell, K., & Womble, S. (2021). #Mathathome during the COVID-19 pandemic: Exploring and reimagining resources and social supports for parents. *Education Sciences*, 11(2), 60. <https://doi.org/10.3390/educsci11020060>
- Hoskins, K., & Wainwright, E. (2025). "I am not a teacher!" The challenges of enacting home schooling during the COVID-19 pandemic among low-income families of primary-aged children. *Education 3–13*, 53(4), 600–614. <https://doi.org/10.1080/03004279.2023.2210590>
- Jay, T., Rose, J., & Simmons, B. (2018). Why is parental involvement in children's mathematics learning hard? Parental perspectives on their role supporting children's learning. *SAGE Open*, 8(2), 215824401877546. <https://doi.org/10.1177/2158244018775466>
- Kalogeropoulos, P., Roche, A., Russo, J., Vats, S., & Russo, T. (2021). Learning mathematics from home during COVID-19: Insights from two inquiry-focussed primary schools. *Eurasia Journal of Mathematics, Science and Technology Education*, 17(5), Article em1957–16. <https://doi.org/10.29333/ejmste/10830>
- Knapp, A., Landers, R., Liang, S., & Jefferson, V. (2017). We all as a family are graduating tonight: A case for mathematical knowledge for parental involvement. *Educational Studies in Mathematics*, 95(1), 79–95. <https://doi.org/10.1007/s10649-016-9741-4>
- Metzger, S. R., Sonnenschein, S., & Galindo, C. (2019). Elementary-age children's conceptions about mathematics utility and their home-based mathematics engagement. *The Journal of Educational Research*, 112(4), 431–446. <https://doi.org/10.1080/00220671.2018.1547961>
- Muir, T. (2018). Using mathematics to forge connections between home and school. In V. Kinnear, M. Y. Lai, & T. Muir (Eds.), *Forging connections in early mathematics teaching and learning* (pp. 173–190). Springer Nature. <https://doi.org/10.1007/978-981-10-7153-9>
- Murphy, S., Danaia, L., Tinkler, J., & Collins, F. (2023). Parents' experiences of mathematics learning at home during the COVID-19 pandemic: A typology of parental engagement in mathematics education. *Educational Studies in Mathematics*. <https://doi.org/10.1007/s10649-023-10224-1>
- Palfy, K., McFeetors, P. J., & McGarvey, L. M. (2020). Mathematics curriculum change: Identifying parental expectations. *Journal of Research in Science, Mathematics and Technology Education*, 3(2), 51–72. <https://doi.org/10.31756/jrsmte.322>
- Queensland Department of Education. (2020). *Learning@home*. Queensland Government. Retrieved 8 October 2021 from <https://education.qld.gov.au/curriculum/learning-at-home>
- Sheldon, S. B., Epstein, J. L., & Galindo, C. L. (2010). Not just numbers: Creating a partnership climate to improve math proficiency in schools. *Leadership and Policy in Schools*, 9(1), 27–48. <https://doi.org/10.1080/15700760802702548>
- Stake, R. E. (2003). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Strategies of qualitative inquiry* (2nd ed., pp. 134–164). Sage Publications.
- Wadham, B., Darragh, L., & Ell, F. (2019). Tensions in mathematics home-school partnerships. In G. Hine, S. Blackley, & A. Cooke (Eds.), *Mathematics Education Research: Impacting Practice. Proceedings of the 42nd annual conference of the Mathematics Education Research Group of Australasia, Perth* (pp. 731–738). MERGA.

Making Multiplication Structure Visible: Spatial Artefacts, Instrumental Genesis, and Emerging Distributive Reasoning

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This study extends instrumentation theory by examining how spatial artefacts support Year-3 students' development of multiplicative thinking through composition and decomposition. Drawing on classroom observations, we trace students' instrumental genesis—how they reconfigure artefacts (instrumentalisation) and develop utilisation schemes (instrumentation). The findings show a trajectory toward deliberate product composition and flexible decomposition, laying the groundwork for distributive reasoning. We introduce *spatially mediated instrumental genesis* to characterise this spatially grounded scheme development.

Introduction

Understanding multiplication as more than repeated addition remains a central challenge in the early years mathematics (Fosnot & Dolk, 2001). A key step is the development of *decomposition strategies* (e.g., 5×12 as $5 \times 10 + 5 \times 2$), which support flexible computation and students' emerging appreciation of distributive reasoning (Fosnot & Dolk, 2001; Kinzer & Stanford, 2013). Spatial representations—including arrays, area-based rectangles, and bar-type models—are commonly used to foreground such composition and partitioning (Barmby et al., 2009; Harries & Barmby, 2006), yet their mathematical potential depends on how learners take them up as resources for thinking and justifying rather than as display forms alone (Fosnot & Dolk, 2001; Putrawangsa & Patahuddin, 2022). From the instrumental approach, introducing an artefact does not itself ensure mathematical productivity. An artefact becomes an instrument only through learners' developing utilisation schemes. However, how students' scheme development is shaped by *spatial structuring work*—particularly when learners move across related spatial artefacts—remains under specified (Rabardel, 2002).

This paper analyses Year-3 students' engagement with two spatial artefacts—*multiplication bars* (pre-made rectangular tiles labelled $a \times b$, physically arranged to compose and decompose products - see Figure 1) and a *bar model* (a drawn rectangular/tape representation flexibly partitioned to express multiplicative structure - see Figure 5)—within a three-task classroom intervention. Using instrumentation theory, we examine how students' ways of acting with and reasoning through spatial artefacts support their composition and decomposition of multiplication, and how this activity contributes to emerging distributive reasoning. We argue that making spatial mediation explicit extends instrumentation theory for domains in which conceptual development is tightly linked to spatial structuring. This paper addresses the following research question: *How do spatial artefacts mediate Year-3 students' instrumental genesis for decomposition strategies and emerging distributive reasoning in multiplication? In other words*, we elaborate spatially mediated instrumental genesis by specifying how students' utilisation schemes depend on aligning, matching, and partitioning spatial measures across spatial artefacts.

Theoretical framework

The instrumental approach distinguishes artefacts from instruments. An artefact is the material or symbolic object made available for activity, whereas an instrument is a mixed entity comprising the artefact and the utilisation schemes through which a user organises and carries out goal-directed action (Rabardel, 2002; Shvarts et al., 2021). This distinction foregrounds that learners must construct ways of using tools and representations if these are to become mathematically functional for learners—for example, to support the composition and decomposition of products and the justification of distributive properties.

Instrumental genesis refers to the process through which an artefact becomes an instrument (Artigue, 2002; Rabardel, 2002). Rabardel (2002) characterises instrumental genesis as a dual, intertwined process: instrumentation (processes oriented toward the subject, involving the emergence and evolution of schemes) and instrumentalisation (processes oriented toward the artefact, involving selection, transformation, regrouping, and attribution of functions). For analysing students' work with spatial representations, this duality foregrounds both how artefacts shape activity and how students progressively invest the artefacts with mathematical functions (e.g., by selecting multiplication-bar tiles whose side lengths match the intended factors, aligning tiles edge-to-edge to compose a target product, splitting a composed arrangement into sub-rectangles that correspond to a chosen decomposition $12 = 10 + 2$), and re-representing that spatial structure as a drawn bar model by marking partitions and labelling the associated partial products.

To trace learning in tool-mediated activity, the instrumental approach commonly draws on the notion of *scheme*, defined as a relatively stable organisation of activity for a class of situations (Shvarts et al., 2021; Vergnaud, 1998). In classroom settings, schemes are inferred from students' techniques and from recurring patterns in their interactions with artefacts. These techniques may serve both pragmatic and epistemic functions: pragmatic in producing results, and epistemic in supporting mathematical meaning-making and justification. This epistemic function, however, often depends on careful task design and classroom institutionalisation (Artigue, 2002; Shvarts et al., 2021).

In this paper, the focal artefacts are spatial—representational and manipulable resources whose mathematical potential depends centrally on spatial organisation and transformation (e.g., covering, composing, partitioning, recombining). Building on our prior work on spatialised instrumentation (Putrawangsa & Hasanah, 2020) and research examining mechanisms that support students' mathematical understanding during spatial reasoning interventions (Lowrie & Logan, 2023; Lowrie et al., 2020), we propose spatially mediated instrumental genesis as an extension of instrumentation theory that foregrounds how scheme development is mediated by spatial structuring operations. Analytically, we trace (a) instrumentation through the emergence of utilisation schemes for coordinating the two factor dimensions (e.g., keeping one factor fixed while partitioning the other to support decomposition), and (b) instrumentalisation through students' spatial actions on the artefacts (e.g., rotating, swapping, composing multiplication bars; creating and labelling partitions on the bar model).

Method

This study reports a focused qualitative analysis situated within a broader design-based research (DBR) project on spatialising multiplication learning for Year-3 students in Indonesia. DBR blends theory-driven design with systematic study of learning in authentic settings, aiming to contribute to both practice and theory (Design-Based Research Collective, 2003).

The classroom experiment involved 24 Year-3 students (aged 9–10) in an Indonesian primary classroom. The intervention comprised three sequenced tasks using spatial artefacts

(i.e., multiplication bars and a bar model). In Task 1 (compare areas), students used a set of multiplication bars (from 1×1 to 5×5 , see Figure 1) to compare the size of two rectangles (A and B, see Figure 2). Students first predicted which rectangle was larger and then used bar configurations to validate their prediction inferring dimensions and products.

Figure 1

A set of multiplication bars

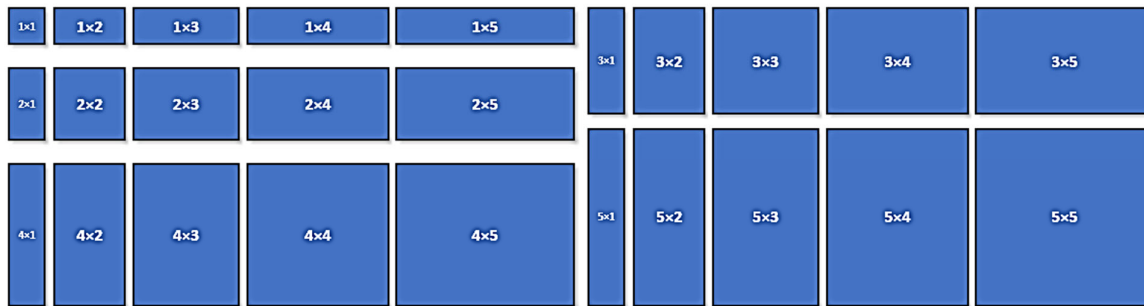
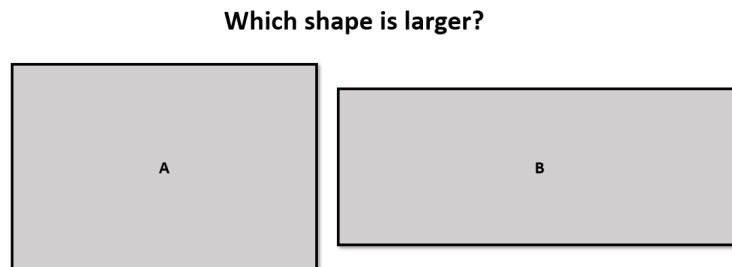


Figure 2

Two rectangles for comparison task



Following the first task, Task 2 (composing a multiplication) asked the students to generate a rectangle by combining at least two multiplication bars and to determine the dimensions and product of the resulting composite rectangle. In the final task, Task 3 (decomposing a multiplication), students solved 5×12 without multiplication bars by partitioning a bar model into two or three parts, computing partial products, and recombining them.

Data were collected through classroom observation conducted during the intervention, including video-recorded classroom interaction and detailed transcripts capturing teacher prompts and student explanations.

Data were analysed qualitatively to characterise students' instrumental genesis as they worked with multiplication bars and the bar model to develop and justify decomposition strategies. Drawing on the instrumental approach, we coded episodes for evidence of instrumentation (the emergence and evolution of utilisation schemes shaped in part by spatial constraints and affordances) and instrumentalisation (students' purposeful transformations and reconfigurations of the artefacts). This analysis focused on how students used spatial structuring to reason about decomposition and to articulate emerging distributive principles. The data points presented in this paper represent the predominant strategies employed by the students in each task.

Findings

Across the three tasks, students' activity illustrates spatially mediated instrumental genesis: that is, the spatial artefacts (multiplication bars and the bar model) progressively became instruments for coordinating dimensions, composing products, and justifying whole products

as sums of partial products, thereby fostering decomposition strategies and emerging distributive reasoning in multiplication.

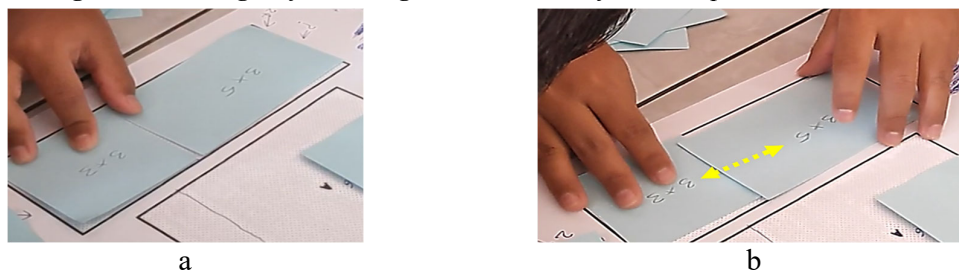
Task 1: Relating factor pairs through composed multiplications

Task 1 was designed to support understanding that different factor pairs can yield equivalent products and that such products can be interpreted as compositions of related multiplications (e.g., 4×6 as $4 \times 5 + 4 \times 1$; 3×8 as $3 \times 5 + 3 \times 3$). During the comparison task, initially, students justified their comparisons perceptually (“wider” vs “taller”), suggesting that factor relations were not yet their primary basis for judging equivalence. Through working with the multiplication bars, however, students began to coordinate factor relations explicitly and to use composition strategies to infer dimensions and determine the rectangles’ products.

In terms of instrumentalisation, students initially undertook substantial technical work with the bar set—scanning available bars, selecting candidates, testing placements, and repeatedly rotating, swapping, and replacing pieces to construct a complete covering configuration for each rectangle (Figure 3a). This instrumentalisation was mathematically consequential because it produced configurations that instantiated a common factor (a shared dimension) and a composed factor (a sum of lengths), thereby making factor relations perceptible and available for reasoning.

Figure 3

Tracing the height and the length of a rectangle with the aids of the multiplication bars



Instrumentation and an emerging utilisation scheme were evident during this task. Once a configuration was established, students developed a scheme called *common-factor coordination* consisting of epistemic techniques such as (1) identifying the common factor from the shared dimension of the bars; (2) composing the other factor by adding the length labels; (3) interpreting the whole product as the sum of partial products; and (4) restating the whole as a single multiplication. For Shape B, for instance, they used 3×5 and 3×3 , composed 8 as $5 + 3$ with the common factor 3, and concluded that 3×8 equals 24 (Figure 3b). The bars thus shaped students’ activity toward recognising equivalence through composed multiplications.

From a learning-process perspective, students’ instrumental genesis was evident. By the end of Task 1, multiplication bars functioned as an instrument for relating two multiplications through factor composition. Here, students could justify that 3×8 was equal to $3 \times 5 + 3 \times 3$ by expressing 3×8 as a composition of two related products sharing a common factor, providing an early form of distributive reasoning tied to factor relations.

Task 2: Composing a multiplication from shared factor multiplications

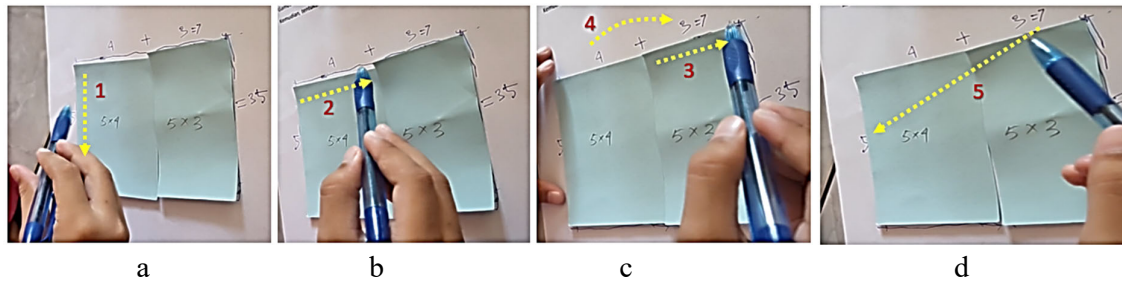
Task 2 aimed to make explicit that a new multiplication can be composed from two known multiplications that share a common factor (e.g., 5×7 from 5×4 and 5×3). This shifted students from recognising composed multiplications in Task 1 to intentionally producing them.

Instrumentalisation was witnessed as the students actively built new composite artefacts by selecting bars with the same width, aligning them side-by-side, and attributing the composite rectangle a single multiplication identity (Figure 4). In doing so, they repurposed the bar set

from ‘measurement’ to ‘construction’—a clear instrumentalisation move oriented to generating a new product.

Figure 4

Composing 5×7 from 5×4 and 5×3



Instrumentation and an emergent utilisation scheme became evident as students engaged with the task. They developed a stable *compose-and-name scheme* linking factor composition to product composition. The scheme led students to (1) keep the common factor invariant (e.g., the shared width); (2) add the remaining factor values to form the composed factor; (3) state the resulting multiplication; and (4) connect it to a sum of related products. For example, one student justified that 5×7 can be generated from $5 \times 4 + 5 \times 3$ by summing 4 and 3 to obtain 7 while holding 5 invariant as the shared factor (Figure 4). This foregrounded multiplication composition as a structural operation rather than merely an isolated arithmetic fact.

In this task, the instrumentalisation and instrumentation jointly supported students' instrumental genesis. By the end of Task 2, multiplication bars functioned as instruments for composing products from related products: students could coordinate a common factor and justify how factor addition ($b+c$) corresponds to product addition ($ab + ac$), preparing the conceptual ground for distributivity.

Task 3: Decomposing a multiplication on a bar model

Task 3 aimed to promote decomposition of a multiplication by partitioning one factor while keeping the other factor invariant, and to justify the total product as a sum of partial products (e.g., 5×12 as $5 \times 9 + 5 \times 3$ or as $5 \times 5 + 5 \times 4 + 5 \times 3$). Importantly, physical multiplication bars were removed to test whether students could apply the scheme using a bar model as a spatial record.

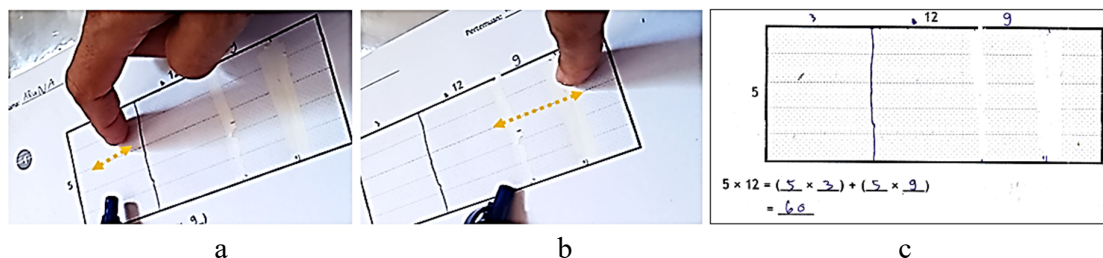
Instrumentalisation was noted as students created partitions on the bar model (two-part and three-part splits), chose accessible addends, labelled each segment with a multiplication, and organised calculations to recombine partial products. These actions show instrumentalisation of the bar model: students transformed a generic bar representation into a structured decomposition artefact tailored to their chosen factor split (Figure 5 and 6).

Instrumentation and an emergent utilisation scheme were clearly observable. The bar model shaped students' activity toward a *partition–compute–recompose scheme* as they (1) ensured the partitioned addends sum to the original factor (e.g., $9+3=12$); (2) computed each partial product with the invariant factor 5; and (3) recombined partial products to justify the whole product. Students explicitly checked the sum of partitions and confirmed that recomposition ($15+45=60$) matches 5×12 , treating recomposition as justification (Figure 5). Three-part decompositions further indicated flexibility and generality of the scheme (Figure 6).

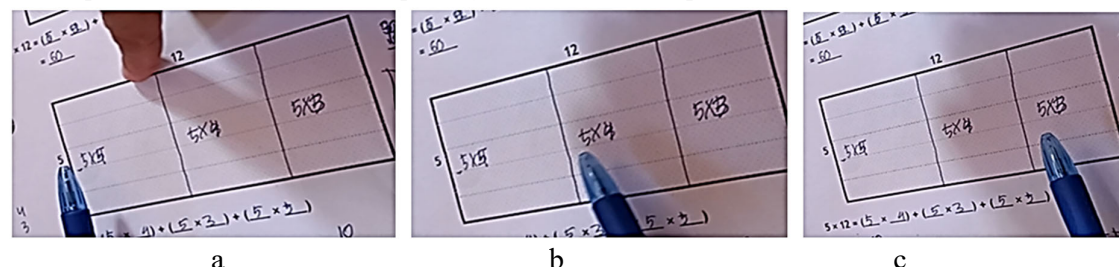
In relation to students' instrumental genesis, by the end of Task 3, the bar model functioned as an instrument for multiplication decomposition: students could independently partition a factor, coordinate it with an invariant factor, and justify the product as a sum of partial products, articulating an emerging distributive principle without reliance on physical bars.

Figure 5

A decomposition strategy for 5×12 , splitting the bar into two partitions, 5×9 and 5×3 .

**Figure 6**

A decomposition strategy for 5×12 , splitting the bar into three partitions, 5×5 , 5×4 , and 5×3 .



Overall, the three-task sequence supported a progression from noticing equivalence (Task 1), to intentionally composing new products (Task 2), to independently decomposing and justifying products on a bar model (Task 3). By the end of the sequence, students could coordinate a common factor with composed or partitioned factors and justify product equivalence as a sum of related products—an emergent, spatially grounded form of instrumental genesis for distributive reasoning.

Discussion and Contribution

This paper extends instrumentation theory by foregrounding spatially mediated instrumental genesis for early multiplication. The findings demonstrate that multiplication bars and a bar model did more than support ‘area reasoning’; they mediated the genesis of instruments for specific multiplication concepts: (i) relating factor pairs of two products, (ii) composing a product from related products sharing a factor, and (iii) decomposing a product into partial products to justify an emerging distributive principle.

Interpreted through Rabardel’s framework, the observed progress reflects the development of instruments as artefact-plus-scheme entities (Rabardel, 2002). Across tasks, instrumentalisation was visible in students’ selection, rearrangement, composition, and partitioning of spatial artefacts, while instrumentation was visible in the emergence of utilisation schemes, such as *common-factor coordination scheme* (Task 1), *compose-and-name scheme* (Task 2), and *partition–compute–recompose scheme* (Task 3) that coordinated common factors and composed or decomposed the other factor.

Artigue (2002) cautions that technical work can remain implicit, and that the epistemic value of techniques does not arise automatically. To this point, the mathematical significance of the process goes beyond the operational—and in fact may not arise automatically from practice. Rather, the instrumental genesis emerges from the intersection of how an artefact shapes the user’s activity and how the user shapes the artefact. The present sequence made technical work visible and progressively mathematical by requiring students to (a) express a bar configuration as a single multiplication, (b) justify a composed multiplication using relations among known multiplications, and (c) justify decomposition through recomposition.

Purposeful task design and teacher mediation supported students' technical work in progressively mathematical ways.

In Artigue's terms, these tasks supported a shift from *pragmatic* technique, where students focus on fitting, aligning, or adjusting bars to achieve the desired configuration, to *epistemic* technique, in which the bar model becomes a genuine *instrument* for establishing and justifying multiplicative relationships. Through this instrumental genesis, techniques gained theoretical value: they became not merely ways of acting on artefacts but ways of *knowing*, *explaining*, and *justifying* mathematics (Artigue, 2002). Theoretically, spatially mediated instrumental genesis enhances the instrumental approach for settings where mathematical content is tightly connected to spatial structuring operations. The construct highlights how spatial artefacts can stabilise common-factor structure and make factor composition/decomposition actionable, thereby shaping the trajectory of scheme development for distributive reasoning (Artigue, 2002; Rabardel, 2002).

Practically, the three-task sequence offers a compact design heuristic for supporting Year 3 students' understanding of multiplication as a structure, particularly as a foundation for distributive reasoning (Fosnot & Dolk, 2001; Kinzer & Stanford, 2013). For classrooms similar to the study site, the findings suggest that distributive reasoning can be developed through carefully sequenced tasks that keep one factor invariant, invite students to compose the other factor, and connect multiplication expression to sums of related products. This has an important classroom implication: rather than introducing the distributive property only as a formal rule, teachers can foster students' structural understanding through task design, shared representations, and discussion that makes factor-product relationships explicit. The findings also indicate that classroom institutionalisation is critical. Teachers need to compare strategies, highlight invariant relationship across tasks, and establish shared mathematical language so that distributive reasoning becomes a recognised and reusable classroom technique rather than a local strategy. In this sense, the sequence supports instrumental genesis toward distributive reasoning as an endorsed mathematical technique (Artigue, 2002).

These implications should, however, be considered in light of the study's limitations. The study was conducted in a single classroom context and focused on one short instructional episode. As such, the findings are best interpreted as offering a situated account of how distributive reasoning may emerge through task sequencing, rather than as evidence of broad generalisability. Future research should examine whether the developed schemes transfer to new numbers, problem contexts, and representations, and how classroom institutionalisation practices support consolidation of distributive reasoning over time.

Conclusion

By analysing a three-task sequence through instrumentation theory, this paper showed how spatial artefacts can mediate students' instrumental genesis for understanding multiplication through composition and decomposition. Multiplication bars and a bar model supported the emergence of utilisation schemes that relate factor pairs, compose products from related products, and decompose products into partial products, grounding an emerging distributive principle. The notion of spatially mediated instrumental genesis offers a theoretically grounded way to describe and design for these developments in early multiplication learning. The findings of this study build on a focused body of research literature that explores the (potential) mechanisms for supporting students' mathematics understandings during spatial reasoning interventions (Lowrie & Logan, 2023; Lowrie et al., 2020).

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References

- Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. *International Journal of Computers for Mathematical Learning*, 7(3), 245-274. <https://doi.org/10.1023/A:1022103903080>
- Barmby, P., Harries, T., Higgins, S., & Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Educational Studies in Mathematics*, 70(3), 217-241. <https://doi.org/10.1007/s10649-008-9145-1>
- Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, 32(1), 5-8. <https://journals.sagepub.com/doi/10.3102/0013189X032001005>
- Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Heinemann. <https://www.heinemann.com/products/e00354.aspx>
- Harries, T., & Barmby, P. (2006). *Representing multiplication* The British Society for Research into Learning Mathematics, <https://bsrlm.org.uk/publications/proceedings-of-day-conference/ip26-3/>
- Kinzer, C. J., & Stanford, T. (2013). The distributive property: The core of multiplication. *Teaching Children Mathematics*, 20(5), 302—309. <https://doi.org/10.5951/teacchilmath.20.5.0302>
- Lowrie, T., & Logan, T. (2023). Spatial Visualization Supports Students' Math: Mechanisms for Spatial Transfer. *Journal of Intelligence*, 11(6), 127. <https://www.mdpi.com/2079-3200/11/6/127>
- Lowrie, T., Resnick, I., Harris, D., & Logan, T. (2020). In search of the mechanisms that enable transfer from spatial reasoning to mathematics understanding. *Mathematics Education Research Journal*, 32(2), 175—188. <https://doi.org/10.1007/s13394-020-00336-9>
- Putrawangsa, S., & Hasanah, U. (2020). Mathematics Education in Digital Era: Utilizing Spatialized Instrumentation in Digital Learning Tools to Promote Conceptual Understanding. *Journal of Physics: Conference Series*, 1657(1), 012086. <https://doi.org/10.1088/1742-6596/1657/1/012086>
- Putrawangsa, S., & Patahuddin, S. (2022). Embodied task to promote spatial reasoning and early understanding of multiplication In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.). *Mathematical Confluences and Journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia) pp. 458-465. Launceston: MERGA
- Rabardel, P. (2002). *People and technology*. Université Paris 8, pp.188. hal-01020705. <https://hal.science/hal-01020705v1>
- Shvarts, A., Alberto, R., Bakker, A., Doorman, M., & Drijvers, P. (2021). Embodied instrumentation in learning mathematics as the genesis of a body-artifact functional system. *Educational Studies in Mathematics*, 1—23. <https://doi.org/10.1007/s10649-021-10053-0>
- Vergnaud, G. (1998). A comprehensive theory of representation for mathematics education. *The Journal of Mathematical Behavior*, 17(2), 167-181. [https://doi.org/10.1016/S0364-0213\(99\)80057-3](https://doi.org/10.1016/S0364-0213(99)80057-3)

Pre-Service Teachers' Conceptual Confidence in Mathematics and Science: Initial Tool Development

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Teacher education programs have a responsibility to develop teachers' confidence as teachers in both mathematics and science, reducing the barriers known to affect the efficacy of teaching of these curriculum areas in the primary classroom. However, Pre-Service Teacher (PSTs) often lack the confidence in teaching mathematics and science. We propose an initial tool using the Cartesian Plane to assess PSTs confidence with specific concepts and how to teach these concepts in mathematics and science. We anticipate that the tool will inform teacher education programs, pedagogical practices and have further implication in the teaching of mathematics and science.

Introduction

Mathematics is a high-status discipline and success in the mathematics strongly correlates with future economic success (Ernest, 2018). Many people, including many primary Pre-Service Teacher (PSTs), have negative learning experiences, shaped by personal experiences and exposure to social images of mathematics (Ernest, 2018). This can result in attitudes towards mathematics that include low levels of confidence and mathematics anxiety (Ernest, 2018). There are also calls to improve the quality of science instruction in classrooms across the globe (Naidoo & Naidoo, 2023) with specific concerns about teachers in primary/elementary years (F-6) due to the lack of core content knowledge training in teacher education courses. These concerns include low levels of science teaching confidence, which may stem from a lack of interest in science, science anxiety, and/or experiences of effective science teaching, science identity (Menon & Sadler, 2016; Naidoo & Naidoo, 2023). Limited scientific content knowledge, combined with the responsibility for teaching multiple strands, poses challenges for preparing teachers for classroom realities.

Furthermore, practicing and pre-service teachers may be unaware of gaps in their own content knowledge. As Brodie and Sanni (2014) argue, content knowledge is foundational to pedagogical content knowledge. However, as Shulman (1986) cautions, "assumptions of ignorance" have led to an overreliance on formal assessment of what teachers do not know (p.1).

If primary PSTs retain these negative attitudes and a lack of confidence with concepts as they move forward into their teaching career, it is likely to impact their mathematics and science teaching practice, and through their teaching practice, impact the attitudes and images of

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 337–344). MERGA.

mathematics and science carried by their future students. Teacher education programs have a great opportunity and responsibility to develop teachers' confidence. As teacher educators with a focus on both mathematics and science, we aim to reduce the barriers known to affect the quality teaching of these curriculum areas.

Literature Review

This literature review draws together three interconnected areas of research that inform the focus and design of this study. First, literature examining PSTs' diversity and the ways in which they navigate, resist or conform to expected norms provides a lens for understanding how confidence and identity are shaped in teacher education contexts. Second, research on teachers' and PSTs' confidence in mathematics and science highlights the affective dimensions of learning. Finally, an examination of how common metaphors and the connotations associated with these metaphors are used to capture teachers' confidence.

Pre-service teachers' diversity and conforming to expected norms

Hilton and Saunders (2024) note the expansion of definitions of diversity, and we follow their definition, considering diversity to include "difference in sex and gender; disability; beliefs and religion; social classes and socio-economic status; as well as race, ethnicity and culture" (p. 1066). Included under the umbrella of disability is neurodiversity/neurodivergence. PST diversity is integral to understanding confidence in mathematics and science as success in education must not be viewed as individual in nature. Ernest (2018) highlights the role of cultural capital in mathematics achievement, with implications for mathematical confidence. Education, including mathematics and science education is structured by dominant norms of saying, doing and relating (Grootenboer & Edwards-Groves, 2019). These norms privilege standardised, seemingly neutral forms of knowledge and often positioning alternative ways of thinking and knowing as irrelevant or inferior (Louie et al., 2021). Such dynamics contribute to persistent imbalances in achievement and confidence between dominant and marginalised groups (O'Keeffe & Paige, 2021). In science, PST diversity encompasses variation in cultural and religious beliefs, which can influence confidence in teaching topics perceived as controversial (Mavuru & Pila, 2021). In addition, PSTs also bring their history of science learning with them into their Initial Teacher Education (ITE) program, with PSTs often identifying teaching practices in high school and university as a contributing factor to their confidence in teaching science to their learners (Mavuru & Pila, 2021). PSTs may also bring preconceived stereotypes about students and schooling contexts into the classroom, highlighting the complex interplay between identity, confidence and diversity in teacher education (Chen & Mensah, 2018).

Primary teacher and preservice teachers' confidence in mathematics and science

This study is focused on primary undergraduate and postgraduate PSTs who are generalists, with the majority teaching across most learning areas. This contrasts with secondary PSTs, who have chosen specialisations. This means that the majority of primary PSTs will be required to teach mathematics and science, whereas there is a greater possibility that the secondary PSTs who will teach mathematics and science have chosen to specialise in these areas, meaning they are more likely to have a greater interest and confidence in the concepts within their chosen specialisation. The prevalence of anxiety towards mathematics and mathematics teaching experienced by primary PSTs is a significant factor that impacts their future teaching of mathematics (Grootenboer, 2008; Larkin, 2016). Menon and Sadler (2016) find there is a similar issue with primary pre-service teachers within the science learning area, additionally noting a possible correlation between PSTs' science content knowledge and science teaching self-confidence.

Confidence is a term that is commonly used within research studies but often not defined. Confidence connects to an individual's belief in their own competence to undertake a task successfully (Lau, 2022). Teacher confidence or anxiety is a key factor in the development of effective mathematics pedagogical knowledge by PSTs (Larkin, 2016). However, Larkin (2016) notes that the increase in asynchronous learning experiences, with the rise in online learning options within the Initial Teacher Education setting, creates a challenge in developing PST confidence with and understanding of mathematical content. This is because of the challenge of retaining the dynamic interaction between learners and educators in an asynchronous teaching model (Kim, 2011).

Teacher's lack of confidence in the subjects that they teach can negatively affect students' engagement, achievement and interest in the subject (Setlatentoa, 2018). Most PSTs identify as "non-science" people, reporting negative school experiences and poor attitudes towards the subject and the capacity to effectively teach science (Mulholland & Wallace, 2002). Whilst PSTs may feel confident in teaching some science topics, they report lower confidence in other topics (Mavuru & Pila, 2021). PSTs level of confidence reduced in meaningful engagement with the content, particularly abstract concepts that are difficult to represent concretely for learners (Mavuru & Pila, 2021).

Metaphorically speaking: Examining common visual metaphors in education.

In an attempt to develop a tool to ascertain pre-service teachers' confidence levels in mathematics, we explore different analogies associated with mathematics. Initially, the learning pit came to mind. In this metaphor, a learner is depicted descending into a "pit" of confusion or uncertainty, working through struggle before ascending the other side and thereby reaching new heights in understanding. We could see links between the learning pit and what pre-service teachers experience when they are engaging in mathematics curriculum courses at the tertiary level and the prospect of teaching mathematics. That is experiencing uncertainty before experience success. In discussing this metaphor as a research team, there was great discussion about the usefulness and the underlying meaning of the metaphor. This led to considering the metaphor of inverting the pit into a mountain. Both metaphors have been extensively used in education to depict the learning process. However, when critically examining these metaphors, we realised that while there are positive associations with both, there are also negative connotations.

James Nottingham conceptualised the Learning PitTM in the mid-2000s as a metaphorical tool to introduce cognitive discomfort and productive struggle that can occur when learners grapple with challenging concepts. Drawing on constructivist traditions, Nottingham framed the pit as a visual heuristic to help students recognise the value of uncertainty, confusion, and the eventual emergence of insight. Since its introduction, the Learning PitTM has been widely adopted in schools, professional learning programs, and growth-mindset initiatives, partly due to its accessibility and resonance with Dweck's (2006) theorisation of challenge and persistence. Despite its widespread adoption, the Learning PitTM metaphor carries several negative connotations that warrant careful consideration. By depicting learning as a descent into a 'pit', the model may evoke imagery of failure, entrapment, or deficit, potentially reinforcing anxiety rather than fostering resilience for some learners. The visual image of being *stuck* or *in a pit* can amplify feelings of helplessness, particularly for students with a history of low academic confidence or those who interpret struggle as a sign of inability rather than growth (Dweck, 2006). It can be argued that the metaphor implicitly individualises responsibility for progress, suggesting that escaping the pit depends primarily on personal effort. As such obscuring the role of instructional design, pedagogical approaches and systemic inequities that shape learners' experiences of difficulty (Wright et al., 2022). The linear upward trajectory depicted in many versions of the Learning PitTM can also oversimplify the nonlinear and

iterative nature of understanding, positioning successful learning as a singular moment of “climbing out” rather than an ongoing process that may involve repeated cycles of uncertainty (Abcouwer et al., 2016). Consequently, while the Learning Pit™ aims to normalise productive struggle, its metaphoric framing may unintentionally reinforce deficit perspectives and narrow conceptions of what meaningful learning entails.

In contrast, Boehm (1959) suggest that learning, and in particular, mathematics education, is like climbing a mountain. Boehm (1959) described mathematics as being for people “who like to climb intellectual mountains” (p. 7). While this metaphor has endured in mathematics education discourse, its implications also warrant further examination. The notion of mathematics as a mountain foregrounds challenge, perseverance, and the satisfaction of ascending the peak of the mountain. This imagery aligns with traditional narratives of rigour and intellectual conquest (Dweck, 2006; Gutiérrez et al., 2023). A contrasting examination of the imagery reveals the metaphor reinforcing elitist and exclusionary views of mathematics as a domain reserved for the exceptionally capable, able or determined (Boaler, 2015). The mountain metaphor may inadvertently position learners at the base of an intimidating ascent, leaving them feeling overwhelmed, unable to see a path forward, and overemphasising struggle. Research on student attitudes suggests that metaphors framing mathematics as inherently arduous can discourage participation and negatively shape self-concept, particularly among students who already feel marginalised in the discipline (Andersson et al., 2015; Erdogan & Yemenli, 2019; Gutiérrez et al., 2023; Larkin & Jorgensen, 2016). While evocative, the ‘intellectual mountain’ and Learning Pit™ analogies must be interrogated for the ways they shape expectations of difficulty, success, and who is perceived as belonging in mathematics.

This left us in a quandary in terms of developing a visual tool to ascertain pre-service teachers’ confidence levels in mathematics and science education. So, perhaps not surprising, we explored how the Cartesian Plane can be used as a visual metaphor to document PST confidence with mathematical and science concepts. In doing so, developing a tool that can be used to answer the research question (RQ): *How do we capture PSTs’ confidence in specific concepts as documented in the Australian Curriculum (F-6) mathematics and science?* Building on this RQ, we seek to answer the following: *How do we use PST’s confidence data about specific concepts to inform the responsive teaching and learning of mathematics and science education?*

Theoretical Framework

This study is situated at the intersection of sociocultural theories of education and affective theories of learning. Together, these perspectives frame PSTs confidence in mathematics and science not as an individual trait or deficit, but as a socially produced, culturally mediated, and historically situated phenomenon.

Sociocultural theories of learning foreground the idea that knowledge, identity, and confidence are constructed through participation in socially and culturally patterned practices (Grootenboer & Edwards-Groves, 2019). From this perspective, mathematics learning is not neutral or universal but is embedded within dominant cultural norms about what counts as legitimate mathematical knowledge, ways of knowing, and ways of being a ‘successful’ mathematics learner. Drawing on Bourdieu’s notion of cultural capital, Ernest (2018) argues that school mathematics privileges the cultural resources, dispositions, and experiences of dominant groups. PSTs who do not possess or recognise these dominant forms of capital may experience mathematics and science as alienating, leading to lower confidence and a diminished sense of belonging in the discipline. Within this framework, PST confidence in mathematics and science is understood as relational rather than individual. That is, it emerges through alignment (or misalignment) between PSTs’ identities, histories, and ways of knowing, and the norms embedded in mathematics education and ITE programs. Within this study, PST

confidence is therefore conceptualised as a social justice issue, shaped by systemic structures, assessment practices, and discourses of ability that determine who is positioned as capable, legitimate, and confident in mathematics.

Affective theories of learning emphasise the role of emotions, beliefs, and self-perceptions in shaping engagement and achievement (Dweck, 2006). Ernest (2018) highlights how negative mathematical and science experiences often result in poor confidence, and anxiety, patterns that are particularly prevalent among primary PSTs. Importantly, these affective responses are not isolated psychological phenomena. Rather, they are deeply intertwined with learners' identities and prior experiences within institutional and cultural contexts. PSTs enter ITE programs carrying embodied histories of mathematics and science learning that influence how they interpret challenge, struggle, and success. This study positions PST confidence as both an outcome of past experiences and a mediating factor in future teaching practice, reinforcing the cyclical nature of affect, identity, and equity in mathematics education.

Methodology

Metaphors are powerful cognitive and cultural tools that shape how individuals conceptualise abstract experiences such as learning (Lakoff, 1987). In education, metaphors such as the Learning Pit™ and mountain climbing are not merely illustrative but actively construct meanings about struggle, success, and belonging. In response to the limitations of dominant metaphors, this study explores the Cartesian plane as a representational tool for capturing PST confidence in mathematics and science. Unlike linear or hierarchical metaphors, the Cartesian plane allows for multidimensional, non-linear representations of confidence across mathematics and science concepts. This aligns with sociocultural and critical perspectives by resisting deficit framings and enabling PSTs to locate their confidence relationally rather than hierarchically. By positioning confidence as variable, contextual, and dynamic, the Cartesian plane offers a conceptual alternative that foregrounds diversity of experience and challenges narrow, standardised conceptions of mathematical competence.

The Cartesian Plane is used to map confidence levels (x axis) against levels of understanding for specific mathematics and science concepts (y axis) using a ten point scale. Costello et al. (1991) define a concept as “a group or set of items which all share a property” (p. 28). While the definition by Costello et al. is situated in mathematics education, we argue that the definition transcends into science education.

Initial Findings: Tool Development

In pursuit of developing a tool to capture PSTs' confidence levels in understanding specific mathematics and science concepts were identified which is presented as initial findings in this paper. We acknowledge from the outset the considerable depth and breadth of possible concepts across both learning areas. To ensure the tool was both feasible and practical, the research team collaboratively identified core concepts for primary mathematics and science. The process was guided by the Australian Curriculum, drawing on the six mathematical strands and the four science understandings. Table 1 presents the initial concepts identified for mathematics and science. While these concepts are presented in list form for clarity, we acknowledge that such representation may imply discreteness. In practice, the concepts are interrelated. The identification of concepts was the first step to answer the RQ: *How do we capture PSTs' confidence in specific concepts as documented in the Australian Curriculum (F-6) mathematics and science?*

Table 1:

Initial concept identification for mathematics and science

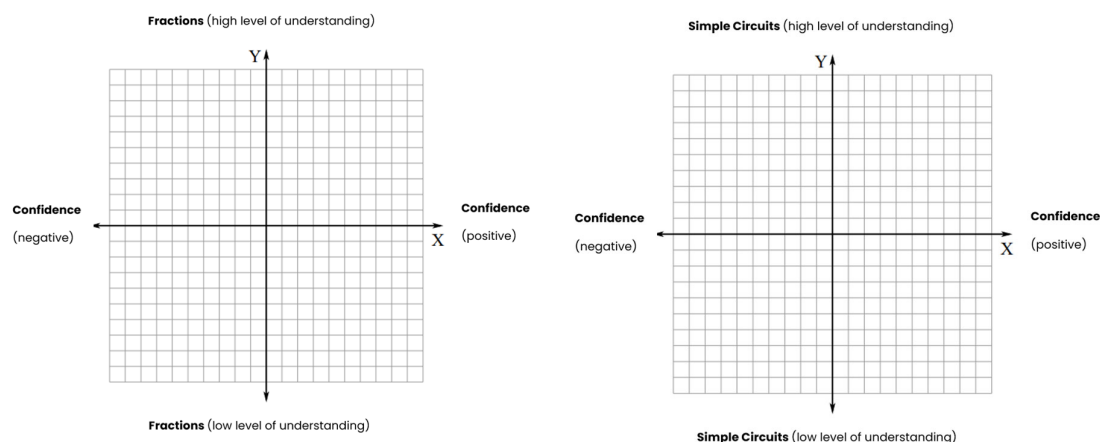
Mathematics	Science
Number - Place Value - Fractions, decimals, percentages - Multiplication and Division - Addition and Subtraction	Physical Science - Movement/push pull energy - Force – gravitational, friction and magnetism - Light and colour (and shadows) - Simple circuits - Renewable energy - Energy transformation
Algebra - Repeated patterns - Additive patterns - Growing patterns - Unknowns in number sentences - Equality/balance/equivalence - Expressions and Equations - Order of operations	Earth and Space Sciences - Solar system – day/night/seasons - Movement of earth and other planets - Rocks/soils/mud/minerals - Water cycle - Weathering
Measurement - Informal units of measurement - Formal units of measurement - Attributes to be measured (Length, Area, Volume, Capacity, Time)	Biological Science - Plants and Animals - Form and features of Living things/Non-living things - Life cycles, Habitats and Food chains - Survival and Adaptations - Ecosystems
Space (Geometry) 2D shape 3D solids - Cartesian Plane - Location and mapping - Transformation (reflection, rotation, translation, enlargement)	Chemical Science - Mixing/mixtures - States of matter/solid, liquid, gas - Properties of Materials
Statistics - Data collection - Data Representation - Data analysis	
Probability - Likelihood - Comparison - Prediction simulations or models - Probability events and outcomes	

In analysing existing visual metaphors of learning, we identified that visual metaphors of the learning pit and mountain are unidimensional and do not provide scope for mapping the intersection between conceptual knowledge and confidence. The Cartesian Plane will act as a visual metaphor whereby PST will map their confidence levels with their level of understanding for each of the concepts shown in Table. It is intended that PSTs will mark on the Cartesian plane a single co-ordinate indicating their confidence for each concept. The data can be used to ascertain relatively quickly both an individual and a group of PSTs' confidence levels. For example, a coordinate placed in quadrant 1 indicates that a PST is confident in and has a high level of understanding in a particular concept. Cartesian planes for each PST will then be

combined into a single visual representation, providing an indication of their confidence levels for both mathematics and science. The combined representations acting as a bidimensional visual metaphor of PST's confidence and understanding. It is also intended that results from a cohort of PSTs will be combined into a single plane mapping a group of PSTs' confidence levels with a given concept. Doing this provides a way of determining a group of PSTs' confidence levels at the beginning of a workshop allowing for responsive teaching practices by teacher educators. Figures 1 and 2 show items for mapping PSTs confidence levels in fractions and simple circuits.

Figures 1 and 2:

Sample Cartesian model for mapping PSTs confidence levels in mathematics and science concepts



Please explain why you place yourself in this quadrant.

Please explain why you place yourself in this quadrant.

An additional section of the tool will capture PSTs' demographic information with questions in this section ascertaining PSTs' i) Schooling experience in mathematics and science; ii) Tertiary education (undergraduate or postgraduate); iii) Work related experience (student support work, tutor, other); iv) Success and challenges in mathematics and science; v) Daily experiences in mathematics and science; vi) Family, peer, personal and social messages and images of mathematics and science. This section brings together PSTs confidence with their social capital and diverse backgrounds. Combined, the Cartesian Plane data and PST background information will provide a holistic picture of the complexities of PSTs' confidence in mathematics and science.

Implications and Conclusion

We anticipate the proposed model to be an innovative approach to capture PSTs confidence in specific mathematics and science concepts. In using the Cartesian Plane, we aim to gather data pre and post workshop to be able to track PST confidence levels. The collection of this data will assist in the identification of areas of strength and challenge with the PSTs' confidence in their knowledge of content and teaching within primary mathematics and science. This has implications for the PSTs undertaking mathematics and science courses to develop their self-awareness and confidence in their own conceptual understanding and in doing so has the potential to strengthen their knowledge. By collecting this data during workshops, it will support teacher educators to enact responsive teaching practices and thereby considering the diverse backgrounds of PSTs. The collection tool is visual and quick to interpret, allowing for modifications to short- and long-term planning to be made as part of a responsive teaching approach.

References

- Abcouwer, T., Smit, B., & Takács, E. (2016). De-linearizing learning. *Proceedings of the AIS SIGED*.
- Andersson, A., Valero, P., & Meaney, T. (2015). "I am [not always] a maths hater": Shifting students' identity narratives in context. *Educational Studies in Mathematics*, 90(2), 143-161.
- Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching* (1st ed.). John Wiley & Sons.
- Boehm, G. A. W. (1959). *The new world of math*. New York: Dial Press.
- Brodie, K., & Sanni, R. (2014). 'We won't know it since we don't teach it': Interactions between teachers' knowledge and practice. *African Journal of Research in Mathematics, Science & Technology Education*, 18(2), 188-197.
- Chen, J. L., & Mensah, F. M. (2018). Teaching contexts that influence elementary preservice teachers' teacher and science teacher identity development. *J. Science Teacher Education*, 29(5), 420-439.
- Costello, P., Horne, M., & Munro, J. (1991). *Sharing maths learning with children: A guide for parents, teachers and others*. Victoria, Australia: ACER.
- Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.
- Erdogan, F., & Yemenli, E. (2019). Gifted students' attitudes towards mathematics: A qualitative multidimensional analysis. *Asia Pacific Education Review*, 20(1), 37-52.
- Ernest, P. (2018). The ethics of mathematics: Is mathematics harmful? In P. Ernest (Ed.), *The Philosophy of Mathematics Education Today* (pp. 187-216). Springer International Publishing.
- Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education*, 11(6), 479-497. doi:10.1007/s10857-008-9084-x
- Grootenboer, P., & Edwards-Groves, C. (2019). Learning mathematics as being stirred into mathematical practices: An alternative perspective on identity formation. *ZDM*, 51(3), 433-444.
- Gutiérrez, R., Myers, M., & Kokka, K. (2023). The stories we tell: Why unpacking narratives of mathematics is important for teacher conocimiento. *The Journal of Mathematical Behavior*, 70, 101025.
- Hilton, A. L., & Saunders, R. (2024). LANTITE's impact on teacher diversity: Unintended consequences of testing pre-service teachers. *The Australian Educational Researcher*, 51, 1063-1082.
- Kim, J. (2011). Developing an instrument to measure social presence in distance higher education. *British Journal of Educational Technology*, 42(5), 763-777.
- Lakoff, G. (1987). Image metaphors. *Metaphor and Symbolic Activity*, 2(3), 219-222.
- Larkin, K. (2016). Course redesign to improve pre-service teacher engagement and confidence to teach mathematics. *International Journal for Mathematics Teaching and Learning*, 17(1).
- Larkin, K., & Jorgensen, R. (2016). "I hate maths: why do we need to do maths?" using iPad video diaries to investigate attitudes and emotions towards mathematics in Year 3 and Year 6 students. *International Journal of Science and Mathematics Education*, 14(5), 925-944.
- Lau, W. W. F. (2022). Predicting pre-service mathematics teachers' teaching and learning conceptions: The role of mathematical beliefs, mathematics self-efficacy, and mathematics teaching efficacy. *International journal of science and mathematics education*, 20(6), 1141-1160.
- Louie, N., Adiredja, A. P., & Jessup, N. (2021). Teacher noticing from a sociopolitical perspective: the FAIR framework for anti-deficit noticing. *ZDM – Mathematics Education*, 53(1), 95-107.
- Mulholland, J., & Wallace, J. (2002). Navigating border crossings: How primary teachers learn to teach science. *Australian Science Teachers' Journal*, 48(2), 12-19.
- Naidoo, K., & Naidoo, L. J. (2023). Designing teaching and reflection experiences to develop candidates' science teaching self-efficacy. *Research in Science & Technological Education*, 41(1), 211-231.
- Mavuru, L., & Pila, O. K. (2021). Pre-service teachers' preparedness and confidence in teaching life sciences topics: What do they lack? *Education and New Developments*. <https://doi.org/10.36315/2021end023>
- Menon, D., & Sadler, T. D. (2016). Preservice Elementary Teachers' Science Self-Efficacy Beliefs and Science Content Knowledge. *Journal of Science Teacher Education*, 27(6), 649-673.
- O'Keeffe, L., & Paige, K. (2021). Re-highlighting the potential of critical numeracy. *Mathematics Education Research Journal*, 33(2), 285-299.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.
- Setlalentoa, W. (2018). Experiences of pre-service teachers on Life Sciences topics and learning. What makes learning difficult and effective? 40th International Academic Conference, 219-229.
- Wright, P., Fejzo, A., & Carvalho, T. (2022). Challenging inequity in mathematics education by making pedagogy more visible to learners. *International Journal of Mathematical Education in Science and Technology*, 53(2), 444-466.

Capturing the use of Key Word Sign in Early Childhood Mathematics

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This paper examines the use of Key Word Sign (KWS) as an inclusive strategy for early mathematical learning. Reporting on phase three of a multi-stage project, it explores how staff and children enacted KWS in an early learning centre. Two main approaches were identified in how children and staff used KWS in mathematics: 1) intentional and deliberate, and 2) spontaneous and responsive. These approaches were used during structured activities and play, enhancing access to mathematical language, strengthening spatial reasoning and promoting multimodal communication. Findings position KWS as a valuable strategy for inclusive embodied mathematics pedagogy.

Introduction

Supporting children's access to early mathematics remains a persistent challenge, with longstanding research highlighting disparities in children's opportunities to engage meaningfully with mathematical ideas, language, and practices (Lee & Ginsburg, 2009). These challenges are amplified by variation in educators' confidence and pedagogical content knowledge (MacDonald et al., 2023). In addition, children bring diverse communication needs (Brebner et al., 2016). As a result, children's early encounters with mathematics can differ significantly in quality and accessibility, shaping their developing identities, dispositions, and participation (Lee & Ginsburg, 2009).

The development of children's mathematical language is central to their emerging mathematical understanding, as language is seen as a tool through which young learners make sense of quantity, space, pattern, and relationship (Verdine et al., 2017). Early mathematical language supports children to articulate their thinking, participate in mathematical dialogue, and engage in mathematical situations (Verdine et al., 2017). Research consistently shows that children's exposure to rich mathematical language varies widely, with significant implications for later achievement (LeFevre et al., 2010). Consequently, supporting children's mathematical language development requires intentional, inclusive pedagogies that recognise communication as embodied, relational, and deeply intertwined with conceptual growth.

Key Word Sign (KWS) is an embodied communication approach that formalises the use of gesture as a core component of meaning-making (Quane et al., 2025). In the Australian context, KWS draws on signs from Australian Sign Language (Auslan) together with natural gestures to visually emphasise key words within spoken sentences. As such, KWS is a widely recognised form of unaided Augmentative and Alternative Communication (AAC). Research demonstrates that combining manual signs with speech can strengthen communication and support speech development for individuals with developmental disabilities (Cruz Leon & Zorn, 2024). Importantly, the benefits of integrating gesture with spoken language extend beyond specialised contexts, with evidence showing positive outcomes for typically developing children, including gains in literacy, foreign language learning (García-Gámez & Macizo, 2023), and mathematical (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 345–352). MERGA.

understanding and retention (Kersey et al., 2024). Together, speech, sign, and gesture form a semiotic bundle referred to as a multimodal repertoire that enhances children's agency and supports the expression and consolidation of conceptual understanding. KWS has long been used to support children with diverse communication needs. A clear gap remains in understanding its broader pedagogical potential, particularly in early mathematics. By reimagining KWS as part of everyday teaching practice, we foreground its capacity to broaden access to core mathematical concepts, strengthen inclusive learning environments (Cologon & Mevawalla, 2018), and honour longstanding traditions of embodied, gesture-rich communication that have always been central to how young children make meaning (Kim et al., 2011). In the early years, where foundational mathematical concepts are formed, the integration of KWS can deepen conceptual understanding while ensuring equitable access to emerging mathematical language.

In this paper, we report on phase three of a multistage project. The initial phase identified 20 KWS that had a mathematical focus on location, specifically position and direction (Quane et al., 2025). Phase one investigated the context of the Early Learning Centre, gaining insights into the mathematics program, exploring teachers' initial understandings of KWS, and documenting their current use of KWS in everyday practice. Phase two focused on supporting teachers' implementation of KWS in mathematics through targeted professional learning and resources. Phase three examined the enactment of KWS in practice, with a specific focus on teachers' and children's use of KWS related to position and direction. This final phase explored how KWS supported children's engagement with spatial language and concepts, capturing both pedagogical practices and children's multimodal mathematical communication. Findings from Phase three are the focus of this paper, answering the following research question: *How do teachers, educators and children use Key Word Sign, and in what situations do they use KWS to communicate their mathematical ideas?*

Spatial awareness, reasoning, and embodied cognition in the early years.

Spatial awareness and reasoning are foundational components of early mathematical development, supporting children's ability to interpret, navigate, and make sense of their world. It is widely recognised that spatial awareness and reasoning underpin later mathematical achievement (Sarama & Clements, 2009). Furthermore, the specific vocabulary related to position and direction has been found to support the formation of mental models that scaffold later mathematical learning (Pruden et al., 2011). Research consistently highlights that spatial thinking emerges early in life and develops through rich sensory-motor experiences, interactions with objects, and engagement in spatial language (Verdine et al., 2017). In the early years, spatial awareness encompasses children's understanding of location (e.g., *in*, *on*, *under*), and direction (e.g., *forward*, *around*, *towards*), forming the basis for more complex skills such as map reading, and problem solving (Sarama & Clements, 2009). Everyday play activities, such as block building, puzzles, construction, and navigating spaces, provide critical opportunities for children to develop flexible spatial representations and make predictions about objects and movement. Educators' use of spatial language during these activities has been shown to significantly enhance children's conceptual development, with specific vocabulary related to position and direction supporting the formation of mental models that scaffold later mathematical learning (Pruden et al., 2011). Within early childhood education, intentional teaching of location, specifically position and direction, plays a vital role in strengthening children's spatial reasoning and supporting equitable access to mathematics (Lowrie et al., 2017). Furthermore, research emphasises the importance of embodied and multimodal learning approaches, where children use gesture, physical movement, and manipulation of materials to externalise and refine their thinking (Kim et al., 2011; Way & Ginns, 2024).

Theoretical Framework

Universal Design for Learning (UDL) is an educational framework that guides the proactive design of learning experiences. UDL is organised around three core principles: 1) multiple means of engagement; 2) multiple means of representation and 3) multiple means of action and expression. These principles are further organised into ways to; a) increase access, b) to support the learning process and c) support learners' executive functioning (CAST, 2024). Each principle is divided into specific guidelines with accompanying considerations that provide more detailed guidance. UDL challenges a one-size-fits all approach by advocating for a flexible, inclusive learning designs that respond to students' varied strengths, interests, and needs (Navaitienė & Stasiūnaitienė, 2021). UDL is not widely used in the early childhood sector. However, clear links between UDL and the Early Years Learning Framework 2.0 (EYLF2.0) (AGDE, 2022) are seen. UDL's emphasis on multimodality aligns with semiotic perspectives that view learning as mediated through signs such as images, gestures, symbols, and tools (Radford, 2003). By providing multiple representational and expressive pathways, UDL acknowledges that learners construct meaning differently based on available semiotic resources and lived experiences (Morgan, 2006). This alignment is reinforced by EYLF Outcome 4, which positions children as confident learners who use diverse modes and materials to explore and communicate ideas (AGDE, 2022).

Methodology

Three types of observations were conducted in the preschool setting, drawing on the observational framework developed by Quane (2025). Descriptive, focused, and selective observations were undertaken throughout the project, with attention to both pedagogical practices and the integration of KWS. Broad descriptive observations of the preschool environment were initially conducted to capture the physical layout, available materials, planned activities, participants, and routines. These sessions, carried out before and after focused observations, occurred outside of children's attendance to allow accurate documentation of contextual variables and collaboration with staff in clarifying details. Field notes and a structured observational protocol supported systematic recording (Quane, 2025). Focused observations were conducted at regular intervals and documented teacher practices, with particular attention to the use of KWS during instruction and interactions. These observations also recorded the objects, artefacts, and mathematical resources used, alongside key events such as the teacher modelling signs like through and around to support children's comprehension (Spradley, 2016). Selective observations were employed to capture specific children's engagement, documenting the actions they performed, the materials they used, and how children responded to teachers' signing and verbal prompts (Quane, 2025; Spradley, 2016). Observations were conducted by three members of the research team. An observational protocol and field notes were used to record the observations. Photographs documented situations where staff and children used KWS in mathematics. The observations reported in this paper, arise from the three types of observations occurring in one session lasting three hours.

Learning Stories

A learning story attempts at capturing the curiosity and knowledge of a learner, through the eyes of a teacher. It begins with a narration of a child's initiative in a given context followed by an analysis of the possible understandings and learning that one or more children are engaged. Carr and Lee (2012) identified the following four design principles in developing a learning story: 1) position learners with agency; 2) include multiple voices; 3) provide navigation markers for learners, both backward looking and forward looking, and 4) integrate dispositional and subject knowledge and practice. A key aspect of learning stories is that they "resist deficit positioning" (p. 138). Learning stories comprehensively documents a moment in time and

multiple learning stories can demonstrate growth as attained by a child. Thereby, making the child's learning visible through skilful observations, sample photographs and analytical reflections. At the root of any learning story is a genuine interest in understanding a child's lived experiences and the process of articulating and documenting this as evidence of learning. As such, learning stories can be quite subjective wherein a teacher can focus on strengths and abilities of a child and provide comprehensive detail about a child's initiated or unplanned sequence of events and its impact on the student's own learning. Learning stories gives educators a licence to add value by incorporating their interpretation of a child's thoughts, emotions, and body language (Carr & Lee, 2012). The learning stories document two key approaches to using KWS in mathematics. First intentional and deliberate. Intentional refers to actions that are purposeful and planned, guided by clear goals or learning intentions (AGDE, 2022, P.17, 18, 48). Deliberate practice refers to conscious, systematic and conscious actions to achieve a specific purpose. The second approach has been classified as spontaneous and responsive use of KWS. Spontaneous refers to actions that occur naturally and unplanned in the moment, arising without prior intention or structured preparation. Teachers optimise spontaneous, teachable moments that arise from learners' need in play, comments, actions and everyday routines to scaffold children's mathematical learning (AGDE, 2022, p. 17). Being responsive to these moments is an important basis for curriculum decision-making in response to children's evolving ideas and interests (AGDE, 2022) and involves noticing, wondering, interpreting and acting on not only what, but how children communicate.

Research context and participants

The study was situated in a single ELC in a regional city in Southeast Queensland. Data reported in this paper was drawn from one kindergarten room enrolling three to four-year-old children. Two teachers, Grace, the room leader, and Jane, had both been at the ELC for seven years. Both teachers had engaged in targeted professional learning focused on the use of KWS to support the development of mathematical language. Room one at the ELC provides a foundational early childhood learning environment for children aged three to four years, designed to consolidate emerging competencies and foster positive dispositions toward formal education. Positioned as a critical stage within the centre's broader play-based curriculum, the ELC prioritises exploration, creativity, and interpersonal development, recognising that cognitive, social, and emotional capacities are interwoven during the early years. Children in room one engage in open-ended experiences that foster curiosity and self-expression, supported by strong educator-child relationships. Learning is embedded in everyday social interactions and an integrated approach that honours individual development while supporting smooth transitions to kindergarten. Learning stories are used to document children's learning.

Findings

Observations were conducted in November 2025 on a Friday morning. Staff and children were beginning to prepare for end of year Christmas celebrations and preparing a concert item. The observations began after a morning tea break. Video and photographs of the children were not permitted due to recent critical issues in the sector around child protection. In analysing the observational data, two key approaches were identified through which teachers and educators went about incorporating KWS in mathematics in activities. In presenting the findings pertaining to children we use learning stories as a mechanism to identify the context and how children use KWS to communicate mathematical ideas and understanding. Our intention in sharing these learning stories is to document and illustrate two different examples, intentional and spontaneous, of how KWS in mathematical learning occurs in an ELC. Our intention is not to provide deep analyse of the learning.

Intentional and deliberate use of KWS

In preparing for the concert, the teacher led the children in a sign and sing session involving a song created by Sunshine Sign and Sing (authors 4 and 5). The song ‘At the zoo’ used the KWS for open and close, forward and backward, under and over, in and out, through and around. The song also used KWS of animal including crocodile, snake, monkey and gecko. The teacher intentionally planned for the signing and singing session to occur after the break with the goal of providing an opportunity for children to rehearse the concert performance. KWS used by the teachers and children have been *italicised*. The following learning story captures deliberate use of KWS:

A learning story about out, outside and inside

The children are sitting on the mat facing the teacher who is sitting on a chair. The teacher leads the children in two songs that use KWS. After signing and singing, the teacher gives instructions about going outside: “When I say get your hats *out* of your bag... then we will go *outside*. Some children begin to stand and move towards the door. The teacher redirects the children and again uses KWS in her instruction “not yet, we are *inside*”. The teacher indicates the area where the children will go, and a child asks why that area. The teacher responds with “the *babies* in the baby room will be sleeping *inside*.” As the teacher signed the KWS for out, outside and inside, several children also signed the KWS. Two children used a pointer finger rather than a flat hand when signing the KWS for outside and inside.

Analysis of Learning

The use of KWS enhanced the teacher’s instructions, providing a visual cue for the children. As children listened to the teacher’s instructions, some copied the KWS, providing them with an opportunity to practice the signs and see their application. As children signed the KWS for out, outside and inside, variation was noted in the production of the sign, indicating that there may be a progression in how children use KWS particularly regarding handshape and orientation.

Spontaneous and responsive use of KWS

KWS used by the teachers and children have been *italicised*. Spontaneous and responsive use of KWS in mathematics occurred for both teachers and children as captured in the following learning story:

A learning story about up, down, under, above, through and around

Three boys are playing in the block corner (a collection of wooden blocks of various shapes and sizes) and are constructing a wall. One boy says, “let’s build the wall up”. Nearby, another group of children are with an educator constructing a marble run (between the television and the block wall, figure 1). The three boys co-constructed their wall and began to add a tower. One boy says, “let’s make it higher,” and adds a block to the tower. The educator, observing the boys, asks, “is it *above* your head?”. The boy who expressed the desire to make it higher stands in front of the wall and says “no”. The boy pauses, looks at the tower and places his hand on top of his head and measures from his head to the tower. As he measures, his hand moves in an upward and not level direction to his head. He looks over at the educator who intervenes stating, “you need to bring it *down*” signing the KWS *down*. The boy removes the block. One of the other boys suggests “let’s make a ramp to go *down*”. The three boys find a wooden truck with a long tray base, experimenting with different configurations of the ramp and rolling cars down it. The boys continue to experiment with the incline of the ramp and decide to make to flat. The educator asks, “What can you find to go *under* it at this end?” pointing to the lower end of the ramp. The boys roll cars across the ramp, watching them stop midway. They decided to change the incline of the ramp. They place a block under one end of ramp. One boy says,

“we need another block *under* the ramp”, pointing to the first block. He goes to the block storage area and finds another block of the same size (Figures 1 and 2). The two other boys place cars on the ramp, watching them roll down the ramp and fall off at the end of the wall.

Figures 1 – 3

Children’s block play construction



Figure 1: Rear view of the constructed block wall



Figure 2: Side view of the constructed block wall



Figure 3: Front view of the constructed block wall

As the two boys continue to roll cars down the ramp, the third boy begins playing with a truck, driving it through the space in the wall (Figure 1). The boy moves to the front of the wall (Figure 3) and continues to drive the car around on the floor. He continues driving the truck through the space and around the space in front of the wall. “Look, I can drive the truck *through* here”, gesturing by using his hand (pointer finger and moving his hand in a forward direction). Meanwhile, the two boys have found a basket and have placed it at the end of the wall (Figure 3). They place cars at the top of the ramp, watching as they roll down and then fall into the basket.

Analysis of Learning

This learning story captures rich moments of collaboration, problem-solving, and multiple modes of communication as the boys engage in construction and experimentation. The educator’s intentional use of an open-ended question and KWS supports both language development and conceptual understanding. The educator modelled the use of signs (*up*, *above* and *under*) during the play. Through parallel play, the children explored spatial awareness using appropriate mathematical language to describe the position and direction of the cars, the ramp and the basket. The children’s persistence in adjusting the ramp’s height, identifying suitable materials, and testing outcomes highlights developing dispositions of curiosity, persistence, and cooperation.

Discussion

The observations illustrate how Key Word Sign (KWS) functioned as an embedded, multimodal pedagogical resource for supporting young children’s mathematical thinking, particularly spatial language related to position, direction, and movement. Two complementary examples of practice were evident: intentional and deliberate use of KWS within structured activities, and spontaneous, responsive use during child-initiated play. Together, these patterns demonstrate how KWS can operate across planned and emergent moments in an ELEC, strengthening continuity between adult-led and play-based mathematics learning.

In the structured context of concert preparation, the intentional integration of KWS within a familiar sign-and-sing routine provided repeated, embodied exposure to spatial concepts. Embedding mathematical language within music and movement aligns with research emphasising the role of rhythm, gesture, and repetition in supporting young children’s conceptual understanding (Shilling, 2002). Importantly, this intentional use positioned KWS

not as an add-on, but as a naturalistic part of a planned program. Equally significant was the spontaneous and responsive use of KWS during block play, where children and the teacher co-constructed mathematical meaning in real time. The learning story highlights how the teacher's strategic modelling of signs, combined with an open-ended question, supported children to reason about height, and spatial relationships. Children's own gestural communication, using signs, hand movements, and embodied measurement, demonstrated emerging spatial reasoning and agency in expressing mathematical ideas. The movement between talk, sign, gesture, and action reflects how children draw on multiple semiotic resources to make sense of relational and dynamic concepts.

From a UDL perspective, these findings illustrate how KWS provided multiple means of representation and expression, supporting variability in children's communication and understanding. The teacher's responsive use of KWS reduced linguistic barriers while amplifying opportunities for engagement, persistence, and collaboration. Collectively, the observations suggest that when KWS is used both intentionally and responsively, it can enrich everyday mathematical interactions, deepen spatial reasoning, and support inclusive participation, particularly during complex, open-ended play where language, movement, and problem-solving intersect.

Conclusion

This study contributes new insights into the use of Key Word Sign (KWS) as an inclusive pedagogical strategy for supporting early mathematical learning, particularly in relation to spatial language and concepts of position and direction. The findings highlight the value of embodied approaches for strengthening mathematical understanding and underscore the importance of both intentional and deliberate, as well as spontaneous and responsive, planning approaches to using KWS in mathematics application. Although limited to a single site, the study points to the potential for greater emphasis on spatial reasoning within early mathematics programs and calls for system-level investment in professional learning and mathematics resources to support equitable access to mathematical language and learning across early childhood contexts.

Acknowledgments

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References

- Australian Government Department of Education [AGDE] (2022). *Belonging, being and becoming: The Early Years Learning Framework for Australia* (V2.0). Australian Government Department of Education for the Ministerial Council. <https://www.acecqa.gov.au/sites/default/files/2023-01/EYLF-2022-V2.0.pdf>
- Brebner, C., Jovanovic, J., Lawless, A., & Young, J. (2016). Early childhood educators' understanding of early communication: Application to their work with young children. *Child Language Teaching and Therapy*, 32(3), 277-292. <https://doi.org/10.1177/0265659016630034>
- Carr, M., & Lee, W. (2012). *Learning stories: constructing learner identities in early education*. SAGE.
- CAST. (2024). *Universal Design for Learning Guidelines version 3*. <https://udlguidelines.cast.org/>
- Cologon, K., & Mevawalla, Z. (2018). Increasing inclusion in early childhood: Key Word Sign as a communication partner intervention. *International Journal of Inclusive Education*, 22(8), 902-920. <https://doi.org/10.1080/13603116.2017.1412515>
- Cruz Leon, M., & Zorn, I. (2024). Picture book videos incorporating key word signing for children who require AAC and their families: design and evaluation of a multimedia-enhanced setting. *Augmentative and Alternative Communication*, 1-13. <https://doi.org/10.1080/07434618.2024.2370821>
- García-Gámez, A. B., & Macizo, P. (2023). Gestures as Scaffolding to Learn Vocabulary in a Foreign Language. *Brain Science*, 13(12). <https://doi.org/10.3390/brainsci13121712>

- Kersey, A. J., Carrazza, C., Novack, M. A., Congdon, E. L., Wakefield, E. M., Hemani-Lopez, N., & Goldin-Meadow, S. (2024). The effects of gesture and action training on the retention of math equivalence. *Frontiers in psychology*, 15. <https://doi.org/10.3389/fpsyg.2024.1386187>
- Kim, M., Roth, W.-M., & Thom, J. (2011). Children's gestures and the embodied knowledge of geometry. *International Journal of Science and Mathematics Education*, 9(1), 207-238.
- Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, 34(4), 37-45. <https://doi.org/10.1177/183693910903400406>
- LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: longitudinal predictors of performance. *Child Development*, 81(6), 1753-1767. <https://doi.org/10.1111/j.1467-8624.2010.01508.x>
- Lowrie, T., Logan, T., & Larkin, K. (2017). The "math" in STEM practices: The role of spatial reasoning in the early years. Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia.
- MacDonald, A., Deehan, J., & Lee, P. (2023). Relations between early childhood educators' qualifications and experience and their beliefs about mathematics education for babies and toddlers. *Australian Journal of Education*, 67(3), 253-269. <https://doi.org/10.1177/00049441231193776>
- Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics*, 61, 219-245. <https://doi.org/10.1007/s10649-006-5477-x>
- Navaitienė, J., & Stasiūnaitienė, E. (2021). The goal of the Universal Design for Learning: Development of all to expert learners. In A. Galkienė & O. Monkevičienė (Eds.), *Improving inclusive education through Universal Design for Learning* (pp. 23-57). Springer International Publishing. https://doi.org/10.1007/978-3-030-80658-3_2
- Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: does talk about the spatial world matter? *Developmental Science*, 14(6), 1417-1430. <https://doi.org/10.1111/j.1467-7687.2011.01088.x>
- Quane, K. (2025). Observing mathematical learning experiences in the primary years to examine how attitudes towards mathematics are enacted in the classroom. *Education Sciences*, 15(4), 417. <https://www.mdpi.com/2227-7102/15/4/417>
- Quane, K., Trewartha, B., Gaunt, L., Twose, Z., & Stigwood, E. (2025). *The use of Key Word Sign to support early mathematical experiences* Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia, Canberra: Canberra, Australian Capital Territory.
- Radford, L. (2003). Gestures, Speech, and the Sprouting of Signs: A Semiotic-Cultural approach to students' types of generalization. *Mathematical Thinking and Learning*, 5(1), 37-70. https://doi.org/10.1207/S15327833MTL0501_02
- Sarama, J., & Clements, D. (2009). *Early childhood mathematics education research: learning trajectories for young children* (First edition. ed.). New York: Routledge. <https://doi.org/10.4324/9780203883785>
- Shilling, W. A. (2002). Mathematics, music, and movement: Exploring concepts and connections. *Early Childhood Education Journal*, 29(3), 179-184. <https://doi.org/10.1023/A:1014536625850>
- Spradley, J. (2016). *Participant observation*. Waveland Press, Inc.
- Verdine, B., Golinkoff, R., Hirsh-Pasek, K., & Newcombe, N. (2017). Spation skills, their development, and their links to mathematics. *Monographs of the Society for Research in Child Development*, 82, 7-30. <https://doi.org/10.1111/mono.12280>
- Way, J., & Ginns, P. (2024). Embodied learning in early mathematics education: Translating research into principles to inform teaching. *Education Sciences*, 14(7), 696. <https://www.mdpi.com/2227-7102/14/7/696>

Seeing the Invisible: Exploring the Use of Spatial Reasoning in Mathematics Problem-Solving Beyond Geometry at the Secondary School Level

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Spatial reasoning has been widely studied and strongly associated with students' mathematics achievement, particularly in geometry and early grades. This study aimed to explore the use of spatial reasoning in solving mathematics problems beyond geometry. An algebra test was administered to six eighth-grade students from three different schools in Depok, Indonesia, followed by semi-structured interviews. The findings showed that the students use spatial reasoning to understand the problem context, model the problem in representations, and plan problem-solving strategies. Spatial reasoning supported problem comprehension and strategic planning, but procedural skills remained crucial for accurate problem solving.

Spatial reasoning refers to the ability to use spatial skills, including understanding, representing, and manipulating spatial properties of objects and spatial relations (Lowrie et al., 2019; Bruce et al., 2017). In this study, spatial reasoning is evaluated through three constructs: spatial visualization, mental rotation, and spatial orientation (Ramful et al., 2017). This recent study focuses on the use of spatial reasoning in mathematical problem-solving beyond geometry, referring to tasks in the algebra domain. Spatial reasoning is a specific cognitive ability that also strongly appears in the area of mathematics education. The development emerged as most research showed that mathematics performance and spatial reasoning are highly correlated. (Mulligan & Woolcott, 2015). Moreover, numerous studies have shown that spatial reasoning interventions significantly impact students' mathematics performance, as shown in the meta-analysis study conducted by Uttal et al. (2013).

Several studies indicated that spatial reasoning has a robust and consistent relationship with solving geometric problems, evidenced by the high correlation between spatial ability and geometry problem-solving performances (Buckley et al., 2019) and the effect of spatial training on students' geometry and measurement performances (Adams et al., 2023). For instance, students who have good spatial abilities tend to be able to demonstrate solving open geometric problems (Wang & Chiew, 2010).

Since the relationship between spatial reasoning and geometry is well established, it is necessary to investigate its relationship with domains beyond geometry. The domains include numbers, algebra, data, and uncertainty, which are studied at the secondary level based on the Indonesian mathematics curriculum (Ministry of Education and Culture, 2018). The studies about the relationship between spatial reasoning and mathematics are dominated by the geometry domain, but the investigation of the relationship between spatial reasoning and domains beyond geometry is still under research. In line with Newcombe et al. (2019), who stated that several algebraic materials may be related to 3D spatial concepts, such as function graphs, but the relationship between them has not been tested.

In addition, several studies related to the relationship between spatial reasoning and problem-solving beyond geometry were mostly conducted at the primary and early school level. For example, spatial visualization and mental rotation ability can predict numeracy skills in preschool to elementary school children (Gunderson et al., 2012). Research has also found that mental rotation training can improve arithmetic ability, especially in math problems involving missing numbers (Cheng & Mix, 2014). In addition, spatial orientation also correlates positively with students' abilities on number lines in elementary school (LeFevre et al., 2013). The main

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 353–360). MERGA.

focus of this study is to explore in what ways students use spatial reasoning in solving mathematics problems beyond geometry at the secondary mathematics level.

Spatial Reasoning and Its Constructs

Spatial reasoning is a combination of several different skills divided into several constructs (Hawes & Ansari, 2020). Although spatial reasoning is a complex collection of skills, the relationships within it are not clear (Lowrie & Logan, 2018). Despite the recent end, there have been attempts to classify spatial skills into different dimensions, such as static and dynamic or intrinsic and extrinsic. However, this classification is not sufficient to accommodate the broad range of applications of spatial skills (Mix et al., 2018). Furthermore, this study employs three spatial reasoning constructs, including spatial visualization, mental rotation, and spatial orientation, as used in a study by Ramful et al. (2017) at the middle level. Spatial visualization refers to the ability or skill to mentally manipulate or transform an object's spatial properties (Lowrie et al., 2017). Meanwhile, spatial orientation requires an egocentric transformation of imagining a change in one's own perspective (Hegarty & Waller, 2004). Then, mental rotation is a specific type of object-based transformation, often distinguished from measures of spatial visualization in factor analytic studies (Mix & Cheng, 2012).

The Relationship between Spatial Reasoning and Mathematics Problem-Solving Beyond Geometry

In the case of secondary level, the relationship between spatial reasoning and mathematics varies and is not consistent across mathematics domains. Research by Bremigan, (2005) investigated the association between students' spatial ability performance and advanced mathematics. The results showed that spatial ability was significantly correlated with performance in advanced mathematics. However, the latest research by Adams et al. (2023) stated that spatial training can support achievement in certain contexts, such as measurement and geometry, but not in numbers and algebra.

Spatial reasoning plays a significant role in mathematics problem-solving beyond geometry. Although these domains are not directly related to the concept of space, as in geometry. However, it allows individuals to visualize and connect elements in an imaginary space. For example, algebra materials often involve lines and curves in their concepts (Newcombe et al., 2019).

The mechanism of how spatial reasoning is used in problem-solving processes beyond geometry has been widely investigated at the early levels, and it is still unclear to what extent it is used at the secondary school level. A study by Empson and Turner (2006) stated that multiplicative thinking requires an understanding of how parts relate to the whole to recognize patterns at an early level. In addition, some component skills in algebra (and likely other facets of algebra that are yet untested), 3-D spatial visualization may mediate the relation between working memory and mathematics achievement (Tolar et al., 2009). In this study, the role of spatial reasoning in the process of solving mathematics problems beyond geometry at the secondary mathematics level still needs to be explored further.

Methodology

The qualitative study uses a case study design to elaborate on how and to what extent spatial reasoning is used in solving mathematics problems beyond geometry (Creswell, 2015). This design emphasizes an in-depth analysis of the case within the context of the use of spatial reasoning by students in mathematics problem solving. The participants were six Grade 8th students from three secondary schools in Depok, Indonesia. After completing the 30-item Spatial Reasoning Instrument (SRI) developed by Ramful et al. (2017), the students were classified into several categories: two students were categorized at the high level ($\geq 75\%$ correct answers), two students were at the intermediate level (50%–74% correct answers), and two

students were classified at the low-intermediate level (25%–49% correct answers). The selection of these levels ensured that the students had sufficient spatial reasoning performance to be used in solving problems beyond geometry. In addition, the variation in spatial reasoning levels and school districts was intended to capture diverse perspectives on students' problem-solving approaches.

In developing the instrument, this study identified two aspects that serve as the main criteria for determining mathematics problems that elicit spatial reasoning, as they reflect the cognitive demands involved in solving problems. First, the problems require students to construct mental representations to encourage spatial reasoning before performing calculations (Marikyan, 2023) and support in modelling the structure of the problem accurately and transforming objects within the problem (Sorby et al., 2022). Second, the problems are non-routine or contextual, requiring students to mentally interpret relationships among information. In this process, spatial reasoning becomes important for working with number lines, graphs, and internal visual models (Harris et al., 2021).

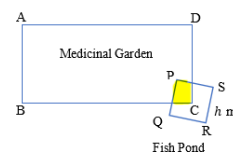
This study selected algebra problems because that domain is newly introduced at the secondary level, making it a relevant domain for representing problem solving beyond geometry in secondary mathematics. After the tasks were designed, the instrument was validated by two expert validators. The revision process focused on refining the spatial reasoning components and the structure of the problem-solving tasks in the instrument. Therefore, the instrument developed in this study is presented in Table 1.

Table 1

Algebra Problem-Solving Administered to 8th Grade Students

Description	Problems
Narrative	Family Medicinal Garden
	Mr. Dimas is a retired doctor who spends his days tending a garden planted with various medicinal plants such as ginger, turmeric, and curcuma. He has a fairly large, rectangular family medicinal garden behind his house.  Sources: id.pinterest.com
Initially, Mr. Dimas only knew that the length of his garden was 6 meters longer than its width (as shown in the picture) because he hadn't measured it directly. Over time, as Mr. Dimas's garden yields increased, he wanted to expand his garden by adding 2 meters to each of its length and width to grow more plants and help more people in need. Therefore, Mr. Dimas wanted to increase the size of his garden while maintaining its shape and planned to make several modifications to it.	
Part a (Focus: SV)	Mr. Dimas tries to make a new sketch of his garden's dimensions. He knows that the perimeter of the new garden is 60 m, measured using a measuring tape that only shows the final result. <u>Determine the area of Mr. Dimas's garden with the new dimensions!</u>
Part b (Focus: SO)	Then, Mr. Dimas plans to create a T-shaped path (from his perspective), with the front on one of the wide sides of the garden, as follows: • One path runs vertically from the front to the back of the garden, right in the middle. • Another path runs horizontally, 1 m wide, behind the garden, along the wide side. If the area of the walkway is 20.5 m², determine the width of the vertical pathway!
Part c (Focus: MR)	Later, Mr. Dimas's medicinal garden experienced a drought due to the medicinal plants being sensitive to changes in humidity. Therefore, Mr. Dimas planned to build a rectangular fish pond to help maintain soil moisture in the medicinal garden. Mr. Dimas sketched a rectangle to represent the medicinal garden, as shown in the following figure.

The pool is designed to extend slightly into the garden area, while most of it remains outside the garden. It is known that each vertex of the pool is at the same distance from the corner point on the edge of the garden (point C). To achieve this design, Mr. Dimas directly measured the area of the fish pond that lies within the garden, obtaining 16 m² (as shown in the yellow shaded region). Determine the length of the pond's side (h)!



Note: SV = Spatial Visualization; MR = Mental Rotation; SO = Spatial Orientation

Findings

This section presents the findings on the extent to which six eighth-grade secondary school students used spatial reasoning in solving mathematics problems beyond geometry, as illustrated in Table 2. The findings show that spatial reasoning (spatial visualization, spatial orientation, and mental rotation) helped students understand the context, model the problem, and plan solution strategies; however, the accuracy of their answers was still strongly determined by the accuracy of algebraic modelling and procedural skills.

Table 2

Findings on Students' Use of Spatial Reasoning in Algebra Problem Solving

SR constructs (function to PS)	Level of SR	The example of excerpts	Outcomes of PS (S)	Description of PS
SV (Mentally modelling the problem).	Developed	"The shape of the garden? It's a rectangle, but its length and width aren't the same." (S1)	Correct (S1, S3)	Found the correct area of the garden using an algebraic equation.
	Developed	"In my opinion, it's still a rectangle but a bit larger in size." (S5)	Incorrect (S2, S4, S5, S6)	Made an error in the algebraic modelling.
SO (Understanding the problem context)	Developed	"So, I imagined that the front sides of the pathway were on the width side. I drew the square so that the width side would be in the front." (S1)	Correct (S1)	Determined the width of the pathway by using algebraic modelling and applying algebraic operations.
	Developed	"For question b, it's like I positioned myself in the middle as if I were standing on the path... the horizontal part would be behind me." (S3)	Incorrect (S2, S3, S4, S5, S6)	Made an error in algebraic modelling and in applying algebraic procedures.
MR (Planning a problem-solving strategy)	Developed	"If the distance from the corner point to point C is the same, then it represents the diagonal of a square. Therefore, the square was rotated to make it perpendicular to the garden. Consequently, the shaded region is a square because its area is 16 m ² . Thus, the side length of the small square is 4 m" (S1)	Correct (S1, S2)	Performed a transformation to determine the targeted construction, which supports planning the strategy.
	Not Developed	"Not really, if I were to rotate it" (didn't think that far)" (S4)	Incorrect (S3, S4, S5, S6)	Not identifying the appropriate construction led to an incorrect problem-solving strategy.

Note: SR=Spatial Reasoning; SV = Spatial Visualization; MR = Mental Rotation; SO = Spatial Orientation; PS = Problem-Solving; S = Student

In solving Problem a (spatial visualization), these excerpts indicated that the students perceived changes in the size of a shape as transformations that do not alter its initial form. The students stated that although the object undergoes widening or enlargement, its geometric characteristics are preserved. This suggests that the students effectively used spatial visualization to understand that the garden's dimensions would change while the form of the shape remained the same. In this case, spatial visualization supported the student's initial understanding and modelling of the problem situation.

However, there was a misunderstanding by the students in modelling this problem to algebraic variables, which led to errors in applying the algebraic procedure. As illustrated in one of the students' answers (see Figure 1), the student was able to recognize the relationship between the new length and width of the garden and the perimeter procedure. Figure 1 shows that the student began by writing the perimeter formula $K = 2(p + l)$ and correctly substituting the value $K = 60$. Nevertheless, the student did not fully understand the problem, assuming that the length and width had the same value, " $p + l = 30$, therefore $p = l = 15$ ". This response suggests that although the student was able to visualize the change in the rectangle's dimensions, an accurate mathematical model was not constructed, resulting in incorrect length and width values and an inaccurate solution.

Figure 1

S5's Answer: Performing Algebra Equations in Problem a (the box on the right provides the English translation of the student's answer)

a. Dik: k taman = 60 m
 Dit: L taman?
 Jwb: $K = 2 \times (p + l)$
 $K = 60 \text{ m}$ maka $p + l = 30$
 $p = \text{panjang}$ luas taman:
 $l = \text{lebar}$ $l = p \times l = 15 \times 15 = 225 \text{ m}^2$
 jadi luas taman Pak Dimas
 dengan ukuran yg baru adalah 225 m^2

$p = l = 15$
 karena $15 + 15 = 30$

Translation:

Given: perimeter of the garden = 60 m

Asked: area of the garden?

Answer: $K = 2 \times (p + l)$

$K = 60 \text{ m}$

$p = \text{length}$

$l = \text{width}$

$p = l = 15$

because $15 + 15 = 30$

so $p + l = 30$

Area of the garden:

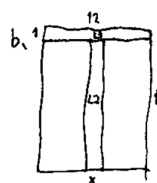
$A = p \times l = 15 \times 15 = 225 \text{ m}^2$

Thus, the area of Mr. Dimas's garden with the new dimensions is 225 m^2

In solving Problem b (spatial orientation), these excerpts indicated that the student actively used spatial orientation to determine an object's position before representing it in the form of a drawing. By imagining the front side of the pathway as being on the width side, the student engaged in perspective taking to adjust the orientation of the shape to match the chosen viewpoint. This response shows that the student attempted to mentally reposition the pathways in order to understand their arrangement within the garden. This statement showed that the student was able to accurately recognize the positions of the pathways within the garden to understand the context of the problem.

Figure 2

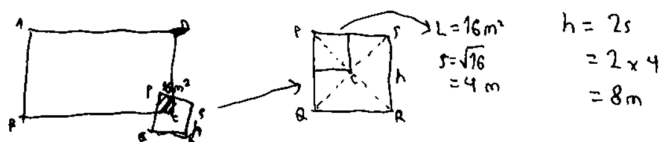
S1's Answer: Determining the Width of the Vertical Walkway using Algebraic Equation in Problem b

b. 
 $L = L_1 + L_2$
 $20,5 = 1 \times 12 + 1 \times x$
 $1 \times x = 20,5 - 12$
 $= 8,5$
 $x = 8,5$
 $= 0,5 \text{ m}$

From this understanding, the students were able to visualize the shape of the pathway clearly. Some students even created drawings to help them remember it more easily and to identify the spatial relationships among the objects in the problem, as illustrated by Figure 2, realized that each pathway was also rectangular in shape. Consequently, they used the formula for the area of a rectangle to calculate the areas of the two pathways. They also recognized that there was an overlap between the two paths, so they proceeded to use the total area formula: $L=L_1+L_2$, so $20.5=1\times 12+17x$. After forming these equations, the student applied the correct algebraic calculations to obtain the value $x=0.5$ m. Finally, in solving Problem c (mental rotation), the excerpts indicate that students understood rotation as a spatial transformation affecting orientation but not shape or size, enabling them to interpret the problem and plan an appropriate strategy to determine the intended geometric shape.

Figure 3

S1's Answer: Determining the Side Lengths of the Fish Pond in Problem c



The students with high problem-solving ability were able to think of the appropriate procedure to find the area of the shaded square. The students were able to solve the problem correctly because they used the appropriate strategy, namely applying the formula for the area of a square, as shown in Figure 3. The students easily used the square area formula (area of the fish pond), $L = 64 m^2$, and therefore determined that the shaded area was $L = 16 m^2$, which is one-fourth of the total area of the fish pond.

Discussion and Conclusion

The findings of this study suggest that spatial reasoning may assist the students to understand the problem situation by transforming verbal and symbolic information into relational structures that can be visualized and mentally manipulated. This is consistent with Lowrie and Logan (2018) who stated that to solve and understand mathematical problems, students must extract information from the problem and transform it into a form of representation referred to as information encoding. Furthermore, Zhu et al. (2024) added that when working on open-ended problems, students use spatial skills to process spatial information and employ analogies to understand spatial relationships.

In addition, utilizing spatial reasoning appears to support the students in constructing mental models of the problem situation to represent relationships among objects, positions, shapes, and orientations, so that the problem context is no longer understood as a collection of separate pieces of information, but as a meaningful whole (Duffy et al., 2024). This process enables students to organize complex information, identify relevant relationships, and bridge contextual situations with appropriate mathematical representations, thereby making their understanding of what is happening in the problem clearer and allowing important information to be comprehended effectively (Choi & Kim, 2021).

Once this spatial structure is established, students can identify relevant quantities (such as length, area, angle, or volume) and the relationships among them, and then represent these relationships in appropriate mathematical symbols, variables, and expressions (Maher & He, 2022). Thus, mathematical models may emerge as abstractions of spatial relationships that students have already understood making the translation from context to symbolic form more meaningful. This aligns with Hawes and Ansari (2020), who stated that spatial models help

translate problems into symbolic form. Before students write equations, functions, or mathematical symbols, they first structure the problem visually, representing its components.

Other findings also suggest that spatial reasoning may help the students identify strategies for solving mathematics problems beyond geometry. Through mental transformations, the students may explore and manipulate mental models of the problem situation while performing preliminary reasoning before engaging in formal calculations (Lowrie et al., 2016). Spatial reasoning may allow students to anticipate possible subsequent solution steps and to foresee the impact of each step on the overall structure of the problem. This ability supports students in selecting and integrating visual and mathematical strategies, thereby making problem-solving more effective.

In conclusion, the findings suggest that students may use spatial reasoning to support three problem-solving processes: understanding the problem context, modelling the problem, and planning a solution strategy. Furthermore, students who were able to develop spatial reasoning tended to understand the problem, plan strategies, or both, more effectively. However, this does not necessarily guarantee correct mathematical solutions, because other cognitive abilities may also play an important role during the procedural stage. Lastly, these interpretations should consider that this study is exploratory and involved a small number of students; therefore, the findings should be viewed as preliminary insights highlighting the potential role of spatial reasoning in solving mathematical problems beyond geometry. Further studies need to integrate spatial reasoning with students' mathematical procedural skills in interventions to explore new approaches to mathematics problem solving beyond geometry.

Acknowledgement

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References

- Adams, J., Resnick, I., & Lowrie, T. (2023). Supporting senior high-school students' measurement and geometry performance: Does spatial training transfer to mathematics achievement? *Mathematics Education Research Journal*, 35(4), 879–900. Scopus. <https://doi.org/10.1007/s13394-022-00416-y>
- Bremigan, E. G. (2005). An analysis of diagram modification and construction in students' solutions to applied calculus problems. *Journal for Research in Mathematics Education*, 36(3), 248–277.
- Bruce, C. D., Davis, B., Sinclair, N., McGarvey, L., Hallowell, D., Drefs, M., Francis, K., Hawes, Z., Moss, J., Mulligan, J., Okamoto, Y., Whiteley, W., & Woolcott, G. (2017). Understanding gaps in research networks: Using “spatial reasoning” as a window into the importance of networked educational research. *Educational Studies in Mathematics*, 95(2), 143–161. <https://doi.org/10.1007/s10649-016-9743-2>
- Buckley, J., Seery, N., & Canty, D. (2019). Investigating the use of spatial reasoning strategies in geometric problem solving. *International Journal of Technology and Design Education*, 29(2), 341–362. <https://doi.org/10.1007/s10798-018-9446-3>
- Cheng, Y.-L., & Mix, K. S. (2014). Spatial Training Improves Children's Mathematics Ability. *Journal of Cognition and Development*, 15(1), 2–11. <https://doi.org/10.1080/15248372.2012.725186>
- Choi, J., & Kim, M. K. (2021). An analysis of spatial reasoning ability and problem solving ability of elementary school students while solving ill-structured problems. *The Mathematical Education*, 60(2), 133–157. <https://doi.org/10.6331/mathedu.2021.60.1.133>
- Creswell, J. W. (2015). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Pearson. <https://thuvienso.hoasen.edu.vn/handle/123456789/12789>
- Duffy, G., Sorby, S., & Bowe, B. (2024). Exploring the role of spatial ability in the mental representation of word problems in mathematics. *Frontiers in Education*, 9, 1346474. <https://doi.org/10.3389/educ.2024.1346474>
- Empson, S. B., & Turner, E. (2006). The emergence of multiplicative thinking in children's solutions to paper folding tasks. *The Journal of Mathematical Behavior*, 25(1), 46–56.
- Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology*, 48(5), 1229–1241. <https://doi.org/10.1037/a0027433>

- Harris, D., Logan, T., & Lowrie, T. (2021). Unpacking mathematical-spatial relations: Problem-solving in static and interactive tasks. *Mathematics Education Research Journal*, 33(3), 495–511. Scopus. <https://doi.org/10.1007/s13394-020-00316-z>
- Hawes, Z., & Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin & Review*, 27(3), 465–482. <https://doi.org/10.3758/s13423-019-01694-7>
- Hegarty, M., & Waller, D. (2004). A dissociation between mental rotation and perspective-taking spatial abilities. *Intelligence*, 32(2), 175–191.
- Hegarty, M., & Waller, D. (2005). Individual differences in spatial abilities. *The Cambridge Handbook of Visuospatial Thinking*, 121–169.
- LeFevre, J.-A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S.-L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology*, 4. <https://doi.org/10.3389/fpsyg.2013.00641>
- Lowrie, T., Logan, T., & Ramful, A. (2016). Spatial Reasoning Influences Students' Performance on Mathematics Tasks. *Mathematics Education Research Group of Australasia*. <https://eric.ed.gov/?id=ED572328>
- Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students' mathematics performance. *British Journal of Educational Psychology*, 87(2), 170–186. <https://doi.org/10.1111/bjep.12142>
- Lowrie, T., & Logan, T. (2018). The Interaction Between Spatial Reasoning Constructs and Mathematics Understandings in Elementary Classrooms. In K. S. Mix & M. T. Battista (Eds.), *Visualizing Mathematics* (pp. 253–276). Springer International Publishing. https://doi.org/10.1007/978-3-319-98767-5_12
- Lowrie, T., Logan, T., & Hegarty, M. (2019). The influence of spatial visualization training on students' spatial reasoning and mathematics performance. *Journal of Cognition and Development*, 20(5), 729–751. <https://doi.org/10.1080/15248372.2019.1653298>
- Maher, E. M., & He, J. (2022). Supporting Spatial Reasoning: Identifying Aspects of Length, Area, and Volume in Textbook Definitions. *International Journal on Studies in Education*, 5(1), 64–87. <https://doi.org/10.46328/ijonse.100>
- Marikyan, G. (2023). Teaching Mathematics with Visuals Gohar Marikyan. *Athens Journal of Sciences*, 10(4), 231–240. <https://doi.org/10.30958/ajs.10-4-3>
- Ministry of Education and Culture. (2018). *Regulation No. 37 of 2018 on competency standards for senior secondary school graduates*. <https://peraturan.go.id/>
- Mix, K. S., & Cheng, Y.-L. (2012). Chapter 6 - The Relation Between Space and Math: Developmental and Educational Implications. In J. B. Benson (Ed.), *Advances in Child Development and Behavior* (Vol. 42, pp. 197–243). JAI. <https://doi.org/10.1016/B978-0-12-394388-0.00006-X>
- Mix, K. S., Hambrick, D. Z., Satyam, V. R., Burgoyne, A. P., & Levine, S. C. (2018). The latent structure of spatial skill: A test of the 2times 2 typology. *Cognition*, 180, 268–278.
- Mulligan, J., & Woolcott, G. (2015). What lies beneath? Conceptual connectivity underlying whole number arithmetic. *Ors: Xuhua Sun, Berinderjeet Kaur, Jarmila Novo*, 220.
- Newcombe, N. (2017). *Harnessing spatial thinking to support stem learning*. https://www.oecd-ilibrary.org/education/harnessing-spatial-thinking-to-support-stem-learning_7d5dcae6-en
- Newcombe, N. S., Booth, J. L., & Gunderson, E. A. (2019). Spatial skills, reasoning, and mathematics. *The Cambridge Handbook of Cognition and Education*, 100–123.
- Ramful, A., Lowrie, T., & Logan, T. (2017). Measurement of Spatial Ability: Construction and Validation of the Spatial Reasoning Instrument for Middle School Students. *Journal of Psychoeducational Assessment*, 35(7), 709–727. <https://doi.org/10.1177/0734282916659207>
- Sorby, S. A., Duffy, G., & Yoon, S. Y. (2022). Math instrument development for examining the relationship between spatial and mathematical problem-solving skills. *Education Sciences*, 12(11), 828.
- Tolar, T. D., Lederberg, A. R., & Fletcher, J. M. (2009). A structural model of algebra achievement: Computational fluency and spatial visualisation as mediators of the effect of working memory on algebra achievement. *Educational Psychology*, 29(2), 239–266. <https://doi.org/10.1080/01443410802708903>
- Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, 139(2), 352–402. <https://doi.org/10.1037/a0028446>
- Wang, Y., & Chiew, V. (2010). On the cognitive process of human problem solving. *Cognitive Systems Research*, 11(1), 81–92. <https://doi.org/10.1016/j.cogsys.2008.08.003>
- Zhu, C., Klapwijk, R., Silva-Ordaz, M., Spandaw, J., & De Vries, M. J. (2024). Investigating the role of spatial thinking in children's design ideation through an open-ended design-by-analogy challenge. *International Journal of Technology and Design Education*, 34(5), 1733–1762. <https://doi.org/10.1007/s10798-024-09877-7>

Design Principles of Mathematics Placements that Support Beginning Teachers to Flourish

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School-based placements play a central role in preservice teachers' professional preparation and identity formation. Drawing on self-determination theory, a qualitative longitudinal study with 98 participants identified four placement features that supported secondary mathematics preservice teachers' flourishing identities through the development of autonomy, competence, and relatedness. These features are conceptualised as design principles for effective secondary mathematics placements that support flourishing, highlighting how placement environments can be intentionally designed to support preservice secondary mathematics teachers' development.

Introduction

Teacher education programs aim to prepare teachers to effectively support diverse students in varied classroom environments. Within initial teacher education (ITE), preservice teachers (PSTs) develop pedagogical knowledge and form an emerging mathematics teacher identity through participation in university coursework and school placements (Darragh, 2016; Shulman, 1987). Placements are central to teacher preparation and identity formation as they allow PSTs to engage in authentic teaching practice (Pendergast et al., 2020).

During placements, PSTs observe experienced teachers, interact with students, and gradually assume responsibility for planning and delivering lessons. These experiences provide opportunities to connect theoretical ideas about teaching with classroom practice and play an important role in shaping PSTs' emerging identities as teachers (Loughran, 2013). In mathematics education, placements are particularly significant. Teaching mathematics involves complex pedagogical decisions. These decisions include selecting representations, responding to student misconceptions, and designing tasks that support conceptual understanding (Shulman, 1987). As PSTs engage in instructional decision-making, they begin to form perspectives about the nature of effective mathematics teaching and their capacity to succeed in the profession. However, placement experiences can vary substantially. While placements are intended to support PST development, some environments provide opportunities for experimentation and reflection, whereas others limit PSTs' roles to observing or replicating established teaching routines (Zeichner, 2010). Despite the recognised importance of placements within teacher education, relatively little attention has been given to identifying placement features that support secondary mathematics PSTs in developing positive identities.

In this study, we examine secondary mathematics PSTs' placement experiences using the concept of *flourishing* teacher identity. Drawing on self-determination theory (SDT), flourishing is understood in relation to the extent to which environments support PSTs' needs for autonomy, competence, and relatedness. The study addresses the following research question: **What features of secondary mathematics teaching placements facilitate environments that support preservice teachers' flourishing identities?** To answer this question, PSTs' experiences of placements were examined to understand the ways in which they impacted their identity, specifically in relation to their sense of belonging, autonomy, and competence.

Conceptual Framing: Flourishing Teacher Identities

In coining the term *flourishing identity* for mathematics teachers (Reid O'Connor & Zunica, 2024), we draw on SDT to conceptualise this as a positive mathematics teacher identity characterised by PSTs' sense of autonomy, competence, and relatedness (Ryan & Deci, 2017). Consistent with contemporary research, identity is understood as a dynamic state that is shaped through participation in professional contexts such as school-based placements (Darragh, 2016). The term *flourishing* is deliberately used to emphasise developing "in a healthy or vigorous way, especially as the result of a particularly congenial environment" (Oxford English Dictionary, 2023). Within SDT, the psychological needs for autonomy, competence, and relatedness are strongly associated with job enjoyment, professional wellbeing, and career retention (Hobson & Maxwell, 2017; Reid O'Connor & Zunica, 2024). These associations make the satisfaction of these needs particularly relevant for understanding mathematics teachers' career trajectories. Teaching placements are critical sites in which these psychological needs may be supported or constrained, shaping PSTs' identity and future work lives.

Flourishing identity in this study is examined through PSTs' experiences of autonomy, competence, and relatedness during school-based placements. Drawing on Ryan and Deci's (2017) work, autonomy is understood as PSTs' opportunities to exercise meaningful pedagogical agency in mathematics classrooms, including making instructional decisions. Competence refers to PSTs' developing confidence in their capacity to teach mathematics effectively, supported through feedback, guided experimentation, and reflection, with experiences of struggle framed as integral to learning rather than as failure. Relatedness is understood as a sense of professional belonging, shaped through mentoring relationships and departmental cultures in which PSTs are recognised and included as legitimate members of mathematics teaching communities. Growth mindset (Dweck, 2006) is additionally positioned within this framework as a *protective mechanism* that shapes how PSTs interpret challenge, feedback, and uncertainty during placements. A growth mindset reflects a disposition through which PSTs persist when encountering pedagogical difficulty (competence), engage in instructional experimentation (autonomy), and seek feedback and re-engage following setbacks (relatedness). Through an SDT lens, a flourishing mathematics teacher identity is also positioned as situated and relational, formed through PSTs' interactions with mentors, students, and the broader school community. This shifts attention from whether PSTs flourish as a result of individual dispositions alone, to the intentional design of placement environments that support flourishing. In this study, we therefore treat flourishing identity as a *design target* and focus on identifying placement features that support its development.

Methodology

As flourishing identity is understood as situated and relational, and shaped through PSTs' interpretations of their placement contexts, identifying placement features that support flourishing requires close attention to PSTs' lived experiences during placement. Guided by the conceptual framing, a qualitative, longitudinal design was employed to examine PSTs' placement experiences and answer the research question. Data were collected between 2023 and 2025 with two cohorts of secondary mathematics PSTs from one Australian university ($n=98$). Surveys, qualitative reflective tasks, and interviews were utilised to capture PSTs' experiences of autonomy, competence, and relatedness during placements, as well as the ways these experiences shaped their developing identities. Not all participants contributed to each data source; survey and reflective task data were collected at single time points, while interviews followed a subset of participants longitudinally across placements.

Placement experiences were examined qualitatively through responses to four open-ended survey items ($n=27$) asking participants: "what did you enjoy most about placement?" and "what experiences influenced your anticipated interest in teaching mathematics/autonomy in

their future teaching role/and willingness to try new mathematics teaching approaches?" ($n=27$). In addition, participants completed an open-ended, reflective task which asked them "how did placement experiences influenced your conceptions of mathematics teaching?" ($n=60$). These were administered after participants first placement experience (midway through their degrees). Finally, to capture placement experiences across an entire secondary mathematics ITE program, interviews were conducted with 11 participants (who are given pseudonyms) across all three placements to form case studies. Analysis for all data sources was conducted thematically by first inductively coding responses before interpreting patterns through the conceptual lens of autonomy, competence, and relatedness (Clarke & Braun, 2017). Triangulation across multiple data sources and longitudinal data supported the trustworthiness of findings, enabling examination of cross-cohort patterns across diverse contexts.

Findings and Discussion

Across all data sources, PSTs were highly reflective about placement experiences and their developing mathematics teacher identities. One PST reflected on "realising how the way you teach is far more important than what you teach". Another noted "my understanding of a secondary mathematics classroom has not changed...what has changed is my understanding of myself as a mathematics teacher", highlighting placement's critical role in identity formation. Analysis of surveys, reflective tasks, and interviews identified four recurrent themes, presented as design principles, characterising placement environments that support PST flourishing.

Design Principle 1: Structured Autonomy in Mathematics Teaching

Placement environments differed in the level of instructional autonomy PSTs were provided. PSTs flourished when placements balanced independence with guidance, allowing responsibility to increase as confidence and competence developed. In early placements, PSTs developed confidence when expectations for lesson planning, observation, and participation were clearly communicated. At the same time, reflective task responses highlighted the importance of giving PSTs sufficient autonomy to develop their individual teaching approach, with a PST noting "what I expected going into placement is trying to be someone like Eddie Woo, but the reality I needed to face is I need my own personal teaching style".

Interview data illustrated how structured autonomy operated in practice. Mich's first placement provided prescriptive lesson templates alongside clear expectations for reflection and feedback, enabling autonomy to emerge within supportive constraints. Mich noted "we are provided resources; she doesn't pressure us to use them". Similarly, Tal's first placement gave scaffolded autonomy in creating worksheets and choosing examples. In the latter half of the placement, Tal was able to take pedagogical risks with supervisory support, and it encouraged them to adopt "our own style". Gradual release of responsibility meant that Tal developed confidence throughout placement two ("I gain[ed] more confidence in the classroom"), and by placement three felt prepared to implement diverse pedagogies ("I enjoyed being able to do my own thing...and like do lots of games"). Contrastingly, PSTs emphasised that autonomy without structure was a stressor. Lem described uncertainty when expectations for observation and participation were unclear, resulting in them not knowing "what to take away" from lessons and hesitant to engage with students. In contrast, Peri noted that knowing "exactly what was expected of us" from the outset reduced anxiety and allowed them to focus on their teaching.

Placements with clear expectations, regular feedback, and gradual release of responsibility enable PSTs to transition from observation to supported teaching and ultimately independent practice (Byth, 2026; Collet et al., 2025). From a SDT perspective, structured autonomy supported PSTs' emerging sense of agency and ownership, allowing for the development of their own teaching identity even when lessons were imperfect. Autonomy was experienced as developmental rather than absolute, with supervisors needing to evaluate PSTs' growing

competence across placements to enact suitable supports. Allowing autonomy to make key instructional decisions is formative for PSTs' mathematics teacher identity, as identity develops through agentic participation in teaching practice (Beauchamp & Thomas, 2009).

Design Principle 2: Mathematics-specific Mentoring

PSTs' developing sense of competence was associated with mathematics-specific mentoring. Whilst PSTs typically entered placements primarily concerned with behaviour management, mentoring that directed attention towards students' mathematical understanding, representations, and misconceptions was associated with increasing their teaching confidence and identity. When mathematics-specific mentoring occurred, PSTs described shifts in how they understood the nature of mathematics teaching, recognising it was "not about teaching facts" but "helping students develop a way of understanding the language of mathematics", and that "effective mathematics teaching follows a clear structure".

Interview data demonstrated how mathematics-specific mentoring supported PSTs in practice. Tal described a supervising teacher who consistently connected practice to theory, and reflected that it was "fulfilling to see the practical and the theoretical kind of combine and merged". This approach enabled Tal to understand both the *how* and *why* of pedagogical decisions. Peri similarly described receiving mathematics-specific mentoring that expanded their pedagogical repertoire, such as using a trundle wheel to measure real spaces when exploring area, noting that they "would have never thought that was even a possibility". Several PSTs described supervisor feedback that directed attention to improving mathematical explanation. Dain reported encouragement to strengthen lesson summaries so students understood the conceptual purpose of lessons, while Lem received mentoring on how to articulate theorems clearly. Cece likewise highlighted mentoring conversations about sequencing content for mixed-ability classes, and how to anticipate student misconceptions.

PSTs noted that without mathematics-specific mentoring they felt unsure of how to develop practice, even with positive supervisory support. Limited opportunities to engage in pedagogical discussions resulted in a negative shift in PSTs' mindsets, with some noting "classroom management is more important than teaching mathematics". Without mathematics-specific mentoring, PSTs defaulted to replicating supervisors' pedagogy ("prac[ticum] students have a lot less autonomy...most of my lessons plans were catered towards my supervising teacher's usual pedagogy") and felt constrained in developing their competence ("I could not teach how I wanted", "you don't have much control over your own actions").

These findings highlight the importance of supervisor mentoring that focuses on sustained engagement with pedagogical decisions to develop PSTs' mathematical pedagogical content knowledge (Ball et al., 2008; Hudson, 2013; Shulman, 1987). Mathematics-specific mentoring additionally supports PSTs' competence by positioning mathematics teaching as learnable (i.e., a growth mindset; Dweck, 2006). As a result, such mentoring approaches position placements as developmental learning opportunities that centre students' mathematics learning.

Design Principle 3: Visible Belief in Learnability of Mathematics Teaching

PSTs' interpretations of instructional difficulty were mediated by supervisors' growth-oriented beliefs about the learnability of mathematics teaching. These messages shaped how PSTs interpreted challenges, influencing whether difficulties were seen as learning opportunities. With supervisor support, PSTs learned to persist through instructional difficulty, emphasising flexibility ("I have learnt that sometimes not all things will go the way you planned"), being prepared with multiple explanations ("you have to persist and try multiple explanations to help students understand"), and being adaptive based on students' needs ("I realised I must constantly try new things because students and days are all individual").

Interviews showed how growth-oriented interpretations of challenges were produced in situ. In Tal's first placement, their supervising teacher explicitly normalised the developmental and ongoing nature of becoming a competent teacher: "he told me that he didn't feel like a proper teacher until he was 4 or 5 years into the job" and that "on the drive home he would think and self-reflect on what could have gone differently". Tal's supervisor also communicated psychological safety around imperfection, telling PSTs to "focus on absorbing as much as we can", and emphasising that when they "stumbled...it's okay". Consistent with this positioning of teaching as learnable, Tal became more willing to seek challenge, noting they hoped future placements would be "completely different style[s] of school just so that I can learn a lot". While some placement accounts aligned with growth-oriented beliefs about teaching, others reinforced fixed narratives. Dain's case illustrated how placements could constrain PSTs' beliefs about what was possible in "real classrooms" when mentoring did not reframe these assumptions. Dain described viewing engaging or innovative pedagogy as "less possible" after placement, culminating in views such as "there's literally no way I can make polynomial division interesting". Other PSTs reported mathematics teaching as "very limiting and mundane" after placement. In these cases, "realism" discourses narrowed pedagogical possibilities, with challenges interpreted as confirmation that alternative approaches were not feasible. These contrasting trajectories were less about whether PSTs encountered challenging lessons (they all did), and more about how supervisors and faculty framed those lessons.

These placement accounts demonstrate that when supervisors framed struggle as expected and productive, PSTs were more likely to persist, experiment, and engage in reflective refinement of their practice (Nalipay et al., 2021). Growth mindsets act as a protective condition that sustains PST engagement and confidence, supporting their transition into a profession in which ongoing development is both necessary and inevitable (McCormack et al., 2006).

Design Principle 4: Relational Safety to Take Pedagogical Risks

PSTs' willingness to take pedagogical risks was connected to relational conditions within placement environments, particularly the extent to which they experienced belonging, trust, and psychological safety. One PST noted that "I'd be more likely to try things if I am in a safe environment...where I don't feel like I will be reprimanded if something goes wrong", highlighting how trust enabled risk-taking. Belonging was foundational for PSTs, with 25 of 27 survey responses referencing rapport with students and staff or feeling part of the school community as the most enjoyable components of placement.

Interviews illustrated how relational safety enabled pedagogical risk-taking in practice. Mich described their supervisor encouraging experimentation prior to giving feedback, allowing them to recognise limitations of lesson designs through experience. Similarly, Tal reported being encouraged to develop their own teaching style rather than imitate their supervisor, which supported experimentation with conceptual and hands-on approaches. Tal's supervisor told them "add your own personal touch...there's no point in you trying to mimic everything I do". Peri also reflected "my supervisor kept trying to stress...that not everything is perfect...don't let it get the better of you". Where supervisors modelled and encouraged pedagogical experimentation and a growth mindset towards teaching, PSTs reported greater willingness to trial new approaches. One PST noted that their supervisor "showed me new methods for certain topics as well as incorporating games into the lesson," strategies they intended to adopt in the future. Contrastingly, damaged relational trust and school environments that discouraged diverse pedagogical strategies had negative consequences for PSTs' willingness to experiment. For example, one PST reported "my supervising teacher constantly correcting me in front of students and peers damaged my confidence...I now am unsure about a career in teaching". Another noted "I would love to learn to do more inquiry tasks, but many teachers I have talked to were not in favour of them and found them difficult to run".

While design principle 3 focuses on how supervisors frame instructional challenges as part of the learnable nature of teaching, this design principle emphasises the relational conditions that enable PSTs to act on growth-oriented beliefs through pedagogical risk-taking (Hobson & Maxwell, 2017). When mistakes are treated as learning opportunities (Dweck, 2006), PSTs are more willing to trial alternative pedagogical approaches and refine their practice through reflection (Hudson, 2013; Schön, 1995; Shulman, 1987).

Practical Implications for Placement Design

Four key features (design principles) were identified in placement environments that supported preservice secondary mathematics teachers' flourishing identities. Taken together, these four design principles provide a framework for structuring secondary mathematics placements that support PST flourishing. The practical implications for different stakeholders (supervising teachers, placement schools, and ITE providers) are discussed below. Implications for PSTs are also addressed, as how PSTs engage within these environments matters given teacher identity is enacted through active participation in practice (Darragh, 2016).

Implications for Supervising Teachers

Supervising teachers play a critical role in creating placement environments that support PST flourishing. Mentoring should deliberately involve PSTs in the instructional decision-making processes of mathematics teaching (*design principle 1*) through a gradual release of responsibility approach involving co-planning opportunities where PSTs make bounded pedagogical decisions followed by reflective debriefs (Collet et al., 2025). This approach normalises, permits, and supports autonomous experimentation in mathematics teaching, all important in the agentic development of identity (Darragh, 2016), supporting PSTs' confidence (Byth, 2026). Supervisors should also prioritise mathematics-specific mentoring (*design principle 2*) that directs feedback towards instructional decisions shaping students' mathematical understanding, which supports the development of PSTs' pedagogical content knowledge (Ball et al., 2008). We suggest a protocol that focuses supervisory feedback on the mathematical goal of the lesson, how student thinking was facilitated, the mathematical representations used and their effectiveness, the mathematical explanations given and their clarity, how misconceptions were addressed, and how students' mathematical learning was evaluated. When feedback targets specific instructional moves, teaching becomes visible as a practice that can be refined through reflection and iteration, communicating the learnability of teaching (*design principle 3*) and reinforcing a growth orientation towards teaching practice (Dweck, 2006). Finally, supervisors can support pedagogical risk-taking by establishing relational safety (*design principle 4*), for example by welcoming PST questions, treating mistakes as learning opportunities, providing feedback respectfully, and meaningfully including PSTs in departmental conversations and routines (Beauchamp & Thomas, 2009).

Implications for Schools Hosting Placements

Schools hosting placements shape the structural conditions that enable PSTs to develop flourishing mathematics teacher identities. Satisfaction of the basic psychological needs in SDT depends on PSTs' interaction with environmental factors (Hobson & Maxwell, 2017), including whether they feel they belong to the school community as this impacts their perceived autonomy (Pendergast et al., 2020). To support structured autonomy (*design principle 1*), schools should avoid expectations that PSTs mimic supervisor's pedagogy and instead make space for PSTs to make autonomous decisions within syllabus-aligned scope and sequences. Allowing PSTs to access mathematics-specific mentoring (*design principle 2*) also requires schools to consider mentor selection carefully, prioritising teachers with strong pedagogical content knowledge (Hudson, 2013). Mentoring should ideally be recognised as a collective departmental responsibility, enabling PSTs to engage in richer professional conversations about mathematics

teaching and further develop their competence. Schools can further reinforce the learnability of mathematics teaching (*design principle 3*) by cultivating cultures in which observing, trialling approaches, revising lessons, and re-teaching are viewed as normal aspects of professional practice. This is particularly important in mathematics, where some student misconceptions are predictable, and instruction benefits from iterative refinement through ongoing professional learning (McCormack et al., 2006). Finally, fostering relational safety (*design principle 4*) requires PSTs to be treated as legitimate members of the mathematics teaching community, for example by including them in planning meetings and resource sharing where possible.

Implications for ITE Providers

ITE providers play an important role in preparing PSTs, supervisors, and schools for placement environments that support flourishing mathematics teacher identities. To support structured autonomy (*design principle 1*), PSTs should be prepared within ITE courses to make and justify instructional decisions in mathematics lessons. Developing this capacity requires explicit attention to pedagogical content knowledge within coursework (Ball et al., 2008; Shulman, 1987) and reflective practice skills (Schon, 1995). Ideally, PSTs should enter placements with an understanding that teaching develops through reflection and refinement (*design principle 3*). Preparing PSTs with reflective tools (Schön, 1995) that frame difficulty diagnostically can reinforce a growth-oriented view of teaching (Dweck, 2006). Mathematics-specific mentoring (*design principle 2*) requires ITE providers to support supervisors' capacity to lead productive pedagogical conversations by providing mathematics-specific briefing materials and feedback protocols. ITE providers should also encourage supervisors and schools to facilitate psychologically safe environments where PSTs engage in structured "risk cycles" (*design principle 4*), where they plan, teach, receive feedback, and re-teach lessons. Such cycles allow PSTs to refine pedagogical approaches through iterative practice.

Implications for Preservice Teachers

While placement environments and mentoring structures play a central role in supporting flourishing identities, PSTs are active participants in these environments and identity is enacted, not just supported (Darragh, 2016; Hobson & Maxwell, 2017). Therefore, it is important to articulate key agency-enabling moves that reinforce and complement the obligations of supervisors, schools, and ITE. PSTs can support their own development by engaging purposefully with the opportunities provided during placement. For example, when given opportunities to make autonomous instructional decisions (*design principle 1*), PSTs can articulate and justify choices and actively seek feedback on the effectiveness of these decisions. Engaging actively in mathematics-focused post-lesson conversations (*design principle 2*), for instance by asking targeted questions about student thinking, representations, or misconceptions etc., can further support PSTs in developing their competence. PSTs may also benefit from approaching moments of instructional difficulty as opportunities for learning (*design principle 3*), using feedback to reflectively plan future instructional moves. Finally, where relationally safe environments are established (*design principle 4*), PSTs can take bounded pedagogical risks, such as trialling alternative representations or discussion structures, and use subsequent feedback and debrief conversations to refine practice.

Conclusion

Placement conditions shape secondary mathematics PSTs' identities. These findings extend research on secondary mathematics teacher identity development by demonstrating that flourishing identities are cultivated through the design of placement environments, rather than existing as individually held dispositions. Consistent with SDT, secondary mathematics PSTs' sense of autonomy, competence, and relatedness (Ryan & Deci, 2017) were shaped through opportunities to participate meaningfully in instructional decision-making, receive

mathematics-specific mentoring, interpret instructional challenges as part of the learnable nature of teaching, and experience relationally safe environments that support pedagogical experimentation. When placements supported these conditions, PSTs were able to engage reflectively with their practice, develop confidence in their capacity to teach mathematics, and see themselves as legitimate members of the mathematics teaching community. These findings demonstrate the importance of intentionally designed placement environments that support the development of flourishing mathematics teachers.

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References

- Ball, D., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, 59(5), 389–407.
- Beauchamp, C., & Thomas, L. (2009). Understanding teacher identity: An overview of issues in the literature and implications for teacher education. *Cambridge Journal of Education*, 39(2), 175–189.
- Byth, A. (2026). Rethinking the gradual release of responsibility in ITE mentoring: an andragogical perspective. *The Australian Educational Researcher*, 53(1), 21.
- Clarke, V., & Braun, V. (2017). Thematic analysis. *The Journal of Positive Psychology*, 12(3), 297–298. <https://doi.org/10.1080/17439760.2016.1262613>
- Collet, V. S., Gragg, S., & Leggett, A. (2025). Making Moves: Use of the Gradual Increase of Responsibility Model for Mentoring Student Teachers in Residency. *Journal of Teacher Education*, 76(5), 474–487.
- Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, 93, 19–33.
- Dweck, C. S. (2006). *Mindset: The new psychology of success*. New York, NY: Random House.
- Hobson, A. J., & Maxwell, B. (2017). Supporting and inhibiting the well-being of early career secondary school teachers: Extending self-determination theory. *British Educational Research Journal*, 43(1), 168–191.
- Hudson, P. (2013). Strategies for mentoring pedagogical knowledge. *Teachers and Teaching, Theory and Practice*, 19(4), 363–381. <https://doi.org/10.1080/13540602.2013.770226>
- Loughran, J. (2013). *Developing a pedagogy of teacher education: Understanding teaching & learning about teaching*. Routledge.
- McCormack, A., Gore, J., & Thomas, K. (2006). Early Career Teacher Professional Learning. *Asia-Pacific Journal of Teacher Education*, 34(1), 95–113. <https://doi.org/10.1080/13598660500480282>
- Nalipay, Ma. J. N., King, R. B., Mordeno, I. G., Chai, C.-S., & Jong, M. S. (2021). Teachers with a growth mindset are motivated and engaged: the relationships among mindsets, motivation, and engagement in teaching. *Social Psychology of Education*, 24(6), 1663–1684.
- Oxford English Dictionary. (2023). *Flourish* (v.), sense 1.1.b. <https://doi.org/10.1093/OED/3228912951>.
- Pendergast, D., Ronsley-Pavia, M., & Dewhurst, Y. (2020). A systematic quantitative literature review of pre-service teachers' sense of belonging during school-based experience. *International Journal of Home Economics*, 13(2), 59–79.
- Reid O'Connor, B., & Zunica, B. (2024). Flourishing Mathematics Teachers: The Effect of School-Based Placements on Preservice Secondary Mathematics Teachers Anticipated Job Enjoyment. In J. Visnovska, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education. Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 368–375). Gold Coast: MERGA.
- Ryan, R. M., & Deci, E. L. (2017). *Self-determination theory: Basic psychological needs in motivation, development, and wellness*. Guilford Publications.
- Schön, D. A. (1995). *Reflective practitioner: How professionals think in action (New ed.)*. Aldershot, England: Arena. doi:10.4324/9781315237473
- Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1–23. <https://doi.org/10.17763/haer.57.1.j463w79r56455411>
- Zeichner, K. (2010). Rethinking the connections between campus courses and field experiences in college- and university-based teacher education. *Journal of Teacher Education*, 61(1-2), 89–99.

Professional Noticing Journal for Evidence-Linked Decisions and Teacher Learning

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This paper introduces the Professional Noticing Journal (PNJ), a learner-centred artefact that keeps attention on students and enables professional noticing. Drawing on two lesson study teams, we show how PNJ entries anchored teaching decisions in student evidence and interpretations, surfaced recurring issues (tool use, units and language, and “answers without explanation”), and prompted targeted responses. Teams converged on portable routines—brief tool onboarding, visible anchors, just-in-time reviews, a Result–How–Meaning talk frame, and a non-tech route. The PNJ also served as a boundary object, coordinating roles and post-lesson discussions.

Introduction

Improving mathematics instruction depends on teachers’ capacity to notice and respond to student thinking. Research on professional noticing conceptualises this work as attending to strategies, interpreting understandings, and deciding how to respond (Jacobs et al., 2010; Sherin et al., 2011). Professional development has advanced methods for sharpening attention and interpretation, often with video or structured discussion (Santagata, 2011; van Es & Sherin, 2002, 2008). Yet a persistent gap remains: translating noticing into clear, content-specific next moves during live lessons and documenting how those moves arise from evidence. In Lesson Study (LS), post-lesson discussions also tend to drift from student data toward general advice unless talk is anchored by shared artefacts (Fernandez & Yoshida, 2004; Lewis et al., 2009).

In this paper, we propose a simple artefact to address this gap. The Professional Noticing Journal (PNJ) is a one-page artefact that asks observers to write about learners across four linked components: Observation, Interpretation, Decision, Reflection (O–I–D–R). Conceptually, the PNJ is designed to (a) support a simple traceability check from observation and interpretation to the proposed move, and (b) record brief carry-forward notes in the Reflection column that can inform subsequent planning. We frame the PNJ as a boundary object intended to coordinate roles and phases within LS while keeping attention on students (Akkerman & Bakker, 2011; Star & Griesemer, 1989).

To analyse what kinds of teacher knowledge, appear in PNJ entries, we use Mathematical Knowledge for Teaching (MKT) as an interpretive lens (Ball et al., 2008) to examine whether and how knowledge-in-use becomes visible.

This study examines the PNJ in two mathematics research lessons from the Philippine curriculum—Grade 7 discounts/percentages and Grade 9 quadratic functions in vertex form—as the first analysed cases in a larger programme on artefact-mediated LS. The study addresses the following research questions:

1. How does the PNJ surface student thinking and teachers’ knowledge-in-use in written entries and in the post lesson analysis and discussion (PLAD)?
2. What carry-forward notes do teachers record, and how are these used in post-lesson discussion?
3. To what extent do written decisions explicitly cite prior observations and interpretations?

Theoretical Framing

Professional Noticing of Students Mathematical Thinking

Professional noticing conceptualises in-the-moment teacher expertise as three interrelated skills: attending to students' strategies, interpreting what those reveal about understanding, and deciding how to respond (Jacobs et al., 2010; Sherin et al., 2011). Much of this work has used video- or artifact-based tasks to cultivate selective attention and sense-making (van Es & Sherin, 2002, 2008) and to structure PD routines that sharpen analyses of student thinking (Santagata, 2011). A persistent challenge is turning noticing into traceable, warranted pedagogical moves in live lessons and collaborative settings such as Lesson Study. Tools that require written observation, interpretation, and decision (O-I-D) can externalise tacit thinking, discipline post-lesson talk, and build shared repertoires. The PNJ extends this literature by adding an explicit Reflection column to capture portable principles or routines and by making the O→I→D linkage inspectable.

Mathematical Knowledge for Teaching (MKT)

MKT frames the knowledge used in the work of teaching mathematics (Ball et al., 2008) and is employed here as an interpretive lens for the PNJ entries and post-lesson discussion. The analysis attends to Subject Matter Knowledge (SMK)—including common, specialised, and horizon aspects where relevant—and to Pedagogical Content Knowledge (PCK), comprising Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT), as well as Knowledge of Content and Curriculum (KCC) (Hill et al., 2008). Coding marks instances where these components are made visible in the written/spoken record and notes sequences in which one component appears to inform another (e.g., evidence about student thinking informing the selection or sequencing of representations), without presupposing which components will be most salient. The framework is used descriptively—to characterise knowledge-in-use as it becomes public through PNJ records and team talk—rather than to rate quality or quantity of knowledge.

Lesson Study as Context for Generating Practical Knowledge

Lesson Study is a collaborative, iterative process in which teachers study, plan, enact, and analyse research lessons to improve instruction (Fernandez & Yoshida, 2004; Lewis et al., 2009). Because Lesson Study aims to centre discussion on evidence of student thinking and to accumulate sharable instructional knowledge across cycles, it provides the collaborative context in which PNJ records are generated and then used to inform subsequent teaching. That is, by design, the PNJ targets this need: it serves as a learner-centred record during observation and as a scaffold for focused, evidence-linked post-lesson discussion.

Boundary Objects and Design-Based Research

Tools that travel across roles and settings must be robust enough to be shared and flexible enough to be locally adapted, qualities captured by boundary objects (Akkerman & Bakker, 2011; Star & Griesemer, 1989). We therefore position the PNJ as a boundary object—a portable, refinable artefact that coordinates attention across observers, teachers, and facilitators via the shared structure Observation → Interpretation → Decision → Reflection. Methodologically, the work aligns with Design-Based Research (DBR), which iteratively co-designs, tests, and refines mediational tools in naturalistic settings to generate practical solutions and theoretical insight (Cobb et al., 2003). DBR guides the design choices that keep the PNJ learner-centred, require interpretive warrants for decisions, and capture reusable routines in the Reflection column.

Method

This study forms part of a larger programme on teacher learning in artefact-mediated LS, using a qualitative multiple-case design. We report two illustrative LS cycles in mathematics to examine how the PNJ works in practice. Specifically, we analyse how the PNJ (a) surfaces evidence of student thinking, (b) makes pedagogical decisions traceable to that evidence and its interpretations, and (c) captures brief “carry-forward” routines in reflections.

The two cases presented are purposively selected for pedagogically distinct topics—Grade 7 discounts/percentages and Grade 9 quadratic functions (vertex form)—to probe the PNJ’s functioning across grade levels, content demands, and representational resources.

The participants

Data were collected in ordinary school settings during two research lessons in the Philippines: Case 1 (Grade 7; discounts/percentages) and Case 2 (Grade 9; quadratic functions in vertex form). For analytic purposes, cases are labelled using a two-part code indicating case number and grade level (eg., 1-G7 denotes Case 1 LS in Grade 7; 2-G9 denotes Case 2 LS in Grade 9). Case 1 involved an LS team of 8 mathematics teachers, while Case 2 involved 4 mathematics teachers from nearby schools. All teams completed an introductory training course on doing LS, including an orientation to using the PNJ during observation and discussion. The trainers served as a ‘knowledgeable other’ who provides expert guidance during the LS process, and facilitator. No identifying information about individuals or schools is reported.

The Two Case Lessons

The Grade 7 lesson unfolded in five episodes. First, a learner facilitated a quick drill on basic arithmetic using teacher-made cards (e.g., $6+4$, $30-2$), with volunteers responding and the teacher confirming correctness. Second, the class briefly recalled decimal–percent equivalents; again, students supplied answers and the teacher verified without extended explanation. Third, students worked with a GeoGebra applet *Discount Activity 1* (geogebra.org/m/kmsmndvn), in which sliders controlled the original price (0–1200) and discount rate (0–50%). The applet displayed corresponding bar representations of the original and discounted prices, allowing students to observe how changes in discount rate affected the selling price. In groups, they populated a table (original price, discount rate, selling price), then addressed follow-up questions aimed at noticing patterns (e.g., how selling price changes with discount rate). Selected entries were recorded on the board before discussion. Fourth, the teacher introduced concise definitions of discount and selling price, with students reading them aloud. Fifth, students used the same applet to complete a second table tied to images with given prices and discount rates—each group handled one row—then answered prompts leading to conjectures for computing discount amount and selling price. Groups presented their results on the board, and the teacher facilitated a brief whole-class consolidation to formulate the general procedure.

The Grade 9 lesson introduced transformations of the quadratic parent function $y=a(x-h)^2+k$ by examining the parameters a , h , and k . It opened with real-world parabolic images to motivate the topic, followed by a brief review of the parent function $y=x^2$ and its invariants (upward opening, vertex at (0,0), symmetry). Students then worked in groups on shared tablets using Desmos across three focused episodes: vertical shifts (k) with $y=x^2+k$, noting the vertex moves up/down while shape and symmetry remain; horizontal shifts (h) with $y=(x-h)^2$, identifying vertex movement to $(h,0)$ and the direction rule from the sign; and the multiplier a with $y=ax^2$, concluding $|a|$ narrows, $0<|a|<1$ widens, and negative a reflects across the x -axis. Groups recorded observations in small tables. The lesson closed with a synthesis task in which students, without technology, predicted and sketched graphs in vertex form $y=a(x-h)^2+k$, identifying the vertex at (h,k) and articulating how a , h , and k each affect the graph.

Data sources

The study drew on two streams of evidence from the two LS teams: (a) the Professional Noticing Journals completed individually by the team members (6 for Grade 7 and 5 for Grade 9) during the research lesson and immediately afterward, and (b) verbatim transcripts of the teams' post-lesson analysis and discussion (PLAD). All artefacts were anonymised. The PNJ is a one-page template that structures Observation → Interpretation → Decision → Reflection (O→I→D→R) and requires entries about learners, not the teacher. Prompts ask observers to: (1) record what students did/said, (2) infer what this suggests about understanding, (3) state a pedagogical decision, and (4) note a brief principle or routine to carry forward. Figure 1 shows an excerpt from one of the PNJ's that we analysed.

Figure 1

Verbatim Professional Noticing Journal entry from case 1

PROFESSIONAL NOTICING JOURNAL				
Name: [REDACTED]		Date: August 11, 2025		
Lesson/Topic: Solve Money Problems Involving Percentages		School of Implementation: [REDACTED]		
EPISODES	(#1) OBSERVING <i>What caught your attention about what the learner(s) said and did in this episode?</i>	(#2) INTERPRETING <i>What could this (your entry in #1) mean in relation to their understanding?</i>	(#3) DECISION MAKING <i>What pedagogical decision may be made in relation to #1 and #2?</i>	(#4) REFLECTING <i>What insights have you gained based on your answers in #1, #2, and #3?</i>
Episode 1 (Activating Prior Knowledge)	The students were unable to cite decimals correctly. For example, 23.5% was expressed as zero point 123 rather than 123 thousandths.	If this misconception is not corrected right away, the learners will bring it with them, and what they remember will be used repeatedly.	Numeracy and literacy go hand in hand. It is extremely crucial in Mathematics to utilize mathematical words.	Learners will only recall what their teachers have taught them. Improvement in numeracy is dependent on how we teach and communicate knowledge to students.
Episode 2 (Task 1)	Learners initially worked independently exploring the applet. Their expressions reflect curiosity and excitement. They ignore the instructions and proceed directly to explore the applet. There was no discussion on what their final answer would be. They are excited to write down their own findings in the activity sheet.	Exploring the applet for the first time and observing what happens when they move the slider draws learners' interest in using the gadget and discovering how it works. It's just that they only move the slider for discounts, thus their original prices are all ₱1,200.	Because this is the first time the students will use the applet, the teacher must thoroughly explain the three procedures before instructing them to form groups. Activity sheets must be distributed first so that students figure out what tasks must be performed before being told to bring in their gadget.	Clear instructions and an activity sequence are critical to learners' understanding of the work at hand. Because each learner is unique, the facilitators play an important role in their success.

Analytic strategy

We conducted qualitative thematic analysis in two coding cycles across both data sources (PNJ entries and PLAD transcripts) using a single, shared codebook.

Cycle 1: Structural and open coding. For PNJ sheets, each entry was treated as four linked components—Observation (O), Interpretation (I), Decision (D), Reflection (R)—and inductively open-coded within component to capture salient features of what teachers noticed, how they made sense of it, what moves they proposed, and what principle/routine they recorded. For PLAD, transcripts were segmented into idea units (speaker turns or clauses that advanced a distinct point). Each unit was tagged O, I, D, or R when it functioned as an observation about students, an interpretation of understanding, a proposed/past teaching move, or a generalisable takeaway. The same open-coding scheme used for PNJ components was applied to PLAD units.

Cycle 2: Theme development. First-cycle codes from both sources were clustered into shared, practice-proximal themes to support cross-case comparison: tools/onboarding, prior knowledge, language & units/representations, reporting & discourse, and access/logistics. Themes were refined through iterative memoing and constant comparison.

Horizontal alignment (within-source coherence). For PNJ, we examined horizontal alignment across O→I→D→R within each entry, marking explicit textual links (e.g., a

Decision that cites or paraphrases a recorded Observation/Interpretation) and classifying linkages as present/absent. For PLAD, we analysed local discourse segments for O→I→D→R sequences, noting whether later turns explicitly took up earlier observations (e.g., “Given what we saw about % vs ₱...”). Alignment summaries were produced at the case level.

Traceability of pedagogical moves. For every Decision in both PNJ and PLAD, coders checked whether the move explicitly referenced (a) at least one prior Observation (D→O) and (b) an Interpretation where present (D→[O+I]). Counts and proportions were computed separately for PNJ and PLAD to describe warranting practices, and representative excerpts were selected.

To enhance credibility and dependability, two researchers co-developed the O/I/D/R codebook and thematic tag definitions using a small PNJ subset, iteratively refining examples and decision rules. A sample of PNJ lines and adjacent PLAD segments was independently double-coded and then reconciled through discussion; disagreements were used to clarify boundaries between codes and to revise the guide. For decisions, we conducted a traceability audit that checked whether each recorded move was explicitly warranted by prior observations and, where present, interpretations, verifying links against the original PNJ text and corresponding PLAD excerpts. Triangulation was supported by convergence between written PNJ entries and spoken references in PLAD, and analytic transparency was maintained through a running log of codebook revisions, rationale for merges/splits, and exemplar selection. All artefacts were anonymised; the focus of analysis remained on aggregated patterns of professional learning rather than evaluation of individuals.

Ethical considerations

Clearance was granted by the Schools Division of Pasig who requested the training. Prior to participation in the training course, teachers provided informed consent for the use of data generated during the training and its associated Lesson Study activities. For students present during classroom observations, parental consent was secured in advance. All PNJs and post-lesson discussion transcripts were anonymised, and no student names or other identifying details are reported. The analysis focused on teachers’ professional learning and aggregated patterns across cases rather than on the evaluation of individual teachers or students.

Results

Surfacing student thinking and teachers’ knowledge-in-use in PNJs (RQ1)

Results show that the journal consistently surfaces evidence of students’ mathematical thinking alongside teachers’ knowledge-in-use. Across both cases, teachers attended to how learners engaged with representations, tools, and explanations, and interpreted these observations in ways that informed content-specific pedagogical decisions.

In Case 1 (Grade 7 percentages/discounts), PNJs frequently documented learners’ correct numerical responses that were not supported by explanations, confusion between decimal and percentage representations, and difficulty manipulating the GeoGebra applet used to explore different discount contexts. These observations were interpreted as indicators of weak conceptual grounding and limited familiarity with digital tools. As detailed in the Figure 1 excerpt, the Grade 7 entry documents weak conceptual grounding, such as misnaming 23.5% as “zero point 123”, and limited tool familiarity when students distractedly played with the applet rather than following task instructions. These specific observations directly informed the teacher’s pedagogical decisions, which reflected teachers’ knowledge-in-use, such as planning brief pre-task onboarding for tool use, clarifying terminology through examples, and restructuring tasks such that learners are required to explain not only the results but also meanings.

In Case 2 (Grade 9 quadratic functions in vertex form), PNJs highlighted learners' reliance on Desmos to identify features of graphs, partial recall of properties of the parent function, and occasional mistakes in naming or describing parameters. These were interpreted as signs that geometric representations supported access to key ideas but that prerequisite knowledge needs reinforcement. Pedagogical decisions emphasised retaining dynamic representations supplemented with explicit recall of function properties and non-technology tasks to consolidate understanding.

Table 1 summarises the recurring themes in the PNJs across the two cases (coded as 1-G7 for Case 1, Grade 7, and 2-G9 for Case 2, Grade 9), linking observed student thinking to interpretations and corresponding pedagogical moves proposed by observers. The table represents recurring patterns across entries rather than exhaustive enumeration of all PNJs.

Table 1

Recurring observations, interpretations, decisions, and reflections (O–I–D–R) across cases

Observation/Surfaced student thinking	Interpretation of understanding	Pedagogical Decisions	Carry-forward Reflections	Case Code
Correct answers given without explanation	Computation detached from meaning	Structure reporting using a Result-How-Meaning frame	Structure discourse that foregrounds explanations	1-G7
Difficulty manipulating GeoGebra sliders	Limited tool familiarity constraining access to concepts	Micro-onboarding prior to task engagement	Tool fluency supports conceptual access	1-G7
Confusion between decimal and percent forms	Weak conceptual grounding in representations	Clarify terminology through examples	Precision in mathematical language (naming) matters	1-G7
Reliance on Desmos to identify graph features	Digital (visual) tools support access but mask gaps in recall	Retain dynamic representations with explicit recall	Representations should be paired with sense-making	2-G9
Incorrect naming of parameters and properties	Partial understanding of function features	Reinforce prerequisite knowledge and language	Prior knowledge must be activated purposefully	2-G9

Carry-forward routines articulated in PNJ reflections (RQ2)

As summarised in Table 1, reflection entries in the PNJs articulated carry-forward routines that abstracted across specific lesson episodes. Rather than recounting isolated observations or decisions, these reflections named principles that addressed recurring instructional challenges, such as the importance of structuring classroom discourse to foreground explanations, and micro-onboarding of digital mathematics tools particularly when they are central to the lesson. Reflections also highlighted the importance of reviewing prerequisite knowledge and skills aligned with the main lesson, and purposeful recall activities that are directly tied to the object of learning.

In post-lesson analyses and discussion, the LS team members and the observers took up these routines as shared reference points when considering revisions and subsequent implementations. Taken together, the reflection entries show how the PNJ functions not only

as an artifact for noticing and decision-making but also as a mechanism for carrying forward reusable instructional routines beyond the immediate lesson.

Traceability of pedagogical decisions across PNJ entries (RQ3)

To address RQ3, PNJ entries were examined for horizontal alignment across Observation, Interpretation, and Decision components only. Alignment focused on whether pedagogical decisions meaningfully took up recorded observations of learner activity and corresponding interpretations of understanding. Reflection entries were excluded from this analysis because they are designed to capture carry-forward generalisations.

Table 2

Horizontal alignment of PNJ entries by case

Alignment category	Definition	Case 1 (G7) n=24	Case 2 (G9) n=16
Fully aligned	Observation and interpretation are explicitly taken up in the pedagogical decision	16 (66.7%)	15 (93.8%)
Partially aligned	Observation and interpretation are coherent, but the decision addresses only part of the interpreted issue	0 (0%)	0 (0%)
Weakly aligned	Observation is present, but the decision is generic or weakly connected	0 (0%)	0 (0%)
Misaligned	Observation and interpretation do not meaningfully inform the decision	8 (33.3%)	1 (6.2%)

As shown in Table 2, most PNJ entries in both cases demonstrated full alignment across O–I–D, indicating that pedagogical decisions were coherently grounded in documented student evidence and interpretations. In Case 1 (Grade 7), two-thirds of entries were fully aligned, while the remaining entries were classified as misaligned, reflecting decisions that were generic or not clearly informed by the specific observations and interpretations recorded. In Case 2 (Grade 9), nearly all entries were fully aligned, with only one entry classified as misaligned.

Discussion and Conclusion

Across both lessons, the PNJ helped teachers keep their focus on students. Entries paired what learners did and said with what that might mean, and then named a teaching move to try next. In Grade 7 (percentages/discounts), teachers often saw correct answers with no explanation, mix-ups between decimals and percents, and struggles with the GeoGebra applet. Their decisions followed from this: give a short tool onboarding before the task, use concrete examples to fix language, and ask groups to report Result → How → Meaning. In Grade 9 (vertex form), teachers noted reliance on Desmos, partial recall of the parent function, and slips in naming parameters; they chose to keep the dynamic graphs but add quick recall prompts and a no-tech follow-up so ideas would stick. This pattern fits what the noticing literature aims for—attend, interpret, decide (e.g., Jacobs et al., 2010)—and, viewed through Mathematical Knowledge for Teaching, shows KCS → KCT links: evidence about student thinking leading to content-specific moves (Ball et al., 2008).

The horizontal alignment check supports this reading: most entries were fully aligned from Observation to Interpretation to Decision (Table 2). As illustrated in Figure 1, teachers translated immediate observations, into targeted decisions and reusable, carry-forward routines, recognising that “clear instructions and an activity sequence are critical to learners’

understanding”. Where entries were misaligned (more in Grade 7), decisions tended to be general next steps rather than a direct take-up of the recorded evidence. A simple fix is to model a one-sentence warrant (“Because we saw X and think it means Y, we will do Z”) and to add a PNJ prompt, “Which student evidence triggered this decision?”

There were two features of the PNJ that added clear value. First, the Reflection column captured short, reusable routines—brief tool onboarding, visible anchors for language/units or parameter–feature links, just-in-time reviews, the Result–How–Meaning frame, and a non-tech route—that teams re-used in post-lesson discussions and future lessons. Second, the PNJ acted as a shared tool that kept our conversations anchored to what students did and what we should do next, addressing a common LS challenge of drifting away from student evidence.

We studied two cases and worked from written and spoken records, so we did not track enactment or learner outcomes; this limits how far we can generalise. Even so, we invite teams to adopt the PNJ, open debriefs with a brief O–I–D check, and institutionalise the common routines above. Our future research will extend the analysis to additional topics, grade bands, and schools; use a comparative design to contrast PNJ-guided debriefs with conventional protocols; and trace enactment by following the uptake of carry-forward routines into subsequent lessons across cycles. We will also test whether stronger O–I–D alignment in PNJs predicts improvements in the quality of student explanations and representations over time, using common indicators and inter-rater reliability checks. Finally, we will examine when SMK and KCC become visible (planning vs. PLAD) and how facilitation practices mediate the PNJ’s work as a boundary object.

References

- Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research*, 81(2), 132–169. <https://doi.org/10.3102/0034654311404435>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407. <https://doi.org/10.1177/0022487108324554>
- Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9–13. <https://doi.org/10.3102/0013189X032001009>
- Fernandez, C., & Yoshida, M. (2004). Lesson study: A Japanese approach to improving mathematics teaching and learning. Lawrence Erlbaum.
- Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, 26(4), 430–511. <https://doi.org/10.1080/07370000802177235>
- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children’s mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202. <https://doi.org/10.5951/jresmetheduc.41.2.0169>
- Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study. *Educational Researcher*, 38(3), 129–141.
- Santagata, R. (2011). From teacher noticing to a framework for analyzing and improving classroom lessons. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers’ eyes* (pp. 152–168). Routledge.
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: Seeing through teachers’ eyes*. Routledge. <https://doi.org/10.4324/9780203832714>
- Star, S. L., & Griesemer, J. R. (1989). Institutional ecology, “translations,” and boundary objects. *Social Studies of Science*, 19(3), 387–420. <https://doi.org/10.1177/030631289019003001>
- van Es, E. A., & Sherin, M. G. (2002). Learning to notice. *Journal of Technology and Teacher Education*, 10(4), 571–596.
- van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers’ learning to notice. *Teaching and Teacher Education*, 24(2), 244–276.

Teachers' Praxis in Mediating Student Understanding Using Authentic Contexts

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This study examines how teachers mediate upper secondary students' understanding of measurement through the strategic use of authentic real-life contexts, situated within the praxis of enacted classroom pedagogy. Measurement is a foundational mathematical domain, yet low-performing, disengaged students consistently struggle to develop transferable conceptual understanding. Attending to how teacher intention translated into enacted praxis, ethnographic methods captured moment-by-moment pedagogies through which real-life contexts were actualised revealing the conditions under which authentic contextualisation produced genuine mathematical engagement.

Introduction

Measurement is a key area of study for secondary mathematics through which students develop numeracy and mathematical competence for life beyond school. Despite its practical significance, low-performing upper-secondary students may have trouble connecting formal measurement with concepts their application in life and work (Bolstad, 2023). These difficulties may be exacerbated by classroom practices that privilege abstract procedures and textbook learning over meaning-making through tools, real-life representations, and situated problems. This paper reports on an aspect of a larger study investigating what teachers and students value when a numeracy model (Goos et al. 2012) is enacted in the classroom. The study employed a person-oriented methodology, using ethnographic classroom observations and interviews in six upper secondary mathematics classrooms over a 14-month period in Australia.

Authenticity in mathematics education has historically been determined by those who design curriculum, write textbooks, and produce classroom resources. As a result, teachers in practice may not always meaningfully select authentically contextualised materials (Gravemeijer et al., 2017). Conceptual and analytical frameworks developed by scholars such as Goos et al. (2012), FitzSimons (2013) and Gravemeijer et al. (2017), have provided valuable tools for defining and categorising authentic tasks, and for examining how teachers approach their design and selection. Yet, analytical frameworks of this kind, however theoretically robust, do not adequately address the question of how tasks are perceived and experienced by students themselves, how authenticity is felt, negotiated, and valued in the lived reality of the classroom. This study aims to shift the focus, relocating authenticity from the domain of curriculum design to the domain of enactment, and re-centring the perspectives of students and teachers as arbiters.

Drawing on numeracy model (Goos et al., 2012), this study examines how students valued the mathematics concepts that were learnt when teachers and upper-secondary students engaged meaningfully with authentic contexts. The Goos et al. numeracy model has four key components: contexts, dispositions, mathematical knowledge, and tools, and this is considered with a critical orientation (2012). Previous studies have considered the importance of using authentic tasks that are drawn from life (Frankenstein, 2009; Geiger et al., 2023; Sakurai & Goos, 2023) and from the workplace (Fitzsimons, 2013). However, few studies have looked at how mathematics is enacted in the classroom and the impact of teacher praxis. This is defined in this study as the reflexive and purposeful interaction and theoretical knowledge between teachers and students (Grootenboer, 2013; Kemmis et al., 2014). Addressing authenticity through the lens of praxis moves the question beyond task design toward the realities of enactment. Our conception of authenticity therefore is situated, relational and concerned with

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 377–384). MERGA.

how students themselves perceive the enactment of mathematical tasks and what values they and their teachers bring to that encounter. It is this intersection of teacher intention, student perception, and enacted praxis that the present study sets out to examine. Hence, the research question is, how does the numeracy model (Goos et al., 2012) support mathematics learning for students who experience disengagement? And furthermore, what is the role of authentic contexts when using the numeracy model?

Literature Review

Numeracy and mathematical literacy are widely endorsed as central aims of secondary mathematics education, yet their operationalisation in senior secondary classrooms remains theoretically and pedagogically contested, with evidence suggesting that teachers' enactment is often fragmented and reduced to procedural fluency rather than rich mathematical practice (Geiger, Goos, & Forgasz, 2015). Internationally, mathematical literacy is framed as the capacity to formulate, employ, and interpret mathematics in real-world contexts, requiring judgement, reasoning, and communication (OECD, 2019). Australian senior secondary curricula similarly emphasise application, modelling, and interpretation, positioning numeracy as a general capability that extends beyond procedural competence (ACARA, 2022). However, these policy aspirations may sit uneasily alongside entrenched classroom practices that continue to privilege abstraction, efficiency, and symbolic manipulation.

Numeracy is frequently treated as an implicit outcome of mathematics learning rather than an explicit object of teaching (Geiger et al., 2015; Goos et al. 2012; Gravemeijer et al., 2017). This assumption is particularly pronounced in upper secondary mathematics, where curriculum density and assessment practices encourage a narrowing of pedagogical focus. As a result, numeracy risks being marginalised or reduced to superficial contextualisation, rather than enacted as a form of situated mathematical practice. This suggests that simply aligning tasks with curriculum language is insufficient; rather, numeracy must be theorised and examined as a distinct form of mathematical activity.

Authentic task design has been explored as a means of engaging students in learning the mathematics. Liljedahl (2015) found that authentic tasks moved the students from abstraction to concrete understanding. In designing and using tasks, teachers may shift their pedagogical practices to accommodate contexts that encompass real life. However, teacher informed praxis, involving careful contextualisation of the mathematics may be hard for teachers to sustain in the face of abstraction in textbooks and teacher resources.

Low-performing students of mathematics found in upper secondary classrooms, often have not had success with traditional approaches to teaching. Abstraction often requires the removal of contexts and a restructure extracting the essential algorithms or patterns to generalise the mathematics into broad universal rules (Liljedahl, 2015). Yet, in the classroom reality, much of the student thinking happens in a concrete space before abstraction can occur (Gula & Lovric, 2024).

Teaching itself can be understood as a form of praxis, a morally complex activity with enduring "affective, intellectual and social consequences" (Grootenboer & Edwards-Groves, 2014, p. 277), a definition drawn from Aristotle. In this framing, education serves a dual purpose: to meet the needs of individuals and to fulfil the needs of society (Grootenboer, 2013) and is practiced across domains that are simultaneously socio-political, cultural-discursive, and material-economic (Grootenboer & Peter-Koop, 2024). Kemmis et al. (2014) similarly positions education as a means of helping "people live well in a world worth living in" (p. 2). Central to this view is the recognition that mathematics should not exist in a vacuum, devoid of the real-world contexts from which it is derived. Mathematical practices are embedded in everyday life, in shopping, in driving, and in countless other activities. Yet these are rarely recognised in classroom settings as authentic experiences. Instead, mathematics education tends

toward abstract or contrived contexts, and the mathematics encountered in real life is often perceived as less important than traditional school mathematics. (Lave, 2019) reinforces this concern, arguing that learning should be situated in real life and not to be separated from it.

Mathematics teaching is inherently value-laden and carries significant moral, ethical, and caring dimensions (Grootenboer, 2013). When teaching fails to honour these dimensions, students risk becoming disengaged from the subject. Grootenboer's (2013) study identified the fragile nature of some students, and the tension teachers face in attending to their social and emotional needs while simultaneously engaging them with abstract or complex mathematical ideas. Grootenboer and Peter-Koop (2024) further suggest that situating mathematics teaching within authentic practice may support students in transferring their knowledge from the classroom to real life, affirming the broader utility of mathematics beyond schooling. The present study draws on these perspectives, employing authentic and relevant contexts to support students' emotional and social engagement, grounded in the understanding that such contexts are situated within the spheres of life students already inhabit.

Research Gap

Goos et al. (2012) provide a conceptual analytical theoretical framework for the teaching and learning of mathematics concepts (Geiger et al., 2015; Goos et al., 2020). Their framework has provided a way for researchers and teachers to see themselves as taking the primary role in situating the mathematics in the real world, bridging the gap between classroom and workplaces (Geiger et al., 2015; Goos et al., 2020). Important though numeracy frameworks are, researchers and teachers may still be seen as the primary arbiters of what is authentic. Hence, the need to expand these frameworks to help understand how students perceive and what factors support them to engage with authentic mathematics in the classroom.

A substantial body of literature has established the pedagogical value of real-life contexts in mathematics education, documenting their capacity to enhance motivation, support conceptual understanding, and render abstract content meaningful to students (Geiger et al., 2015; Goos et al., 2020). Analytical and conceptual frameworks have made significant contributions to understanding how authentic tasks might be defined and how teachers approach their design and selection. Yet as Grootenboer (2014) and Goos et al. (2012) have noted, the field has attended more closely to the theoretical rationale for contextualised mathematics than to the conditions of enactment. Crucially, these frameworks do not adequately address the question at the level of the student: how do students themselves perceive, recognise, and engage authentically with mathematical tasks, and what social, emotional, and cognitive values they bring to that encounter (Seah, 2016).

Numeracy development in upper secondary measurement cannot be assumed to arise from context-rich tasks alone. Attention should be focussed on the conditions in which students experience tasks as genuinely meaningful rather than merely contextually dressed. Drawing on the numeracy framework in which learning is understood as situated and contextually constituted, (Goos et al., 2012), this study sought to examine how authentic contexts work in practice, how they are selected, orchestrated, and received, and what students and teachers value in the pedagogical practices. This paper aims to respond to calls from Geiger et al. (2015), Goos et al. (2020), and Grootenboer & Peter-Koop (2024) for research that attends to the praxis of contextualised mathematics teaching and extends that work by treating student perception and value as the primary focus.

Research Question

Given that using real world contexts for authentic task design in numeracy lessons is well accepted in the literature, it is important to examine how these tasks are enacted with low-performing students who are also likely to exhibit disengagement from the mathematics. This

paper seeks to explore how mathematics teaching and learning is enacted, and what is valued by the teachers and low-performing mathematics students when using the numeracy model.

Methodology

This study employed qualitative data collection methods including ethnographic classroom observations (Mills & Morton 2013) and semi-structured interviews with teachers and students using person-oriented approaches (POA) (Bergman & Wångby, 2014). POA allows for deep study of a small number of individuals within the classroom environment (Sakurai et al., 2023), considering all individuals as unique, and that on a global level, patterns may emerge to provide insights (Bergman & Wångby, 2014). POA acknowledges that people are complex with multiple layers of interconnected systems of cognition, social, cultural and emotional domains. Given this study seeks to examine how values were enacted in foundational mathematics classrooms of low-performing upper-secondary students, small numbers of subjects were selected to allow for richer, more nuanced insights to emerge.

Six classes from five experienced teachers with a total of 40 students were studied to determine what values were enacted in upper-secondary classes of low performing students with recurring patterns of poor behaviour consistent with high disengagement from learning. Approximately eight classroom observations were conducted per class over a 14-month period, with individual interviews for teachers and small group interviews for students at the end of each observation period. Hence, POA approaches allowed the researcher to be embedded in the classroom as an accepted presence by the students and their teachers. Classes were drawn from metropolitan and regional Victoria, Australia and from a mix of government and private schools. Classes were determined by the teacher to be low performing with students studying a foundational mathematics program. Narrative ethnographic notes (Cohen et al., 2018; Moschkovich, 2019) using the numeracy framework (Goos et al., 2012) provided a detailed account of how values were enacted within the classroom, capturing the lived experiences of both teachers and students. These notes served as a primary lens through which the praxis of validation techniques could be observed and interpreted within the naturalistic setting of a foundational mathematics classroom (Cohen et al., 2018; Moschkovich, 2019).

Measurement activities in this study were deliberately designed by teachers around authentic familiar contexts to make measurement concepts accessible and meaningful to low-performing upper-secondary students and situated in the numeracy model (Goos et al., 2012). This contextualisation provided authentic pedagogical opportunities and supported the decision to situate the data collection and analysis in the Goos et al. (2012) Numeracy model.

Data analysis was conducted using POA methodology (Vargha et al., 2015). A systematic interpretation was undertaken of narrative field notes to distil individuals' behaviour to uncover patterns of teacher and student behaviour within the foundational mathematics classrooms. The ethnographic narratives were imported into ATLAS.ti. Codes were developed and refined in alignment with the numeracy model (Goos et al., 2012), allowing themes to emerge that reflected how values, contexts, and mathematical knowledge were enacted. Trustworthiness was established through "credibility, transferability, dependability, and confirmability" (Cohen et al., 2018, p.319), and operationalised via member checking, factual reporting, over-inclusion, an ATLAS.ti audit trail, and researcher reflexivity in relation to the guiding numeracy model (Cohen et al., 2018).

Results

Specifically, this study considered how teacher intention translates into enacted praxis; the moment-by-moment pedagogical work through which real-life contexts are actualised in the classrooms of low-performing upper-secondary students. All six observed classrooms enacted authentic, real-life numeracy tasks consistent across the dataset. The following results

demonstrate how students engaged in mathematical reasoning and what that engagement reveal about their values and motivations.

Teacher A deliberately selected an image of a bus partially submerged in floodwater which was sourced from contemporary media and presented it without mathematical framing. The intention was to elicit spontaneous quantitative engagement. The lesson was characterised by rapid, overlapping student contributions which were competitive in tone, iterative in structure, and self-sustaining until a verifiable answer was collectively reached, as shown in Table 1.

Table 1

Student discussion about the height of a bus in floodwater

Student	Conversation
A	It's not a coach, is a public transport bus
B	1.5m
A	No
B	2m
C	NOOO
A	No-one is as tall as a bus. 1.5m is a normal height
C	Nah, it's the roof mate 1.5m!
TEACHER	I'm 1.65m
A	But you're short!
B	She's taller than the bus!
A, B, C	SHOUTING
A	How big is a bus?
B	2m?
A	3m?
C	No way!!!!
A	7 foot, that's over 2m. Let's look it up.... 3.25m
C	Bro, do you think he's the size of a bus, he's half a bus

This discussion unfolded as students examined the teacher's carefully chosen image of a bus submerged in Queensland floodwater. Although no formula had been given and no mathematical task was explicitly set, what followed constituted mathematical reasoning: estimation, proportional reasoning, and verification. The exchange progressed from student conjecture (Teacher: I'm 1.65m) to externally verified knowledge (let's look it up), resulting in a collective answer. The teacher's pedagogical design was deliberate: by withholding explicit mathematical framing, thereby creating the conditions for students to consider the problem and participate in an intrinsically motivated activity. This discourse considers what conditions do students elect to engage in mathematical reasoning.

Students across this study consistently distinguished between knowing a formula and understanding what it means in practice. For many, the formula as traditionally used in the classroom was an obstacle to learning: "I can know a formula but sometimes I still won't know the answer. I can have the formula in front of me and the numbers, but the formula can still be just more confusing." Another put the same insight more plainly: "I reckon the formula is useless, unless you understand what you're doing or like how to use it."

When understanding was absent, students exhibited a characteristic withdrawal: “it just didn’t really click for me, and I didn’t have much interest as well”; “I lose my **** and don’t bother doing it”. The teachers in this study referred to this by firstly embedding the formula in an experience. One teacher introduced $s = d/t$ not as an equation to be memorised but through a running drill in which students measured their own speed, another teacher had students consider why the triangle was the strongest shape for truss bridges. As Teacher A noted with the bus task, their students were considering a genuine real-life question (how tall is that bus?) before they realised the mathematics underpinning the task. Understanding and engagement were simultaneous in these enactments.

Relevance was a value that students cited consistently. However, students were not deceived by contextual packaging: “even when they put algebra into a real-life situation it still doesn’t have much relevance to me... just because they put it into a real-life situation ... it has to have importance.” Importance, for these students, was material and specific. Percentages mattered because “when I’m out shopping... find something worth 35 bucks, you get 20 percent off, just chuck the formula in the calculator.” Measurement mattered because “you can’t waste anything, so you make sure you use the correct pieces”, and more urgently, “when I’m at work if I’m gonna cut some timber... you do it again, then you waste, waste some money.”

By contrast, content experienced as relevant only to school provoked strong critique such as two students considered that Pythagoras appears in an exam but nowhere else. They reported “you’re still held to such a high standard to expect to know that... it’s not required to know that outside of getting that score... school forces you to know it.” The language of compulsion using words like *forced*, *required*, *held*, described a relationship with mathematics shaped entirely by obligation. The emotional consequent was explicit: “causing stress over something that really just doesn’t add up.” The students were not, however, outright rejectors of the mathematics, “I know I say like this is stupid, this is stupid, but it’s like it could actually help us in the long run”. This statement suggests that the value of mathematics should be made visible. This is supported by Grootenboer and Peter-Koop (2024) demonstrating that when the mathematics values are inherent, students may engage more strongly with the learning.

The teachers in this study responded precisely to this demand, and they looked for mathematics latent in situations students were already attending to. The bus task was one such moment. The teacher did not construct the scenario but recognised its potential and fostered students’ curiosity as a pedagogical tool. Another example asked about the height of the McDonald’s sign. “How many of you work at McD’s? How many times have you seen the sign out front, have you ever considered how tall it is? And they go, What.” The teacher used the “What?” as the engagement tool, linking the mathematics to an authentic and familiar context as supported by (Grootenboer & Edwards-Groves, 2014). The teacher revealed the mathematics in objects so familiar they had become invisible: “it’s about incorporating everyday objects that they know but never thought about. Humanising the mathematics.” A parallel strategy involved designing open-ended practical tasks oriented toward students’ vocational aspirations. Teacher B was direct about their reasoning: “[these tasks] mostly work as most of these kids want to be tradies, want to work”, and the tasks were constructed accordingly, giving students “a realistic understanding of what they’re working towards.” The open-endedness is deliberate; “they’re free to explore”. This teacher valued autonomy and relevance as central to engagement, not incidental to it. Students corroborated this: “hands on makes me want to learn”; “if it matters to you then that influences you towards learning it.”

What the data reveals is that students may not be disengaged from mathematics itself, but a particular, institutionalised form of it. The bus exchange showed students estimating, arguing, and self-correcting. That is doing exactly what mathematics education sets out to. The gap may not be between students and mathematics, but between *mathematics-as-lived* and *mathematics-as-schooled* as noted by Grootenboer and Edwards-Groves (2019).

Discussion and Implications for Practice

The findings of this study consider the praxis conditions where real-life contexts may produce genuine mathematical engagement among disengaged upper secondary students. Consistent with the work of Geiger et al. (2015), Goos et al. (2020), and Grootenboer and Peter-Koop (2024), the data confirms that contextualised tasks may hold significant pedagogical potential. Yet they also substantiate the field's emerging recognition that context alone is insufficient. The quality of enactment may support student engagement, this may include teacher intention, pedagogical design, and critically, the degree to which students perceived the task as carrying authentic consequence. This aligns with Seah's (2016) contention that student engagement with mathematics is fundamentally value-laden, and shaped by social, emotional, and cognitive orientations that analytical frameworks focused on task design alone cannot capture. The findings echo calls by Grootenboer et al. (2024) for research that attends to the enacted, relational dimensions of contextualised mathematics teaching.

Methodologically, this study is constrained by its ethnographic scope and localised sample. To study deeply the issues of praxis, to uncover the implicit values of teachers and students, means that only a small number of individuals can practicably be studied. Additionally, POA methodology is time consuming, and the issue of temporality dictates that the data collection period is protracted. It is recommended more in-depth case studies may further this knowledge.

All five teachers in this study were observed using authentic contexts pedagogically when teaching measurement concepts. The transferability of measurement concepts for students was most evident when students were required to make assumptions about the mathematics in situations that carried genuine consequence in the world. The students demonstrated the capacity to perceive and act on mathematical structure beyond the classroom, as supported by Gravemeijer et al. (2017) and Grootenboer and Edwards-Groves (2014). This stands in contrast to students' accounts of conventional pedagogical approaches, where mathematics may appear to the students as a set of formulae to be held in memory until the exam, which they characterised as an obligation rather than mathematical agency. Students positioned within authentic contexts reached autonomously for mathematical tools. The bus-in-floodwater task demonstrated this dynamic that the literature has long identified as a condition of deep understanding (Goos et al., 2020; Lave, 2019). The teacher elicited spontaneous mathematical reasoning from the students and engaged students who in the past self-reported that they rarely encountered this type of mathematics in formal classrooms.

The deliberate integration of authentic contexts has positive effects on students observed and self-reported engagement. In authentic tasks, mathematical skills of estimation, proportional reasoning and verification arise as necessary rather than imposed. An open-ended, praxis-grounded pedagogy may well provide a more robust model for how teacher intentions can be translated into enacted classroom experience that students perceive as meaningful and relevant to their lives. These findings reinforce broader calls for curriculum and pedagogical reform that creates space for contextualised, process-oriented mathematical activity (Goos et al., 2020; Grootenboer & Peter-Koop, 2024) in upper secondary mathematics classrooms.

Ethics approval 2024-29481-54364-4 was granted by The University of Melbourne, and participating students and teachers gave informed consent.

References

- Bergman, L. R., & Wångby, M. (2014). The person-oriented approach: A short theoretical and practical guide. *Eesti Haridusteaduste Ajakiri. Estonian Journal of Education*, 2(1), <https://doi.org/10.12697/eha.2014.2.1.02b>
- Bolstad, O.H. (2023). Lower secondary students' encounters with mathematical literacy. *Mathematics Education Research Journal* 35(1), 237–253 <https://doi.org/10.1007/s13394-021-00386-7>
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge.

- Fitzsimons, G. E. (2013). Doing mathematics in the workplace: A brief review of selected literature. *Adults Learning Mathematics: An International Journal*, 8(1), 7–19.
- Frankenstein, M. (2009). Developing a critical mathematical numeracy through real real-life word problems. In L. Verschaffel, B. Greer, W. Van Dooren, & S. Mukhopadhyay (Eds.), *Words and worlds: Modelling Verbal Descriptions of Situations* (Vol 16, pp. 111–130). Brill Sense.
- Geiger, V., Gal, I., & Graven, M. (2023). The connections between citizenship education and mathematics education. *ZDM*, 55(5), 923–940. <https://doi.org/10.1007/s11858-023-01521-3>
- Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: A survey of the state of the field. *ZDM*, 47(4), 531–548. <https://doi.org/10.1007/s11858-015-0708-1>
- Goos, M., Dole, S., & Geiger, V. (2012). Numeracy across the curriculum. *Australian Mathematics Teacher*, 68(1), 3–7.
- Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2020). *Numeracy across the curriculum: Research-based strategies for enhancing teaching and learning*. Routledge.
- Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education* 15(1), 105–123.
- Grootenboer, P. (2013). The praxis of mathematics teaching: Developing mathematical identities. *Pedagogy, culture and society*, 21(2), 321–342. <https://doi.org/10.1080/14681366.2012.759131>
- Grootenboer, P., & Edwards-Groves, C. (2014). *Mathematics teaching as praxis* [Paper presentation]. 37th Annual Meeting of the Mathematics Education Research Group of Australasia (MERGA), Sydney, New South Wales, Australia. <https://eric.ed.gov/?id=ED572643>
- Grootenboer, P., Peter-Koop, A. (2024). Practice theory in mathematics education research. In: Clements, M., Kaur, B., Lowrie, T., Mesa, V., Prytz, J. (eds) *Fourth International Handbook of Mathematics Education*. Springer International Handbooks of Education. Springer, Cham. https://doi.org/10.1007/978-3-031-51474-6_13
- Gula, T., & Lovric, M. (2024). Numeracy tasks: Inspiring transfer between concrete and abstract thinking spaces. *Numeracy*, 17(1). <https://doi.org/10.5038/1936-4660.17.1.1447>
- Kemmis, S., Wilkinson, J., Edwards-Groves, C., Hardy, I., Grootenboer, P., & Bristol, L. (2014). Praxis, practice and practice architectures. In S. Kemmis, J. Wilkinson, C. Edwards-Groves, I. Hardy, P. Grootenboer, & L. Bristol (Eds.), *Changing practices, Changing education* (pp. 25–41). Springer. https://doi.org/10.1007/978-981-4560-47-4_2
- Lave, J. (2019). *Learning and everyday life*. Cambridge University Press.
- Liljedahl, P. (2015). Numeracy task design: A case of changing mathematics teaching practice. *ZDM*, 47(4), 625–637. <https://doi.org/10.1007/s11858-015-0703-6>
- Mills, D., & Morton, M. (2013). *Ethnography in education*. SAGE Publications Ltd. <https://doi.org/10.4135/9781446251201>
- Moschkovich, J. N. (2019). A naturalistic paradigm: An introduction to using ethnographic methods for research in mathematics education. In G. Kaiser & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education* (Vol. 59, pp. 59–79). Springer.
- Sakurai, J., & Goos, M. (2023). Revisiting tools in numeracy learning: The role of authentic digital tools. *Frontiers in Education*, 8. <https://doi.org/10.3389/feduc.2023.1291407>
- Seah, W. T. (2016). Values in the mathematics classroom: supporting cognitive and affective pedagogical ideas. pedagogical research, *Lectito Journals* 1(2), <http://dx.doi.org/10.20897/lectito.201653>
- Vargha, A., Torma, B., & Bergman, L. R. (2015). ROPstat: A general statistical package useful for conducting person-oriented analyses. *Journal for Person-Oriented Research*, 1(1–2), 87–98. <https://doi.org/10.17505/jpor.2015.09>

Beyond Methodological Boundaries Examining Mathematics Leadership: The Potential of Developmental Work Research

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Reports of mathematics leadership in MERGA conference research papers have relied on exploratory methodologies, leaving a gap in interventionist approaches. We argue for Developmental Work Research, within the cultural-historical activity theory tradition, as a methodology for examining mathematics leadership during a time of educational change. We outline how DWR can support mathematics leaders and researchers in collaboratively surfacing, examining, and addressing leadership practice problems. We illustrate this through our current project with several mathematics leaders who are experiencing practice problems stemming from policy change.

This paper is not a report of empirical findings, nor is it a theoretical paper in the conventional sense typically presented at MERGA conferences. Instead, we offer a methodologically oriented paper that articulates an approach for examining mathematics leadership using a methodology known as *Developmental Work Research* (Engeström, 2015). Drawing on work within that field, along with insights from the first author's ongoing research program (Sexton, 2025a), we propose a case for using Developmental Work Research (DWR) to create knowledge about mathematics leadership during times of educational change.

We begin with a concise overview of methodologies reported in MERGA conference research papers about primary school mathematics leadership. We identify a gap in the use of interventionist approaches and argue that addressing this limitation is timely given the shifting policy landscape in Victoria, Australia. Then, we outline the nature and function of DWR as a way of working beyond the methodological boundaries that have framed research about mathematics leadership thus far, drawing on examples from our current project to show its application. Given MERGA paper constraints, broad information that prioritises DWR's methodological positioning is presented over empirical claims that are yet to be known.

Historicising Methodologies Reported in MERGA Conference Papers

We focus on primary school mathematics leadership research papers published in MERGA conference proceedings over the past 10 years. As a field of inquiry, mathematics leadership is relatively new to MERGA (Sexton, 2025b). Our review of papers published between 2016 and 2025 found 17 papers focused on primary school mathematics leadership. We broadly attend to methodologies and methods, identifying patterns reported in those papers.

Across the papers, all mathematics leadership studies were conducted in Australia, with a concentration in Victoria. Studies drew on several conceptual and theoretical frames, including teacher professional growth models (e.g., Clarke & Hollingsworth, used by Roche & Gervasoni, 2017), leadership and educational change perspectives (e.g., Fullan, used by Driscoll, 2022), and sociocultural theories including situated learning theory (e.g., Driscoll & Cheeseman, 2022) and cultural-historical activity theory (Sexton, 2025b; Sexton & Lamb, 2017).

Findings were generated through predominantly qualitative methodologies, with case study the most employed research design (e.g., Driscoll, 2017; Copping, 2022), although mixed-methods design has been used (Vale et al., 2021). Studies typically used semi-structured interviews (e.g., Marshman et al., 2024), observations (e.g., Lomas, 2022), and document retrieval (e.g., Sexton & Lamb, 2017) as primary data generation methods. Thematic analysis, conducted inductively or deductively, was the dominant analytical approach (Roche &

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 385–392). MERGA.

Gervasoni, 2017; Sexton, 2025b). Collectively, the methodological aims and patterns reported in the research papers suggest a largely exploratory inquiry focus.

Given the emergence of the field (Sexton, 2025b), this exploratory orientation is not surprising. However, a gap exists in interventionist approaches to mathematics leadership research, suggesting the need to move beyond the exploratory methodological boundaries that have characterised the field thus far. This is timely in Victoria where the introduction of the *Victorian Teaching and Learning Model 2.0* (Department of Education Victoria, 2025) has mediated tensions for several mathematics leaders navigating the model's mandates.

Articulating Reasons for DWR with Mathematics Leaders in Victoria

The Victorian Teaching and Learning Model 2.0 (VTLM 2.0), officially launched in 2024, has stimulated a change in the Victorian educational policy landscape. The VTLM 2.0 mandates that by 2028, teachers working in Victorian government schools will use explicit teaching across all curriculum areas (Department of Education Victoria, 2025). The VTLM 2.0 expects explicit teaching before opportunities for reasoning and problem solving, representing a departure from student-focused inquiry approaches.

The first author has worked with several mathematics leaders in a leadership program in Victoria since 2023. The program has focused on developing leadership practice of mathematics teaching aligned with student-focused inquiry approaches (e.g., English, 2023) to meet expectations of the Victorian Curriculum (VCAA, 2023). This historical focus now sits in tension with the VTLM 2.0 mandate, creating a practice problem for the mathematics leaders. The following comments from two leaders illustrate this tension:

The move towards explicit teaching including what classifies as explicit teaching is a massive issue. There has also been a move towards retrieval practices and what classifies as feedback and assessment. But what's missing is the how and why...and how that relates to learning maths.

I'm still getting my head around "Vic Curric 2.0", especially the new processes. But something tells me that you can't teach the mathematical processes if you only use explicit teaching. Aren't the processes about inquiry and helping the students to find out using maths? I'm not sure lately.

We interpret these accounts as experiences of a genuine practice problem about how to reconcile policy mandates with the mathematics leaders' historical leadership focus. This tension is not one of simply implementing the VTLM 2.0. It has the leaders grappling with questions about pedagogy and leading development in mathematics teaching. We see this space as a research-rich opportunity that demands an interventionist approach. An aim of our current project is to support the mathematics leaders as they navigate complexities of VTLM 2.0 implementation, while simultaneously creating knowledge about the enactment of mathematics leadership in contexts of policy-driven change.

As an interventionist methodology, Developmental Work Research (DWR) engages participants in surfacing, examining, and addressing practice problems with researcher support (Engeström, 2015; Nuttall, 2013). DWR positions participants as co-researchers who analyse workplace activity and develop new solutions to practice problems. The situation experienced by several Victorian mathematics leaders represents the kind of historically embedded practice problem that DWR is designed to surface, examine, and address.

Situating DWR Within a Cultural-Historical Activity Theoretical Frame

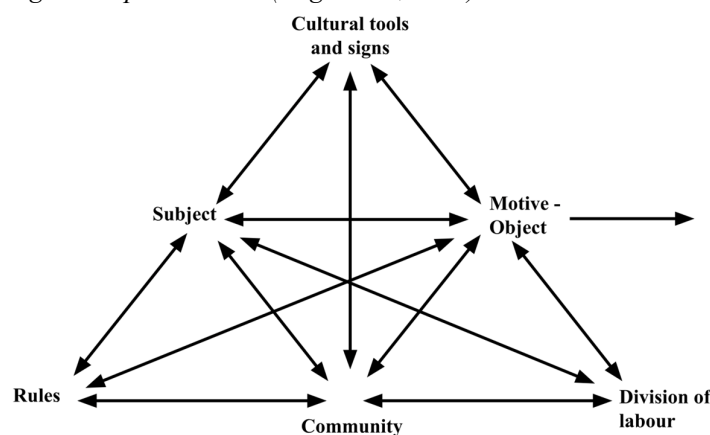
To understand how DWR is used to respond to practice problems, it is necessary to first situate it within its theoretical home in cultural-historical activity theory (CHAT). CHAT stems from the work of Vygotsky and his collaborators in early twentieth century Russia. In the late 1980s, Engeström further developed CHAT through his theorisation of the *activity system* (Engeström, 2015). As a theory of development, CHAT explains the relationship between human thought and action, focusing on how humans develop internal activity, understood as

cognition, and external activity which is materialised through forms of behaviour (Nuttall, 2013). CHAT proposes that both psychological and practical activity are simultaneously drawn forward through mediation processes that shape relationships between individuals and socially shared *motive-objects* of activity. Motive-objects are the teleological, collectively oriented purposes and tasks that give activity its direction and meaning (Leont'ev, 1978; Nuttall, 2013).

Building on Vygotsky's foundational ideas, Leont'ev theorised that human activity is inherently collective in nature and always oriented toward motive-objects that sustain both cultural and social life (Leont'ev, 1978). Engeström (2015) further expanded this by arguing that the activity system is a collectively organised, mediated formation oriented towards shared motive-objects that are transformed into culturally valued *outcomes*. Engeström's idea of the activity system is depicted by the well-known triangular representation (Figure 1).

Figure 1

Activity System Triangular Representation (Engeström, 2015)



The activity system representation explains the dynamic relations within human activity and the mediational means through which activity is organised. These include *cultural tools and signs*, *rules* guiding shared activity, and the *division of labour* which explains role, responsibility, and power distributions within and across the system (Engeström, 2015). The activity system also includes the *community*, the people who share and contribute to achievement of motive-objects. A defining feature of activity systems is that of *contradictions*, understood not only as practice problems, but as productive drivers of development and change (Engeström, 2015). Contradictions surface from misalignments between the mediational means within and beyond activity systems. When brought to people's consciousness, contradictions provide the driving force for *expansive learning* (Engeström, 2015; Nuttall, 2013). DWR harnesses this developmental process methodologically by using interventions to surface and examine contradictions within a professional community and to address them collectively through new, expanded forms of activity (Engeström, 2015).

Explaining Developmental Work Research

DWR moves beyond action research approaches that emphasise individual reflective practice. Within DWR, participants work alongside researchers, collectively examining systemic contradictions in their workplace to develop more robust understandings of their work activity (Engeström, 2015). With researcher support, participants also work together to adopt new or adapt existing mediational means with the aim of transforming the activity system in response to the surfaced and examined contradictions (Engeström, 2015; Nuttall, 2013).

Expansive Learning

Central to DWR is the concept of expansive learning (Engeström, 2015). Within DWR, learning is not merely the acquisition of new knowledge and skills. Instead, expansive learning is understood as a collective process of transformation of the entire activity system in which the participants work. Engeström (2015) reported the nature of expansive learning as:

- *Contradiction-driven transformation*: Expansive learning is initiated when participants encounter systemic contradictions that cannot be resolved through existing ways of working. Contradictions are historically accumulated, and tracing their roots is central to understanding why current tensions exist. When contradictions become consciously felt by participants as a *double bind* (an experience of being pulled in irreconcilable directions), they provide the driving force for expansive learning (Engeström, 2015; Kerosuo et al., 2010; Nuttall, 2013).
- *Motive-object expansion*: Through expansive learning, participants fundamentally redefine the motive-object of activity, constructing a new shared motive-object that redirects collective activity toward outcomes not previously possible within the existing activity system (Engeström, 2015; Kerosuo et al., 2010).
- *Double stimulation*: Expansive learning is grounded in Vygotsky's principle of double stimulation, whereby participants simultaneously work on contradictions while engaging with theoretical, conceptual, and practical tools as a second stimulus. This dual engagement produces the deeper conceptual shifts that distinguish DWR from other methodologies like action research (Engeström, 2015; Nuttall, 2013).
- *Collective and collaborative activity*: Expansive learning is not an individual pursuit but rather a social process through which participants collectively analyse their activity system, trace its historical development, and work collaboratively toward new, expanded forms of practice (Engeström, 2015; Kerosuo et al., 2010).

Figure 2 illustrates how these expansive learning characteristics are enacted through actions that mediate participants' transformation of activity with researchers.

Figure 2

Expansive Learning Cycle (adapted from Engeström, 2015; Nuttall et al., 2024)

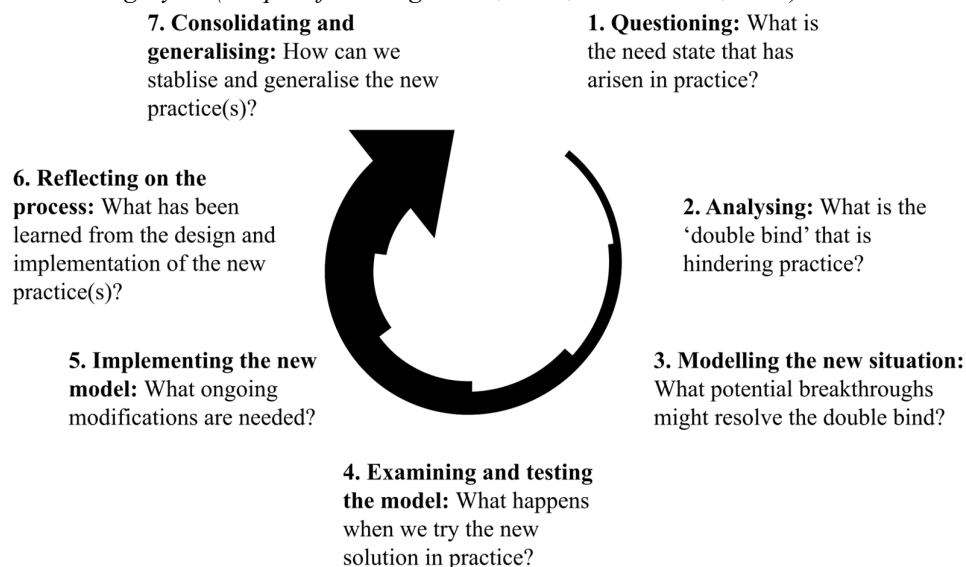


Table 1 illustrates each expansive learning action and their anticipated application in Change Laboratories (Change Labs), the spaces in which we will work with the leaders during our project.

Table 1

Expansive Learning Actions and Their Anticipated Application to our Current Project

Expansive learning action	Description	Anticipated application to our project
1. Questioning	Participants begin to question their prevailing activity, surfacing a state of need that signals contradictions within the activity system that require attention (Engeström, 2015).	Mathematics leaders have begun questioning how to reconcile their historical leadership practice on developing student-focused inquiry practices with the VTLM 2.0 mandate for explicit teaching.
2. Analysing	Participants analyse the historical roots and current state of contradictions within the activity system, surfacing <i>double binds</i> that act as drivers of change (Kerosuo et al., 2010).	Through initial interviews and document analysis, we generate mirror data and co-identify historically accumulated tensions between mathematics pedagogy, change driven by policy, and mathematics leadership.
3. Modelling the new situation	Participants collectively model new solutions to identified contradictions, creating new and/or reconfigured motive-objects, adopting/adapting mediational means, and practices within the activity system (Engeström, 2015).	Change Lab workshops support mathematics leaders to use theoretical, conceptual, and practical tools to model new approaches for leading mathematics teaching that reconcile student-focused inquiry and policy mandates for explicit teaching.
4. Examining and testing the model	New model is examined and tested within the activity system, with adjustments made as the model is enriched through practice (Kerosuo et al., 2010).	Leaders trial new approaches between Change Lab workshops, reporting back on experiences, change in leadership, and impacts for collective examination.
5. Implementing the new model	New model is implemented within the activity system, with participants taking increasing responsibility for its development at the level of practice (Engeström, 2015).	Leaders implement new model of leading mathematics teaching within schools, navigating the complexities of VTLM 2.0 implementation by adapting/adapting mediational means within the activity system.
6. Reflecting on the process	Participants evaluate and stabilise the process of expansive learning, consolidating new understanding of their activity system (Kerosuo et al., 2010).	Leaders collectively reflect on the expansive learning process, evaluating new leadership actions developed through the Change Lab workshops.
7. Consolidating and generalising	New practice is consolidated and generalised, with participants embedding new ways of working within and potentially beyond their activity system (Engeström, 2015).	New model of mathematics leadership is developed and enacted within leaders' schools, and new knowledge about mathematics leadership is created.

Whilst the expansive learning cycle provides a coherent framework for how DWR unfolds, in practice, the cycle is responsive rather than linear (Nuttal et al., 2024). The cycle's actions are adaptive as participants work through contradictions and trial new action models (Kerosuo et al., 2010). We anticipate that responsiveness will feature in our work with leaders as they navigate VTLM 2.0 implementation complexities whilst making sense of the theoretical, conceptual, and practical tools offered to them within the Change Laboratory workshops.

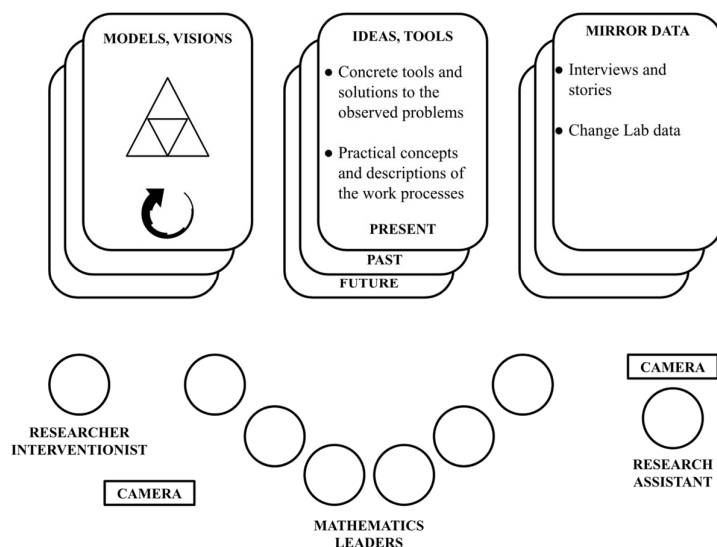
Change Laboratories

Change Laboratories (Change Labs) are the site where expansive learning is enacted (Engeström, 2015; Nuttall, 2013). They consist of at least four workshops in which researchers and participants co-analyse and plan ways to address contradictions within the activity system. Workshops typically run for about three hours (Nuttall et al., 2024). Before the first Change Lab, researchers interview participants and analyse documents to identify workplace contradictions. Analysis insights are presented as mirror data in the initial workshop, reflecting experiences of contradictions back to the participants (Nuttall, 2013).

Change Labs are structured environments, and Figure 3 illustrates the anticipated physical arrangement of our project's Change Labs. This includes the placement of the *mirror data*, *ideas and tools*, and *models and visions* wallboards. It shows the positioning of participants, research team, and recording equipment used to capture data. The setup moves between past, present, and future timeframes, enabling participants to surface and trace the historical roots of contradictions, examine their current manifestations, and address them through envisioned and new forms of practice. The participants' dialogue and actions about the adoption and adaptation of mediational means (e.g., theoretical tools like the “Engeström triangle”) form the data that evidence expansive learning enactment (Nuttall, 2013). In our project, we plan to use four Change Labs across the 2026 school year to allow time and space for the mathematics leaders to enact the expansive learning cycle (Engeström, 2015).

Figure 3

Anticipated Layout of Change Laboratory Workshops (adapted from Kerosuo et al., 2010)



The mirror data wallboard presents participants with evidence of contradictions with their activity system. The models and visions wallboard provides the theoretical and conceptual tools, including the activity system model (Figure 1) and the expansive learning cycle (Figure 2). These tools support participants to co-analyse contradictions and co-develop new forms of action. Within DWR, these tools function as more than explanatory diagrams. They become cultural tools, introduced as a second stimulus, that mediate the analysis of contradictions and the planning of new forms of activity. The ideas and tools wallboard captures the practical tools, new solutions, and agreed approaches collectively generated by participants as they work through the expansive learning cycle. This wallboard externalises emerging forms of practice, making them available for collective reflection, critique, and further development (Engeström, 2015; Kerosuo et al., 2010). In our project, this wallboard will document a new model of leadership that participants co-develop as they navigate VTLM 2.0 implementation.

Change Labs act as a professional learning and research methodology which means they simultaneously generate rich data about educational leadership as a form of emergent, contested, and developmental practice (Nuttall, 2013). It is this dual purpose that positions DWR as an approach that works beyond the methodological boundaries reported in previous mathematics leadership research.

Acknowledging Strengths and Constraints of DWR

DWR offers several affordances that make it suited to examining mathematics leadership during a period of educational reform. First, DWR simultaneously functions as a professional learning approach and a research methodology, positioning mathematics leaders as co-researchers rather than as passive subjects of inquiry (Nuttall, 2013). Second, DWR creates the conditions for mathematics leaders to enact professional agency at a time of mandated reform, supporting them to collectively surface, examine, and address contradictions between their historical leadership practice and new policy mandates. Third, the longitudinal and interventionist nature of DWR generates rich data about leadership as emergent, contested, and developmental (Nuttall et al., 2024) directly addressing the methodological gap identified in previous mathematics leadership research published by MERGA.

DWR is not without its constraints. It is labour-intensive, requiring sustained commitment across pre-Change Lab workshop data collection, Change Lab workshop facilitation and data generation, between-workshop analysis, and the planning of follow-up workshops (Kerosuo et al., 2010; Nuttall, 2013). Change Labs are also not scalable, meaning claims generated from them are context specific (Nuttall et al., 2024). A further tension exists in honouring the collective and individual dimensions of participants' experience, which is an ongoing challenge within expansive learning research more broadly (Engeström & Sannino, 2010). Despite these constraints, the affordances of DWR make it a methodology for examining mathematics leadership in the context of policy-driven change.

Conclusion

We have argued for DWR as a CHAT-informed methodology that moves beyond the exploratory research approaches reported in MERGA conference proceedings. DWR offers a way of examining the tensions mathematics leaders face during periods of policy-driven reform. By positioning mathematics leaders as co-researchers and simultaneously functioning as a professional learning and research methodology, DWR offers a means of not only examining mathematics leadership but transforming it. The tensions facing several Victorian mathematics leaders navigating VTLM 2.0 implementation are the kinds of historically embedded contradictions that DWR is designed to address, making it a fit-for-purpose methodology for our research project.

Whilst this paper has prioritised methodological positioning over empirical claims, our project aims to generate data about mathematics leadership as an emergent, contested, and developmental form of middle leadership practice. We do not propose DWR as a replacement for existing methodologies, but as a timely addition to methodological conversations about mathematics leadership research within the MERGA community.

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References

Department of Education Victoria. (2025). *Victorian teaching and learning model policy*. <https://www2.education.vic.gov.au/pal/victorian-teaching-learning-model/policy>

- Copping, K. (2022). Perceptions of the role of primary mathematics leaders. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7, pp. 146–153). MERGA.
- Driscoll, K. (2017). Primary school mathematics leaders' views of their mathematics leadership role. In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning!* (Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia, July 2–6, pp. 213–220). MERGA.
- Driscoll, K. (2022). Primary school mathematics leaders' actions that facilitate effective mathematics planning and support teachers' professional learning. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7, pp. 178–185). MERGA.
- Driscoll, K., & Cheeseman, J. (2022). School mathematics leaders' support of primary teachers' professional learning in meetings. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7, pp. 186–193). MERGA.
- Engeström, Y. (2015). *Learning by expanding: An activity-theoretical approach* (2nd ed.). Cambridge University Press.
- Engeström, Y., & Sannino, A. (2010). Studies of expansive learning: Foundations, findings and future challenges. *Educational Research Review*, 5(1), 1–24. <https://doi.org/10.1016/j.edurev.2009.12.002>
- English, L. D. (2023). Ways of thinking in STEM-based problem solving. *ZDM*, 55(6), 1219–1230.
- Kerosuo, H., Kajamaa, A., & Engeström, Y. (2010). Promoting innovation and learning through Change Laboratory: An example from Finnish health care. *Central European Journal of Public Policy*, 4(1), 110–131.
- Leont'ev, A. N. (1978). *Activity, consciousness, and personality*. Prentice-Hall.
- Lomas, L. (2022). Preparing job-embedded primary mathematics specialists to lead in Australian schools. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 346–353). MERGA.
- Marshman, M., Ross, E., Bennison, A., Goos, M. (2024). Defining the problem in a changing landscape: How leaders plan for and address mathematics curriculum change. In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education*. (Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia pp. 367–374). MERGA.
- Nuttall, J. (2013). The potential of Developmental Work Research as a professional learning methodology in early childhood education. *Contemporary Issues in Early Childhood*, 14(3), 201–211.
- Nuttall, J. G., Henderson, L., Wood, E., & Martin, J. (2024). *Leadership in early childhood education: a cultural-historical theory of practice development*. Springer.
- Roche, A., & Gervasoni, A. (2017). Perceived changes in teachers' knowledge and practice: The impact on classroom teachers from leader participation in whole-school reform of mathematics teaching and learning. In A. Downton, S. Livy, & J. Hall (Eds.), *Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 213–220). MERGA.
- Sexton, M. (2025a). Methodological considerations using cultural-historical activity theory to research mathematics leadership at times of educational reform. In P.J. White, R. Tytler, J.P. Ferguson, J. Cripps Clark, J.P. Brown (Eds.), *Methodological approaches to STEM education research* (Vol. 6, pp. 147–168). Cambridge Scholars.
- Sexton, M. (2025b). Conceptualising primary school mathematics leadership as poly-motivated activity. In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 405–412). MERGA.
- Sexton, M., & Lamb, J. (2017). Using activity theory to understand a mathematics leaders' motivations and use of mathematical knowledge for teaching. In A. Downton, S. Livy, & J. Hall (Eds.), *Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia* (pp. 466–473). MERGA.
- Vale, C., Roche, A., Cheeseman, J., Downton, A., Gervasoni, A., Kalogeropolous, P., Livy, S., & Russo, J. (2021). Leading mathematics: Doings of primary and secondary school mathematics leaders. In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & S. L. Chin (Eds.), *Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 401–408). MERGA.
- Victorian Curriculum and Assessment Authority. (2023). *Victorian Curriculum F–10 Version 2.0*. <https://victoriancurriculum.vcaa.vic.edu.au/>

Nudging Pathways: The Role of Teacher Recommendations as Strategic Interventions in Senior Mathematics and STEM Extension Choices

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This paper examines the role of teacher recommendations in shaping students' participation in advanced mathematics subjects and STEM extension programs. Such recommendations are often assumed to be influential but rarely theorised or empirically examined. Drawing on findings from two studies conducted in Victoria, Australia, and unpacked through the lens of Nudge Theory and Choice Architecture, the discussion highlights how teacher recommendations function as a part of educational choice architecture across both formal curriculum decisions and optional enrichment pathways.

Introduction

Concerns about student participation in advanced mathematics have been widely documented in mathematics education research, particularly regarding declining enrolments, inequitable access, and the long-term implications for STEM pathways (Department of Industry, Science and Resources, 2025). In senior secondary schooling, decisions about whether to enrol in advanced or intermediate mathematics subjects represent pivotal moments that can shape students' post-school options and identities as mathematical learners. These decisions are influenced by a range of factors, including perceived ability, future aspirations, institutional requirements and social influences (Hine et al., 2024; Holmes et al., 2025).

Within this decision-making landscape, mathematics teachers are frequently positioned as *significant others* (Henschel et al., 2023) whose recommendations are expected to guide students towards appropriate pathways. However, despite its presumed importance, the role of teacher recommendations has received relatively limited empirical examination. In particular, there is little clarity about the extent to which recommendations influence students' decisions relative to other factors, especially when it comes to significant and high-stakes choices.

This paper addresses this gap by examining teacher recommendations through the lens of Choice Architecture and Nudge Theory. Drawing on two empirical studies conducted in Victoria, Australia, the paper investigates the extent to which teacher recommendations function as subtle but powerful influences on students' engagement with higher-level mathematics and extension STEM opportunities. By conceptualising recommendations as nudges within the context of choice architecture, the paper offers a reframing with implications for equity, responsibility, and professional practice in mathematics education.

Literature Review

Research on senior secondary mathematics subject choice has identified a complex set of interacting influences, including students' self-perceptions of mathematical ability, enjoyment, future career aspirations, and institutional constraints such as university prerequisites (Hine et al., 2024; Holmes et al., 2025). Students' beliefs about whether a subject is 'for them' have been shown to play a particularly strong role, often shaped by prior achievement and classroom experiences (Hine et al., 2024).

Social influences also feature prominently in the literature. Parents, peers, and teachers have all been identified as important sources of guidance and information during subject selection processes (Holmes et al., 2025). Among these, teachers are often described as trusted experts

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 393–400). MERGA.

with specialised knowledge of both the curriculum and pedagogy, and can influence students through their pedagogies and enthusiasm for mathematics (Holmes et al., 2025). The influence of teachers as significant others or socialisers (Eccles & Wigfield, 2024) encouraging female participation in mathematics is well-known in the mathematics education gender literature (Forgasz & Markovits, 2018; Henschel et al., 2023). However, much of the existing research treats teacher advice as one among many influences, rather than as a distinct phenomenon warranting focussed examination.

Importantly, there is limited theoretical framing of how teacher recommendations or advice exert influence in terms of mathematics and STEM pathways. Recommendations are often assumed to operate through persuasion rather than prescription. However, such explanations risk oversimplifying the subtle ways in which advice shapes students' perceptions of risk, suitability, opportunity, and challenges. This gap suggests the need for alternative theoretical perspectives that can account for the direct and context-dependent nature of teacher recommendations.

By including both senior mathematics subject selection and STEM extension participation, the nudging effects can be examined across differing choice environments. In mathematics, choices are constrained by prerequisites and university requirements (Hine et al., 2024), whereas STEM extension programs typically involve open access with minimal formal barriers (Ihrig et al., 2018). If teacher recommendations exert influence in both contexts, this suggests a robust nudging mechanism that extends beyond discipline-specific structures.

Nudge Theory and Choice Architecture in Education

Nudge Theory, originating in behavioural economics, describes how small changes in the way choices are presented can significantly influence decision-making without restricting freedom of choice (Thaler, 2018; Weijers et al., 2021). Nudges operate by shaping the psychological, social, and physical environment surrounding the decisions, the *choice architecture* (Weijers et al., 2021), often by making certain options more salient, normative, or cognitively accessible. Importantly, nudges do not charge individuals to act in a particular way. Instead, they steer behaviour while preserving autonomy.

Within educational contexts, nudges have been applied to understand how institutional structures, expectations, and messaging influence student behaviour (e.g., see a discussion on the application of nudges in education by Decuypere & Hartong, 2023). Various nudge strategies have been studied, such as apps and online platforms for career development (Lent, 2025), opting out of high school advanced courses instead of opting in (Austin et al., 2022), and student and parent interventions to increase female uptake of STEM subjects at UK's Advanced Level (Department for Education, Behavioural Insights Team, 2020). The "Nudging to STEM Success" project (O'Hara & Sparrow, 2019) uses a series of text messages to nudge students to re-enrol in STEM programs. The initial randomised controlled trial involving more than 2700 students and the subsequent implementation involving more than 9500 students found that nudges increased students' persistence in participating in STEM courses (Soricone & Endel, 2019).

Weijers et al. (2021) described two types of nudges. Type 1 nudges influence behaviour through automatic, low-effort cognitive processes (System 1 thinking). They alter the decision-making environment or context (choice architecture) so that students make better decisions without needing deliberate reasoning. Examples in the mathematics classroom might include having the default calculator mode set to exact values and degrees or radians, having a poster of key formulae or problem-solving steps placed at eye level on the walls, worksheets designed with sufficient space for students to write their workings, and beginning mathematics exercises with easy questions that are similar to worked examples to encourage students to participate. Type 2 nudges target reflective, deliberate thinking (System 2 thinking). They prompt students

to pause, think, plan, or reflect before making a choice. Examples include metacognitive question prompts and scaffolding prompts for students to explain their solutions, set goals, or analyse their errors. Printed exit tickets collected at the door could involve both Types 1 and 2 nudges. Nudges can have a positive or negative impact, depending on how they are framed and perceived (e.g., mathematics is hard versus this concept can be challenging, and that is why we will break it down together) (Sunstein, 2017; Weijers et al., 2025).

Teacher recommendations can be conceptualised as a form of deliberate and explicit/transparent nudge, in which trusted authority figures subtly signal what choices are appropriate, achievable, expected, or aspirational (Decuypere & Hartong, 2023). Such recommendations may influence students' perceptions of their own competence and the legitimacy of particular pathways, even when students retain the formal freedom to choose otherwise.

Viewing teacher recommendations as nudges through the lens of choice architecture shifts attention away from questions of persuasion or pressure, and towards the ethical and practical implications of choice design. This perspective is particularly relevant in mathematics education, where decisions about advanced mathematics study are often framed as difficult, high-risk and consequential. It also complements existing motivational theories, such as Eccles and Wigfield's (2024) Situated Expectancy-Value Theory, in that it provides a micro-lens to theorise the contextual processes and mechanisms of how teachers, as socialisers, impact students' expectancy for success. In the following two studies, we explore the potential of applying Nudge Theory as a lens to frame the role of teachers' recommendations and advice on students' mathematics and STEM pathways.

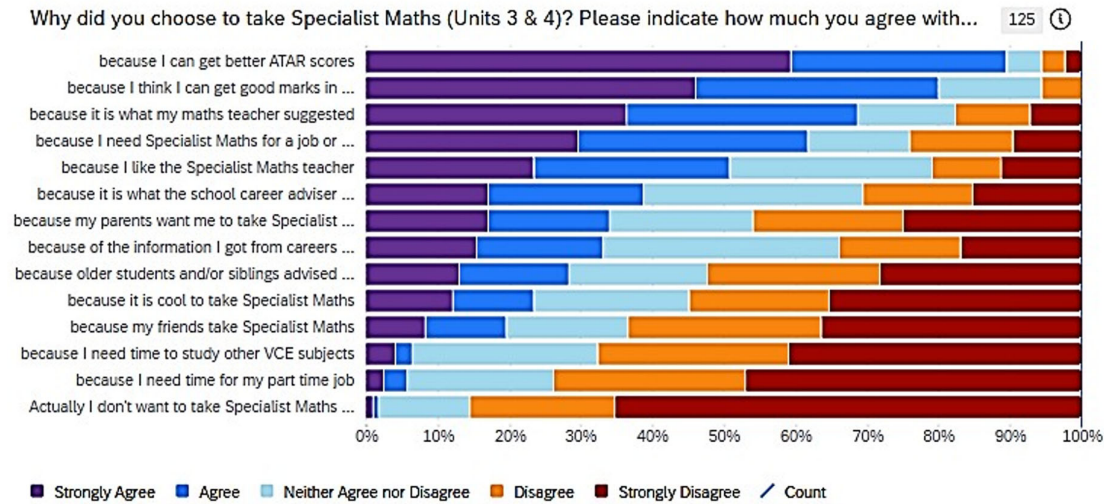
Study 1: Senior Secondary Mathematics Subject Choices

The first study investigated senior secondary students' reasons for choosing or not choosing advanced and intermediate mathematics subjects in Victoria, Australia. Data were collected through an anonymous online survey with 838 respondents (after cleaning) enrolled in senior secondary schooling (Years 10-12), recruited via Facebook advertisement approach (Forgasz et al., 2018). The survey included items addressing a range of potential influences on senior secondary mathematics subject choices, including perceived suitability, university requirements, teacher recommendations, peer influence and personal interest, informed by the instrument used by McPhan et al. (2008).

Of the 164 students who reported taking the Victorian (Australian) Specialist Mathematics Units 3 and 4 (equivalent to senior secondary Year 12 advanced mathematics), 125 (60 males, 64 females, 1 missing) responded to an item in which they indicated their level of agreement with a list of potential factors (see Figure 1). The survey data indicated that teacher recommendations were the third most influential reason for choosing advanced mathematics (36% Strongly Agree and 32% Agree). The first two most influential factors were that students could maximise their university entrance score (59% SA + 30% A) with the high scaling of Specialist Mathematics and that they could achieve good marks in Specialist Mathematics (46% SA + 34% A). Notably, teacher recommendations ranked above peer influence and other social factors, and liking their Specialist Mathematics teacher was ranked fifth (23% SA + 27% A). This is different from the Western Australian context, where parental advice was the most frequently cited source of influence of the 20 students interviewed (Kirkham et al., 2020), and the scaling effect was only perceived by half of the 1633 students surveyed as adequate to motivate them to choose advanced mathematics (Hine et al., 2024).

Figure 1

Participants' Reasons for Enrolling in Advanced Mathematics



This finding suggests that teacher recommendations function alongside systemic factors relating to tertiary entrance, reinforcing students' perceptions of the suitability and establishing the legitimacy of their subject choice. From a Nudge perspective, teacher recommendations can be seen as expert advice and may help reduce uncertainty and perceived risk associated with enrolling in the higher-level mathematics subject, particularly for students who are ambivalent about their capabilities.

Study 2: STEM Extension Programs Enrolment

The second study examined students' enrolment in a suite of STEM extension programs in various STEM disciplines (e.g., nanotechnology, astrophysics, bioinformatics) delivered online during school hours. The programs are available at a subsidised fee to students from metropolitan, regional and rural government schools, and have been conducted for several years. An anonymous online survey was administered to alumni of the STEM extension programs, and of more than 500 alumni who were sent the survey, 65 responded (23M, 42F). One of the survey items was "What made you decide to join the STEM extension course?", with an open textbox for students to complete. The 65 qualitative responses were coded and analysed using thematic analysis (Braun & Clarke, 2019). Table 1 shows the types of reasons cited by the students, in total and according to subgroups (location during the time of study, and gender).

A high proportion of students, particularly females (2M, 14F), reported being recommended by their teacher or principal (16 out of 65 students, or 25%). This was the second most frequently cited reason (and the highest frequency for female students) for enrolling in the STEM extension, just behind having a strong interest in STEM (19/64 or 30%). This category excludes five students (4M, 1F) who said they were recommended but were not explicit in stating who recommended them. If we assumed that the recommendations were made by teachers and schools, then teacher/principal recommendations would also be the second-most frequently cited reason by male students. Several students' responses made reference to teacher or principal recommendation:

My teacher broached the subject with a handful of students truly committed to school and who were fairly switched on when it came to science. (Male, Rural or Regional)

My principal came to me hoping I would be one of the people to try it out and I gladly took up my own time to do this course. (Female, Rural or Regional)

Table 1.*What made students decide to join the STEM extension program*

Reason*	Total number and percentage of students (n=65)	No. (%) of Males (n=23)	No. (%) of Females (n= 42)	No. (%) of students from metropolit an areas (n=31)	No. (%) of students from rural or regional areas (n=33)
Strong interest in STEM	20 (30%)	8 (35%)	11 (26%)	10 (32%)	10 (30%)
Recommended or selected by teacher or principal	16 (25%)	2 (9%)	14 (33%)	7 (23%)	9 (27%)
Interest in specialised subjects	14 (22%)	6 (26%)	8 (19%)	9 (29%)	4 (12%)
Wanted extension in science/maths	13 (20%)	3 (13%)	10 (24%)	5 (16%)	8 (24%)
Preparation for future studies or career	12 (18%)	2 (9%)	10 (24%)	5 (16%)	6 (18%)
Recommended (by unknown)	5 (8%)	4 (17%)	1 (2%)	3 (10%)	2 (6%)
General interest	4 (6%)	2 (9%)	2 (5%)	2 (6%)	2 (6%)
Challenge myself	3 (5%)	0 (0%)	3 (7%)	1 (3%)	2 (6%)
New learning experience	2 (3%)	1 (4%)	1 (2%)	0 (0%)	2 (6%)
Recommended by friend	2 (3%)	0 (0%)	2 (5%)	0 (0%)	2 (6%)

Note. Reasons are not mutually exclusive and thus column percentages add up to more than 100%. Percentages are calculated within gender and location categories in the respective columns.

Other reasons included interest in specialised subjects (as opposed to mainstream science or mathematics subjects), preparation for future studies in senior secondary or university, general interest, a desire to challenge themselves, or a combination of these factors. These findings indicate that teacher and principal recommendations played a vital role in students' (particularly females) awareness of and participation in the STEM extension program. This influence may be pronounced in rural and regional contexts, with students reporting few alternative sources of information about extension opportunities (consistent with Ihrig et al., 2018) ("living in a rural area it is quite difficult to learn more about science", Female, Rural or Regional). In this setting, school leaders and teachers appeared to function as key information channels to opportunity.

The findings highlight the role of recommendations not only in shaping choices among known options, but also in determining which options become visible to students in the first place. From a Nudge perspective, this represents a powerful form of choice architecture, where the availability of options is itself mediated through institutional actors. Unlike mathematics subject selection, participation in the STEM extension program was not constrained by curriculum sequencing or formal prerequisites (such as the need to maximise university entrance scores). The prominence of teacher and principal recommendations in this context suggests that nudging effects operate even in the absence of systemic or structural limits, underscoring the broader influence of educators as choice architects.

Discussion: Teacher Recommendations as Educational Nudges

In the context of senior secondary subject selection, students face a decision characterised by high stakes (future career options) and imperfect information (uncertainty about their own capacity for advanced work). This is a classic scenario where cognitive biases thrive and where a well-placed nudge can be highly effective and strategic (Weijers et al., 2021). The teacher has a *messenger effect* of nudging (Weijers et al., 2025). A teacher's recommendation acts as a

powerful **informational nudge** due to their expertise and epistemic authority (Fried, 2020). This leverages the **authority heuristically** (Weijers et al., 2025), where individuals rely on the judgment of an expert to simplify a complex decision (Fried, 2020). For the student, the cognitive load of evaluating their own aptitude, comparing it to peers, evaluating risks of loss or failure, and projecting future success, is significantly reduced. The teacher's advice effectively transforms the decision from an open-ended question (e.g., Which mathematics subject should I take at senior secondary level?) into an easier confirmation (Should I accept my teacher's recommendation?). The high level of trust students place in their teachers, which is a prerequisite for the nudge's effectiveness, is a critical element of the choice architecture.

In senior secondary mathematics subject selection, students face a high-stakes choice under imperfect information about their own readiness for advanced work, a context that heightens reliance on heuristics and susceptibility to cognitive biases (Weijers et al., 2021). In such conditions, well-designed nudges can structure the choice environment without limiting autonomy, and education studies increasingly document their promise and design considerations (Weijers et al., 2025). Within this choice architecture, the teacher functions as a high-credibility messenger: qualitative evidence from vocational and higher education shows teachers routinely, and often implicitly, deploy nudges that shape students' learning behaviours and choices (Decuyper & Hartong, 2023), while mathematics education research has long observed the salience of teacher authority in classroom interactions (Fried, 2020). This reduces cognitive load by shifting the task from open-ended complex decision-making to a confirm-or-decline evaluation of the teacher's recommended option, which is precisely the kind of cognitive streamlining seen when individuals use heuristics under uncertainty (Weijers et al., 2021). The effectiveness of this nudge depends on trust: students' willingness to treat teachers as legitimate authorities, recognised in mathematics education as a field with distinctive authority dynamics, underwrites the persuasive force of the recommendation (Fried, 2020). Importantly for senior secondary enrolment patterns, Australian research indicates that mathematics teacher leaders' attributions and actions measurably influence students' uptake of higher-level mathematics, aligning with the messenger-effect account of teacher recommendations guiding subject choice.

Overcoming Cognitive Biases: Status Quo and Loss Aversion

The teacher's nudge is particularly effective because it helps students overcome two key cognitive biases that often lead to sub-optimal educational choices:

1. **Status Quo Bias:** Students often prefer to stick with the familiar or easier option, which in this context may be a lower-level mathematics subject or simply not applying for an extension program. The teacher's recommendation disrupts this inertia by creating a new, expert-endorsed status quo.
2. **Loss Aversion:** The decision to take a higher advanced level mathematics subject is often framed by students as a potential loss (e.g., loss of free time, loss of a high grade if they struggle). The teacher's recommendation can reframe the decision, emphasising the **gain** (e.g., access to university courses, a challenging and rewarding experience) and providing a form of **risk mitigation** by assuring the student of their capability.

The data from Study 1, in which the recommendation by mathematics teachers was cited more often than recommendations from other sources (friends, career advisors, family), suggest that the teacher's recommendation exerts a powerful influence. This nudge might steer them toward a logically advantageous choice, helping to overcome possible risks or confidence issues related to selecting the advanced mathematics subject.

The Role of the Nudge in Equity and Access

The findings from Study 2, which highlighted the critical role of recommendations in rural and regional schools (and for females), underscore the importance of teachers' or principals' nudges in promoting **educational equity**. In rural and regional areas, the institutional and informational infrastructure for advanced STEM opportunities is often weaker than in metropolitan schools (Ihrig et al., 2018). A teacher's personal recommendation serves as a **compensatory mechanism**, ensuring that students who might otherwise be unaware of opportunities or unsure of their capabilities are actively guided towards them.

A principal's recommendation, often cited by students from rural or regional schools, acts as a powerful **institutional nudge**. This top-down endorsement provides a strong signal of the program's value and the student's suitability, which can be particularly persuasive in smaller, tightly-knit school communities. This demonstrates that the choice architecture can be manipulated at multiple levels, the personal, relational level (teacher) and the institutional level (principal), to achieve the desired behavioural outcome of increased participation in advanced STEM pathways.

The Ethical Dimension of the Nudge

A key ethical consideration in Nudge Theory is the concept of **libertarian paternalism** (Decuyper & Hartong, 2023). The intervention must be non-coercive and preserve the individual's freedom of choice. A teacher's recommendation meets this criterion, and the strong influence observed across both studies highlights the responsibility teachers hold as choice architects. This influence makes it essential that recommendations are well-informed, equitable, and tailored to individual students, particularly given that in Study 1 the recommendation aligns closely with the student's perceived suitability for the subject.

Across both the mathematics and STEM extension contexts, teacher recommendations acted not simply as supportive advice but as nudges shaping how students interpreted their readiness and the legitimacy of particular pathways. In the mathematics study, they reinforce decisions at key transition points, while in the STEM extension program study, they open opportunities that might otherwise be invisible. Rural and regional data suggest ethical concerns: uneven access to information can make the lack of recommendations act as gatekeepers, foreclosing opportunities even when formally available. Viewing recommendations as nudges foregrounds their ethical impact. They can expand access but also reinforce inequities if influenced by biases related to gender, background, ability, or likelihood of success. Since nudges are not inherently positive and may backfire (Sunstein, 2017; Thaler, 2018), thoughtful and transparent recommendation practices are essential. This framing positions teacher recommendations as a significant yet under-acknowledged mechanism shaping participation and access across mathematics and STEM education systems.

Conclusion

We have argued that mathematics teacher recommendations play a significant and under-examined role in shaping students' participation in advanced mathematics and extension STEM opportunities. Drawing on two studies from Victoria, Australia, and using Nudge Theory and Choice Architecture as an analytic lens, the paper highlights how recommendations operate as subtle but powerful influences on student decision-making. Recognising recommendations as nudges invites renewed attention to their ethical, pedagogical, and policy implications, particularly in relation to equity and access in mathematics education.

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References

- Austin, M., Backes, B., Goldhaber, D., Li, D., & Streich, F. (2022). *Leveling up: A behavioral nudge to increase enrollment in advanced coursework*. CALDER Working Paper No. 271–1022. National Center for Analysis of Longitudinal Data in Education Research.
- Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative Research in Sport, Exercise and Health*, 11(4), 589–597. <https://doi.org/10.1080/2159676X.2019.1628806>
- Decuyper, M., & Hartong, S. (2023). Edunudge. *Learning, Media and Technology*, 48(1), 138–152.
- Department for Education, Behavioural Insights Team. (2020). *Applying behavioural insights to increase female students' uptake of STEM subjects at A Level*. Department for Education. <https://www.gov.uk/government/publications/applying-behavioural-insights-to-increase-female-students-uptake-of-stem-subjects-at-a-level>
- Department of Industry, Science and Resources, Australian Government. (2025). *STEM equity monitor* <https://www.industry.gov.au/publications/stem-equity-monitor>
- Eccles, J. S., & Wigfield, A. (2024). The development, testing, and refinement of Eccles, Wigfield, and colleagues' Situated Expectancy-Value Model of achievement performance and choice. *Educational Psychology Review*, 36, 51. <https://doi.org/10.1007/s10648-024-09888-9>
- Forgasz, H., & Markovits, Z. (2018). Elementary students' views on the gendering of mathematics. *European Journal of Educational Research*, 7(4), 867–876. <https://doi.org/10.12973/eu-jer.7.4.867>
- Forgasz, H., Tan, H., Leder, G., & McLeod, A. (2018). Enhancing survey participation: Facebook advertisements for recruitment in educational research. *International Journal of Research & Method in Education*, 41(3), 257–270. <https://doi.org/10.1080/1743727X.2017.1295939>
- Fried, M. N. (2020). Authority and mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 69–72), Springer. <https://doi.org/10.1007/978-3-030-15789-0>
- Henschel, S., Jansen, M., & Schneider, R. (2023). How gender stereotypes of students and significant others are related to motivational and affective outcomes in mathematics at the end of secondary school. *Contemporary Educational Psychology*, 73. Article 102161.
- Hine, G., Forlin, C. & Chivers, P. (2024). Influences on student decisions to enrol in higher-level mathematics courses. *Mathematics Education Research Journal*, 36, 695–713.
- Holmes, K., Mackenzie, E., Thompson, M., O'Connor, B. R., & Anderson, J. (2025). Increasing student interest in secondary mathematics: Student perspectives. *Mathematics Education Research Journal*.
- Ihrig, L. M., Lane, E., Mahatmya, D., & Assouline, S. G. (2018). STEM excellence and leadership program: Increasing the level of STEM challenge and engagement for high-achieving students in economically disadvantaged rural communities. *Journal for the Education of the Gifted*, 41(1), 24–42.
- Lent, R. W. (2025). Choice architecture and the potential to nudge career development at scale. *Journal of Career Assessment*, 33(4), 716–738. <https://doi.org/10.1177/10690727241298688>
- McPhan, G., Morony, W., Pegg, J., Cooksey, R., & Lynch, T. (2008). Maths? Why not? Final report prepared for the Department of Education, Employment and Workplace Relations (DEEWR). Australian Government.
- O'Hara, R. E., & Sparrow, B. (2019). A summer nudge campaign to motivate community college STEM students to reenroll. *AERA Open*, 5(3), 1–10. <https://doi.org/10.1177/2332858419875715>
- Soricone, L., & Endel, B. (2019). *Nudging to STEM success: Implementation report*. Jobs for the Future (JFF) and Persistence Plus. <https://www.jff.org/idea/nudging-stem-success/>
- Sunstein, C. R. (2017). Nudges that fail. *Behavioural Public Policy*, 1(1), 4–25. <https://doi.org/10.1017/bpp.2016.3>
- Thaler, R. H. (2018). Nudge, not sludge. *Science Magazine*, 361(6401), 431. <https://doi.org/10.1126/science.aau9241>
- Weijers, R. J., de Koning, B. B., & Paas, F. (2021). Nudging in education: From theory towards guidelines for successful implementation. *European Journal of Psychology of Education*, 36, 883–902. <https://doi.org/10.1007/s10212-020-00495-0>
- Weijers, R. J., de Koning, B. B., Klatte, E., & Paas, F. (2025). How do teachers in vocational and higher education nudge their students? A qualitative study. *European Journal of Higher Education*, 15(2), 282–330.
- Wilkie, K. J. & Tan, H. (2019). Exploring mathematics teacher leaders' attributions and actions in influencing senior secondary students' mathematics subject enrolments. *Mathematics Education Research Journal*, 31, 441–464. <https://doi.org/10.1007/s13394-019-00264-3>

An Ontological and Epistemological Interrogation of Explicit Teaching in Mathematics

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Much is claimed about the efficacy and efficiency of Explicit Teaching in schools. Recommendations for pedagogy emphasising explicit instruction have been widely adopted by education systems across Australia, commonly citing evidence from the Australian Education Research Organisation. But what does Explicit Teaching say about ontological and epistemological questions such as learning, knowledge and how knowledge is developed? This paper uses an imagined dialogue between Critical Realism and Explicit Teaching to seek to articulate the philosophical underpinnings of Explicit Teaching in mathematics.

Much is claimed about the efficacy and efficiency of Explicit Teaching in school mathematics. Proponents of Explicit Teaching advocate classroom practices that include articulating clear learning intentions and success criteria, modelling procedures and gradual release of responsibility through approaches such as “I do, We do, You do” (Ashman, 2020). It is not the purpose of this paper to provide a commentary on the empirical evidence cited in support or otherwise of Explicit Teaching. Nor is it the purpose of this paper to discuss the efficacy or otherwise of pedagogical approaches recommended by proponents of Explicit Teaching. Rather, the paper seeks to articulate the ontological and epistemological underpinnings in the literature and advocacy of Explicit Teaching in mathematics.

In this paper I use the capitalised form Explicit Teaching with reference to an approach that not only describes classroom interactions but is also a de facto organising principle in that it structures and in some cases scripts lessons (e.g. Ochre Education, n.d.). This distinguishes Explicit Teaching in its capitalised form from explicit teaching (lower case) that forms a part of many, if not most, mathematics teaching episodes. Indeed, if one interprets the term “explicit” using its etymology—*ex* meaning *out* and *plicare* meaning to *fold*—then guided inquiry or other approaches criticised in the literature cited by proponents of Explicit Teaching (e.g. Kirschner et al., 2006) could be considered highly explicit as they aim to “fold out” the mathematics inherent in a situation (Hmelo-Silver et al., 2007).

In my interrogation of the ontological and epistemological underpinnings of Explicit Teaching I draw on the theoretical framework of critical realism (Bhaskar, 2008) and its articulation of the nature of the material and social world, and of how humans come to know and use our understanding of the world. I make no claims regarding the value or truth of critical realism as opposed to other ontological or epistemological positions; rather, I use it to frame questions that might contribute to a more nuanced view of the philosophical underpinnings of Explicit Teaching in mathematics. While other philosophical positions in the sciences or social sciences could equally well be employed, I use critical realism because it simultaneously addresses questions of ontology, epistemology and the central importance of rationality. I also use critical realism as it originated as a philosophy of science and social science that seeks to question both the positivist view of science as monistic in its development and deductive in its structure and a relativist view of science that denies the existence of universal principles. As Explicit Teaching claims to enact recommendations based on what is often termed the science of learning (or reading, or mathematics), and hence to claim a scientific basis for its recommendations, it is reasonable to interrogate it using a philosophy of science.

I frame the paper using the rhetorical device of an interrogation in order to ask close, specific and direct questions, rather than presenting a more open-ended discussion or analysis. In this paper, the interrogator is an imagined personification of critical realism (CR), and the interrogee is the imagined personification of Explicit Teaching (ET). The personification of CR is based on the three principal tenets of critical reasoning outlined below, while the personification of Explicit Teaching is drawn from the Australian Education Research Organisation (AERO) website (Australian Education Research Organisation, n.d.-a), which is, in turn, widely advocated in the media and by some education systems (e.g. Victoria Department of Education, n.d.; WA Department of Education, n.d.) as the evidence-based pedagogy that *ought* to be adopted in all schools. Again, it is not my intention to discuss normative questions of whether or not Explicit Teaching ought to be the recommended or mandated organising principle and pedagogy in schools; rather, in seeking to uncover the ontological and epistemological underpinnings of Explicit Teaching, I seek to shed light on how Explicit Teaching might shape our experience and understanding of mathematics in and beyond school and in the material and social world.

Current policy efforts to improve mathematics outcomes in Australian schools has cast increased attention on pedagogical approaches. Inquiries such as the Quality Initial Teacher Education Review, together with reports providing a “guaranteed” method for boosting student learning in primary school mathematics (Hunter et al., 2025) have, as noted above, led to the advocacy of Explicit Teaching by systems and the media. At the same time, groups such as the Mathematics Education Research Group of Australasia have argued that one narrow approach cannot address the full range of aims and proficiencies in the Australian Curriculum: Mathematics v9 (2026) such as mathematical modelling, computational thinking or statistical investigations (Mathematics Education Research Group of Australasia, 2023). Such debates therefore take on both an educational and a political dimension, making an examination of the philosophical underpinnings of Explicit Teaching both timely and apposite.

Critical realism – ontological reality, epistemological relativism and judgmental rationality

The interrogator in the imagined interrogation, critical realism, makes a clear distinction between what Bhaskar (1989) describes as the transitive world of knowing (epistemology) and the intransitive world of being (ontology). In education, and particularly in educational research and school curriculum, where knowledge adopts primary importance, this distinction is crucial. When, for example, measurable outcomes such as scores on standardised tests are assumed to fully capture the reality of teaching methods or educational innovations, this reduces the reality of schooling (ontology) to what we can know about it (epistemology) and commits what Bhaskar terms the *epistemic fallacy* (e.g. “if we can’t see it, it’s not real”). By contrast, the *ontic fallacy* ignores the cognitive and social mechanisms by which knowledge is produced leaving only a set of apparently objective facts (as if “the facts for themselves”). For example, when questions such as how standardised tests are constructed, what they measure, how they are administered or how the data are analysed, are ignored or glossed over, test scores then becoming objectified as a direct and unmediated representation of reality.

A failure to clearly articulate epistemological and ontological positions in educational research or curriculum then runs the risk of committing such mutually reinforcing fallacies (Scott, 2010). Critical realism claims to address these fallacies by combining and reconciling three principal standpoints: ontological reality, epistemological relativism and judgmental rationality. First, ontological reality refers to the nature of the material and social world. Critical realism holds that phenomena in the material and social world exist independently of our ability to explain them. In the case of mathematics, we could also add phenomena in the conceptual world (Thornton, 2022). Second, epistemic relativism refers to how we come to know the world.

It acknowledges that all our concepts are historically and socially generated, hence are relative to a particular perspective and subject to change. As our conceptual toolkit changes, either/both individually and collectively as a society, so does our understanding of the world. Third, judgmental rationality asserts that science is not arbitrary and that there are rational criteria for judging some theories as better or more explanatory than others. Different research methods, for example, reveal different aspects of the world and can be more or less descriptive or explanatory depending on the context.

According to Scott (2010), critical realism therefore:

“possesses a superior grasp of:

- the relationship between ontology and epistemology central to an understanding of the social world, and in particular, education;...
- the constitution of knowledge and its formation, and the implications of both of these for the school curriculum; and
- the need to adopt a critical approach to both education and the way it is understood. (p.11-12)

Approach

As noted above, the imagined personification of Explicit Teaching is drawn from papers publicly available on the Australian Education Research Organisation (AERO) website (Australian Education Research Organisation, n.d.-a). While I acknowledge that explicit instruction is only one element of AERO’s published model of teaching and learning, I claim that, taken as a whole, the model represents what is typically understood as Explicit Teaching in its broader and structuring sense. For example, the four elements of “Teaching that aligns with how students learn” are headed: “Enabling, Planning, Instruction and Gradual Release” (Australian Education Research Organisation, 2023c), which then become a universal structure for the illustrative lesson plans on the AERO website. Furthermore, the model has been “widely adapted and taken up by jurisdictions in their own teaching and learning models” and “AERO has taken a deliberate and structured approach to advancing evidence-based teaching, including explicit instruction—one of the most effective ways to improve student learning” (Australian Education Research Organisation, n.d.-b, p. 7,14).

I do not claim to have completed an extensive meta-analysis of the evidence for Explicit Teaching, nor a systematic review of literature on the AERO website. Such endeavours may be opportunities for further research. Rather, I searched papers of specific types on the AERO website containing specific terms that might shed light on ontological and epistemological perspectives. I chose to focus on a subset of the materials available on the AERO website that would be most likely to explain the basis for its teaching and learning recommendations: these were Articles, Explainers, Practice guides, Research reports, and Research summaries. Practice guides are included as they purport to synthesise “the most rigorous and relevant evidence-based practices from meta-analyses, systematic reviews and literature reviews” (Australian Education Research Organisation, n.d.-a).

Table 1 shows the number of materials sourced from the AERO website by search term and by resource type. Of course, many materials are referenced twice in the below table.

Table 1: Materials obtained from the AERO website by search term and resource type.

Search term	Articles	Explainers	Practice guides	Research reports	Research summaries	TOTAL
Learning	65	24	83	31	16	219
Evidence	66	23	57	35	16	197
Knowledge	37	19	52	12	23	143
Explicit	22	15	33	8	5	83
Mathematics	4	8	10	7	2	31
Reality	2			3		5
Judgment		1		1		2

Having identified materials likely to shed light on questions of ontology or epistemology, I identified specific instances in the materials that I felt most clearly captured the ontological or epistemological thrust of the materials taken as a whole. I then used these as the responses in the imagined interrogation.

Interrogating Explicit Teaching

In the imagined interrogation that follows, the questions asked by Critical Realism (CR) are framed by the three key principles of critical realism: ontological reality, epistemological relativism, and judgmental rationality. As the AERO website claims to enact both a knowledge-rich and evidence-based approach, two questions are asked relating to each of these principles: the first relates to the view of knowledge and learning underpinning Explicit Teaching, the second to the evidence in support of the pedagogical practices of Explicit Teaching. Table 2 shows the framework through which the questions were generated.

Table 2: Questioning framework for the interrogation

	Ontology	Epistemology	Judgmental rationality
Knowledge and learning	?	What is knowledge?	How do students learn to become critical users of mathematics?
Evidence	What evidence supports Explicit Teaching?	How do you decide what counts as valid evidence?	How do teachers exercise judgment?

To ensure that the responses provided by the imagined personification of Explicit Teaching (ET) are not simply my interpretations, all responses are direct citations from materials sourced from the AERO website. As noted above, these citations represent a best approximation and clearest representation of the general ontological and epistemological thrust of the AERO materials taken as a whole.

Rather than adding an interpretation of the interrogatory exchanges following each question, I invite the reader to consider the responses as a whole. Some tentative conclusions and implications follow the exchange.

On ontological reality

CR: AERO's stated vision is "for Australia to achieve excellence and equity in educational outcomes for all children and young people through effective use of evidence" (Australian Education Research Organisation, 2025b, p. 6). Specifically, AERO's constitution states that its specific purpose is to "improve *learning* outcomes for all children and young people" (Australian Education Research Organisation, 2025b, p. 2, italics added). What is *learning*?

ET: Learning is a change in long-term memory (Australian Education Research Organisation, 2023c).

Learning happens when new knowledge is acquired, processed and retained in memory for later use (Australian Education Research Organisation, 2023b, p. 1).

There are 2 types of memory that manage information during learning:

- working memory, which processes information
- long-term memory, which stores information for later use (Australian Education Research Organisation, 2025a, p. 2).

CR: The AERO website tagline is “Evidence and resources educators can trust” (*Australian Education Research Organisation*, n.d.-a). What evidence is used to support Explicit Teaching?

ET: To uphold Australia’s commitment to educational excellence, equity and lifelong learning – as articulated nationally by the Mparntwe Declaration—curriculum design *must* be underpinned by the *cognitive science* on how students learn (Lonsdale et al., 2024, p. 21, italics added)

Understanding the science behind learning and memory can help teachers understand why spacing and retrieval practice are so effective in helping students learn and remember information (Australian Education Research Organisation, 2022a)

Explicit instruction is a very efficient strategy for helping students learn because it suits how the brain processes, stores and retrieves information (Australian Education Research Organisation, 2022b).

On epistemic relativism

CR: Explicit Teaching asserts that knowledge is central to learning, arguing for a “knowledge-rich curriculum” (Lonsdale et al., 2024). What do you understand by knowledge, who decides what knowledge is important to learn and how does knowledge develop?

ET: Knowledge is the subject-specific information, concepts and principles that students are *expected* to know and understand at each stage of their schooling (Lonsdale et al., 2024, p. 6, italics added).

Explicit instruction...involves fully explaining and effectively demonstrating what students *need* to learn (Australian Education Research Organisation, 2023a, p. 1, italics added).

Knowledge is acquired in a logical and structured way when learning follows predictable progression patterns, moving from understanding foundational concepts to more complex applications (Australian Education Research Organisation, 2025c, p. 3).

CR: AERO asserts that “[g]reater uptake of evidenced-based teaching practice improves learning outcomes in schools” (Australian Education Research Organisation, 2025b, p. 5). How do you decide what evidence is valid, reliable and important to implement?

ET: At the Australian Education Research Organisation (AERO), we believe research evidence and context are important teammates... ‘Evidence-based practices’ are backed up by research evidence. This means there is broad consensus from rigorously conducted evaluations that they *work* (Australian Education Research Organisation, 2022c, italics added).

AERO’s Standards of Evidence establish our view on what constitutes rigorous and relevant evidence...*Rigorous* evidence is defined as evidence produced using research methods (whether qualitative, quantitative or mixed methods) that isolate the specific impact of a particular educational approach. *Relevant* evidence is defined as evidence produced in contexts that are similar to one’s own (Australian Education Research Organisation, 2024a, p. 1, italics in original).

On judgmental rationality

CR: The Australian Curriculum Mathematics v9 (2026) includes many priorities that go beyond acquisition of knowledge and require students to make judgments. These include the proficiencies of problem solving and reasoning described in the key considerations for mathematics, mathematical modelling, computational thinking, the statistical investigation process, probability experiments and simulations, and the use of digital tools. How are these priorities developed through Explicit Teaching?

ET: Curriculum development should sequence learning appropriate to the hierarchical, cumulative nature of maths. Teaching and learning plans should first explicitly teach related skills and knowledge and give opportunities to practise. Open and complex tasks with many possible paths to solutions should *follow* (Australian Education Research Organisation, 2024b, p. 5, italics added).

As students progress towards mastery, they can demonstrate their understanding in increasingly sophisticated ways. Automating knowledge in long-term memory enhances their ability to solve complex, unfamiliar problems and generate new insights that reflect deep learning. These skills support

lifelong learning and success both in and beyond the classroom (Australian Education Research Organisation, 2025d, p. 4).

CR: The Australian Association of Mathematics Teachers position statement on Pedagogy in Mathematics (2025) states:

No single pedagogical approach can meet the needs of every mathematics learner. Effective mathematics teaching relies on a range of intentional and high-quality practices that can be tailored to specific contexts, learners and purposes...Teachers are encouraged to develop and deploy a diverse repertoire of strategies, grounded in both research and practical insights, to create responsive mathematics learning environments for all students (p.1).

How do you respond to that statement?

ET: Evidence is important, but what is more important is [...] teacher expertise and professionalism [to] make better judgments about when, and how, to use research (Gleeson et al., 2022, citing Wiliam, 2019)⁴.

AERO has developed Standards of evidence and Evidence decision-making tools that aim to guide teachers and educator's assessment of the confidence they should have that a particular practice will 'work' based on the amount, rigour and contextual relevance of research evidence available (Australian Education Research Organisation, 2022, p. 18).

Conclusion and further directions

My reading of the materials relating to Explicit Teaching on the AERO website through the lens of critical realism reveals three foundational ideas:

3. *Knowledge is the cornerstone of the advocacy of Explicit Teaching.* However, who decides what knowledge is worth acquiring and why remains unquestioned, resulting in simplistic statements such as “the knowledge that students *need* to learn” as cited above. Consequent on this view of knowledge is the equating of learning with a change in long-term memory and a model of the brain as a storage device, which in turn suggests that a collection of facts or concepts residing in the individual mind is a faithful representation of reality. But this is not the only potential conception of the human mind. As Donald (2001, p. xiii) argues “[T]he human mind has a collective counterpart: culture...The key to understanding the human intellect is not so much the design of the individual brain as the synergy of many brains”. And “questions about knowledge always take us back to some of our most basic assumptions about what it is to be educated or to educate someone; they are, in the broadest sense, philosophical and political questions about who we are and what we value” (Young, 2008, p. xvii).
4. *We acquire knowledge in a consistent and predictable way through clear explanation, modelling, practice and intervention as required.* The logical consequence is that we share a common view of the world that can be transmitted through structured Explicit Teaching. Again, this is by no means the only defensible epistemological position. Brandom (2000), for example, argues for an epistemology founded on inference rather than reference (that is, in making a conceptual claim one is explicitly endorsing a set of inferences that articulate its conceptual content). “For a commitment to become explicit is for it to be thrown into the game of giving and asking for reasons as something whose justification, in terms of other commitments and entitlements, is liable to question” (Brandom, 2000, p. 76).

⁴ Gleeson et al. is a report commissioned by AERO. The citation from Wiliam (2019) is from an opinion piece headed “Teaching not a research-based profession”, in which he says, “Classrooms are just too complicated for research ever to tell teachers what to do”.

5. *We make judgments following acquisition of knowledge.* For students, this means learning foundational knowledge prior to engaging in higher order tasks involving critical thinking, synthesis or evaluation. For teachers, judgment is pragmatic. While being encouraged to assess whether a particular practice will *work*, to assess the *usability* of research evidence and to make judgments about *when*, and *how*, to use this research, there is little scope for teachers to consider the potential contribution of research to questions beyond those of improved outcomes on standardised measures of achievement.

Taken together, these foundational ideas suggest what I would term a realist but blinkered ontology, an absolutist epistemology, and a judgmental pragmatism. However, these conclusions are almost entirely implicit. The AERO website is entirely silent on questions of ontology and epistemology—no results were returned in a search of the website for these terms. In its silence on these matters, Explicit Teaching then becomes not only a pedagogy and an organising principle, but a de facto self-referential philosophy of mathematics education, perhaps best expressed as “The purpose of education is to acquire knowledge; knowledge is the set of skills and concepts that students need to learn; learning happens when those skills and concepts are stored in memory”.

The absence of any external referent creates what I suggest is an ontological and epistemological chasm in the advocacy of Explicit Teaching. This then places this self-referential philosophy on very shaky ground. Indeed, it is difficult to see how Explicit Teaching, as advocated on the AERO website, can address deeper questions of purpose and culture espoused in the rationale for mathematics education described in curriculum documents such as the Australian Curriculum v9 (2026). This is not to say that questions of purpose and culture cannot be addressed in Explicit Teaching, rather it seems that they have yet to be asked.

In fairness one could argue that most published mathematics education research is silent on questions of epistemology and ontology (Thornton et al., 2016). Hence, it may be timely to reframe the rhetoric around “explicit teaching”. The dichotomous view and apparent incompatibility of inquiry or problem-based approaches to teaching mathematics and Explicit Teaching as advocated by the sources discussed above, is neither helpful nor productive. I would argue that even a blended or balanced approach suggesting that there is a place for both fails to adequately address the apparent impasse. Rather, it is timely to consider questions of ontology, epistemology and rationality, and hence to re-examine teacher and student roles in an articulation of “explicit” that captures its origins as “unfolding”.

References

- Ashman, G. (2020). The power of explicit teaching and direct instruction. SAGE Publications Ltd.
- Australian Association of Mathematics teachers Ltd. (2025). *Pedagogy in Mathematics*. <https://aamt.edu.au/wp-content/uploads/2025/01/AAMT-Pedagogy-in-Mathematics.pdf>
- Australian Curriculum Assessment and Reporting Authority. (2026). *Mathematics | V9 Australian Curriculum*. <https://www.australiancurriculum.edu.au/curriculum-information/understand-this-learning-area/mathematic>
- Australian Education Research Organisation. (n.d.-a). <https://www.edresearch.edu.au/>
- Australian Education Research Organisation. (n.d.-b). *Australian Education Research Organisation Annual report 2025*. <https://www.edresearch.edu.au/other/agency-publications/annual-report-2025>
- Australian Education Research Organisation. (2022a). *Spacing and retrieval*. <https://www.edresearch.edu.au/summaries-explainers/explainers/spacing-retrieval>
- Australian Education Research Organisation. (2022b). *Explicit instruction*. <https://www.edresearch.edu.au/summaries-explainers/explainers/explicit-instruction>
- Australian Education Research Organisation. (2022c). “*But that would never work here*” – Does context matter more than evidence. <https://www.edresearch.edu.au/other/articles/would-never-work-here-does-context-matter>

- Australian Education Research Organisation. (2023a). *Explicit instruction optimises learning*. <https://www.edresearch.edu.au/summaries-explainers/explainers/explicit-instruction-optimises-learning>
- Australian Education Research Organisation. (2023b). *Knowledge is central to learning*. <https://www.edresearch.edu.au/sites/default/files/2023-09/knowledge-is-central-to-learning-aa.pdf>
- Australian Education Research Organisation. (2023c). *Teaching for how students learn: A model of learning and teaching*. <https://www.edresearch.edu.au/model-learning-teaching>
- Australian Education Research Organisation. (2024a). *AERO – Standards of evidence*. <https://www.edresearch.edu.au/other/agency-publications/evidence/standards-evidence>
- Australian Education Research Organisation. (2024b). *Developing maths proficiency*. <https://www.edresearch.edu.au/sites/default/files/2024-09/developing-maths-proficiency-aa%20%281%29.pdf>
- Australian Education Research Organisation. (2025a). *AERO Explainer – Attention and focus*. <https://www.edresearch.edu.au/sites/default/files/2025-04/attention-and-focus-explainer-aa.pdf>
- Australian Education Research Organisation. (2025b). *AERO Strategic plan 2025–2027*. <https://www.edresearch.edu.au/other/agency-publications/strategic-plan-2025-27>
- Australian Education Research Organisation. (2025c). *Knowledge and memory*. <https://www.edresearch.edu.au/sites/default/files/2025-04/knowledge-and-memory-explainer-aa.pdf>
- Australian Education Research Organisation. (2025d). *Mastery and application*. <https://www.edresearch.edu.au/sites/default/files/2025-04/mastery-and-application-explainer-aa.pdf>
- Bhaskar, R. (1989). *Reclaiming reality: a critical introduction to contemporary philosophy*. Verso.
- Bhaskar, R. (2008). *A Realist Theory of Science*. Routledge.
- Brandom, R. (2000). *Articulating Reasons*. Harvard University Press.
- Donald, M. (2001). *A mind so rare: The evolution of human consciousness*. Norton
- Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist*, 42(2), 99–107. <https://doi.org/10.1080/00461520701263368>
- Hunter, J., Haywood, A., Parkinson, N., & Petrie, D. (2025). *The Maths Guarantee: How to boost students' learning in primary school*. www.grattan.edu.au/donate.
- Gleeson, J., Cutler, B., Larsen, S., Parker, B., Ridgway, K., Rickinson, M., Walsh, L., People, J., & Marshall, P. (2022). *Supporting teachers to use research evidence well in practice*. https://www.edresearch.edu.au/sites/default/files/2022-12/supporting-teachers-to-use-research-evidence-aa_1.pdf
- Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching. *Educational Psychologist*, 41(2), 75–86. https://doi.org/10.1207/s15326985ep4102_1
- Lonsdale, E., Griffiths, K., Lind, R., & Marslen, T. (2024). *A knowledge-rich approach to curriculum design*. Australian Education Research Organisation.
- Mathematics Education Research Group of Australasia. (2023). *Response to the Teacher Education Expert Panel Discussion Paper*. https://www.education.gov.au/system/files/documents/submission-file/2023-07/TEEP_Mathematics%20Education%20Research%20Group%20of%20Australasia.pdf
- Ochre Education. (n.d.). <https://ochre.org.au/>
- Scott, D. (2010). *Education, epistemology and critical realism (1st ed.)*. Routledge.
- Thornton, S. (2022). (Re)asserting a knowledge-building agenda in school mathematics. *Mathematics Education Research Journal*, 34(1), 69–85. <https://doi.org/10.1007/s13394-020-00322-1>
- Thornton, S., Kinnear, V., & Walshaw, M. (2016). A Philosophical Gaze on Australasian Mathematics Education Research. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in Mathematics Education in Australasia 2012-2015* (pp. 29–48). Springer Singapore.
- Victoria Department of Education. (n.d.). *Victorian Teaching and Learning Model 2.0*. <https://www2.education.vic.gov.au/pal/victorian-teaching-learning-model/policy>
- WA Department of Education. (n.d.). *Quality Teaching Strategy*. <https://www.education.wa.edu.au/quality-teaching-strategy>
- William, D. (2019). *Teaching not a research-based profession*. <https://www.tes.com/magazine/archive/dylan-william-teaching-not-research-based-profession>
- Young, M. F. D. (2008). *Bringing knowledge back in: from social constructivism to social realism in the sociology of education*. Routledge.

Halving and Doubling in Multi-Digit Multiplication: How Students' Spatial Structuring of Arrays Supports Structural Understanding

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This paper examines how students' spatial structuring of arrays supported their reasoning about halving and doubling in multi-digit multiplication. Drawing on a teaching experiment in two Year 5 classrooms, the analysis focused on how students organised and transformed arrays when reasoning about multiplicative relationships. The findings suggest that arrays supported students not simply by representing multiplication visually, but by enabling them to test different transformations of the array. In particular, students recognised that halving and doubling required transformations that preserved equal groups and the coordinated row-and-column structure of the array.

The capacity to reason using a deep conceptual understanding of multiplicative structure is crucial to students' mathematical development (Clark & Kamii, 1996). Mathematical structure involves recognising general properties that are instantiated in particular situations as relationships between elements (Mason et al., 2009). In multiplicative contexts, this includes recognising properties such as distribution, associativity, and commutativity as generalisable relationships. It is important that students develop understanding of the mathematical properties that underlie multiplication as this knowledge forms the basis for algebraic thinking (Shifter et al., 2008).

Compared to single-digit multiplication, multi-digit multiplication requires a substantial shift in students' reasoning, as they must coordinate the multiplicative relationships inherent in place value with the distributive structure of multiplication (Izsák, 2004). Although substantial research has examined single-digit multiplication, comparatively few studies have focused on multi-digit multiplication (Hickendorff et al., 2019), and much of this work has emphasised the distributive property. Less attention has been given to how students develop understanding of the associative property in multi-digit multiplication, particularly in relation to halving and doubling, or to the role that spatial reasoning may play in supporting this understanding (Antonides et al., 2025; Battista, 2004).

This study responds to these calls by examining the role of students' spatial structuring of arrays in supporting their understanding of halving and doubling as an application of the associative property in multi-digit multiplication. Halving and doubling is treated here as an application of associative reasoning because it involves decomposing one factor and regrouping the multiplicative structure while preserving the product. The study was guided by the following research question: *How does students' spatial structuring of arrays support their reasoning about halving and doubling in multi-digit multiplication?*

Literature Review

The array is a powerful representation for multiplication because it makes the underlying spatial structure of multiplication visible (Barmby et al., 2009; Battista et al., 1998). As a two-dimensional representation, the array draws attention to equal groupings and to the coordination of composite units, enabling learners to perceive how multiplicative relationships are organised to produce a whole (Steffe, 1994). For this reason, several studies have examined students' spatial structuring when working with arrays in early multiplication and area contexts.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 409–416). MERGA.

Battista et al. (1998, p. 503) describe spatial structuring as mentally organising objects by identifying their parts, combining these parts into larger units, and recognising the relationships between them. In array contexts, spatial structuring is essential because it enables students to organise and enumerate spatial units and composite units, coordinate rows and columns as composite units, and recognise the multiplicative relationships represented in the array (Battista, 2004; Battista et al., 1998). Battista et al. (1998) identified three levels of sophistication through which students' progress when spatially structuring arrays. At the first two levels, students tend to focus on individual units or attend to rows and columns separately. Only at the most sophisticated level do students coordinate rows and columns simultaneously, enabling them to perceive the array as a structured multiplicative whole rather than as a collection of individual units. Consistent with this perspective, Outhred and Mitchelmore (2000) found that students often begin by covering rectangles with individual units and only gradually organise these into rows and columns that reflect the two-dimensional structure of the array.

Several studies have shown that arrays afford students access to important understandings in multi-digit multiplication. Larsson et al. (2017) argue that arrays help students move beyond additive interpretations of multiplication by making multiplicative relationships between factors visible. In particular, arrays can make the distributive property visible (Barmby et al., 2009; Izsák, 2004). Barmby et al. (2009) reported that when working with arrays, students demonstrated an intuitive sense of the distributive property.

Importantly, existing studies of students' understanding of the associative property have not examined students' reasoning in relation to arrays or considered how a representation such as the array might support students to make sense of halving and doubling. Downton et al. (2022), for example, examined students' understanding of the associative property across three applications: noticing, halving and doubling, and place value. Their findings showed that although many students could identify halving and doubling as an efficient strategy, few could explain why halving one factor and doubling another preserves the product, and students also demonstrated limited understanding of associative reasoning in place value contexts. These findings align with other research indicating that students' understanding of the associative property is often underdeveloped (Kanbir et al., 2018). Barnett and Ding (2019) similarly found that classroom teaching often left the associative property implicit and provided limited support for students to understand it as a structural property rather than as a computational rule.

Taken together, this literature suggests that while arrays support reasoning about multiplicative structure, their role in supporting understanding of halving and doubling remains underexplored. In particular, little is known about how students' spatial structuring of arrays may support understanding of halving and doubling as an application of the associative property in multi-digit multiplication. This paper addresses that gap by examining how students' spatial structuring of arrays supports their reasoning about halving and doubling in multi-digit multiplication.

Theoretical Framework

This study was informed by Realistic Mathematics Education (RME), which shaped both the design of the instructional sequence and the interpretation of students' learning. RME is grounded in the view that mathematics should be learned through students' own activity, with meaningful contexts and representations supporting the development of increasingly formal mathematical understanding (Freudenthal, 1973; Gravemeijer, 1994). From this perspective, mathematical learning involves students organising and reorganising situations mathematically, with models emerging first as representations of situated activity and later becoming tools for more general reasoning. In the present study, the array functioned in this way. Initially, it operated as a model of students' reasoning about cupcake arrangements and area, but through

classroom discussion and repeated use it also became a model for reasoning about multiplicative relationships, including the associative property.

To examine how students reasoned with the array, the study drew on Battista et al.'s (1998) notion of spatial structuring. In array contexts, spatial structuring enables students to coordinate rows and columns as composite units and to perceive the multiplicative relationships represented in the array (Battista, 2004; Battista et al., 1998). In this study, spatial structuring provided the primary analytic lens for examining how students used arrays to reason about halving and doubling. Particular attention was given to the actions students performed on arrays, such as partitioning, translating, decomposing, joining, and rotating, and to whether these actions preserved equal groups, conserved the total number of units, and maintained the coordinated row-and-column structure of the array. This framework supported analysis of how students used the array to develop more generalised reasoning about multiplicative structure.

Methods

The research reported in this paper was situated within a larger project focused on articulating and refining a learning trajectory for multi-digit multiplication (Tripet, 2019). Design research methods, as articulated by Cobb and Gravemeijer (2008), were employed to enable first-hand observation of students' mathematical activity as instruction unfolded.

Context

The teaching experiment phase of the research was conducted in two Year 5 classrooms (students aged 9–11 years) in Sydney, with 23 students in Class 1 and 22 students in Class 2 (45 students in total). The classes were selected on the basis that they had not yet received formal classroom instruction in solving two-digit by two-digit multiplication problems, although some students had encountered the standard algorithm through tutoring or at home.

During the teaching experiment phase, I adopted the role of teacher, which allowed me to control the learning environment and the teaching episodes across both classes while experiencing first-hand the events of the classroom. The same instructional sequence was implemented in both classrooms over a two-week period. The sequence comprised four teaching episodes, each spanning two or three one-hour lessons. Each episode began with a problem for students to solve, followed by a whole-class discussion and time for students to explore and build on strategies shared during the discussion. This paper reports on the tasks that were the focus of the second and third teaching episodes from the teaching experiment.

Data Collection and Analysis

Three forms of data were collected: student work samples produced during the teaching episodes, transcribed video recordings of whole-class and small-group activity, and field notes recorded during and immediately after each lesson.

Consistent with teaching experiment methodology, analysis occurred in two phases. During the teaching experiment, ongoing cycles of design and analysis were used to interpret students' emerging reasoning and to inform subsequent instructional decisions. The analysis reported in this paper draws primarily on a retrospective analysis of the second and third teaching episodes.

The retrospective analysis was conducted through a spatial structuring lens. Video transcripts and student work samples were examined to identify how students organised and reorganised units and composite units within arrays when reasoning about halving and doubling. Analysis focused particularly on the spatial actions students used, drawing on the action categories identified by Antonides et al. (2025). The actions most relevant to this study are presented in Table 1. These actions were analysed in relation to whether they preserved equal groups, conserved the total number of units, and maintained the coordinated row-and-column structure of the array.

Strategies were also coded for the multiplicative reasoning they revealed, including correct and incorrect applications of halving and doubling and related alternative conceptions. The dataset was organised both by individual student and by solution method, enabling patterns to be identified within and across students. The examples presented in the results were selected as illustrative cases of broader patterns observed across the two classes.

Table 1

Coding for students spatial structuring actions (Antonides et al., 2025)

Spatial structuring actions	Descriptions
Decomposing	Dividing a spatial region into separate, disjointed, and potentially incongruent parts
Joining	Placing spatial objects adjacent to each other so that external parts of those objects coincide
Measurement-based comparison	Using numbers as measurements of spatial attributes to directly or indirectly compare two spatial objects
Numerical structuring	Constructing a coherent structure or form for a set of computations, including those involving numerical or algebraic expressions
Partitioning/Subdividing	Breaking a spatial object into separate, equal-size parts
Rotating/Orienting	Turning an object about a point, possibly in relation to a frame of reference
Spatial-numerical linked structuring	Spatial structuring that is conceptually linked to a numerical structuring
Translating	Mental act of moving an object in space while preserving its size, shape, and orientation

Results

The instructional sequence followed the narrative of a bakery, with different forms of the array used to model a range of contexts. In the first teaching episode, students calculated the total number of cakes in a 24×8 array. Most students used the array to model their strategies, indicating confidence with the representation in its various forms. The results reported here focus on the second and third teaching episodes, where students' reasoning with arrays became the focus of analysis. The selected examples illustrate patterns observed across both classes. In each case, students' spatial structuring involved multiple actions that became visible through their interactions with the array.

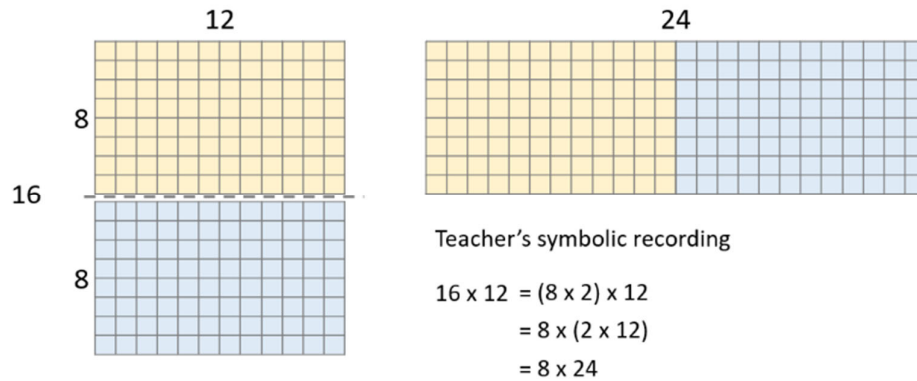
In the second teaching episode, students were presented with an image of 16 filled cupcake boxes arranged in a 4×4 array, with each box holding 12 cupcakes. Although the individual cupcakes were not visible, students calculated the total number of cupcakes and noted that 16×12 was equivalent to a calculation from the first teaching episode, 24×8 . Explaining this equivalence became the focus of investigation.

Ryan and Dylan (Class 1) constructed an array to represent the problem. Rather than representing the 16 boxes alone, they used one square unit to represent each individual cake within the boxes. In doing so, they recognised that 16 could be halved to make 8 and that 12 could be doubled to make 24. They then cut their grid array in half (partitioning) and rearranged it (translating) to create 24×8 array (Figure 1). Through these actions, they preserved both the total number of units and the coordinated row-and-column structure of the array (spatial-numerical linked structuring). When Ryan and Dylan later shared their work in the whole-class discussion, the associative property was introduced and explicitly linked to their halving and doubling strategy. Students were then given a series of multiplication problems to explore when

halving and doubling might be useful. Through this work, they concluded that the strategy was particularly effective when it produced a multiple of ten or one hundred.

Figure 1

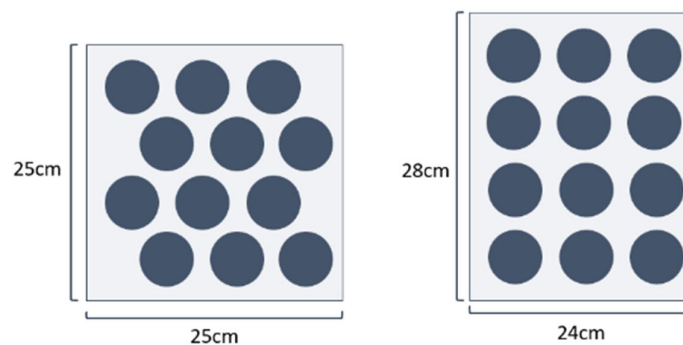
An illustration of Ryan and Dylan's representation with the teacher's symbolic recording



In the third teaching episode, students were shown an image of two cupcake trays inside cupcake boxes, one measuring 28 x 24cm and the other 25 x 25cm (Figure 2). The class discussed why one arrangement appeared skewed while the other did not, and students were asked to predict whether a 28 x 24 tray would have a larger area than a 25 x 25 tray. In both classes, some students proposed that taking 2 from 28 and adding it to 24 would produce 26 x 26, suggesting an incorrect extension of the earlier halving and doubling strategy. These responses shifted the focus of the investigation, and students were asked to test whether this reasoning was correct.

Figure 2

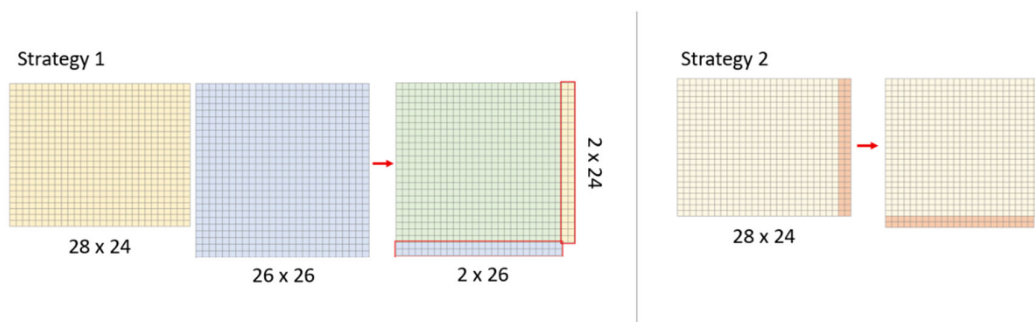
The third teaching episode and the trays that were the focus of investigation



Two significant strategies were observed across the classes (Figure 3). In the first, Mitchell and Tom (Class 1) constructed 28 x 24 and 26 x 26 arrays on grid paper and overlaid them to compare their areas directly (measurement-based comparison). Because the overhanging regions differed in size, they concluded that the two products were not equal, and that 28 x 24 was greater than 26 x 26 (spatial-numerical linked structuring). In the second, Amelie (Class 2) constructed a 28 x 24 array, cut two columns from the 28 (decomposing), and repositioned them as two additional rows beneath the existing 24 rows (joining and rotating). This revealed a 2 x 2 gap in the bottom corner, leading her to conclude that 28 x 24 must be 4 less than 26 x 26 (spatial-numerical linked structuring). In explaining her strategy, Amelie noted that the rows “don’t match up”, recognising that her transformation disrupted the coordinated row-and-column structure of the array.

Figure 3

An illustration of Mitchell and Tom's measurement-based comparison (Strategy 1) and Amelie's use of decomposition and rejoining (Strategy 2)



These strategies were later shared in whole-class discussion, after which students explored them further using other multiplication problems. Through area-based comparison and through decomposing and repositioning parts of arrays, students' spatial structuring supported their understanding of the distinction between additive and multiplicative compensation. This distinction was made explicit in Samar's explanation:

Samar: Last time we halved and doubled. When you halved, the two parts were the same and so you could move the parts of the array. But when you take just two off, the parts are not the same and so they won't match. So, I think that you can divide but not subtract.

Teacher: What do you mean that you can 'divide'?

Samar: You can divide by 2 and then double or divide by 3 or divide by 4...and you can keep going. But you can't just subtract 2 or 3 or 4. You need the parts [of the array] to be the same.

Over the following three one-hour lessons, the use of the associative property, particularly the use of factors to decompose numbers, became taken-as-shared in the class. This was illustrated by one student's use of factors to solve 32×25 :

Louisa: 25 is a friendly number because you just multiply it by 4 to get 100, so you divide the 32 by 4 to get 8, so it is just the same as 8×100 .

Discussion and Implications

These findings suggest that students' spatial structuring of arrays provided a useful basis for reasoning about halving and doubling as an application of the associative property in multi-digit multiplication. Downton et al. (2022) found that although many students could identify halving and doubling as an efficient strategy, few could explain why halving one factor and doubling another preserves the product. The present study points to a possible way of addressing this difficulty. Working with arrays in which individual units were explicitly represented appeared to provide students with a more concrete basis for reasoning about why halving and doubling preserves equivalence.

Across the examples, students chose to represent each individual unit, suggesting that explicit unit representation was important for making relationships between the factors visible. However, the findings indicate that constructing an array alone was not sufficient. What mattered was how students transformed the array and whether those transformations maintained the coordinated row-and-column structure of the array and preserved the product. Amelie's actions of (decomposing), (rotating), and (joining) disrupted this structure and did not preserve the product. This was also reflected in Mitchell and Tom's measurement-based comparison, which showed that 28×24 was not equivalent to 26×26 as the overhanging regions were different in size. By contrast, Ryan and Dylan's (partitioning) and (translating) preserved equal groups and maintained the total product. Samar picked up on this idea when she explained that halving (partitioning) produced equal-sized parts, whereas subtraction in this case did not.

Together, these examples suggest that students were better positioned to reason about halving and doubling when they could see that the product remained unchanged only when the array was partitioned using equal groups and so maintaining the overall multiplicative structure.

The findings also highlight the importance of making these structural relationships explicit (Shifter et al., 2008). Barnett and Ding (2019) found that classroom instruction often overemphasised computational strategies for obtaining an answer without making explicit links to the underlying properties that explain why those strategies work. They noted that teaching often left the associative property implicit and provided limited support for students to understand it as a structural property rather than as a computational rule. They also noted that teaching did not always adequately connect the abstract notion of the associative property to concrete forms of justification. The present study offers one example of how this might be addressed in the classroom.

In contrast, the present study explicitly connected students' strategies to the associative property and encouraged them to test halving and doubling across multiple cases. Students were afforded time to investigate their conjectures and to see why the product was preserved by making visible their spatial structuring actions, and specifically if these actions preserved the coordinated row-and-column structure of the array. Importantly, students also explored examples and non-examples of halving and doubling, which drew attention to the importance of partitioning into equal-sized groups. This focus on what remained invariant across cases may have supported students to recognise the strategy as an instance of a general property of multiplication (Mason et al., 2009), as reflected in Louisa's final solution to 32×25 .

Through this teaching experiment, the array provided a concrete means through which students could test and justify halving and doubling as a viable strategy. More importantly, this suggests that teaching for understanding of the associative property should involve more than naming the property or demonstrating an efficient method. Students need opportunities to work with representations that make multiplicative structure visible and to examine which transformations preserve that structure and which do not.

Conclusion

Overall, this study suggests that the value of the array in teaching halving and doubling lies not simply in representing multiplication, but in enabling students to test whether transformations preserve multiplicative structure. Students' reasoning appeared to become more structurally grounded when they recognised that preserving the product depended on maintaining equal groups and the coordinated row-and-column structure of the array. In this way, the findings offer a more precise account of how arrays may support understanding of the associative property in multi-digit multiplication.

Acknowledgements

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References

- Antonides, J., Norton, A., Battista, M. T., & Zwanch, K. (2025). Spatial Structuring and Units Coordination: Building and Illustrating a Coordinated Theoretical Framework. *Cognition and Instruction*, 1–44. <https://doi.org/10.1080/07370008.2025.2571509>
- Battista, M. T. (2004) Applying Cognition-Based Assessment to Elementary School Students' Development of Understanding of Area and Volume Measurement, *Mathematical Thinking and Learning*, 6(2), 185–204. https://doi.org/10.1207/s15327833mtl0602_6
- Battista, M. T., Clements, D., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education*, 29(5), 503–532.

- Barmby, P., Harries, T., Higgins, S. & Suggate, J. (2009). The array representation and primary children's understanding and reasoning in multiplication. *Educational Studies in Mathematics*, 70, 217–241. <https://doi.org/10.1007/s10649-008-9145-1>
- Barnett, E., & Ding, M. (2019). Teaching of the associative property: A natural classroom investigation. *Investigations in Mathematics Learning*, 11(2), 148–166. <https://doi.org/10.1080/19477503.2018.1425592>
- Clark, F., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1-5. *Journal for Research in Mathematics Education*, 27(1), 41–51. <https://doi.org/10.2307/749196>
- Cobb, P., & Gravemeijer, K. (2008). Experimenting to support and understand learning processes. In A. Kelly, R. Lesh, & J. Baek (Eds.), *Handbook of Design Research Methods in Education: Innovations in Science, Technology, Engineering, and Mathematics Learning and Teaching* (pp. 68–95). Routledge.
- Downton, A., Russo, J., & Hopkins, S. (2022). Students' understanding of the associative property and its applications: noticing, doubling and halving, and place value. *Mathematics Education Research Journal*, 34(2), 437–456. <https://doi.org/10.1007/s13394-020-00351-w>
- Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning*, 62(2), 105–128. https://doi.org/10.1207/s15327833mtl0602_3
- Hickendorff, M., Torbeyns J. & Verschaffel L. (2019), Multi-digit addition, subtraction, multiplication, and division strategies. In Fritz A., Haase V.G., Räsänen P. (Eds.) *International Handbook of Mathematical Learning Difficulties*. Springer. doi:10.1007/978-3-319-97148-3_32.
- Izsak, A. (2004). Teaching and Learning Two-Digit Multiplication: Coordinating Analyses of Classroom Practices and Individual Student Learning. *Mathematical Thinking and Learning*, 6(1), 37–79. https://doi.org/10.1207/s15327833mtl0601_3
- Kanbir, S., Clements, M. A. (Ken), & Elleton, N. F. (2018). *Using design research and history to tackle a fundamental problem with school algebra*. Springer. <https://doi.org/10.1007/978-3-319-59204-6>.
- Larsson, K., Petterssen, K., & Andrews, P. (2017). Students' conceptualisations of multiplication as repeated addition or equal groups in relation to multi-digit and decimal number. *Journal of Mathematical Behavior*, 48, 1–13. <https://doi.org/10.1016/j.jmathb.2017.07.003>.
- Mason, J., Stephens, M. & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, 21, 10–32. <https://doi.org/10.1007/BF03217543>
- Outhred, L. N., & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education*, 31(2), 144–167. <https://doi.org/10.2307/749749>
- Schifter, D., Monk, S., Russell, S. J., & Bastable, V. (2008). Early algebra: what does understanding the laws of arithmetic mean in the early grades? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 413–448). Erlbaum.
- Steffe, L. (1994). Children's multiplying schemes. In G. Harel & J. Confrey (Eds.), *The Development of Multiplicative Reasoning in the Learning of Mathematics* (pp. 3–39). SUNY Press.
- Tripet, K. (2019). *A journey to understanding: Developing computational fluency in multi-digit multiplication* [Doctoral dissertation, The University of Sydney]. Sydney eScholarship Repository. <http://hdl.handle.net/2123/20763>
- Young-Loveridge, J., & Mills, J. (2009). Teaching multi-digit multiplication using array-based materials. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing Divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (pp. 635–643). MERGA.

Mathematics Education as Mathematical and Educational

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We bring two schools of research thought into a conversation. One engages with mathematics teaching, learning, and instructional design built on Freudenthal's views of *mathematics* as a human activity, where learning emerges in participation. The other is Biesta's philosophy-based perspective, where *education* is existential work in which students encounter the world and come to terms with it. We clarify when activities in classrooms are (or are not) *mathematical*, when they are (or are not) *educational*, and show that these pursuits align. This opens possibilities for clarifying what is involved in designing school mathematics that is both mathematical and educational.

Engaging with the work of Gert Biesta (2022) introduces one to arguments and language for considering education from a perspective where a person's place in, and their relationship to, the world are of key concern. Biesta's ideas resonate strongly with the well-established design research tradition in mathematics education (Cobb et al., 2008; Freudenthal, 1973) that have guided our research journey (e.g., Cortina et al., 2014; Visnovska et al., 2025). We introduce and discuss some of those resonances and elaborate on their significance to the mathematics education research community in grounding decisions about what teaching and learning mathematics should (and should not) entail, and why the difference matters. We illustrate how thinking with Biesta provides orientation for making instructional design decisions and how it can inform thinking about school mathematics at the curricular scale.

Drawing on Biesta's work broadly and on the *World-Centred Education* book specifically, we first introduce the key notions. We attend to how educational work is existential; outline the three domains of purpose in education; clarify the distinction of the student as being object or subject of their education. Our aim is to foreground educational considerations for mathematics education and clarify both the mathematical and educational value of mathematics in school.

Learning from World-Centred Education

Biesta's work is grounded in the position that the work of education is existential, rather than conceptual. For researchers in mathematics education—a field that is, and has been, so prominently about conceptual change and understanding—this perspective on education might be far from obvious. Biesta (2017) maintains that education should not merely be a means for equipping students for their understanding and making sense of the world, but it must be a place for encountering the world and coming to terms with it.

The real educational work ... is about ... bringing children and young people *into dialogue* with the world. It is about turning them towards the world and about arousing their desire for wanting to be *in* the world and *with* the world, and not just with themselves. (p.37, italics in the original)

Biesta draws on Meirieu as he elaborates that education is about exploring what it might mean to exist in the world in a grown-up way, that is, “in the world without occupying the centre of the world” (Meirieu, 2007, p. 96). As Biesta elaborates in *World Centred Education*, educational questions are fundamentally existential questions, that is, questions about how we try to exist as human beings, how we try to live our life in and with a world that is not of our making and that is under no obligation to give us what we want from it or expect from it. (2022, p. 9)

In other words, as the world is not of our making and does not play by our rules, it is often confronting to encounter and to understand it. And it then matters what we choose to do with what we came to understand, and that we come to own the impacts our choices have on the world and on those around us.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 417–424). MERGA.

In mathematics education, Hans Freudenthal (1973) has imagined school as a place where students can encounter mathematics, come to terms with it, and be brought into dialogue with it. He viewed mathematics as a human activity and wished that the students encounter mathematics as such—as an activity of organising the world aimed at being in the world and acting within it. Freudenthal sought and proposed ways for supporting students to progressively become capable in organising the world—and subsequently mathematics—mathematically. We are all aware that mathematics created through such organising is not docile. It is governed by internal necessity and is self-constraining. Once an organised system of relations is established, it “talks back” to its organiser, revealing inconsistencies, and prompting the organiser to reconsider what has been pondered, concluded, or built so far. The *world of mathematics* can be confronting too. And while mathematics education might choose to focus on the conceptual work of how students are gaining mathematical understandings in school, it cannot escape the bind of how much it matters what those students choose to do with their understandings, starting with what they choose to do in the classroom, and how their choices impact those around them.

As mathematics educators, we aim to turn students towards mathematics and arouse their desire for wanting to be in and with mathematics—even though mathematics is under no obligation to give the students what they want or what they might, initially, expect from it. Mathematics education, too, faces questions that are existential. Biesta’s view illuminates the dangers of staying purely in the conceptual when we organise mathematics education. It also indicates how much more can be accomplished in conceptual development when the existential work of education is taken seriously in mathematics classrooms.

Purposes of Mathematics Education

Framing educational work as existential, Biesta (2022) illustrates that education always serves three parallel functions, suggesting three domains of educational purpose, namely, qualification, socialisation, and subjectification. He argues that each of these domains is shaped through schooling, whether this happens by design or incidentally. As we introduce the three notions, we contextualise them for mathematics education, linking them to relevant research.

Qualification

The first of the domains, the educational pursuit of mathematical *qualification*, is the most familiar. It is observable in how curricula list the content to be learned, understandings to be derived, and skills to be developed. Most clearly perhaps, pursuit of qualification is observable in the assessment instruments and practices used to determine who learned mathematics, or how well did they learn it. This purpose of education is rarely challenged, as it aligns with the present broad focus on learning (Biesta, 2015), predominantly viewed as acquisition of knowledge, skills, or proficiencies (Sfard & Cobb, 2014). Qualification purposes determine how mathematics is organised for teaching, and how it is approached in schools today.

One of the consequences of singling out the qualification purposes as the focus of education is the reinforcement of framing education as a cognitive commodity. The students should be personally motivated to acquire such commodity (of facts, skills, and ideas) because those who succeed in its acquisition gain a competitive advantage over those who do not (Sfard, 1998). Under these assumptions, organising the mathematics curriculum as a list of facts, skills, and ideas that need to be handed over to conceivably willing and eager recipients sounds like a straightforward job. The years of earnest efforts at doing so show that many recipients do not tend to be quite so willing, and that even some of the willing ones tend to face considerable difficulties in acquiring what is handed over to them. We return to qualification as the source of these difficulties when we discuss how this purpose interacts with the remaining domains.

Socialisation

Whether by design or not, education inevitably plays a role in “providing the new generation with orientation into [some of] the traditions, cultures, and practices of past and present” (Biesta, 2022, p. 7-8). Biesta considers this role as constitutive of the purposes of the domain of *socialisation*. Which practices students are enculturated into depends greatly on whether the domain of socialisation receives serious consideration in curricular and instructional design.

Biesta’s examples describe the students’ socialisation into practices of the society broadly, highlighting issues like democratic participation and citizenship. Following Freudenthal’s vision of school mathematics, Yackel and Cobb (1996) sought *classroom practices* that made all students’ participation in mathematical discussions viable, revealing the value of socialisation as a means of supporting students’ mathematical learning. Aligned with Biesta’s proposition, they brought attention to the fact that there always are social practices that students learn to enact in their classroom. The fact that students know who is expected to speak and when, or what are the acceptable ways to contribute to a conversation, are some of the indications of practices that have become normative in their classroom. Cobb and colleagues (e.g., 1997) documented that what those classroom social practices are matters both for the teacher’s success in shaping students’ mathematical learning and for the students themselves. By distinguishing between social and social-mathematical norms as constitutive of classroom practices, Cobb and colleagues brought attention to a *domain-specific layer* of socialisation that takes place in classrooms—the layer responsible for what students come to see as mathematics, and which activities, actions, and contributions they come to see as mathematical.

The classroom practices into which students are socialised are consequential to whether students identify with, comply with, or resist, the forms of participation that are expected of them (Cobb et al., 2009). In far too many mathematics classrooms, students are socialised into the traditions and practices of mathematics schooling, which are not only different from those of mathematics, but are incompatible with them. Students may come to learn that being correct and being fast in calculations are the most highly valued attributes. They may come to learn that mathematical correctness, just like correctness in spelling, is arbitrated by external sources—the textbook, the test answer key, or their teacher’s final say—and that to be good in mathematics one must rely on their memory (cf. Boaler, 2000). These student observations are then confirmed in how they are assessed.

Enculturation of this kind is *mathematically* problematic: Accomplished mathematicians do not recognise the classroom practices focused on speed and correctness as being central to mathematics (e.g., Boaler, 2016). Speed and correctness may emerge as side products of doing mathematics with curiosity and dedication for extended time, but chasing these qualities in their own right does not provide a viable path to becoming good at, or interested in, mathematics (Boaler, 2000). The reliance on external arbiters to determine the correctness of a mathematical argument is outright un-mathematical (cf. Bass, 2017). Making inferences and calculations without logical or arithmetic errors is certainly helpful to a mathematician. But the mathematical value of correctness is rooted in the possibility to determine whether something is correct or incorrect by appealing to the authority of mathematics itself, not to its external arbiters. The removal of the possibility (and indeed the necessity) to determine the correctness of one’s mathematical actions through one’s own intellect effectively removes mathematics from activities in mathematics classrooms, and renders those activities un-mathematical.

The images students form of what mathematics is and what it requires of them, as they are socialised into practices of mathematics in their classroom, shape whether they will be willing to take this kind of mathematics on. Only a few students can be the fastest in one classroom. When a competitive speed is a requirement for “doing well in school mathematics”, this systemically convinces large numbers of students that there is something about themselves as a person that prevents them from ever being good at mathematics, in other words, that they are

not “mathematical”. This kind of enculturation embeds what Rancière (1987/1991) refers to as a *stultification* of students: a process of bringing students to view themselves as stupid in the face of what is being required of them in school. In addition to bringing too many students to see themselves as not mathematical, mathematics schooling of this kind also does injustice to those who identify with it and succeed. Sooner or later, these students will be confronted with the mismatch between what they are good at and what is genuinely mathematical.

Lack of proactive attention to what students are being socialised into in schools and in mathematics classrooms has a significant (and far too familiar) moral as well as mathematical cost. On the positive side, the work of Cobb and colleagues, and similar research conducted within the design research tradition in mathematics education, shows how this cost can be avoided. These studies document productive social and social-mathematical norms, how these are proactively negotiated in classrooms, and how—once established—they support both students’ willingness to engage in mathematical activities (cf. Cobb et al., 2009) and their learning. We add with Biesta that socialisation into well-chosen classroom practices has an educational value in its own right, beyond being a means of supporting mathematical learning.

Subjectification

The third, and the most debated of Biesta’s (2022) domains of educational purpose, is *subjectification*. This centres on “bringing the subject-ness of the child ‘into play’” (p. 7) in their school experiences. Biesta evokes this subject-ness by highlighting the *difference* between activities that invite a student’s “arrival of the ‘I’ in the world as *subject* of its own life” and those which construe the student’s ‘I’ “as *object* of forces or desires from ‘elsewhere’” (p. vii, italics added). He refers only to the former type of activities as being *educational*.

For Biesta, “the very appearance of the ‘I’ in the world” (p. vii) is at stake in education, leading him to suggest that subjectification should be its primary focus. Biesta’s suggestion is based on the observation that if, during the years of schooling, students are treated as objects that comply with what others desire of them, then this gives them a wrong indication about how to be in the world and with the world, to the detriment of us all. What students do not get to experience when education objectivises them is the difficult freedom of making their own decisions regarding their acting in the world and experiencing the consequences of those actions as the world speaks back to them. Biesta (2022) suggests that organising education for students’ subject-ness is a major challenge for education and education research.

If we accept Biesta’s challenge, the educational value of mathematics would lie in its potential to help students recognise themselves, first, as subjects of their mathematical activity, rather than as objects of what someone else desires them to do, be, and know. Findings from design research studies that pursue Freudenthal’s vision (e.g., Cobb et al., 1997; Cortina et al., 2014) show that when mathematics is thoughtfully organised for teaching, students can make independent mathematical decisions, can compare, debate, defend, and change them, and in the process experience (mathematical) consequences of their decisions. In Biesta’s terms, students get to enact their subject-ness. We suggest that coming to experience themselves as subjects in relation to mathematics is one layer in an arrival of one’s ‘I’ in the world as a subject—rather than an object—of one’s life. In this way, mathematics in school can provide a path for calling students to attend to the world, to observe it, to make sense of it, and to recognize themselves as subjects who live, act, and remain in continuous relationship with it.

In the discourse of schooling, however, mathematics has been framed as content, techniques, skills, and more recently proficiencies *to be acquired* and *possessed*. Students are envisioned as the ones who comply and engage in these acquisitions. The acquisitions are justified to them almost exclusively in terms of the possible (yet elusive) qualification-related benefits to their distant future selves. Operating under this acquisitionist paradigm (Sfard & Cobb, 2014), school mathematics often asks students to refrain from exercising their subject-

ness, and to agree—even enthusiastically—to become objects of forces and desires from elsewhere for the sake of their and their society’s future. We know too well that when driven to succeed, students comply with such objectivisation processes by suppressing their selves, along with their curiosity and criticality, temporarily or permanently.

Biesta (2022) is deeply concerned about the dangers of equating education with its qualification purposes, primarily when students’ subject-ness is too readily sacrificed in the process of accomplishing their qualification. To make this point, he introduces the Parks-Eichmann paradox, a harrowing illustration of how something that “appears as educational success from one perspective, is actually quite problematic when viewed differently” (p. 26). Rosa Parks’s and Adolf Eichmann’s actions left significant marks on the 20th century. Rosa Parks, fully aware of the possible consequences in 1955 Alabama, refused to obey the bus driver’s order and did not give up her seat in a “coloured” section of the bus to a white passenger. It was through individual actions of Parks and others that a bigger change in the societal order was taking shape. Biesta contrasts Parks’s decision to act in a particular way with the actions of Adolf Eichmann, the high-ranking Nazi SS officer, who successfully completed considerably more schooling. Eichmann, when convicted on crimes against humanity during World War II, admitted arranging the mass deportation of Jews and others to extermination camps. At the same time, he denied responsibility for the consequences of his actions, stating that he was only following orders. Biesta uses this contrast to highlight that it was not the differences in qualification between Parks and Eichmann that mattered the most to the ways in which they acted in the world. It mattered more how they related to their ‘I’ as either a subject of their own life or as an object of (or a tool for enacting) someone else’s desires. Remaining an object allowed Eichmann to not face the difficult decisions in the world as being his own.

In the light of the stark illustration, Biesta maintains that the role of education is not realised if it produces people who are able and willing to follow orders with excellence but without willingness or ability to recognise that they indeed have the difficult freedom to make their own decisions about the orders they are asked to execute.

As Biesta clarifies, teaching to pursue students’ subject-ness is not an easy path for students or for teachers. Refusing to treat students as objects of their schooling may be a good start, but a suggestion of this kind does not clarify what specifically could or should be done in classrooms instead. What is the teacher to do to ‘allow’ the student’s ‘I’ to ‘arrive’ to the world, even if just to the world of mathematics in their classroom? Part of the difficulty lies in the magnitude and severity of the examples like Parks-Eichmann paradox, which illustrate the need to act educationally but do not give any indication of what could or should be done in a classroom on a day-to-day basis. Allowing for such arrival does not mean that the teacher can be passive, but it is also very clear that nothing here can be given to students to acquire.

As hinted earlier, the classroom design studies built from Freudenthal’s line of thought offer practical insights on how to design, and teach, for subject-ness. These researchers did not conceptualise, or pursue, students’ subject-ness—they pursued good mathematics teaching and learning. Is it then true that students’ subject-ness emerges in those classrooms, and if so, how?

Designing for Participation in Classroom Mathematical Practices

Hans Freudenthal is widely credited for presenting a view of what mathematics is, and what it should be about in classrooms, characterising mathematics as a particular type of human activity, in which one increasingly gets to take part (e.g., Freudenthal, 1973). Freudenthal suggested that to teach mathematics well, we must approach it—and organise it for teaching—with this view in mind. We would like to highlight that this approach assumes a perspective of learning which is different from that of an acquisition. Learning here is characterised by participation and measured by documenting changes in forms of participation (Cortina et al., 2014). To Freudenthal, mathematics is a practice, something in which humans engage and

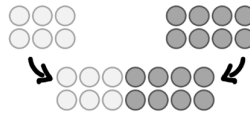
participate together. He clarified that mathematics entails—first and foremost—the act of *mathematising*. For our present purposes, let us define mathematising as an act of gradually generating more mathematics by noticing, exploring, and making clearer the patterns of, and structures within, how things are or how they behave. The ‘things’ here are both the ‘real’ things we encounter in the world, and the mathematical things that came to be of interest through prior mathematising. In this sense, research mathematicians mathematise to develop new insights and theories; and young children mathematise to organise patterns observed around them.

As an example of an activity of mathematising, consider adding different numbers. One may notice that a sum of even numbers brings an even result. But would this always be so? Is this an inevitability, or is this a chance occurrence only true for the numbers we have tried so far? How could we know for sure? Could we check with all the numbers? By seeking answers to these and related questions, the numbers that students already understood previously would come to be understood in a new way (see Figure 1): The classroom may first notice that an even number of cookies can be organised into a double row of cookies. Aligning two such double rows of cookies gives a longer double row, thus again an even number.

The informal patterns of adding even numbers, and the reasons for these patterns, are not just pieces of mathematical knowledge to add to one’s memory. In the classroom, these would become part of what numbers are and what can be done with them or explored about them. The activities may extend into exploring parity patterns in subtraction, and with odd numbers. The habit of concocting and pursuing questions about one’s mathematical activity is being established as itself being deeply mathematical: As we seek answers to our question, we also wonder what else is interesting to ask. In addition, the processes by which one can ascertain or refute that mathematical relations hold are becoming part of students’ mathematical repertoires.

Figure 1

An argument to show that adding two even numbers must yield an even number. Specific amounts illustrate a generalised geometric principle, which can later be formalised using algebraic notation.



If students are to come to associate mathematics with their activity of mathematising, and thus socialise into mathematics in a broad sense, the activity of mathematising must be present in their classrooms. Instead of a ready-made body of knowledge that must—as Biesta says—be brought “into someone” (the student; Biesta, 2017, p. 75), Freudenthal viewed the mathematical ideas and processes as having to be created and re-created anew by each group that takes part in this type of human activity. In this view, students are brought to mathematics (cf. Biesta, 2017) and taught to experience it, and face it, so to speak, on their terms.

We know from Dewey (1916/2018) that children do not spontaneously explore the ideas that, with hindsight, would be of educational value to them (e.g., parity). How, then, can a teacher bring students to mathematics without objectifying them in the process? The design tools that oriented the classroom studies, specifically the instructional design theory of Realistic Mathematics Education (RME; Freudenthal, 1973; Treffers, 1987), offer an insight. The three tenets of RME envision a mathematics education in which students are to encounter and face mathematics. They also provide heuristics for planning such mathematics education.

To Freudenthal (1971), mathematics is primarily “an activity of organizing a subject matter” (p. 414), and to do such organising, a student needs to be placed in situations where organising would occur to them as a useful thing to do. The first of the three central tenets of RME design theory deals with making students’ genuine engagement with mathematics possible in the

classroom. It suggests that for students to have a chance of (re)creating a mathematical idea, they need to encounter this idea in activities in which they could and would want to be active, so that they can notice, interrogate, and eventually establish the mathematical idea or insight. Care is taken in design and enactment of activities to ensure that students develop a reason, and have a mathematical way, to engage in what is brought to their attention. This demand placed on RME-based design and teaching is rarely afforded elsewhere, but it functions to maintain the freedom for each student to engage (or not). It is a responsibility first of design, and then of teaching, not to exclude anyone for lacking a reason or mathematics needed to engage.

The second tenet of RME reminds that the mathematics in the classroom must be capable of gradually evolving into the mathematics of mathematicians, the disciplinary mathematics. It states that the starting point of students' activity must be justifiable in terms of the potential end goal of the mathematical sequence. The starting point needs to allow for mathematics to grow towards this end goal *in a manner that upholds the internal coherence of mathematics*, while gradually transitioning from the initial informal ways of speaking, symbolising, and reasoning towards the sophisticated and increasingly standardised ways (Treffers, 1987). The activities must target those ways of participation that are organically expandable over time into the advanced mathematical practices of mathematicians. At the very least, the earlier ways of engaging in the classroom (including meanings and definitions of the elements of mathematical work) should not create obstacles to how students will need to engage with the discipline as they progress. This functions to maximise the freedom for students to engage over the long run.

The third tenet of RME ensures that the evolution of students' classroom activities towards greater mathematical sophistication *always proceeds by mathematical means*. This demands to sequence mathematical ideas for teaching in a way that would allow students to conjecture, glean, approach the next idea via the mathematical tools that they already have at their disposal. This ensures that the students—like mathematicians—experience how they can progressively mathematise the phenomena and situations at hand, rather than merely observe how the teacher or somebody else does so.

In Conclusion

Thinking with Biesta helps us to name the key strength of RME instructional design theory: RME is not a guide on how to bring mathematics to students. Instead, it is best viewed as a theory that engages with how to bring students to mathematics. The RME tenets overtly guard against objectifying students who engage in the designed activities. The need for students to experience their own mathematising shows that the very arrival of their 'I' in the world of mathematics as a *subject* of their own mathematical activity is inherent to RME. Guiding the classroom activities to be recognisably mathematical (generative and reflective), the activities are well suited to socialise the students into what *mathematical* means and feels like. Using RME in instructional design means designing for all three domains of educational purpose.

Classroom design studies in mathematics education (e.g., Cobb et al., 1997) tend to position socialisation- and subjectification-related parts of the design as the *means of supporting* the specific forms of mathematical reasoning in the classroom (i.e., the qualification purposes). While these studies show that profound classroom mathematical learning is possible, the means that supported this learning have often been critiqued as being unrealistic under current school conditions. In highlighting socialisation and subjectification as essential legitimate purposes of education, Biesta removes the possibility to treat their pursuit as “unrealistic” and encourages us to turn the question around. How can we organise the qualification requirements in mathematics education so that adequate mathematical socialisation and subjectification will become part of teaching and learning in mainstream mathematics classrooms?

We discuss elsewhere (Višňovská & Cortina, 2025) that mathematics content is often organised for teaching so that students experience mathematics as lacking coherence.

Disruptions of mathematical coherence mean that new mathematical ideas at times cannot be developed from previous ideas present in the classroom and must instead be given to the students. We now view curricular sequencing of this kind as omitting the reasoning resources essential to teaching for students' subject-ness. While such curricula may appear adequate when judged in terms of the mathematics content they bring to students, they do not adequately support teachers in the difficult task of bringing students to that mathematics. This limitation is significant as it cannot be overcome by good teaching without changing the curricular sequence.

The mathematics education research community has long been driven by the goal of making students' experiences of school mathematics more meaningful and more mathematical. Our discussion suggests that this goal could be advanced by examining the organisation of mathematics curricula and activities for their capacity to be, in Biesta's terms, educational.

References

- Bass, H. (2017). Designing opportunities to learn mathematics theory-building practices. *Educational Studies in Mathematics*, 95, 229–244. <https://doi.org/10.1007/s10649-016-9747-y>
- Biesta, G. J. J. (2015). Freeing teaching from learning: Opening up existential possibilities in educational relationships. *Studies in Philosophy and Education*, 34, 229–243.
- Biesta, G. J. J. (2017). *Letting Art Teach: Art Education 'After' Joseph Beuys*. ArtEZ Press.
- Biesta, G. J. J. (2022). *World-Centred Education*. Routledge. <https://doi.org/10.4324/9781003098331>
- Boaler, J. (2000). Mathematics from another world: traditional communities and the alienation of learners. *The Journal of Mathematical Behavior*, 18(4), 379–397.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. Jossey-Bass.
- Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first-grade classroom. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 151–232). Lawrence Erlbaum. <https://doi.org/10.4324/9781003064121>
- Cobb, P., Gresalfi, M. S., & Hodge, L. L. (2009). A design research perspective on the identities that students are developing in mathematics classrooms. In B. Schwarz, T. Dreyfus, & R. Hershkowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 223–243). Routledge.
- Cobb, P., Zhao, Q., & Visnovska, J. (2008). Learning from and adapting the theory of realistic mathematics education. *Education et Didactique*, 2(1), 55–73. <https://doi.org/10.4000/educationdidactique.276>
- Cortina, J. L., Visnovska, J., & Zúñiga, C. (2014). Unit fractions in the context of proportionality: Supporting students' reasoning about the inverse order relationship. *Mathematics Education Research Journal*, 26, 79–99.
- Dewey, J. (1916/2018). *Democracy and education: An introduction to the philosophy of education*. Myers.
- Freudenthal, H. (1971). Geometry between the devil and the deep sea. *Educational Studies in Mathematics*, 3, 413–435.
- Freudenthal, H. (1973). *Mathematics as an Educational Task*. Reidel.
- Meirieu, P. (2007). *Pédagogie: Le devoir de résister* [Education: The duty to resist]. ESF Éditeur.
- Rancière, J. (1987/1991). *The Ignorant Schoolmaster*. Stanford University.
- Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, 27(2), 4–13.
- Sfard, A., & Cobb, P. (2014). Research in mathematics education what can it teach us about human learning. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (2nd ed.).
- Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction – the Wiskobas project*. Reidel. <https://doi.org/10.1007/978-94-009-3707-9>
- Višňovská, J., & Cortina, J. L. (2025). Curriculum and mathematical coherence: Exploring the tensions in teaching and learning number. *ZDM Mathematics Education*.
- Visnovska, J., Graven, M., Cortina, J. L., & Vale, P. (2025). Decolonising content narratives in mathematics and science education: The case of reinventing length measurement. *African Journal of Research in Mathematics, Science and Technology Education*, 29(1), 28–41.
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27, 458–477.

Investigating Primary Students' Images of a Whole in Fractions

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This paper aims to investigate what images of a whole that Grade 4 students have when solving problems involving unit, proper, and improper fractions. Data were collected through fraction tasks, post-test on fraction schemes, and interviews. Coding of the data reveals that the students had visual and symbolic images of the whole. However, they have not had a flexible and connected image of the whole across the fractions.

A *whole* is one of the essential concepts for fraction understanding. Students' ability to determine a whole in, for example, $\frac{2}{5}$ as two parts *out of* or *within* five equal parts of a chocolate bar, gives meaning to the fraction as a part-whole relation (Hackenberg, 2013). Yoshida and Sawano (2002) documented that the concept has a significant impact on students' understanding of fraction equivalence and magnitude. For example, when comparing $\frac{2}{7}$ and $\frac{5}{7}$, students who are aware of the whole could likely answer that $\frac{5}{7}$ is greater than $\frac{2}{7}$. Moreover, the whole is essential to the understanding of fraction arithmetic, such as multiplication (Behr et al., 1992). A problem like $\frac{1}{3} \times \frac{1}{4}$ could be perceived as finding one-third of one-fourth *of a whole* chocolate bar. Another critical role of understanding the whole is when encountering improper fractions. In a proper fraction, such as $\frac{2}{5}$, students might observe that the whole is the greater number in the fraction. This observation is misleading when working with improper fractions, such as $\frac{7}{5}$, in which the whole is not the greater number.

Despite its critical role, determining a whole in fraction-related problems is challenging even for teachers (Bednarz & Proulx, 2014). This challenge is magnified by the fact that the concept is not explicitly taught in the mathematics classroom (Yoshida & Sawano, 2002). Furthermore, there is a lack of research on what students perceive as a whole in fractions, especially students' images of a whole in different fractions, for example, from unit/proper fractions to improper fractions. Such research is pivotal to improving fraction teaching and learning. Therefore, this paper aims to investigate the images of a whole that Grade 4 students have when solving problems involving different fractions. The findings of this paper are expected to extend our insights into students' understanding of the concept and, more importantly, to contribute to improving fraction instruction.

Images of a Whole in Fractions

This paper is part of a classroom intervention study that aimed to support Grade 4 students in Indonesia in learning fractions through a spatial intervention. The study design was underpinned by spatial reasoning, growth of mathematical understanding (Pirie & Kieren, 1994), and fraction schemes (Steffe & Olive, 2010). This paper focuses on the images students have about a whole in fraction-scheme-related problems. This section reviews a whole in problems involving different fraction schemes and *images having* (Pirie & Kieren, 1994), one of the stages in the growth of mathematical understanding. The conceptualisation of the whole and the images having underpin this paper.

Scheme theory is one of the theoretical lenses for interpreting students' reasoning about fractions. In this lens, a fraction scheme is a model of students' recurring ways of operating in situations involving fractions (Steffe & Olive, 2010). The scheme consists of three parts: an experiential situation, an activity associated with the situation, and the result of the activity (von Glasersfeld, 1980, cited in Steffe & Olive, 2010, p. 20). For example, when asked to represent $\frac{2}{5}$ in a chocolate bar (a *situation* relating to the *part-whole scheme* involving unit or proper (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 425–432). MERGA.

fraction), a student may partition (*the activity*, also called an *operation*, Hackenberg, 2013) the bar into five equal parts and then shade (*the activity*) two parts of the whole partitioned parts. The term *operation* is used in this paper. The *result* of the activity is the student's constructed representation of $2/5$ or two shaded parts within the chocolate bar. The intervention in the present study provided Grade 4 students with spatial-thinking situations, such as paper folding, that provoked the construction of fraction operations, such as partition.

To come to the result, the student might have identified the whole, which is the five parts, by partitioning the chocolate bar. Determining the whole in other fraction schemes: *partitive* (unit fraction, proper fraction), *reversible partitive* (proper fraction), and *iterative* (improper fraction), requires not only partition but more complex operations, such as iterating and splitting (Hackenberg, 2007; Norton & Wilkins, 2009). Table 1 summarises how students operate to determine the whole. For example, splitting is required to determine the whole in problems involving reversible partitive and iterative fraction schemes. It is the way students partition and iterate simultaneously. An example of a splitting problem is "This is your friend's choco bar, which is three times as long as yours. Draw your choco bar." The problem uses an iterative language, but students need to partition the "your friend's choco" into three equal parts, while, at the same time, iterate the one part to fit the whole choco. Consider Test 10 (Reversible partitive scheme, Table 1), the ability to split is foundational to solving the test; the students partition the given rope into two equal parts ($2/6$) and, at the same time, iterate one part ($1/6$) to make the whole six parts ($6/6$).

Table 1

Operations Involved in Determining a Whole in Different Fraction Schemes

Schemes	Sample problems	Ways of operating
Part-whole	<i>Task 4B</i> : The figure* shown below is a whole block. Draw $1/5$ and $3/5$ of the whole block.	<i>Partition</i> the given figure (the whole block) into five equal parts ($5/5$).
Partitive	<i>Task 4C</i> : The figure* shown below is $1/3$ of a whole block. Draw the whole block.	Copy (<i>iterate</i>) the given figure ($1/3$) three times or <i>add</i> two similar figures to the right to make the whole block (three blocks, $3/3$).
Reversible partitive	<i>Task 4D</i> : The figure* shown below is $2/6$ of a whole block. Draw $6/6$ of the block. <i>Test 10</i> : The rope** shown below is $2/6$ as long as the whole rope. Draw the whole rope	<i>Splitting</i> the given figure/rope ($2/6$, two parts) to make $1/6$ (one part) and the whole ($6/6$, six parts).
Iterative	<i>Task 4E</i> : The figure shown below is $9/7$ of a whole block. Draw the whole block. <i>Test 12</i> : The given rope below is $7/4$ as long as the whole rope. Draw the whole rope.	<i>Splitting</i> the given figure ($9/7$, nine parts) to make $1/7$ (one part) and the whole ($7/7$, seven parts). <i>Splitting</i> the given figure ($7/4$, seven parts) to make $1/4$ (one part) and the whole ($4/4$, four parts).

Notes: *The figure given in each Task is an unpartitioned block/fraction bar. **The rope given in the Test items is unpartitioned

Students with a part-whole fraction scheme might be able to determine the whole in partitive scheme problems by treating them as part-whole relations. For example, a student is likely to consider Task 4C (Table 1) as making $1/3$ within the given figure; therefore, partition it into three equal parts and then determine the whole as three equal parts, or $3/3$. Norton and Wilkins (2010) point out this case, where students could use their part-whole fraction scheme to solve problems involving the partitive fraction scheme. Regarding this, it is likely that the reversible partitive problems (Task 4D, Test 10 in Table 1) will be treated as part-whole relations: partitioning the given figure/rope into six equal parts and selecting the six parts ($6/6$) as the whole.

The student's ability to determine the whole in various fraction-scheme problems, for example, three equal parts (Task 4C), might indicate that the student has an image of the fraction ($1/3$) and the whole within the fraction. Pirie and Kieren (1994) call this *image having* or having a mental model of $1/3$, represented by the given figure, and the whole, which is three copies of the figure. The focus of this paper is on the images of a whole that Grade 4 students have when solving Tasks 4B-4E, Tests 10 and 12.

Martin and Pirie (2003) defined images as "any ideas the learner may have about the topic, any 'mental' representations, not just visuals or pictorial ones (p.173)." In this paper, images are perceived as students' mental representations of the whole in the problems involving different fraction schemes. The whole could be visual or pictorial in the form of partitioned parts of the fraction representation or symbolic fractions (n/n) representing the whole. The mental representations of the whole are observable in students' drawings when answering the problems and in their verbal explanations during the interviews. Based on the model of growth of mathematical understanding (Pirie & Kieren, 1994), the images students have in learning are the results of relevant *image making* activities, such as spatial activities in the present study. The images developed from the activities could be incomplete or incorrect (Pirie & Kieren, 1994), which may indicate the students' understanding. Presmeg (1986) identified different visual images that high school students had in solving math problems, for example, a student had a concrete image to represent trigonometric ratios. For primary students, a whole in fractions may be perceived differently, potentially leading to incomplete images as a result of their learning experiences, including the spatial intervention in the present study. For example, the students might think of the whole as 'always' the greater number in fractions because it is the longer parts if drawn in fraction bars. This image is incorrect because it does not apply to improper fractions. This incorrect image could shed light on (further) improvement of the fraction instruction.

The ability to determine a whole is a fundamental part of fraction understanding. Such an ability depends on students' available operations, such as partition, iteration, or splitting, required to solve fraction scheme problems. The students might see the whole in the fraction problems differently; it could be right or incomplete. Therefore, this paper seeks to answer this question: *What images of a whole did Grade 4 students have when solving problems involving different fraction schemes?*

Methods

Research Context and Participants

The spatial intervention in the present study was delivered to 31 Grade 4 students over 11 lessons by a classroom teacher. The lessons provided students with spatial activities promoting fraction operations, such as partitioning through paper folding (lessons 1-5) and a number of fraction tasks (lessons 6-11). A fraction scheme test (Norton & Wilkins, 2009) was administered before and after the intervention. To solve the test, students need to have developed fraction operations, such as partition, allowing the present study to examine the impact of the intervention. This paper focuses on four tasks and two test items that asked the students to determine or draw a whole (Table 1). Six students (2 boys, 4 girls) with different levels of fraction schemes based on pre-test results were purposively selected. The students were interviewed after lessons containing the fraction tasks and after the pre- and post-test to further understand the impact of the intervention. The whole was not explicitly included in the spatial activities. It was only discussed in the lessons, where the tasks about the whole were given.

Data Collection and Analysis

Data sources for this paper were (1) the six students' answers to the four tasks (Table 1, Tasks 4B-4E, a total of 24 answers) and two post-test items (Table 1, Test 10, Test 12, 12 answers), and (2) their interviews on the tasks and test items (36 transcripts). Each student was interviewed individually, using a semi-structured approach, after the lesson containing the four tasks and the post-test. In the present study, the students' interviews chiefly aimed to investigate their growth of fraction understanding across the lessons and after the intervention, including the promoted fraction operations. For this, an interview protocol was developed following fraction schemes (Steffe & Olive, 2010) and growth of mathematical understanding (Pirie & Kieren, 1994) and used for the interviews. For example, in Task 4B, students were asked, "How did you divide the whole block?" Other sample questions are "Which one is the whole rope?" and "Why is it the whole rope?" for Test 10. These questions could show their images of the whole.

The term 'block' in the tasks and interview transcripts refers to the housing blocks, which served as the learning context. The blocks represent fraction bars. The ways in which the students determine or draw the whole are summarised in Table 1. Task 4C is the reciprocal of Task 4B: given $\frac{1}{3}$ of a block, a whole block was to be drawn. Conversely, in Task 4B, the students need to break the whole to make fractions, for example, $\frac{3}{5}$. What makes Task 4B intriguing is that the students have to partition the unpartitioned block to make the whole, based on the fractions they are asked to draw. All the tasks reported here not only asked the students to draw the whole or the fraction representing the whole, but also other fractions, such as $\frac{4}{6}$ (Task 4D), $\frac{2}{3}$ (Task 4C), or $\frac{3}{7}$ (Task 4E). The other fractions were excluded to focus on students' answers regarding the whole in the fractions.

In the context of the present study, the students' answers to the tasks/tests and interviews were coded referring to a combination of inductive and deductive approaches. The six students' test answers were coded inductively across two coding cycles: process and pattern coding (Saldaña, 2013). For example, the student's answer to Test 12 (Figure 1B) shows that she could partition the given rope into seven equal parts and make an additional four-part drawing representing her answer. The answer was coded as *drawing on the given representation with appropriate equal partition and additional drawing*. The student's interview transcript on the test item was coded deductively based on the conceptualisation of fraction schemes and growth of mathematical understanding. To capture the images the student has, the student's answer and interview on the item were combined. For instance, although her drawing seemed correct (Figure 1B), she identified the whole as 4 parts ($\frac{4}{4}$), but in the interview one part was identified as $\frac{1}{7}$ instead of $\frac{1}{4}$. This was coded as *no indication of three-level unit coordination and visual and symbolic images of the whole*.

Figure 1

Students' Answers



Findings

This part reports the emerging themes in students' images of a whole across different fraction scheme problems, along with supporting evidence. The students' names (Anna, Andi, Hana, Lili, Rian, Zane) reported here are pseudonymous. The main findings are two-fold. First, the six students have visual and symbolic images of the whole. Second, the six students have not had a *flexible and connected image* of the whole. Each finding is elaborated as follows.

Visual and Symbolic Images

The six students seemed to have visual and symbolic images of the whole within the fraction problems. Not only did they have visual representations of the whole, for example, the five partitioned parts within the representation in Task 4B, but they also translated the parts to a symbolic fraction, $5/5$ (T1). Although the students could relate the whole parts within the representations to symbolic fractions, the images they had were not necessarily complete. This will be explained in the following section.

T1 (Transcript 1) R: Researcher Ai: Andi

- 1 R : what block is this? [Task 4B]
- 2 Ai: the whole block
- 3 R : how did you divide this? [pointing to the block]
- 4 Ai: I divided it into five parts [...]
- 5 R : why did you make five?
- 6 Ai: to make $1/5$
- 7 R : what is the whole for $1/5$?
- 8 Ai: $5/5$

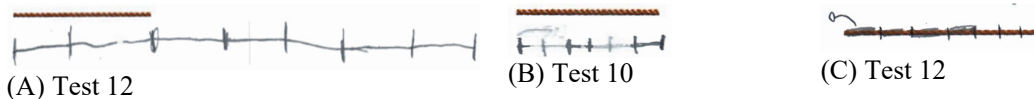
Static, Inconsistent, and Flexible but Unconnected Images

In general, the students have not had flexible, connected images of the whole in the fraction problems. It is flexible if a student can see the change of the whole from unit/proper fractions (partitive, reversible partitive scheme) to improper fractions (iterative scheme); the (whole) partitioned parts or the greater number are only applicable to unit/proper fractions. A connected image of the whole is when the student can coordinate multiple units in improper fractions, for example, $1/4$, $4/4$, and $7/4$. This allows the student to *ignore* the (whole) visible partitioned parts or the greater number in $7/4$. This finding is characterised by three emerging categories of images that the students had, as follows.

First, *static images* of the whole. Two students were in this category. For Anna, the whole was *always longer*. For the tasks involving unit and proper fractions (Tasks 4B-4D), she referred the whole to the denominator. When asked how she solved Test 10 (Figure 1A), Anna said, “*it was asked to draw a whole rope this way (her hand goes beyond the given rope following her answer).*” This indicates that the whole for her is longer than the given rope. Similarly, she perceived the longer parts (the numerator) in improper fractions (T2, Test 12, Figure 2A) as the whole. Andi also had a static image of the whole, which was the (whole) visible parts within the representation he partitioned (Figure 2B, T3). Unlike Anna, he saw the whole as the greater number in all the fractions, which were drawn into all the partitioned parts.

Figure 2

Students' Answers



T2 R: Researcher A: Anna

- 1 R: what about the whole? [Task 4E, she divided the block into nine parts]
- 2 A: nine [nine parts in the block]
- 5 R: why nine?
- 6 A: because it is $9/7$
- [...]
- 7 R: how did you solve it? [Test 12, improper fraction $7/4$]
- 8 A: It is asked to draw a whole rope so I drew it to make seven and then I divided it

T3 R: Researcher Ai: Andi

- 1 R: how did you solve it? [Test 10]
- 2 Ai: I divided it into six parts
- 3 R: why did you divide it into six parts?
- 4 Ai: to make $\frac{2}{6}$ to make it the whole rope [...]
- 5 R: what is the whole rope?
- 6 Ai: $\frac{6}{6}$
- [...]
- 7 R: what is the whole for $\frac{9}{7}$? [Task 4E]
- 8 Ai: $\frac{9}{9}$ [...]
- 9 R: this block is $\frac{9}{7}$ how did you divide it?
- 10 Ai: nine parts

Second, *inconsistent images* of the whole. Rian and Lili were included in this category. In Tasks 4B-4D, both students had the denominator as the whole (T4 lines 1-6). Rian was like Anna, who thought the whole was always longer in representations. This made him draw a longer rope (seven parts), extending the given rope in Test 12 as he had done in Test 10. However, he considered the whole to be the four parts ($\frac{4}{4}$) in the extended seven-part rope, representing the $\frac{7}{4}$. In Task 4E, Lili had her nine partitioned parts as the whole (the numerator), but she answered four parts as the whole for Test 12 (T4 lines 7-12).

T4 R: Researcher Ri: Rian L: Lili

- 1 R: what block is this? [pointing to the given block, Task 4B]
- 2 Ri: a whole
- 3 R: what fraction is this whole block?
- 4 Ri: $\frac{5}{5}$
- 5 R: why? [...]
- 6 Ri: I can see from this [pointing to the denominator of the fractions to draw]
- [...]
- 7 R: how did you solve it? [Test 12]
- 8 L: I divided this [the given rope] into seven parts
- 9 R: why?
- 10 L: because it is $\frac{7}{4}$ and the whole is four [...]
- 11 R: what fraction is the whole?
- 12 L: $\frac{4}{4}$

Third, *flexible but unconnected images* of the whole. In this category, Hana and Zane flexibly perceived the whole as it was expected for the different fractions. The whole was the longer partitioned parts (the denominator) in the unit and proper fractions (Tasks 4B-4D). In the case of $\frac{9}{7}$ (Task 4E), they considered seven parts ($\frac{7}{7}$) as the whole in the interviews, since they initially made seven parts for $\frac{9}{7}$. However, they changed their answers when I had them refer to the problem statement: the figure shows $\frac{9}{7}$ of a block. Hanna responded, “*ooh it is divided into nine and shade seven.*” Despite this intended answer, they did not appear to have a solid image of $\frac{9}{7}$ in the representation (e.g., Hana’s response, “*ooh ...*”). Of the six students, only Hana and Zane seemed to have shifted their images of the whole when encountering improper fractions, $\frac{9}{7}$ and $\frac{7}{4}$. Moreover, they could argue why the whole was four parts in $\frac{7}{4}$ (T5 lines 1-12). Thus, their images of the whole were likely flexible. Lili also had the same image of the whole in Test 12, $\frac{4}{4}$ or four parts. She was excluded from this category because her image of the whole was unchanged from the unit/proper fractions, which was still reliant on the nine partitioned parts she made for $\frac{9}{7}$. Although the two students flexibly perceived the whole across fractions, their images were *unconnected*. When asked what fraction represents

one part within their seven-partitioned rope in Task 12, the answer was $1/7$, instead of $1/4$ (T5 lines 13-16). They might not have developed the ability to coordinate $1/4$, $4/4$, and $7/4$, which is required for working with improper fractions.

T5 Z: Zane H: Hana

- 1 R: why did you make this into four [four parts under the given rope, Figure 1B]?
- 2 Z: because it is asked for $7/4$
- 3 R: it is asked for the whole rope is this the whole rope? [pointing to her drawn answer]
- 4 Z: yes
- 5 R: what fraction is this whole rope?
- 6 Z: $4/4$
- 7 R: why?
- 8 Z: because it is the whole rope
- [...]
- 9 R: why did you mark four? [the four parts, Figure 2C]
- 10 H: because of this [pointing to the denominator in $7/4$ given in the test statement]
- 11 R: what is the whole rope?
- 12 H: four
- 13 R: what fraction is this? [pointing to the most left part of the rope in Figure 2C]
- 14 H: $1/7$
- 15 R: why $1/7$?
- 16 H: because taking one from seven parts

Discussion and Implications

The aim of this paper is to investigate Grade 4 students' images of a whole in different fractions scheme problems: unit and proper fractions (part-whole scheme, partitive scheme), and improper fractions (iterative scheme) after receiving a spatial intervention that promoted the construction of fraction operations, such as partition or iteration.

The data analysis reveals that students had visual and symbolic images of the whole across the different fraction problems. Translating the whole partitioned parts or fraction visual representation to symbolic fractions (n/n) in the fraction problems, for example, $5/5$ in Task 4B (Transcript 1) requires the ability to perform partition and iteration. The students were only presented with an unpartitioned fraction bar and symbolic fractions to be drawn. They partitioned the bar into five equal parts (the whole) and likely iterated $1/5$ to make $5/5$ representing the whole. Prior publication of the results of the present intervention study documented that the six students were able to construct the symbolic fraction n/n from partitioned fraction bars (Wahyu, 2025). This finding indicates the potential of the spatial intervention delivered to students, in which relevant fraction operations, such as partition and iteration, were promoted through spatial activities.

Despite being able to relate the visual to symbolic images of the whole, most students have not developed a complete image of the whole throughout the different fractions. In problems involving unit and proper fractions, all six students perceived the denominator, the greater number in the fractions, or the whole visible parts within a partitioned representation as the whole. For example, three partitioned parts ($3/3$) are the whole for $1/3$ (Task 4C). When encountering improper fractions, the whole for two students remained the same: the greater number or numerator, for example, nine partitioned parts or $9/9$ for $9/7$ (Task 4E). The other two students showed inconsistency: the whole for $9/7$ was 9 partitioned parts, but for $7/4$ it was four partitioned parts ($4/4$). The remaining two students were able to see the correct whole in improper fractions but were unable to determine that one of the seven partitioned parts representing $7/4$ is $1/4$, not $1/7$. This finding confirms what Pirie and Kieren (1994) point out:

the images students have (*image having*) after *image making* activities, such as the intervention in the present study, may be incomplete.

One possible reason for the incomplete image is that fractions, in general, are demanding, especially improper fractions. Research on fraction schemes (Steffe & Olive, 2010) shows that different operations underpin fraction understanding (Table 1). For instance, students need to develop splitting (making partition and iteration simultaneously) and a three-level unit coordination (coordinating $1/4$, $4/4$, and $7/4$) to work with improper fractions (Hackenberg, 2007). To draw the whole for $7/4$ (Test 12), students need to simultaneously partition $7/4$ into $1/4$ (one part) and iterate $1/4$ to make $4/4$ (four parts) and $7/4$ (seven parts). In addition, they need to coordinate $1/4$, $4/4$, and $7/4$ or be aware that the seven parts are not $7/7$. The students may not have developed the required operations, which are very complex, including the two students who identified one of the seven parts as $1/7$. The absence of such complex operations may cause some students to rely on their part-whole understanding of fractions when determining the whole in improper fractions (Norton & Wilkins, 2010), for example, perceiving the whole as the greater number or numerator. The findings of this study imply that the underpinning fraction operations, such as partition, iteration, or unit coordination, need to be explicitly targeted in fraction interventions.

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References

- Bednarz, N., & Proulx, J. (2014). The (relativity of the) whole as a fundamental dimension in the conceptualization of the fraction. *Fields Mathematics Education Journal*, 2, 27-54.
- Behr, M. J., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio, proportion. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296-333). Macmillan.
- Hackenberg, A. (2007). Units coordination and the construction of improper fractions: A revision of the splitting hypothesis. *The Journal of Mathematical Behavior*, 26(1), 27-47.
- Hackenberg, A. (2013). The fractional knowledge and algebraic reasoning of students with the first multiplicative concept. *Journal of Mathematical Behavior*, 32(3), 538-563.
- Martin, L., & Pirie, S. (2003). Making images and noticing properties: The role of graphing software in mathematical generalisation. *Mathematics Education Research Journal*, 15(2), 171-186.
- Norton, A., & Wilkins, J. L. M. (2009). A quantitative analysis of children's splitting operations and fraction schemes. *Journal of Mathematical Behavior*, 28(2-3), 150-161. <https://doi.org/10.1016/j.jmathb.2009.06.002>
- Norton, A., & Wilkins, J. L. M. (2010). Students' partitive reasoning. *Journal of Mathematical Behavior*, 29, 181-194. <https://doi.org/10.1016/j.jmathb.2010.10.001>
- Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, 26, 165-190.
- Presmeg, N. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics*, 6(3), 42-46.
- Saldaña, J. (2013). *The coding manual for qualitative researchers* (2 ed.). SAGE.
- Steffe, L. P., & Olive, J. (2010). *Children's fractional knowledge*. Springer.
- Wahyu, K. (2025). Gesturing partition and fraction understanding: A case of primary students solving partition tasks. In S. M. Patahuddin, L. Gaunt, D. Harris & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 453-460). Canberra: MERGA.
- Yoshida, H., & Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: Equal-partitioning and equal-whole. *Japanese Psychological Research*, 44(4), 183-195.

Exploring Price Conceptions as Part of Financial Numeracy

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This study investigates how third- and fourth-grade students conceive of prices and what challenges and potentials these conceptions present for the development of financial numeracy. Drawing on pre-intervention interviews from a Design-Based Research study aimed at fostering financial numeracy, students' explanations of price formation were analysed using qualitative content analysis. The findings show that many students initially interpret prices as objectively measurable quantities. At the same time, students were able to draw on diverse funds of knowledge when reasoning about pricing.

Introduction and Problem Statement

Financial competencies are widely recognised as essential in the 21st century, as they enable informed decision-making and resilience in an increasingly complex, and dynamic, financial landscape. Furthermore, research has demonstrated the central role of mathematics in financial decision-making, with large-scale studies showing strong positive correlations between mathematical and financial competencies (OECD, 2023). Despite this close connection, financial numeracy in school mathematics is often reduced to a narrow, basic-skills perspective rather than being viewed more broadly as a social practice. In primary classrooms, financial contexts frequently serve merely as artificial contexts for practising arithmetic procedures, rather than as authentic contexts that support students in making informed financial decisions (Savard et al., 2020). Such approaches risk obscuring the complexity of financial decisions, and limit students' opportunities to engage critically with financial situations. Broad definitions of financial numeracy (as suggested by Blue et al., 2018; and Savard & Cavalcante, 2021) emphasise that it should also support students in understanding financial concepts, such as prices, through mathematical investigation and in critically examining systemic implications, including the social and ethical consequences of financial decisions, such as pricing practices. This study explores how German primary school students conceive of prices and examines the challenges and potentials these conceptions pose for the development of financial numeracy.

Theoretical Background

Definition of Numeracy and Financial Numeracy

Numeracy definitions can be understood along a continuum, ranging from a narrow view of numeracy as a set of technical skills to a broader view of numeracy as a social practice. On the technical side, numeracy is viewed either procedurally, emphasising basic mathematical skills, or functionally, stressing its importance for personal well-being and economic productivity. On the social-practice side, numeracy is contextualised, focusing on its relevance across home, work, and civic settings. It also takes a critical perspective, emphasising how numeracy can help individuals interpret data responsibly and challenge social injustice (Goos & O'Sullivan, 2022). The "Numeracy in the 21st Century" model by Goos et al. (2020) reflects this broader view of numeracy and the dimensions it encompasses. It is not limited to mathematical knowledge but also includes dispositions, tools, and the sense-making of extra-mathematical contexts. These dimensions are embedded within a critical orientation, which involves using mathematics to make decisions, build arguments, and evaluate positions. A critical orientation also includes understanding how mathematics is used or misused in social contexts, enabling critical participation in public discourse (Goos et al., 2020).

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 433–440). MERGA.

Financial numeracy is a type of numeracy in which mathematics is applied in financial contexts, involving specific domain demands (Gal et al., 2020). In a broader sense, and in line with numeracy as a social practice, financial numeracy is not limited to developing mathematical knowledge in financial contexts. It also involves understanding financial concepts through mathematical investigation and examining their connections to the economic system and other systems (e.g., politics, ethics), thereby encouraging a critical orientation regarding the social and ethical implications of financial decisions (Blue et al., 2018; Savard & Cavalcante, 2021). While often used interchangeably with financial literacy, the term financial numeracy foregrounds the role of mathematics in financial competence.

Money as a Special System of Measurement

Savard et al. (2020) argue that, in primary school, money should be addressed explicitly as its own system of measurement with the aim of deepening students' understanding of what money represents and how the economic value of products and services is defined and can be questioned. This perspective is particularly relevant to a broader view of financial numeracy, which involves examining financial concepts such as prices through mathematical investigation and critically examining existing financial practices, such as pricing decisions. Regarding prices and pricing, students need to understand that determining the economic value of a good or service differs fundamentally from measuring physical quantities such as length or weight. For instance, deciding how much to charge, or pay, for a loaf of bread requires considering multiple variables. For example, the quantity and type of bread, and the place of sale, all influence its economic value. Moreover, how these variables are weighted is not fixed but is negotiated and agreed upon by the parties involved in the economic transaction (Savard et al., 2020). Winter (1992) formulated similar ideas in mathematics education by distinguishing between different types of modelling. The measurement of physical quantities, such as length, relies on descriptive models. In contrast, everyday activities between people - particularly financial transactions - are based on normative models. Both Savard et al. (2020) and Winter (1992) therefore argue that mathematics lessons should help students understand that setting prices is fundamentally different from measuring physical quantities such as length or weight.

Investigating Children's Funds of Knowledge about Financial Concepts

In Australasian mathematics education research, a Funds of Knowledge approach has been used to examine how children's experiences with money at home and in their communities shape their engagement with financial learning and how they mathematise financial situations. This perspective assumes that classroom learning can be enhanced when teachers understand students' household and community experiences (Moll & González, 2012). For example, Hunter and Sawatzki (2019) found that 10–12-year-old students drew on social and cultural norms when using mathematics to justify financial decisions in a task about paying for a family meal bill, discussing what “value for money” meant to them. Furthermore, Sawatzki and Goos (2018) showed that students' pricing decisions depended on the funds of knowledge they drew upon: some prioritised social considerations and avoided setting prices too high, often resulting in loss-making responses, while others who gave profit-making responses relied on financial or entrepreneurial knowledge, referring to financial concepts such as “cost”, “price”, and “profit” in their proposed solutions. Similarly, Blue et al. (2018) found that Year 4 students needed to connect ethical, social, and mathematical considerations when solving pricing problems.

While several studies in the Australasian context have examined how primary school children perceive financial problem contexts and the financial concepts they contain, there are currently few similar studies in Germany. As a result, relatively little is known about whether German primary school students recognise the special system of measurement behind pricing and what funds of knowledge they draw upon when explaining prices. Addressing this gap, the present study investigates the following research question:

- How do third- and fourth-grade students conceive of prices, and what challenges and potentials do these conceptions pose for the promotion of financial numeracy?

Methodology

Research design and participants

The research question was embedded in a larger Design-Based Research (DBR) study examining how financial numeracy can be promoted in third- and fourth-grade mathematics lessons. The overall project comprised two DBR cycles: cycle 1 (2023) involved 16 students (9 third graders, 7 fourth graders), and cycle 2 (2024) involved a different group of 15 students (10 third graders, 5 fourth graders). A qualitative approach to data collection and analysis was adopted to gain deeper insights into students' thinking and reasoning. Across both cycles, data were collected through semi-structured pre- and post-intervention interviews and participatory classroom observations during the intervention, which took the form of a simulated classroom economy. In this setting, students made financial decisions, enabling the exploration of their financial knowledge, attitudes, and behaviours. Participation in the DBR study was voluntary and took place within an after-school program. The participating children came from two rural schools in Southern Germany. Most children were from middle-class backgrounds with diverse cultural backgrounds. They demonstrated differing levels of mathematical achievement, as indicated by their mathematics grades. Unlike Australia, where financial literacy is a curriculum connection, it is not yet explicitly embedded in the German curriculum, which likely affects how money and finance are addressed in classrooms. In both countries, money is included in the measurement strand of the mathematics curriculum. In cycle 1, the design-research focused on individual purchasing practices, reflecting a narrower view of financial numeracy centred on personal money management. The research focus of cycle 2 shifted to students' understanding of prices and the social and economic implications of pricing practices, to inform the redesign of the intervention towards fostering financial numeracy in its broader **sense**. This paper draws on pre-intervention interviews from cycle 2 to analyse students' initial conceptions of prices. The following sections outline the data collection and analysis procedures in detail.

Data collection

To inform data collection, prior research on children's price conceptions was reviewed and interview questions were adapted from established studies (e.g., Berti & Bombi, 1988; Claar, 1990; Leiser & Halachmi, 2006). Table 1 presents an excerpt of the posed questions.

Table 1

Sample Questions for Exploring Children's Price Conceptions

Key Questions for the Interview
Do you know how much an apple and a bar of chocolate cost at the moment?
An apple currently costs between 30 and 50 cents. How is the price of an apple determined?
Why is the price of a bar of chocolate usually higher than the price of an apple?
Imagine that an apple used to cost 30 cents. Now the price has risen to 40 cents. Can you explain why this might happen? If necessary, ask: Are there any other reasons?

Note. Similar questions were asked about other everyday products. Product pairs for price comparison were selected that were likely to be familiar to the students and that allowed exploration of whether students considered factors beyond physical characteristics (e.g., size or weight) when explaining prices. For example, students compared a sack of potatoes (2.5 kg) with a box of chocolates (100 g) and were asked which product was more expensive and why. Although the sack of potatoes is heavier, it can still be cheaper than the box of chocolates, as other price-determining factors need to be considered.

The 15 children in DBR cycle 2 were interviewed in pairs. This dialogue-based format aimed to create an open, conversational atmosphere and allowed observation of how children interact when explaining and justifying their answers. The entire pre-intervention interview

lasted approximately 30–40 minutes, with the section on pricing taking about 5–10 minutes. All interviews were audio-recorded and later transcribed. Using a semi-structured interview guide allowed the interviewer to react flexibly to students' answers and ask additional questions, enabling a deeper exploration of their thinking and reasoning. Follow-up questions were included to investigate whether children could think of other possible explanations, with the aim of obtaining as comprehensive a picture of their understanding as possible. Depending on the students' responses, further probing questions were asked. For example, to explore whether students recognised that prices cannot be lowered indefinitely due to needing to cover costs, the follow-up question "Can the price of an apple also fall to 10 cents or less? Why or why not?" was posed if children had already mentioned supply and demand as reasons for price changes.

Data analysis

Interview data were analysed using qualitative content analysis (Mayring, 2010). Students' responses to the pricing questions were first coded using a deductive category system derived from Claar's (1990) framework of children's evolving price understanding (see Table 2). Although Claar's study is comparatively dated, it remains one of the few that provides a comprehensive conceptualisation of pricing within a market economy. The categories represent increasingly differentiated and integrated conceptions of price formation, with price-determining criteria becoming, in the latter categories, more general, abstract, and less directly tied to the physical characteristics of products.

Table 2

Categories for Analysis of Children's Price Conceptions, adapted from Claar (1990)

Category	Summary Description of Category
A: Price determined by product characteristics or functionality	Prices are based exclusively on specific characteristics of the product (e.g., size, weight) or on functional criteria relating to the use of the product (e.g., usefulness, durability).
B: Price determined by one cost factor	The amount of work involved in manufacturing and distributing a product is considered as a basis for the price. Only one cost factor (e.g., transportation costs) is mentioned.
B-2: Price determined by several cost factors	The amount of work involved in manufacturing and distributing a product is considered as a basis for the price. Several cost factors (e.g., costs for materials, transportation and advertising) are mentioned.
C: Price determined by supply and demand	In addition to costs, it is considered that supply and demand influence prices. The functional relationships between supply, demand and price can be explained (e.g., when demand exceeds supply, the price increases).
C-2: Price determined by integration of supply and demand with costs	Costs, supply and demand are combined in the reasoning and explanation of price formation. For example, it can be explained that prices cannot be reduced too much because then costs would no longer be covered.
D: Market imperfections included in price explanations	Deviations from a perfect market, such as incomplete market transparency, location advantages or the influence of advertising and brand image, are considered when explaining prices.

In a second step, the analysis focused on the everyday experiences and contextual knowledge students drew upon when explaining prices. These references were interpreted through a Funds of Knowledge lens as indicators of the knowledge sources that informed students' reasoning (e.g., shopping experiences or local debates about prices they witnessed).

Findings

In presenting the findings, pseudonyms are used for all students, and interview excerpts are translated verbatim into English. Most children's pre-intervention price conceptions were coded

as part of category A, where prices are derived solely from product characteristics such as size, weight, or functionality. While such attributes can influence price, relying on them alone to explain price formation does not fully reflect economic reality. For instance, a lettuce is not more expensive than a keyring simply because it is larger. The following excerpt illustrates this, showing how Helena explained the price difference between a bar of chocolate and an apple.

Interviewer: Why do you think the bar of chocolate costs more than the apple?

Helena: Because chocolate is longer/well, it is a little longer.

Helena treated length as the sole price-determining factor, concluding that the chocolate bar was more expensive simply because it was longer.

Weight or size was also frequently cited as the only reason for price differences between products. Some children reasoned that a bag of potatoes (2.5 kg) was more expensive than a box of chocolates (100 g) because the potatoes were heavier. Others argued that the sack of potatoes was more expensive because it was larger. In some cases, the price was also explained by the functionality of the products, for example, arguing that potatoes are more expensive because they can be used to make many different things (e.g., chips or crisps), whereas chocolates are less versatile.

A focus on physical attributes also appeared in explanations of price changes, which many children attributed only to differences in size or weight. Mira, for example, assumed that a price increase for a product was only possible if the properties of the product also changed – in her opinion, apples could only become more expensive if they also became larger.

Interviewer: And then, let's say the apple used to cost 35 cents. And now the apple suddenly costs 40 cents. How can you explain that? How can that happen?

Mira: Maybe it has become heavier.

A similar argument was also found in Helena's and Selina's responses to the interview questions, who said that the price of an apple can only increase if this is accompanied by a change in its weight.

Interviewer: And now the price has suddenly been increased to 40 cents. Why might that be?

Helena: If it's a heavier apple.

Interviewer: So, if it's heavier, it is more expensive? (Helena nods) Could there be other reasons, or can it only be due to the weight?

Selina: It can only be due to the weight.

Some children also mentioned the amount of work involved, expressed as individual cost factors, in the production and distribution of products, which was coded as category B. For example, Ines contradicted Lilia's statements, who justified the price of an apple by reference only to characteristics such as weight and width, and instead referred to production costs, specifically water usage in apple cultivation.

Interviewer: An apple costs between 30 and 50 cents. Do you know how that price is determined?

Lilia: I think it's based on weight and width.

Interviewer: So how big the apple is, you mean?

Lilia: Yes.

Ines: I think it's more to do with production. For example, we recently discussed the topic of water, and abroad an apple needs one litre or so. And in Germany, it only needs 200 or 500 millilitres. Maybe that's what it depends on.

Justus, on the other hand, addressed transportation costs that affect apple prices.

Justus: Well, there are no apples in Germany at this time of year, in winter. So, they have to be transported across the oceans by boat, and now, with the war in Ukraine, the boats may have to travel further. So somehow that has to pay off.

Supply and demand (category C) were rarely mentioned by the children in the pre-intervention interview as possible factors influencing prices. One exception was Nils and Robin who cited a supply shortage as a possible reason for a price increase.

Interviewer: An apple used to cost 35 cents, but now the price is increasing to 40 cents. How can that be? What could make it more expensive?

Nils: Yes, if there aren't as many apples anymore.

Interviewer: So, because there is less supply?

Nils: Yes.

Robin: That there might be a big drought and then there's not as much rain.

Interviewer: So, then there are fewer apples, and then you can make them more expensive, do you think?

Robin: Yes.

In addition to changes on the supply side, Nils and Robin also mentioned individual cost factors – such as transportation costs – as price-determining factors. However, there was no evidence of an integrated argument linking supply, demand, and costs (category C-2). Furthermore, they were only able to explain a price increase by means of a shortage of supply but could not justify a price reduction with changes in supply or demand.

When children discussed supply and demand, they sometimes assumed different functional relationships among supply, demand, and price than those described in economic theory. For example, Loris initially assumed a "more-is-more" logic, according to which a larger supply of apples (with demand remaining constant) leads to higher prices.

The interview discussions provided insights that extend beyond a simple categorisation of students' answers. In particular, they show how students' explanations of prices evolved during the course of the interview and how, in some cases, an emerging critical orientation toward pricing decisions became evident.

In some cases, children noticed a contradiction in their stated price-determining factors over the course of the interview as soon as they tried to apply them to other products and realised that their reasoning no longer applied. For example, Loris and Mira initially argued that size and weight were the only price-determining factors but then came across counterexamples where this explanation no longer seemed to apply. Loris noted from his own consumer experience that there are products that are larger than others but are not more expensive, and that taste is not always the decisive factor in determining price.

Loris: But I don't think it's because of taste, because there are pizzas with pineapple on them, which a lot of people don't like, but they're more expensive than an apple, for sure. And maybe people just don't like pizza.

Interviewer: That's the question, yes.

Loris: And it's not because of how big or small something is either. Because there was a packet of cocoa that was small and cost 3, or 4, or 5 €, I'm not sure.

Interviewer: So, the large one was cheaper than the small one?

Loris: Yes.

Interestingly, without the interviewer prompting them, some children already critically questioned issues of equality and how prices affect others, especially when these issues were part of their lived experiences. However, their suggested solutions indicated that they did not yet fully grasp the systemic implications of pricing decisions within an economic system. For example, Emil had recently noticed farmer demonstrations for fairer payment for their products. Living in a rural area where many people work in agriculture, he suggested that the shopkeepers pay farmers more for their apples:

Because, there's also a dispute at the moment because farmers don't get enough money when they sell their produce. But, for example, if they have children, they sell their fruit and vegetables and

meat. And if they want 3 € for it and people only pay 10 cents, for example, that's why there's a dispute. And I would, if I were a shopkeeper, I would give them 3, maybe 4 € for it, maybe a tip.

When the interviewer asked whether this would in turn require customers to pay higher prices, Emil suggested that the additional paid amount could be treated as a “tip” from the shopkeeper, without seeming to acknowledge that a shopkeeper aims to make a profit and is unlikely to cover such costs personally. His proposal, while well-intentioned, was therefore not a sustainable solution.

Interviewer: But then people would have to buy it for 4 €, right?

Emil: No, I would just charge 3 €, and I meant that the 1-euro coin is a tip.

Discussion

The findings presented above suggest that the majority of the third- and fourth-grade students in this study tended to view prices more as objectively measurable quantities tied to physical product features (e.g., length or weight), and less as subjective and negotiable constructs shaped by broader economic and social factors. This aligns with earlier research on children's economic reasoning, which indicates that children aged 8 to 10 often attribute prices mainly to physical characteristics of products (e.g., Claar, 1990). Such conceptions may present challenges for fostering financial numeracy in a broader sense, particularly with regard to developing a critical orientation (e.g., Goos & O'Sullivan, 2022). When students perceive prices as fixed and objective rather than socially negotiated, it becomes more difficult for them to make informed financial decisions, or to critically examine issues of fairness and inequality related to pricing.

At the same time, the findings suggest that children draw on rich funds of knowledge when reasoning about prices. Their explanations should therefore not be interpreted simply as incorrect conceptions but as working theories that help them make sense of pricing in everyday contexts. For example, explaining prices in terms of weight is not entirely inaccurate, as weight can influence price determination in some cases. Children may have experienced, for instance, when buying fruit or vegetables, that heavier quantities generally cost more. However, such explanations remain incomplete, as weight is only one of several factors influencing prices. The interviews also showed that children reflected on, and refined, their explanations during discussion. Some recognised inconsistencies when applying their reasoning to different products, as illustrated by Loris, while others challenged each other's ideas and proposed alternative explanations, as in the exchange between Lila and Ines. These interactions suggest that classroom discussions can support the development and refinement of students' reasoning about prices.

Furthermore, some children demonstrated an emerging critical orientation toward pricing decisions. For example, Emil explained apple prices by referring to farmers receiving insufficient payment for their products and suggested that shopkeepers should pay them more. His reasoning was primarily grounded in social rather than economic reasoning. Similar patterns have been reported in previous studies, where students' reasoning about financial situations was often shaped by social and ethical concerns (e.g., Blue et al., 2018; Hunter & Sawatzki, 2019; Sawatzki & Goos, 2018). While such responses indicate a sensitivity to issues of fairness and inequality, they also highlight the need to support students in developing a more integrated understanding of pricing practices and in considering how practices might be changed to promote greater fairness by connecting social, economic, and mathematical perspectives.

Conclusion

This study contributes to knowledge by providing insights into how some German primary school children conceive of prices and by discussing both the challenges and potentials these

conceptions may pose for developing financial numeracy. The findings suggest that many third- and fourth graders initially view prices as objectively measurable quantities linked mainly to physical product characteristics but are also able to modify their conceptions through drawing on their funds of knowledge and in discussion with others. Furthermore, some are also able to question existing pricing practices. The findings suggest that students should be supported to extend their knowledge by empowering them to critically explore how prices are formed and how they shape social and economic reality, rather than viewing prices as purely objective, descriptive measures. These findings informed the subsequent design phase of the broader DBR study, in which financial numeracy tasks were developed to support students in connecting social, economic, and mathematical perspectives when analysing price formation and evaluating pricing practices. This study is limited by its small sample size and limited student diversity, which restricts the generalisability of the findings to how primary school students conceptualise prices. In addition, the analysis is based on responses to a limited set of interview questions and therefore provides only a partial view of the knowledge and experiences students may draw upon when explaining prices. Alternative questions or additional data collection methods might have provided further insights into students' reasoning about prices.

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References

- Berti, A. E., & Bombi, A. S. (1988). *The child's construction of economics*. Cambridge University Press.
- Blue, L., O'Brien, M., & Makar, K. (2018). Exploring the classroom practices that may enable a compassionate approach to financial literacy education. *Mathematics Education Research Journal*, 30(2), 143–164. <https://doi.org/10.1007/s13394-017-0223-5>
- Claar, A. (1990). *Die Entwicklung ökonomischer Begriffe im Jugendalter. Eine strukturalistische Analyse*. Springer.
- Gal, I., Grotluschen, A., Tout, D., & Kaiser, G. (2020). Numeracy, adult education, and vulnerable adults: a critical view of a neglected field. *ZDM*, 52(3), 377–394. <https://doi.org/10.1007/s11858-020-01155-9>
- Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2020). *Numeracy Across the Curriculum: Research-based strategies for enhancing teaching and learning*. Routledge.
- Goos, M., & O'Sullivan, K. (2022). Numeracy Across the Curriculum. *Oxford Research Encyclopedia of Education*. <https://doi.org/10.1093/acrefore/9780190264093.013.1530>
- Hunter, J., & Sawatzki, C. (2019). Discovering diverse students' funds of knowledge related to finance: Pāsifika students in New Zealand. *Mathematics Education Research Journal*, 31(4), 419–439. <https://doi.org/10.1007/s13394-019-00259-0>
- Mayring, P. (2010). *Qualitative Inhaltsanalyse. Grundlagen und Techniken*. Beltz Juventa.
- OECD. (2023). *PISA 2022 Results (Volume IV): How financially smart are students?* OECD Publishing. <https://doi.org/10.1787/5a849c2a-en>.
- Leiser, D., & Halachmi, R. B. (2006). Children's understanding of market forces. *Journal of Economic Psychology*, 27(1), 6–19. <https://doi.org/10.1016/j.joep.2005.06.008>
- Moll, L. C., & González, N. (2012). Teachers as social scientists: Learning about culture from household research. In *Race, Ethnicity, and Multiculturalism: Policy and Practice* (pp. 89–114). Taylor and Francis Inc.
- Savard, A., Cavalcante, A., Turineck, L. P., & Javaherpour, A. (2020). Some considerations towards the concept of money in elementary school: money as measurement. *For The Learning of Mathematics*, 40(3), 22–24.
- Savard, A., & Cavalcante, A. (2021). *Financial Numeracy in Mathematics Education*. Springer.
- Sawatzki, C., & Goos, M. (2018). Cost, price and profit: what influences students' decisions about fundraising? *Mathematics Education Research Journal*, 30(4), 525–544. <https://doi.org/10.1007/s13394-018-0241-y>
- Winter, H. (1992). *Sachrechnen in der Grundschule*. Cornelsen.

Mathematical Bibliotherapy for Pre-service Teachers: A Case Study

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Pre-service teachers often have negative memories of learning mathematics that can affect their confidence as teachers of mathematics. In this paper we explore the viability of an approach called *bibliotherapy* that has been used to help pre-service teachers to overcome their challenges with mathematics. We present a case study of one pre-service teacher who engaged in our ‘proof-of-concept’ study. We argue that the use of a fictional story and reflective essay provides the reader with the opportunity to connect with a variety of characters and thus has potential to expand on the bibliotherapy methods used previously in mathematics education.

Many pre-service primary teachers arrive at their compulsory mathematics courses haunted by bad memories of learning the subject. To loosen their stranglehold, instructors often ask students to write about these memories and reflect on them with others (Marmur & Darragh, 2024). Like all good therapy, this can be cathartic. Students gain insight into their experiences, recover a sense of agency, and realise they are not alone. But this kind of writing is not easy. It can be painful to revisit bad memories. Some may be so upsetting that students refuse to recall them, let alone write about them.

A gentler, less threatening alternative is to ask students to read stories about others facing similar challenges. Through reading, they may identify with a character emotionally and become invested in their success. They may experience tension and release as the character confronts and overcomes their challenges. They may learn new insights and apply them to their own problems, which may seem more manageable as a result. This use of literature for therapeutic goals is known as *bibliotherapy* and has been effective in clinical settings to treat youth and adolescents with behavioural issues (McCulliss & Chamberlain, 2013), and psychiatric patients recovering from depression and psychosis (Chamberlain, 2019). It has also been used in mathematics education to reduce mathematics anxiety among students (Hébert & Furner, 1997) and pre-service teachers (Wilson & Thornton, 2005).

The success of this approach is contingent on the choice of reading material. Research suggests that stories are more likely to have therapeutic value if the character is relatable and the problem they face corresponds to readers’ specific needs (Lutovac & Kaasila, 2020; Gerlach & Subramanian, 2016). In this paper, we report on one case study to demonstrate and explore our approach to bibliotherapy. We begin by reviewing research on bibliotherapy and how to select texts that can be used with pre-service primary teachers. We then describe the texts we used in our study, and the methods for collecting and analysing data. We report findings on the viability of our bibliotherapy approach, and end by extending the criteria by Lutovac and Kaasila (2020) for selecting appropriate texts.

What is Bibliotherapy?

Bibliotherapy is the use of literature to support therapeutic goals in the reader. These goals may be affective, such as increased empathy, positive self-image, and positive attitudes, or

cognitive, such as increased critical thinking, self-evaluation, and awareness of human behaviour (McCulliss & Chamberlain, 2013).

The therapeutic benefits of bibliotherapy were initially conceived as occurring over three phases: *identification*, in which the reader identifies with a character from the text; *catharsis*, in which the reader becomes emotionally involved in the story and releases their own emotional tension; and *insight*, in which the reader develops a new awareness of how to approach problems they face in their own lives (Hébert & Furner, 1997). Researchers have expanded these phases to include *involvement*, a primary phase in which the reader becomes interested and invested in the narrative (Heath et al., 2005); *universalism*, in which the reader looks beyond their own problems to realise that others may share or understand their challenges (Heath et al., 2005); and *projection*, in which readers consider what their newfound insights could mean for their future actions (Wilson & Thornton, 2005). Discussion of the text, and other follow-up activities like writing, play a critical role in the bibliotherapy process by promoting sharing and further reflection, which can enhance self-awareness and self-concept (Hébert & Furner, 1997).

In teacher education, bibliotherapy that is used “to help promote prospective teachers’ professional development” is known as *pedagogical bibliotherapy* (Lutovac & Kaasila, 2020, p. 495). Bibliotherapy has been used to help pre-service teachers reflect on the kind of mathematics teacher they want to be and what obstacles might stand in the way (Wilson & Thornton, 2005) and enhance their pedagogical knowledge and practice.

Effective Texts for Bibliotherapy

Lutovac and Kaasila (2020) asked prospective primary teachers to read six case studies of prospective teachers’ experiences with mathematics and write reflections on the case study they identified with most. Based on these reflections, they propose that to encourage identification, reading material for bibliotherapy should meet the following five criteria:

1. be based on the first-person so-called true stories or documents of the common experiences of other (prospective) teachers and making reading as contextualised as possible,
2. portray coping and positive resolution,
3. include turning points portraying the changes in character’s knowledge, beliefs, emotions and actions, which will encourage prospective teachers to reflect on their own changes and may even influence them,
4. have a clear pedagogical message that can benefit prospective teachers’ future practices by promoting feelings of hope, raising (self) awareness, stimulating reflection and action-taking,
5. not only enable relating to the past and present experiences, but also, if possible, enable projection of experiences to allow prospective teachers to reflect on what may happen, loose [sic] fears, provide comfort and confidence, or if needed, take preventive actions. (p. 495)

Although Lutovac and Kaasila’s (2020) study shows that texts meeting these five criteria can engage prospective teachers in pedagogical bibliotherapy, research suggests that other kinds of texts can also be effective. Academic research articles have been used successfully with pre-service teachers (Wilson & Thornton, 2005), and children’s picture books (Heath et al., 2005), fiction (Pierce, 2015) and non-fiction self-help books (Floyd et al., 2004) have been used in other contexts. In this study, we explore the viability of using a fictional short story and reflective essay that draws on research to offer insight into the story. Our research question is: *To what extent can a fictional third-person story paired with a reflective essay support pre-service primary teachers in pedagogical bibliotherapy?*

Methods

Our study is part of a larger project researching the role of story and essay in supporting people (not just pre-service teachers) to reflect on their relationship to mathematics. Here, we report on one participant from our study, using a case study methodology (Yin, 2018).

Participant and Setting

Anna (pseudonym) was enrolled in a primary Initial Teacher Education program in a large New Zealand tertiary institution. At the time of the interview, she was on her final practicum. She had previously worked as an Early Childhood Education (ECE) teacher in NZ, was a mother to a 16-year old son, and had grown up in China (where she completed her secondary school education). Her responses to a questionnaire about her attitudes towards mathematics at the beginning of the study showed that she did not experience mathematics anxiety and viewed mathematics to be a worthwhile and necessary subject.

Data Collection

Anna read a short story called *Ouch* (described below), then wrote answers to some questions about the story, such as which character she identified with the most, and her perspective on certain events in the story. Then, she participated in an individual semi-structured interview with the first author, who asked about her thoughts on the story, what emotions she experienced, and the extent to which it related to her own experiences. These questions were informed by therapeutic questioning norms (see Yoon & Penwarden, 2021). Next, Anna read an essay that discusses *Ouch* and participated in a second round of interview questions, similar to the first. Finally, one week after the interview, she completed the same questionnaire about her attitudes towards mathematics and responded in writing about whether she had thought about the story or essay since the interview.

Data Analysis

Interview data were transcribed and analysed together with the written responses to identify the participant's life details, memories, reflections on the text as a piece of writing, reflections on the story (e.g. characters, plot), and the views on teaching. They were also coded to identify evidence of Anna engaging in the six phases of bibliotherapy, which were operationalised in the coding scheme shown in Table 1. The first author conducted the initial coding, then shared the coding with the co-authors, as well as the coding scheme. The co-authors used the coding scheme to analyse the transcript of another preservice teacher (not reported here), compared coding, and discussed the criteria. This led to a refinement of the coding scheme, and the establishment of a new code that identified evidence of distancing oneself from difficult experiences and emotions. Finally, the first author re-coded the data using the revised coding scheme.

Table 1*Coding scheme for bibliotherapy phases for Data Analysis*

Phase	Indicators of engagement in bibliotherapy phase
Involvement	Expressions of interest, e.g. curiosity, surprise, appreciation Investment in the characters, e.g. imagining what happens to them next Engagement with the text, e.g. wanting to use the texts
Identification	Linguistic markers of identifying, e.g. relate, similar, empathise, know Relating to or recognising <i>relationships</i> between characters
Insight	Expressions of newness or surprise about information Insights into the characters, as well as into their own experiences
Catharsis	Expressions of emotions, attitudes, values, beliefs Tension and release
Universalism	Awareness that others may share their challenges
Projection	Reflections on what makes a good teacher Hopes and challenges for their own teaching Consideration of how their insights can bear on their future actions as teachers

Description of the Texts Used in the Research

Our study uses two interlinked texts, written by the first author:

6. A short story called *Ouch* (1608 words);
7. A reflective essay that analyses the events in *Ouch* (2225 words).

The *Ouch* story and its accompanying essay can be found in Yoon (2026), with an early version of *Ouch* published in Yoon (2025a). The story and essay are part of a larger collection that seeks to engage readers in reflecting on their relationship with mathematics, and to share research insights into mathematical teaching and learning (see Yoon, 2025b for a detailed discussion of the aims). The target audience for these stories and essays is parents, teachers, pre-service teachers, and anyone interested in how children learn mathematics.

The *Ouch* story features a mathematics-averse mother, Jenny, who observes her 4½ year old daughter, Rosie, playing with her friends at the early childhood centre. When Rosie counts some toy figurines incorrectly, Jenny remembers being humiliated as a child for making a similar mistake in front of the class, with her primary school teacher expressing her frustration and impatience. She turns to Rosie's teacher, Sunita, who is with her, watching the children play, and says, "She gets that from me, I was hopeless at maths, too... I don't have a maths brain ... I'm not a maths person." Sunita doesn't laugh. Gravely, she tells Jenny that she must "never say that again." Jenny is taken aback but Sunita is called away to attend to another child before she can explain the reasons for her request. The story ends with Jenny wondering why Sunita, who is usually so gentle and supportive, has told her off.

The essay explores Jenny's *maths brain/math person* speech and possible reasons behind Sunita's request, such as the intergenerational passing down of mathematics anxiety and avoidant behaviour through language that communicates a fixed mindset (Boaler, 2016). It uses the analogy of learning to drive a car to show how mathematics learning is dependent on practice and instruction, not talent alone. It discusses how negative attitudes towards mathematics formed in childhood can be reinforced by culture and society (Epstein et al., 2010), and ends by illuminating principles of counting, such as order-irrelevance and cardinality, as well as why children commonly omit the number 15 while counting due to its unusual construction as *fifteen* rather than *fiveteen* (Fuson, 1988).

Findings

Anna exhibited all six phases of bibliotherapy in her interview and her written responses.

Involvement

Anna was initially confused by *Ouch* (the story), saying she had expected it to be more like a standard research article, containing “goals, objectives, implementation.” Yet she was engaged with the story, describing it as “outside the box ... very narrative and illustrative, like a fiction ... like a novel ... very beautiful.” She called it a “good learning story” about how to help Rosie “find the fun of maths,” and clarified that a learning story was a narrative written by an ECE teacher or family member that describes and reflects on some aspect of a young child’s learning journey (see Carr, 2001). Anna described the essay as “the missing part of a learning story” that explained “all the theory, all the science ... behind learning.” For her, the essay completed the *Ouch* story to create a “perfect learning story now.”

Identification

Anna identified with Sunita, saying she appreciated Sunita’s strength and her ability to protect children from harmful messages, and acknowledged a similarity in having worked as an ECE teacher, too. Anna is also a parent, but she said she “wasn’t like the Mum (Jenny)” in her approach to her child’s learning. Yet despite not identifying with Jenny as a parent, she strongly related to Jenny’s experience as a child, saying the disapproval Jenny received from her primary school teacher “reminded me of other experiences that brought up similar feelings.” She shared a memory of her year 6 teacher who told her, “He never seen a girl like me [who] just had such a bad maths achievement.” Until then, she had enjoyed mathematics and performed well in it, but her teacher’s criticism forced her to reconsider this. She connected this memory to Sunita’s protection of Rosie, thereby relating to Rosie’s vulnerability as a child.

Catharsis

Anna was surprised that the *Ouch* story brought up old memories for her, saying, “I didn’t expect it ... to go back into my, the someone’s personal experience,” changing the reference to herself with the vaguer “someone”. Some of her memories were ones she had repressed: “It’s like something that’s buried deeply in my mind and never touched that. But when I read this story, it also gave me a chance to open a little bit and just dig into [it] and check.” For Anna, there was a strong connection between her emotional experiences and those in the story: “This story is like a path or like a tunnel, just connected to my experience of studying math. And it’s not quite the same, but I think the emotion and feelings [in] that story ... [are] really similar.”

Insight

When Anna initially shared her experience of being criticised by her Year 6 teacher, she defended him as “a great teacher” who had “very high standard professional skills.” But as she reflected on how much she valued Sunita’s intervention to protect Rosie, she came to acknowledge the harmfulness of her Year 6 teacher’s actions, saying, “Maybe I feel sorry. I feel sorry for myself when I was young and had that discrimination. Yeah, something like that. Yeah.” Later, she reflects on the parallel experience Jenny had with her childhood teacher, saying the teacher “just shouldn’t have done that.” She also expressed gratitude for the new insights the essay gave her into mathematics learning, such as why 15 is frequently omitted by children learning to count.

Universalism

In the final reflection one week after the interview, Anna acknowledged the similarity between her own experience and Jenny’s, saying that Jenny “carried trauma from her own

family and school experiences". Anna explained that Jenny's fear of failure stayed with her and was passed on to her daughter, which prompted her to say she was "not a maths person" and tried to justify her daughter's similar struggle. Crucially, she commented that Jenny's struggles with mathematics were not her fault: "She simply didn't have the right resources, teachers, or teaching approaches to support her learning. She should not blame herself for that."

Projection

Throughout the interview, Anna frequently reflected on what makes a good teacher in response to the texts and the memories they evoked. She was critical that Jenny's primary school teacher made her frustration with Jenny's mistakes visible, calling it unprofessional, an over-reaction, and that teachers should express their emotions in ways that don't hurt the child. In addition to reflecting on her Year 6 teacher's harsh criticism, she also mentioned a physics teacher who "gave up on me," a chemistry teacher who "ignored me," and an excellent chemistry tutor whose passion and dedication re-ignited her interest in the subject. In response to the analogy in the essay about learning to drive a car, she talked about a wonderful driving instructor who instilled confidence in her when she was "so scared," and taught her carefully and "step-by-step." She recalled her mother, who was a "genius," not being able to teach her when she asked her a question, as she couldn't understand what her daughter was struggling with. She said, "How to teach is really important," that it can determine a child's path, and that teachers and parents should not get upset with children when they find learning difficult. She said that she wants to be the kind of teacher who can relate to a child's struggle in learning, who can regulate their emotions around children, who can instill confidence, who knows their students' developmental stage, and can teach effectively to fit with their needs. Overall, Anna participated in all six phases of bibliotherapy while reflecting on the *Ouch* story and its accompanying essay.

Discussion

Anna's case study challenges three of Lutovac and Kaasila's (2020) criteria for selecting texts for pedagogical bibliotherapy.

First, *Ouch* is a third-person limited story of fiction with Jenny, a parent, as the protagonist, so it fails the criterion of being a "first-person so-called true [story]" of a "(prospective) teacher." Nevertheless, it offers several opportunities for identification. In Lutovac and Kaasila's (2020) study, the opportunities for identification were individual and conscious. Participants were given six case study biographies of prospective teachers and were asked to choose the one they identified with most. This instruction, together with the closeness of the first-person narrative, potentially created a binary effect where the participant either identified with the protagonist and their experiences or they did not, creating a one-to-one mapping between the chosen biography and the participant. In contrast, *Ouch* introduced Anna to multiple characters, and although Anna identified with the teacher (Sunita) the most, she also related to aspects of the other characters who were not teachers. For example, she did not identify with Jenny as a parent due to differences in their attitudes towards mathematics learning, yet she found herself recalling childhood experiences she had long forgotten that were similar to Jenny's as a child. These memories surprised her and fed into her reflections on the kind of teacher she wanted to be. The variety of characters in *Ouch*, and the more distant third-person point of view may have created more subtle opportunities for Anna to explore repressed memories than she might have otherwise.

Ouch also fails Lutovac and Kaasila's (2020) second and third criteria by not portraying a "positive resolution" or "turning points portraying the changes in character's knowledge, beliefs, emotions and actions" (p. 495). This is because *Ouch* does not follow the classic story structure of 'a hero overcoming a challenge' but rather portrays a 'slice of life' that introduces

a problem without resolving it. The story ends with Jenny aware of a problem with her *maths brain/maths person* speech, but unsure what the problem is, or what to do about it. The essay on *Ouch* opens the possibility of a positive resolution to this problem by inviting the reader to wonder whether Jenny's attitude towards mathematics could change. Yet it does not show this change occurring and focuses on illuminating the problem rather than fixing it. Thus, Anna's case study shows that pedagogical bibliotherapy texts do not have to be first-person nonfiction accounts of teachers who positively resolve a problem and experience turning points in their beliefs and actions.

Anna's case study supports Lutovac and Kaasila's (2020) fourth criterion that texts should have a "clear pedagogical message that can benefit prospective teachers' future practices." However, it fulfils this through two texts, not one. The essay that accompanies the *Ouch* story uses research to unpack questions raised in the story—how negative attitudes towards mathematics are formed, how they can be unintentionally passed down, and how children learn to count. Anna appreciated these pedagogical insights and said they were directly applicable to her own teaching. It would have been possible to communicate these insights in a standalone essay, but such an essay would have missed the emotional resonance created by the link to the story. Similarly, it would have been possible to communicate these insights in a standalone story (for example, by having Sunita teaching Jenny about how children count), but this would have required reducing the information to maintain the credibility of the story. We are also mindful that expecting stories to deliver a message places an unnatural burden on the literary form: "The notion that the story has a message assumes that it can be reduced to a few abstract words, neatly summarised ... if that were true, why would writers go to the trouble of making up plots, scenery ... why not just deliver the message?" (Le Guin, 2005, para. 5-6). For us, having two interlinked texts meant that each could perform the function they were most suited to: the story sparking the reader's curiosity about an imaginary scenario, the essay offering analysis and insight.

Anna's case study provides preliminary evidence that story-essay pairings like *Ouch* can be viable texts for pedagogical bibliotherapy with pre-service primary teachers. Based on our findings, we propose incorporating the following into Lutovac and Kaasila's (2020) criteria for selecting texts for pedagogical bibliotherapy:

- Third-person stories involving multiple characters involved in the mathematics learning context, with a variety of identities (not all teachers);
- Stories that frame or present a problem without necessarily showing a positive resolution to it, nor accompanying changes in the character's beliefs and actions;
- Story-essay pairings in which the essay analyses and offers insight into the problems of learning and pedagogy presented in the story.

More research is needed with a wider variety of texts to expand these criteria further and reframe them in terms of the functions the texts should perform, rather than specifying the form they should take. In our research, we are currently exploring the bibliotherapeutic possibilities of additional story-essay pairings (e.g. *Whiskers* from Yoon, 2025a), with other participants, including parents of young children, practising primary teachers, and pre-service primary school teachers at the beginning of their initial teacher education with a range of attitudes towards mathematics.

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References

- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.
- Carr, M. (2001). *Assessment in early childhood settings: Learning stories*. Paul Chapman.
- Chamberlain, D. (2019). The experience of older adults who participate in a bibliotherapy/poetry group in an older adult inpatient mental health assessment and treatment ward. *Journal of Poetry Therapy*, 32(4), 223–239. <https://doi.org/10.1080/08893675.2019.1639879>
- Epstein, D., Mendick, H., & Moreau, M.-P. (2010). Imagining the mathematician: Young people talking about popular representations of maths. *Discourse: Studies in the Cultural Politics of Education*, 31(1), 45–60. <https://doi.org/10.1080/01596300903465419>
- Floyd, M., Scogin, F., McKendree-Smith, N. L., Floyd, D. L., & Rokke, P. D. (2004). Cognitive therapy for depression. *Behavior Modification*, 28(2), 297–318. <https://doi.org/10.1177/0145445503259284>
- Fuson, K. (1988). *Children's counting and concepts of number*. Springer-Verlag.
- Gerlach, H., & Subramanian, A. (2016). Qualitative analysis of bibliotherapy as a tool for adults who stutter and graduate students. *Journal of Fluency Disorders*, 47, 1–12. <https://doi.org/10.1016/j.jfludis.2015.12.001>
- Heath, M. A., Sheen, D., Leavy, D., Young, E., & Money, K. (2005). Bibliotherapy: A resource to facilitate emotional healing and growth. *School Psychology International*, 26(5), 563–580.
- Hébert, T. P., & Furner, J. M. (1997). Helping high ability students overcome math anxiety through bibliotherapy. *Journal of secondary gifted education*, 8(4), 164–178.
- Le Guin, U. (2005). A message about messages. *CBC Magazine*. https://web.archive.org/web/20051128074549/http://www.cbcbooks.org/cbcmagazine/meet/leguin_ursula_k.html
- Lutovac, S., & Kaasila, R. (2020). How to select reading for application of pedagogical bibliotherapy? Insights from prospective teachers' identification processes. *Journal of Mathematics Teacher Education*, 23(5), 483–498. <https://doi.org/10.1007/s10857-019-09437-0>
- Marmur O., & Darragh L. (2024). Emotional associations with mathematics: Using the lenses of affect and identity to understand pre-service teacher stories. In Evans, T., Marmur, O., Hunter, J., Leach, G., & Jhagroo, J. (Eds.). *Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3). Auckland, New Zealand: PME.
- McCulliss, D., & Chamberlain, D. (2013). Bibliotherapy for youth and adolescents—School-based application and research. *Journal of Poetry Therapy*, 26(1), 13–40. <https://doi.org/10.1080/08893675.2013.764052>
- Pierce, L. M. (2015). The use of bibliotherapy with adolescents and their families. *Journal of Family Psychotherapy*, 26(4), 323–330. <https://doi.org/10.1080/08975353.2015.1097294>
- Wilson, S., & Thornton, S. (2005). I am really not alone in this anxiety: Bibliotherapy and pre-service primary teachers' self-image as mathematicians. In *MERGA 28: Building connections: Research theory and practice. Proceedings of the annual conference held at RMIT Melbourne 7th-9th July; 2005*. (pp. 791–798). Mathematics Education Research Group of Australasia.
- Yin, R. K. (2018). *Case study research and applications* (6th ed.). Sage Publications, Inc.
- Yoon, C. (2025a). East Asian narratives of mathematics in an arts-based research project. In O. N. Kwon & B. Kaur (Eds.), *Proceedings of the 9th ICMI-East Asia Regional Conference on Mathematics Education* (Vol. 1) (pp. 191–208). Seoul, Korea: EARCOME9.
- Yoon, C. (2025b). Treasures from the field: Sharing mathematics education research through story and essay. *Educational Studies in Mathematics*, 120(2), 205–223. <https://doi.org/10.1007/s10649-025-10418-9>
- Yoon, C. (2026). *Listening and responding to mathematics-averse speech* [Manuscript submitted for publication]. Department of Mathematics, University of Auckland.
- Yoon, C. & Penwarden, S. (2021). Kindness in educational critique. *Knowledge Cultures*, 9(3), 169–183.

Integrating Appreciative Education and Funds of Knowledge in Mathematics Classrooms

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Many students see mathematics as disconnected from their lives, which can lead to low confidence and limited engagement. Strength-based approaches such as Appreciative Education (AE) and sociocultural perspectives such as Funds of Knowledge (FoK) offer ways to make mathematics learning more meaningful and relevant. However, these approaches have largely been developed separately, and their relationship in mathematics education has received limited attention. This paper explores how AE and FoK can be understood together and proposes a conceptual framework that connects their key ideas.

Introduction

Many students continue to experience mathematics as disconnected from their lived experiences, which can lead to low confidence and limited engagement, particularly among learners from culturally diverse backgrounds (Moschkovich, 2012). Despite ongoing reform efforts and curriculum developments, mathematics classrooms often remain procedural and textbook-driven, with limited attention to students' cultural knowledge and everyday experiences.

This challenge has become particularly visible in the New Zealand context. International assessments such as the Programme for International Student Assessment (PISA) have revealed a steady decline in mathematics performance, with widening gaps across cultural, linguistic, and socioeconomic groups. The 2022 results showed the lowest mean score since New Zealand began participating in PISA. These patterns have intensified concerns about both achievement and equity in mathematics education.

In Aotearoa New Zealand, educational policy has increasingly emphasised equity, inclusion, and cultural responsiveness, including commitments shaped by Te Tiriti o Waitangi, the 1840 agreement between Māori and the British Crown. In an educational context, the principles of Te Tiriti o Waitangi underpin the partnership between Māori and educational institutions, protection of Māori language, culture, identity, and knowledge, and participation of Māori learners and whānau in education (Orange, 2015). Such commitments call for teaching approaches that acknowledge identity, language, and lived experience as central to learning. Within this context, frameworks such as FoK and AE offer relevant but distinct perspectives. FoK highlights the cultural and intellectual resources embedded in students' home and community practices (Moll et al., 1992), while AE foregrounds relational practice and the affirmation of strengths (Bloom et al., 2013).

Although each framework contributes to ongoing discussions about engagement and equity, research on their integration within mathematics education remains limited. Exploring how relational affirmation and sociocultural recognition might be brought together offers a potential direction for rethinking mathematics teaching in culturally diverse classrooms.

Literature Review

In New Zealand, mathematics education in primary and secondary school classrooms is shaped by national curriculum frameworks and sociocultural diversity (Ministry of Education, 2007). Although the New Zealand Curriculum promotes learner-centred approaches and contextualised learning, classroom practices often remain procedural and textbook-driven (Anthony & Walshaw, 2007). Research indicates that when students' home practices, cultural narratives, and epistemologies are not recognised in the classroom, their motivation and participation in mathematics decline (Hunter & Anthony, 2011). The recent curriculum refreshes prioritised cultural responsiveness, identity development, and inclusiveness, while also emphasising learner progress and the development of mathematical knowledge, skills, and competencies (Ministry of Education, 2023). These priorities align with psychological principles related to autonomy, competence, and relatedness (Deci & Ryan, 1985). In classroom learning, supporting these needs is associated with stronger engagement and well-being (Niemiec & Ryan, 2009). This is relevant to mathematics learning because students with low mathematics self-efficacy may have less confidence in their capacity to solve mathematical problems, while mathematics anxiety is negatively associated with mathematics achievement (Barroso et al., 2021; Pajares & Miller, 1994). Primary and secondary students seek recognition and affirmation in learning contexts (Yu & Zhang, 2021), and appropriate use of appreciation can enhance engagement and confidence (Yang, 2024). These developmental and contextual considerations highlight the importance of relational, culturally responsive approaches in mathematics education.

Appreciative Education

Appreciative Inquiry (AI) was first proposed as a strengths-based approach to organisational and social change (Cooperrider & Srivastva, 1987). AI emphasises identifying and leveraging the existing strengths, successes, and potential within a system and follows a cyclical process known as the 4-D model: Discovery, Dream, Design, and Destiny (Cooperrider & Srivastva, 1987).

Building on these foundational principles, AE has emerged as an educational adaptation of AI, developed to affirm students' strengths, cultivate trust-based teacher–student relationships, and support holistic learner development (Bloom et al., 2013). AE is a framework for delivering high-quality education at both the individual and organisational levels, adapting the principles of appreciative inquiry to educational settings (Bloom et al., 2013). It is rooted in social constructivism, positive psychology, and appreciative inquiry. AE is fundamentally underpinned by social constructivism, which views learning as an active and collaborative process. AE emphasises open-ended questioning, mutual learning, and the recognition of individual strengths, creating a classroom environment where students' existing knowledge is valued and expanded. A second key foundation of AE lies in positive psychology, which emphasises the cultivation of individual strengths, positive emotions, and psychological well-being (Seligman & Csikszentmihalyi, 2000; Seligman, 2011). Rather than focusing on grades or correcting behaviour, AE supports the whole student by fostering a supportive and encouraging environment (Bloom et al., 2013). In mathematics classrooms, such an approach could reduce anxiety, increase student engagement, and create a safe space where students feel confident to try, fail, and grow (Yan & Li, 2022; Yang, 2024).

An essential component of AE is the appreciative mindset (Bloom et al., 2013), which underpins its relational and strengths-based philosophy. Rather than concentrating on students' deficits or shortcomings, this mindset encourages educators to intentionally seek out and affirm students' strengths, potential, and past successes. To operationalize this mindset, AE follows a cyclical process known as the 6-D model: Disarm, Discovery, Dream, Design, Deliver, and Don't settle (Bloom et al., 2013). This cycle includes two additional phases at the beginning

and end compared to the appreciative inquiry framework, emphasising the creation of a trusting environment and ongoing development.

Recent studies have demonstrated AE's impact across various educational contexts, including student advising, classroom instruction, and institutional leadership (Bloom et al., 2013; He et al., 2014; Mather & Smith, 2021). Furthermore, AE has been applied in mathematics education. At the primary level, Yang (2024), in the Chinese context, highlighted the use of stories, games, and real-life situations in primary mathematics teaching to promote students' interest, participation, confidence, and problem-solving skills. Although the study did not explicitly use the AE framework, its focus on positive participation, learner confidence, and meaningful everyday contexts is consistent with AE principles. Most AE studies are situated outside New Zealand, and there appears to be very limited research examining this issue within the context of Aotearoa New Zealand. This gap presents a valuable opportunity to explore how AE can be meaningfully adapted to support culturally responsive mathematics teaching in New Zealand primary and secondary classrooms.

Funds of Knowledge

Early research by Moll et al. (1992) demonstrated how integrating students' everyday family activities and cultural practices into teaching enhances curriculum relevance and fosters equity. González et al. (2005) further showed that cultural heritage, multilingual abilities, and community-based knowledge constitute valuable intellectual resources. These theoretical developments were extended through the concept of funds of identity (Esteban-Guitart & Moll, 2014), which emphasises identity construction and cultural negotiation as central to equitable and meaningful learning.

The application of FoK in mathematics education has yielded significant insights. Civil (1994, 2002, 2007), working largely in elementary and community-based contexts, demonstrated how family practices such as cooking and budgeting contain rich mathematical concepts that enhance classroom engagement when incorporated into instruction. Walkington (2017), in a secondary algebra context, quantitatively validated personalised algebra problems based on students' interests, revealing significant learning gains. Williams et al. (2020) and Gilde and Volman (2021) further illustrated across school settings how integrating FoK can foster inclusivity and equity within mathematics classrooms.

In the New Zealand context, FoK-informed studies have contributed to culturally responsive pedagogy (Cowie et al., 2011; Hogg, 2013, 2016), aligned with national frameworks such as Tātaiako (Ministry of Education, 2011) and Tapasā (Ministry of Education, 2018). Research has shown how integrating Māori and Pacific knowledge can enhance engagement and strengthen learner identities. Across these studies, FoK has been developed through diverse methodological approaches and contextual adaptations, reflecting both its flexibility and its complexity.

At the same time, important theoretical and practical tensions remain. Critical scholarship questions how FoK can be selectively recognised and enacted in classroom practice. Selective enactment may privilege experiences perceived as safe or socially acceptable while marginalising those shaped by inequality or structural injustice (Bylica, 2023). This concern is discussed through the concept of dark funds of knowledge (Zipin, 2009, 2025), which draws attention to lived experiences often overlooked in school contexts yet possessing educative use-value. Scholars have also questioned how far FoK can be sustained within the everyday constraints of schooling (McIntyre et al., 2001; Rios-Aguilar et al., 2011). From a New Zealand perspective, Hogg (2011) notes that engaging seriously with students' community knowledge demands ongoing relational labour and pedagogical judgement that can be difficult to sustain.

Despite these challenges, FoK continues to offer a valuable framework for recognising students' lived experiences as intellectual resources for learning (Moll et al., 1992; González et

al., 2005). By positioning students' cultural and community knowledge as assets rather than deficits, FoK provides important possibilities for creating more inclusive and meaningful mathematics classrooms (Civil, 2007).

Research Gap and Conceptual Method

Two key gaps emerge from the literature reviewed. The first gap concerns the limited conceptual and empirical understanding of how pedagogical practices informed by AE and FoK are enacted in secondary mathematics classrooms. While both frameworks have been widely discussed and documented, particularly in primary contexts, less is known about how they operate within the subject-specific demands of secondary mathematics, including curricular sequencing, assessment pressures, and disciplinary norms.

The second gap relates to a potential decline in student engagement and confidence as they move into secondary school. Although AE and FoK both emphasise students' strengths, agency, and lived experiences, they have rarely been examined together in classroom-based research. As a result, it remains unclear how these approaches might be integrated to support coherent and sustainable practice in secondary mathematics classrooms.

Addressing these gaps may offer several potential benefits for mathematics classrooms. Students may benefit from opportunities to reflect on their learning and recognise their own strengths in mathematics. Incorporating students' cultural knowledge and lived experiences into classroom activities may also support a stronger sense of belonging and engagement. For teachers, integrating AE and FoK may provide insights into strengths-based and culturally responsive teaching strategies. An appreciative approach may also support teachers in recognising students' strengths and creating classroom environments that encourage meaningful participation in mathematics learning. These considerations highlight the need for a framework that connects relational affirmation with culturally grounded knowledge in mathematics learning.

A conceptual methodological approach was used to develop a combined AE and FoK framework. First, the authors conducted a focused review of literature relating to AE and FoK to develop a clear understanding of the core principles, applications, and limitations of each framework. The two frameworks were then mapped side-by-side by identifying their central concepts, focal practices, educational applications, and points of difference and complementarity. Particular attention was given to how AE's emphasis on affirming students' strengths could align with FoK's focus on valuing students' cultural, family, and everyday experiences within mathematics education. Through this comparative analysis, areas of conceptual overlap and complementarity were identified and organized into comparative tables to form the basis of a combined AE-FoK conceptual framework.

Proposed AE–FoK Framework

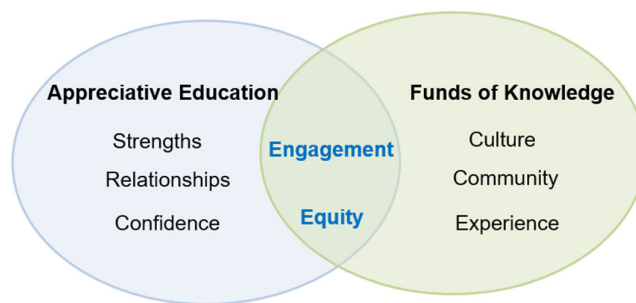
AE and FoK come from different theoretical traditions, yet they share several assumptions that support their integration in mathematics education. Both approaches adopt a strengths-based view of learners and understand learning as socially constructed through interaction and experience. AE draws on social constructivism and positive psychology and emphasises recognising students' strengths through positive dialogue and supportive relationships that nurture motivation and well-being (Bloom et al., 2013). FoK highlights the cultural knowledge and lived experiences students develop within families and communities and treats these experiences as valuable resources for classroom learning (Moll et al., 1992; González et al., 2005). These shared foundations suggest that AE and FoK can work productively together.

Despite these overlaps, the two frameworks emphasise different aspects of teaching and learning. AE emphasises encouragement, trust, and affirmation as central to students' confidence and participation in learning. Previous studies have shown that appreciative

practices can strengthen engagement and foster positive learning environments across educational contexts (He & Hutson, 2018; Mather & Smith, 2021). FoK focuses on the knowledge and experiences students draw from their families, cultures, and communities (Moll et al., 1992). Research in mathematics education shows that connecting instruction to students' everyday mathematical practices can enhance relevance and participation in mathematics learning (Civil, 1994, 2002, 2007; Walkington, 2017). AE is primarily concerned with the relational conditions of learning, while FoK emphasises the knowledge students contribute to the learning process. Together, these perspectives play complementary roles in classroom practice. Figure 1 illustrates the similarities, differences, and complementary relationship between AE and FoK.

Figure 1

Conceptual relationship between AE and FoK



Using AE with the support of FoK, teachers' encouragement has the potential to become more specific and meaningful. Instead of offering general praise, teachers can recognise strengths connected to students' real family and cultural experiences. Appreciation is therefore grounded in identifiable learning resources rather than remaining abstract. The strengths of students become more visible in mathematical activity, enabling teachers to connect encouragement more directly to learning. As a result, AE becomes more meaningful within mathematics classrooms.

When FoK is enacted within an appreciative learning environment, classroom participation potentially could change in noticeable ways. A supportive and affirming atmosphere encourages learners to share experiences that might otherwise remain outside mathematical discussion. As trust develops through ongoing interaction, teachers become more able to recognise forms of knowledge connected to family, cultural, and community practices rather than relying only on surface-level responses. Research in culturally responsive mathematics education shows that participation increases when everyday experiences are treated as legitimate resources for learning (González et al., 2001; Williams et al., 2020). In this context, appreciative practices help create relational conditions that could enable FoK to emerge more fully within mathematics learning.

These processes reveal a reciprocal relationship between the two frameworks. AE supports the discovery of FoK, while FoK grounds appreciative practices in lived experience. Their integration has the potential to support both emotional well-being and engagement in mathematics learning and responds to concerns about declining participation and persistent equity gaps in the New Zealand context.

Although AE and FoK arise from different theoretical traditions, both align with culturally responsive and strength-based approaches to education. AE emphasises relational and emotional support through affirmation and trust-building, whereas FoK highlights learning grounded in cultural and lived experience. Considered together, the two perspectives support a

form of teaching that is both relationally focused and culturally responsive. A comparison between AE and FoK is presented in Table 1.

Table 1

Conceptual comparison of AE and FoK

Dimension	AE	FoK
Core Concept	Uses a strengths perspective to uncover potential in students, teachers, and schools.	Views students' family cultural capital and lived experiences as core teaching resources.
Focus	Students' strengths, potential, and growth narratives.	Students' cultural knowledge, background, family skills, and community experiences
Applications	School-wide culture reform, mental health programs, teacher training.	Multicultural classrooms, family-school partnerships, community-based curriculum.
Limitations	May overlook structural inequalities (e.g., resource disparities).	Requires significant teacher time for family knowledge collection.
Complementarity	AE provides a strengths-based pedagogical orientation.	FoK provides content (cultural resources).

In the Aotearoa New Zealand context, discussions of equity in mathematics education have increasingly drawn attention to both relational teaching practices and the recognition of culturally grounded knowledge. AE and FoK have typically been discussed along these lines separately, with one emphasising relationship and the other emphasising lived experience. Bringing these perspectives together reframes mathematics learning as shaped through their interaction rather than through either dimension alone. The AE–FoK framework proposed in this paper conceptualises engagement in mathematics as emerging through the ongoing interplay between relational affirmation and culturally grounded experience, offering a way to understand participation as relationally and culturally constituted within diverse classrooms. Future research would involve investigating the enactment of AE–FoK integration in secondary mathematics classrooms, particularly how relational affirmation may enable students' FoK to become visible and meaningfully connected to mathematical content.

Conclusion

This paper has proposed an integrated AE–FoK framework that brings together AE and FoK to offer a relational and culturally grounded perspective on mathematics learning. Rather than treating engagement as an individual learner characteristic, the framework conceptualises participation as emerging through the interaction between supportive relationships and lived experience. Appreciative practices help establish trust and recognition, while culturally grounded knowledge provides meaningful connections between students' everyday experiences and mathematical activity. In secondary mathematics classrooms, the framework could guide teachers to start from students' experiences and use appreciative dialogue to connect these experiences with mathematical tasks and discussion.

References

- Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics / Pāngarau: Best evidence synthesis iteration*. Ministry of Education.
- Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A meta-analysis of the relation between math anxiety and math achievement. *Psychological Bulletin*, 147(2), 134–168. <https://doi.org/10.1037/bul0000307>
- Bloom, J. L., Hutson, B. L., He, Y., & Konkle, E. (2013). Appreciative education. *New Directions for Student Services*, 2013(143), 5–18. <https://doi.org/10.1002/ss.20055>

- Bylica, K. (2023). “You can only choose from the things You know”: Engaging with students’ dark and politicized funds of knowledge in the music classroom. *Education and Urban Society*, 55(5), 617–640. <https://doi.org/10.1177/00131245221087995>
- Civil, M. (1994). Connecting the home and the school: Funds of knowledge for mathematics teaching and learning. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA, United States.
- Civil, M. (2002). Everyday mathematics, mathematicians’ mathematics, and school mathematics: Can we bring them together? *Journal for Research in Mathematics Education Monograph*, 11, 40–62. <https://doi.org/10.2307/749964>
- Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In *Improving access to mathematics* (pp. 105–117). Teachers College Press.
- Cooperrider, D. L., & Srivastva, S. (1987). Appreciative inquiry in organizational life. In *research in organizational change and development* (Vol. 1, pp. 129–169). JAI Press.
- Cowie, B., Jones, A., & Otrell-Cass, K. (2011). Re-engaging students in science: Issues of assessment, funds of knowledge and sites for learning. *International Journal of Science and Mathematics Education*, 9(2), 347–366. <https://doi.org/10.1007/s10763-010-9229-0>
- Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behavior*. Springer. <https://doi.org/10.1007/978-1-4899-2271-7>
- Esteban-Guitart, M., & Moll, L. C. (2014). Funds of identity: A new concept based on the funds of knowledge approach. *Culture and Psychology*, 20(1), 31–48. <https://doi.org/10.1177/1354067x13515934>
- Gilde, J. t., & Volman, M. (2021). Finding and using students’ funds of knowledge and identity in superdiverse primary schools: A collaborative action research project. *Cambridge Journal of Education*, 51(6), 673–692. <https://doi.org/10.1080/0305764x.2021.1906845>
- González, N., Andrade, R., Civil, M., & Moll, L. C. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, 6(1–2), 115–132. https://doi.org/10.1207/s15327671espr0601-2_7
- González, N., Moll, L. C., & Amanti, C. (2005). *Funds of knowledge: Theorizing practices in households, communities, and classrooms*. Lawrence Erlbaum Associates. <https://doi.org/10.4324/9781410613462>
- He, Y., Hutson, B. L., & Bloom, J. L. (2014). A Call for action to engage in appreciative education. *Journal of Appreciative Education*, 2(1), 1–9.
- He, Y., & Hutson, B. L. (2018). Exploring and leveraging Chinese international students’ strengths for success. *Journal of International Students*, 8(1), 87–108. <https://doi.org/10.32674/jis.v8i1.153>
- Hogg, L. (2011). Funds of knowledge: An investigation of coherence within literature. *Teaching and Teacher Education*, 27(3), 666–667.
- Hogg, L. (2013). *Applying funds of knowledge in a New Zealand high school: The emergence of team-based collaboration as an approach*. <https://doi.org/10.26686/wgtn.17005948>
- Hogg, L. (2016). Applying funds of knowledge theory in a New Zealand high school: New directions for pedagogical practice. *Teachers and Curriculum*, 16(1), 49–55. <https://doi.org/10.15663/tandc.v16i1.108>
- Hunter, R., & Anthony, G. (2011). Forging mathematical relationships in inquiry-based classrooms with Pasifika students. *Journal of Urban Mathematics Education*, 4(1), 98–119. <https://doi.org/10.21423/jume-v4i1a100>
- Mather, P. C., & Smith, T. M. (2021). Implementing appreciative education in a first-year seminar. *Journal of Appreciative Education*, 7(1). <https://janeway.uncpress.org/jae/article/id/883/>
- McIntyre, E., Rosebery, A. S., & González, N. (2001). *Classroom diversity: Connecting curriculum to students’ lives*. Heinemann.
- Ministry of Education. (2007). *The New Zealand curriculum*. Learning Media.
- Ministry of Education. (2011). *Tātaiako: Cultural competencies for teachers of Māori learners*. <https://teachingcouncil.nz/en/grow-my-practice/professional-learning/tataiako-for-teachers-of-maori-learners>
- Ministry of Education. (2018). *Tapasā: Cultural competencies framework for teachers of Pacific learners*. <https://teachingcouncil.nz/en/grow-my-practice/professional-learning/tapasa-for-teachers-of-pacific-learners>
- Ministry of Education. (2023). *Te Mātaiaho: The refreshed New Zealand curriculum* (draft). <https://www.mgc.school.nz/wp-content/uploads/2023/05/Te-Mataiaho-Draft-NZ-Curriculum-March-2023.pdf>
- Moll, L. C., Amanti, C., Neff, D. L., & González, N. (1992). Funds of knowledge for teaching: Using a qualitative approach to connect homes and classrooms. *Theory Into Practice*, 31(2), 132–141. <https://doi.org/10.1080/00405849209543534>

- Moschkovich, J. N. (2012). Mathematics, the Common Core, and Language: Recommendations for mathematics instruction for ELs aligned with the common core. *Journal of Urban Mathematics Education*, 5(1), 45–57.
- Niemiec, C. P., & Ryan, R. M. (2009). Autonomy, competence, and relatedness in the classroom: Applying self-determination theory to educational practice. *Theory and Research in Education*, 7(2), 133–144. <https://doi.org/10.1177/1477878509104318>
- Orange, C. (2015). *The Treaty of Waitangi*. Bridget Williams Books.
- Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology*, 86(2), 193–203. <https://doi.org/10.1037/0022-0663.86.2.193>
- Rios-Aguilar, C., Kiyama, J. M., Gravitt, M., & Moll, L. C. (2011). Funds of knowledge for the poor and forms of capital for the rich? A capital approach to examining funds of knowledge. *Theory and Research in Education*, 9(2), 163–184. <https://doi.org/10.1177/1477878511409776>
- Seligman, M. E. P. (2011). *Flourish: A visionary new understanding of happiness and well-being*. Free Press. <https://doi.org/10.5860/choice.48-7217>
- Seligman, M. E. P., & Csikszentmihalyi, M. (2000). Positive psychology: An introduction. *American Psychologist*, 55(1), 5–14. <https://doi.org/10.1037/0003-066x.55.1.5>
- Walkington, C. (2017). Design research on personalized problem posing in algebra. In *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (pp. 195–202).
- Williams, J. J., Tunks, J., González-Carriedo, R., Faulkenberry, E., & Middlemiss, W. (2020). Supporting mathematics understanding through funds of knowledge. *Urban Education*, 55(3), 476–502. <https://doi.org/10.1177/0042085916654523>
- Yan, H., & Li, X. (2022). A study on the application of appreciation education in teaching mathematics. *Research and Advances in Education*, 1(1), 62–67. <https://doi.org/10.56397/RAE.2022.07.09>
- Yang, T. (2024). How to create effective problem situations in primary school mathematics teaching. *Journal of Theory and Practice of Contemporary Education*, 4(3).
- Yu, Y., & Zhang, Y. (2021). Personality and developmental characteristics of primary school students' personality types. *Frontiers in Psychology*, 12, 693329.
- Zipin, L. (2009). Dark funds of knowledge, deep funds of pedagogy: Exploring boundaries between lifeworlds and schools. *Discourse: Studies in the Cultural Politics of Education*, 30(3), 317–331. <https://doi.org/10.1080/01596300903037044>
- Zipin, L. (2025). The educative use-value of 'dark funds of knowledge' in dark-age time. *Studies in Psychology: Estudios de Psicología*, 46(1–3), 475–519. <https://doi.org/10.1177/02109395251396930>

Changing Conditions to Enable Student Problem Posing: A Practice Architectures Analysis

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Posing mathematical problems is an inherent part of how we engage with mathematics in our daily lives, yet teaching students to make sense of situations and pose problems is not common practice. This paper explores the implementation of student problem posing across two iterative cycles of implementation. It draws on the Theory of Practice Architectures as an analytical framework to examine how site-specific practice architectures reconfigured across cycles to further enable student problem posing. This paper offers insights into how classroom conditions and reconfigurations make possible particular forms of participation and engagement during mathematical problem posing.

Mathematical Problem Posing (MPP) is defined as “the process of formulating and expressing a problem within the domain of mathematics” (Zhang & Cai, 2021, p.962), and includes both the generation of new problems, as well as the reformulation of existing problems (Silver, 1994). MPP is commonly categorised as free, semi-structured, or structured (Stoyanova & Ellerton, 1996). In free problem posing situations, as enacted in this study, students are invited to generate mathematical problems from a given situation, such as a visual image, without predetermined constraints (Stoyanova & Ellerton, 1996).

More than three decades after Silver’s (1994) seminal paper identified problem posing as a significant instructional shift from teacher-centred to student focused practice, the challenge of implementing problem posing persists. Despite problem posing being an aim of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA, n.d.]), mathematics tasks often remain teacher-centred, prioritising the application of prescribed procedures and the production of correct answers over the exploration of underlying mathematical ideas (Schoenfeld, 2017). This didactic teaching approach has been further reinforced by recent, contested “Science of Maths” movements that foreground direct, explicit instruction over other pedagogical approaches (National Council of Supervisors of Mathematics [NCSM], 2025). Nevertheless, interest in MPP research has continued to increase, reflected in recent journal special issues, books, conferences, review papers, and meta-analyses (see Cai et al., 2024). Drawing on the Theory of Practice Architectures (TPA), this paper examines how student problem posing is shaped by the practice architectures within the site, highlighting that the enactment of problem posing cannot be understood in isolation from the conditions in which it occurs.

Background Literature

Research on MPP is often categorised into three interconnected perspectives, problem posing as a cognitive activity, as a learning goal, and as an instructional approach (Cai et al., 2024). Problem posing as a cognitive activity examines the process of how individuals construct personal interpretations of situations and formulate them into mathematical problems (Stoyanova & Ellerton, 1996), problem posing as a learning goal views the development of problem posing skills as a direct instructional objective, and problem posing as an instructional approach involves teaching mathematics through problem posing (Cai et al., 2024). While each perspective offers a distinct lens for examining problem posing, they are not mutually exclusive and frequently overlap in research and practice (Cai et al., 2024). For example, while problem posing as a learning goal is the primary focus of this article, as the classroom conditions for

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (pp. 457–464). MERGA.

enabling students to problem pose are examined, in the larger doctoral study, problem posing was also used to review and teach particular mathematical concepts (instructional approach), and reveal insights into students' mathematical thinking and understanding (cognitive activity).

Research that positions problem posing as a learning goal focuses on development of students' capacity to generate problems. Early research conducted by English (1998) found that informal contexts, such as stimulus pictures and literature, were more successful in encouraging generation of diverse problem types in primary school than when asked to pose problems from provided number-sentences, or non-goal-specific statements such as "Sarah has five dolls on one shelf in her room and four toy cars on another shelf" (p.88). Lewis and Colonnese (2021) used Three-Act-Tasks to develop Year 2 students' capacity to pose problems, highlighting the use of teacher questioning, modelling, concrete materials, and generation of sequels to the Three-Act-Tasks as supportive to students developing ability. Cai and Hwang (2021) reported findings from a longitudinal study that investigated the impact of professional development on teachers' learning to integrate problem posing into the classroom, and on its impact on students' learning outcomes. Case studies from the longitudinal study provide insight into understanding how teachers learn to integrate problem posing in their classrooms (see Zhang & Cai, 2021). While research investigating how teachers implement MPP continues to grow (Cai et al., 2023), less attention has been given to how site-specific arrangements shape its enactment. Using the TPA, this paper addresses this gap by examining how cultural-discursive, material-economic, and social-political arrangements are reconfigured across two cycles of implementation to further enable student problem posing. The following research question frames this paper: *How are site-specific practice architectures reconfigured across cycles of implementation to further enable student mathematical problem posing?*

Research Design

This study adopted a Critical Participatory Action Research (CPAR) methodology (Kemmis, McTaggart et al., 2014) and drew on the TPA to examine how teacher practices were enabled and constrained during iterative cycles of problem posing and solving sequences. The TPA was employed as an analytical framework to attend to the interrelated cultural-discursive, material-economic, and social-political arrangements that shape practice (Grootenboer & Edwards-Groves, 2023). Through this lens, close analysis of how practices are enacted in particular moments and settings was possible, making visible the conditions that enable or constrain practices and the ways these conditions can be reconfigured over time in relation to the intentions, actions, and purposes of the practice (Grootenboer & Edwards-Groves, 2023).

Theoretical Framework

According to Kemmis and Grootenboer (2008), the TPA proposes that practices are comprised of three entwined, socially constructed activities: sayings, doings, and relating. Practices both shape, and are shaped by, cultural-discursive, material-economic, and social-political arrangements, which are interconnected and held together in practice-arrangement bundles (Grootenboer & Edwards-Groves, 2023). The TPA has been used to examine practices and identify what enables and constrains practices in various fields (e.g., Gibbs et al., 2022; Rönnerman et al., 2023). It is one of many practice theories that focus on "what happens: how life unfolds - and how practices unfold, in the intersubjective space in which we encounter one another and the world" (Kemmis, 2021, p.7). The TPA provides a theoretical way of understanding education practices (e.g., teaching), with the discernment that practices are held in place by practice architectures, the cultural-discursive, material-economic, and social-political arrangements and conditions in the site (Kemmis & Grootenboer, 2008). These

arrangements and conditions enable and constrain the sayings, doings, and relatings that form a practice (Edwards-Groves & Kemmis, 2016). Cultural-discursive arrangements shape the language and discourses through which practices are understood and enacted; material-economic arrangements shape the activities and use of space-time; and social-political arrangements shape relationships, power and solidarity within and across practices (Kemmis et al., 2017). Crucially, it is not these arrangements in isolation that shape practice, but their dynamic interplay within a site (Kemmis, McTaggart et al., 2014). Across sites such as lessons, classrooms, or school communities, practices and practice architectures are understood to “hang together” as part of the project of a practice, encompassing the intentions, actions and purposes that orient practice towards particular ends (Kemmis, Heikkinen et al., 2014).

Context and Participants

This study was conducted in a P-12 independent school in South-East Queensland with three Year 6 teachers: Seth (six years teaching experience), Ella (14 years), Kelly (17 years) (pseudonyms) and their classes. The teaching team was in their fifth year of working together. Kelly participated in the initial phase of the study and contributed to the first CPAR cycle before withdrawing due to personal reasons. Ella had previously participated in a research project exploring instructional strategies to support students’ problem posing (Zorn, 2022). Subsequent informal discussions about the challenges of implementing these practices, particularly in supporting students to pose problems from a prompt, prompted the development of the present study.

Data Analysis

Data Analysis employed Fereday and Muir-Cochrane’s (2006) hybrid approach to thematic analysis to analyse transcript data from planning meetings, classroom observations, teacher reflections, student focus group discussions, and researcher field notes. Following inductive thematic analysis, to identify the teacher practices and inquiry sequence for problem posing and solving, the TPA was applied as a deductive coding framework to analyse how the cultural-discursive, material-economic, and social-political arrangements enabled and constrained implementation across the inquiry phases. Tables of invention (Kemmis et al., 2014) were used as mapping tools to analyse data for each teaching practice and inquiry phase in relation to the practice architectures. To examine changes over time, a comparative table of invention was developed to map shifts from Cycle One to Cycle Two and the dialectical relationship between changes in practice and the practice architectures.

Results and Discussion

This section presents a comparative analysis of Cycle One and Cycle Two, focusing on how changes to classroom conditions enabled student problem posing and shaped the nature and breadth of the questions generated. While a brief description of what occurred in each cycle is provided to establish context, the analysis foregrounds connections between changing conditions, teacher practice, and student problem posing. Drawing on the TPA, the discussion examines how reconfigured cultural-discursive, material-economic, and social-political arrangements interacted with students’ developing capacities to pose mathematical problems. In doing so, this section attends to the problems students posed, as well as how problem posing practices emerged through the interdependence of teaching and learning across cycles, that being how student learning shaped teacher practices, and in return shaped student learning. Findings are presented in chronological order.

Problem Posing Through the Cycles

During the initial cycle, the teachers collaboratively selected a lifeguard service sign image as the problem posing prompt, based on its perceived familiarity to the students. This rationale was articulated by all three teachers, who noted that students “would all be familiar with that, like from living around here”, that “it’s summer” and that the students had explored “beach safety in Year 5”. Cycle One unfolded across multiple sessions spaced throughout the term, with problem posing and solving revisited over time, rather than enacted in a single block. MPP was initially introduced through whole-class modelling, with teachers presenting a real-life object such as a coffee cup and asking students, “what math questions could I possibly ask about a cup?”. While this supported students’ initial engagement with and orientation to MPP, subsequent small group work with the lifeguard service sign revealed limitations in the depth of questions generated. Working in groups of four or five, many students posed surface-level questions, such as, “how many words are there [on the sign]? How many people in the background? How many different colours?”. Seth also reported that his students prioritised “getting it done” by posing a certain number of questions, rather than focusing on depth.

As a result, Cycle One included two sessions in each classroom focused on (1) supporting students to pose questions, and (2) increase the challenge of their questions by connecting them to mathematical concepts and distinguishing between mathematical and non-mathematical questions. Additionally, during student focus group discussions, the students expressed frustration at selecting problems to solve with peer groups. As one student explained, “we have like some people wanting different questions...it was kind of hard to choose one single question”, while another noted, “it was just annoying...we all found good questions that we can choose and then we’re like, we should pick this one, though, but no, actually that one, no I like this one”. Although this challenge was not directly related to the generation of problems, it highlighted tensions in the problem selection phase, which was critical in the progression from problem posing to problem solving. Student-generated and selected problems from Cycle One are presented below (Table 1), grouped into four focus categories.

Table 1

Cycle one student generated problems

Category	Questions
Sign production	How much does it cost to make the sign? How big is the sign? Where did they buy the materials? How much of each material do you need? How many people took [did it take] to make it?
Lifeguard working hours	How many hours are lifeguards on duty? What is the lifeguard’s schedule? How many lifeguards are on duty? How many minutes has one lifeguard been on duty in a week? Do they work longer hours on New Years Eve? How many hours/weeks/months is the beach open? How many hours is the lifeguard working for in a day? Are there any closing times?
Grains of sand	How many grains of sand are in a cubic metre? How big is the average grain of sand? How many grains of sand fit in a centimetre?
Duration of tides	How long does it take from high tide to low tide? Is it different at different times? What is the average time?

In Cycle Two, the problem posing and solving sequence was enacted within a concentrated block of time, completed across consecutive sessions (n=5). In response to student feedback following Cycle One, such as, “give us a variety of photos”, “different options”, and “choice of photos” students independently generated questions using a wider range of eight teacher selected visual prompts positioned around the classroom. Students circulated between the

prompts, posing questions independently for ten minutes, before working collaboratively in groups of 3-4 to identify and cross out questions they considered to be non-mathematical. Students were then paired to select a prompt and problem to pursue, with opportunities to pose further questions and identify additional information required to solve their problem. As predicted by the teachers, “the more often you do it, they will straightaway jump to the mathematical question”, the problem posing phase in Cycle Two was completed within a single session. Student-generated and selected problems from Cycle Two are presented below (Table 2) and have been categorised based on the prompt. While eight prompts were provided, the eighth prompt, a photograph from the recent school camp, was not selected by any student pairs, therefore is not included in table.

Table 2

Cycle Two student generated problems

Prompt	Questions
Watermelon	How many watermelons are in this [image]? What’s the weight of the watermelons in this picture? How heavy is one quarter of a watermelon? How much does one whole watermelon cost?
Bottles of glue	How much glue was there combined in all the bottles? How much glue does one bottle contain? How many bottles are there? What is the viscosity of the glue? How many litres are in 24 bottles? How expensive is a single glue bottle? How much do all the bottles cost? How much slime could you make from all the glue in the picture?
Tray of chocolate	How many cubic centimetres are in a chocolate bar? How many litres of chocolate would it [the tray of chocolate] make?
Sprite bottle stack	How many bottles are there? How much did all of them cost? How many millilitres are in all the bottles? How much do all of them weigh?
Olympic swimming results	How much water is in the pool? (capacity in cubic meters)
Building heights	If the Burj Khalifa was a pyramid [similar to the Pyramid of Giza], how big would the base be? How much did the buildings cost all combined? With inflation how much would it be today? What is the height difference between the B67 (fictional tower from movie interest) and the Burj Khalifa?
Lemons	How many whole lemons are in the picture? How much juice do they make? How much did they cost? What fractions can you make from the lemons?

Changing Conditions

According to the TPA, changing the practice architectures is a fundamental way to change the practices themselves. The practice architectures are the cultural-discursive, material-economic, and social-political arrangements that enable and constrain what is said, done, and how people relate to others and the world (Kemmis et al., 2025). These arrangements prefigure, but do not predetermine, the actions undertaken in a site (Mahon et al., 2018). This section examines how the conditions supporting MPP were reconfigured from Cycle One to Cycle Two. Drawing on the TPA, the analysis focuses on how changes to the practice architectures shaped what became possible in students’ problem posing practices. By comparing the two cycles, the below section illustrates how shifts in prompts, and task structure and participation altered the breadth of problems posed and the ways students engaged with problem posing.

Prompts

In Cycle One, while the single Lifeguard Service Sign prompt afforded some rich problem possibilities (e.g. schedules, tides, materials), these only became visible with teacher support to move students away from initial low-level questions focused on counting words, colours, and people. Reliance on a single context also limited opportunities for students to draw on diverse prior knowledge. The introduction of multiple prompts in Cycle Two expanded students' access to prior knowledge by providing a wider range of contexts through which they could draw on personal experiences and existing mathematical knowledge. This reconfiguration of the material-economic arrangements increased the likelihood that students would encounter a prompt that resonated with their lived experiences, disciplinary knowledge, or interests. For example, the glue bottle prompt enabled some students to draw on interests in slime making to pose related problems, while the building heights prompt supported connections to geometric and proportional reasoning. Similarly, the lemons prompt supported students to connect everyday experiences with food and fractions, generating questions about quantities and juice yield.

These examples illustrate how reconfiguring the material-economic arrangements facilitated a shift in the cultural-discursive arrangements by expanding the range of language and meanings available for problem posing. Also evident was the students' reliance on familiar discursive resources, such as established counting and measurement question structures (e.g. how many/much), reflecting established cultural-discursive arrangements within mathematics lessons in this site. In this way, multiple prompts functioned not simply as additional resources, but as conditions that enabled students to draw on varied prior knowledge and make connections between everyday experiences and mathematical concepts. The use of multiple prompts also positioned students as valuable contributors through their feedback and provided them with choice, altering the social-political dynamics within the practice. This shift supported increased student ownership, responsibility, and agency within the practice, potentially promoting further engagement (Kemmis et al., 2025).

Task structure and participation

In the initial cycle, problem posing was introduced to the students through whole-class teacher modelling and co-construction using familiar, real-life objects (e.g., mug, water bottle). However, the lesson quickly shifted to collaborative group work in groups of four or five, where many students initially struggled to pose year-level appropriate questions and tended to focus on easily accessible concepts, while others focused on quickly generating questions to complete the task. Group brainstorming relies on specific discourses that encourage participation, with shared language and conceptual resources enabling or constraining the breadth and depth of ideas generated (Mahon et al., 2018). In this context, cultural-discursive arrangements prefigured what could be said and shared, while social-political arrangements, including existing power relations within the site, may have led to particular voices being more dominant than others, contributing to limited exploration.

In the second cycle, students began by independently posing problems across multiple prompts, enabling more equitable participation and a wider range of questions being generated. Reconfiguring the timing, structure, and groupings (material-economic arrangements), simultaneously reshaped the social-political and cultural-discursive arrangements, altering how students engaged with problem posing, with one another, and with teachers. Independent work allowed students to draw on their prior knowledge and use language accessible to them, while teachers were able to engage in more targeted dialogue with individual students, for example, "Are those full lemons? If you don't know how many of those pieces make a lemon, what would

the question be?”. Changes to group size supported more efficient problem selection, enabling students to transition more smoothly into the problem-solving phase.

Conclusion and Implications

This study examined the implementation of MPP across two iterative cycles of CPAR. Drawing on the TPA as an analytical lens, the study explored how cultural-discursive, material-economic, and social-political arrangements were reconfigured across cycles to support the implementation of problem posing and solving. It contributes to the growing body of problem posing research by shifting attention from tasks or student products alone to the conditions that make possible particular forms of participation and engagement with mathematical ideas during MPP.

Across the cycles, changes in student problem posing did not occur simply because students became better problem posers but because the practice architectures were progressively reconfigured. Shifting from a single prompt to multiple prompts expanded access to prior knowledge and interests, enabling students to connect personal experiences to mathematical ideas, and reducing reliance on teacher support. Reconfiguring task structures and participation to include independent problem posing before collaborative refinement supported more equitable participation, allowing for a broader range of questions to be shared. Additionally, changes in group size addressed student frustration around problem selection, supporting a smoother transition to the problem-solving phase of the sequence. These changes illustrate the interdependence of teaching and learning practices, demonstrating how adjustments to teaching practices reshaped the conditions for student learning, and vice versa.

While the form of student questions remained predominantly focused on “how many” and “how much”, this should not be interpreted as a lack of development across the two cycles. Even though cultural-discursive arrangements within mathematics classes in the site may implicitly reinforce the narrow idea that mathematics is mainly about numeration, measurement, and numerical outcomes, in this case it enabled the students to engage productively as the conditions supporting problem posing were reconfigured. These familiar question forms supported students to engage with a wider range of mathematical ideas and contexts. This reinforces the importance of interpreting student products in relation to the conditions under which they were generated, rather than solely as indicators of ability.

This study demonstrates the value of the TPA as an analytical framework for examining practice and changes over time. The use of tables of invention to systematically organise, compare, and interpret data across cycles made visible the dialectical relationship between practice and the practice architectures, offering a way to study implementation as a dynamic, site-specific process. In doing so, the study extends problem posing literature by providing a theoretically grounded account of how practices are developed and sustained through ongoing reconfigurations rather than fidelity to predetermined models.

The evidence presented in this study is limited to a single site and a small teaching team, however, it offers insights into how changing classroom conditions can support student problem posing in upper primary mathematics. Future research could explore how similar configurations operate across different year levels and contexts, and how broader practice architectures interact with classroom-level practices.

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References

- Australian Curriculum Assessment Reporting Authority. (n.d.). *Foundation to year 10 curriculum*. <https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-10>
- Cai, J., & Hwang, S. (2021). Teachers as redesigners of curriculum to teach mathematics through problem posing: Conceptualization and initial findings of a problem-posing project. *ZDM – Mathematics Education*, 53, 1403-1416. <https://doi.org/10.1007/s11858-021-01252-3>
- Cai, J., Hwang, S., & Melville, S. (2023). Mathematical problem-posing research: Thirty years of advances building on the publication of “on mathematical problem posing”. In J. Cai, G.J. Stylianides & P.A. Kenney (Eds.), *Research studies in learning and teaching mathematics*, (1-26). Springer.
- Cai, J., Koichu, B., Rott, B., & Jiang, C. (2024). Advances in research on mathematical problem posing: Focus on task variables. *The Journal of Mathematical Behavior*, 76, 1-17.
- Edwards-Groves, C., & Kemmis, S. (2016). Pedagogy, education and praxis: Understanding new forms of intersubjectivity through action research and practice theory. *Educational Action Research*, 24(1), 77-96, <https://doi.org/10.1080/09650792.2015.1076730>
- English, L. (1998). Children’s problem posing within formal and informal contexts. *Journal for Research in Mathematics Education*, 29(1), 83-106.
- Fereday, J. & Muir-Cochrane, E. (2006) Demonstrating rigor using thematic analysis: A hybrid approach of inductive and deductive coding and theme development. *International Journal of Qualitative Methods*, 5(1), 1-11. <https://doi.org/10.1177/160940690600500107>
- Gibbs, L., Cooke, M., Salamon, A. (2022). Unpacking the theory of practice architectures for research in early childhood education. *Australasian Journal of Early Childhood*, 47(4), 260-274.
- Grootenboer, P. & Edwards-Groves, C. (2023). *The theory of practice architectures: Researching practices*. Springer.
- Kemmis, S. (2021). A practice theory perspective on learning: beyond a ‘standard’ view. *Studies in Continuing Education*, 43(3), 280-295. <https://doi.org/10.1080/0158037X.2021.1920384>
- Kemmis, S., Edwards-Groves, C., & Grootenboer, P. (2025). *Reframing learning: Changing practices, sites, histories, lives*. Routledge.
- Kemmis, S. & Grootenboer, P. (2008). Situating praxis in practice: Practice architectures and the cultural, social and material conditions for Practice. In S. Kemmis & T.J. Smith (Eds.), *Enabling praxis: Challenges for education*, (37–64). Sense Publishers.
- Kemmis, S., Heikkinen, H., Fransson, G., Aspors, J. & Edwards-Groves, C. (2014). Mentoring of new teachers as a contested practice: Supervision, support and collaborative self-development. *Teaching and Teacher Education*, 43, 154-164. <https://doi.org/10.1016/j.tate.2014.07.001>
- Kemmis, S., McTaggart, R., & Nixon, R. (2014). *The action research planner: Doing critical participatory action research*. Springer. <https://doi.org/10.1007/978-981-4560-67-2>
- Lewis, W.M., & Colonnese, M.W. (2021). Fostering mathematical creativity through problem posing and three-act-tasks. *Gifted Child Today*, 44(3), 141-150.
- Mahon, K., Francisco, S., & Lloyd, A. (2018) In C. Edwards-Grove, P. Grootenboer & J. Wilkinson (Eds.), *Education in an era of schooling*, (167–181). Springer.
- National Council of Supervisors of Mathematics. (2025). Strengthening research-informed decision making for mathematics education [Position statement]. National Council Supervisors of Mathematics.
- Rönnerman, K., Grootenboer, P., & Edwards-Groves, C. (2023). The practice architectures of middle leading in early childhood education. *International Journal of Child Care and Education Policy*, 11(1), 1-20. <https://doi.org/10.1186/s40723-017-0032-z>
- Schoenfeld, A. H. (2017). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, 196(2), 1-38.
- Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, 14(1), 19–28. <https://www.jstor.org/stable/40248099>
- Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students’ problem posing. In P. Clarkson (Ed.), *Technology in mathematics education* (pp. 518-525). Mathematics Education Research Group of Australasia.
- Zhang, L., & Cai, J. (2021). Teaching mathematics through problem posing: Insights from an analysis of teaching cases. *ZDM-Mathematics Education*, 53, 961-973. <https://doi.org/10.1007/s11858-021-01260-3>
- Zorn, K. (2022). *Student perceptions of engagement and problem-posing during an inquiry-based learning mathematical investigation*. [Master’s thesis, Griffith University]. Griffith University.

Round Tables

‘Success’ as Mathematics Education Researchers: A Community Conversation

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Mathematics education researchers occupy a range of roles across research, teacher education, policy engagement, and partnerships with schools, which must shift and change as all things in education do (e.g. Geijsel et al., 2010; Schoenfeld, 2000). These roles involve differing expectations for influence, beliefs and standpoints, professional recognition, and contributions to the mathematics education community (e.g. Vanderlinde & Van Braak, 2010). Furthermore, despite a shared commitment to improving educational experiences, a persistent research-practice gap remains, particularly within mathematics education (Goos, 2014), including perceptions that research and classroom practice operate within different priorities and systems (e.g. Cai et al., 2018). At the same time, increasing pressures to quantify and measure academic productivity can shape how research contributions are defined and valued (e.g. Ball, 2003). Within these varying roles and narratives, however, what constitutes ‘*success*’ for mathematics education researchers remains largely implicit or presumed rather than explicitly examined.

This round table invites MERGA community members to collectively reflect on how mathematics education researchers conceptualise ‘*success*’ in their work. Participants will engage in reflective, structured small-group discussions about what ‘*success*’ as mathematics education researchers may look like, sound like, and feel like. Extending upon the foundations of a study examining how teachers conceptualise and determine *success* within mathematics classrooms, this session seeks to understand another dimension of *success* as researchers across career stages, professional contexts, and countries share and compare perspectives. With participant consent, activity artefacts and discussion recordings may be collected during the session, with the intention of collating and sharing insights with the MERGA community at a future conference.

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References

- Ball, S. J. (2003). The teacher’s soul and the terrors of performativity. *Journal of Education Policy*, 18(2), 215-228.
- Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., & Hiebert, J. (2018). Reconceptualizing the roles of researchers and teachers to bring research closer to teaching. *Journal for Research in Mathematics Education*, 49(5), 514-520.
- Geijsel, F. P., Krüger, M. L., & Slegers, P. J. (2010). Data feedback for school improvement: The role of researchers and school leaders. *The Australian Educational Researcher*, 37(2), 59-75.
- Goos, M. (2014). Researcher–teacher relationships and models for teaching development in mathematics education. *ZDM*, 46(2), 189-200.
- Schoenfeld, A. H. (2000). Purposes and methods of research in mathematics education. *Notices of the AMS*, 47(6), 641-649.
- Vanderlinde, R., & Van Braak, J. (2010). The gap between educational research and practice: Views of teachers, school leaders, intermediaries and researchers. *British Educational Research Journal*, 36(2), 299-316.

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Mathematics Teacher Education for a Challenging Future: Continuing the Conversation

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We live in a challenging world affected by global issues such as sustainability, social justice, political shifts, and pandemics. At the same time, the demands of teaching continue to evolve rapidly alongside developments in technology and artificial intelligence (Celik et al., 2022). This raises important questions about how to better prepare prospective teachers to enter this ever-changing landscape.

This round table discussion revisits key findings from the discussion group held at the ICME15 Conference in Sydney, Australia, on “Mathematics Teacher Education for a Challenging Future: What counts and what doesn’t?” In conversation with mathematics education researchers, teacher educators, and teachers from 19 countries, including Australia, New Zealand, Singapore, India, and Spain, we explored the demands and challenges in preparing mathematics teachers to enter the workforce and sustaining their professional growth. We reflected on how mathematics teacher education could be reimaged and redesigned to develop the knowledge (Scheiner et al., 2023; Shulman, 1987), competencies (Llinares, 2024), beliefs, and dispositions (Philipp, 2007) necessary and sufficient for prospective mathematics teachers, while also comparing similarities and differences across international contexts. We concluded that there is no one-size-fits-all criterion for initial teacher education courses due to local demands; however, a key priority should be supporting the development of prospective teachers’ identities, professional values, inquiry stances, and well-being (Anthony et al., 2015).

As we continue the conversation, this session presents the key themes that emerged from the discussion group and invites the MERGA community to reflect on and contribute perspectives on what aspects of teachers’ identity, professional values, and well-being should be prioritised in preparing future mathematics teachers. In addition, we will explore possible avenues for future collaboration in mathematics teacher education.

References

- Anthony, G., Hunter, J., & Hunter, R. (2015). Prospective teachers development of adaptive expertise. *Teaching and Teacher Education*, 49, 108-117. <https://doi.org/10.1016/j.tate.2015.03.010>
- Celik, I., Dindar, M., Muukkonen, H., & Järvelä, S. (2022). The promises and challenges of artificial intelligence for teachers: A systematic review of research. *TechTrends*, 66(4), 616-630. <https://doi.org/10.1007/s11528-022-00715-y>
- Llinares, S. (2024). The knowing-doing duality: a characteristic of mathematics teachers’ professional work. *Journal of Mathematics Teacher Education*, 27(6), 879-883. <https://doi.org/10.1007/s10857-024-09663-1>
- Philipp, R. A. (2007). Mathematics teachers’ beliefs and affect. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257-315). Information Age Publishing.
- Scheiner, T., Buchholtz, N., & Kaiser, G. (2023). Mathematical knowledge for teaching and mathematics didactic knowledge: a comparative study. *Journal of Mathematics Teacher Education*. <https://doi.org/10.1007/s10857-023-09598-z>
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 467). MERGA.

Publishing in International Mathematics Education Journals

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This Round Table is an initiative of the Editor-in-Chief and Acting Editor-in Chief of MERJ in support of quality submissions to MERJ, in particular, those from early career researchers. This Round Table is supported by the MERGA VP Publications as part of an ongoing effort to strengthen Australasian mathematics education research.

One question that is of great importance not only, but particularly, to early-career researchers in the field of mathematics education is: Why should I publish in international or prestigious journals? This is followed by a series of further questions – such as how do I recognise a prestigious journal, what does ‘international’ mean, or how do I deal with the outcome of a (non-)successful peer-review process. Publishing both nationally and internationally is by no means a trivial matter, but a necessary prerequisite and foundation for a (successful) academic career. Publishing therefore means more than simply meeting institutional expectations; it is the path through which researchers enter the international academic discourse, situate their work within established debates, and make their ideas visible, citable and influential beyond local and national contexts. At the same time, publishing in academic journals can seem opaque, tedious and highly selective, particularly for those encountering it for the first time. This is precisely what this Round Table, and the participants in this Round Table, will address. The aim of this session is to make the publishing process more transparent, manageable, and targeted.

With a focus on international journals in mathematics education, this round table will discuss how to identify suitable publication opportunities by considering the journal’s subject area, readership, quality indicators and rankings, and how to use such information strategically without losing sight of academic suitability. Using examples such as MERJ and ZDM, we will examine the publication process from submission through the peer-review process to editorial decision-making, including rejection, revision and resubmission, as well as acceptance. Particular attention will be paid to the period following a decision: how to interpret reviewers’ comments, how to draft a convincing revision, when it is justified to remain with the same journal, and when it is better to submit a manuscript elsewhere. The discussion will also address what follows acceptance – proofs, copyright, first-time online publication, and ways to promote published work so that it reaches the audience it deserves.

The roundtable aims not only to explain how the publishing process works, but also to encourage early-career researchers to view publishing as a meaningful and supportive academic practice rather than a mysterious gatekeeping exercise. It is particularly valuable for those wondering where to submit their work, how to respond to peer reviews, and how to stay motivated during the revision and resubmission process. Participants will receive practical guidance, realistic expectations and concrete strategies to navigate the international journaling landscape with greater confidence, resilience and ambition.

Identity and Mathematics Education

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Over a number of years now there has been interest in the concept of ‘identity’ in mathematics education. Interest has been sustained for some time, to illustrate: in 2006 myself and colleagues from Charles Sturt University presented a MERGA symposium on this topic (and one of our 4-page) papers now has over 175 citations; in 2019 Mellony Gravan led a special issue of ZDM on identity; and, at the recent ICME in Sydney there was Topic Study Group 5.1 which also focussed on identity. Furthermore, the notion of identity has been used and conceptualised by researchers from a range of disciplines including the psychological/developmental, the socio-cultural, and the post-structural.

In this round-table discussion I want to present another view—a site-ontological practice perspective of mathematical identity. This perspective sees mathematical identities being formed and sustained through participation in mathematical practices, in ensembles (or communities of practice) of other *mathematicians*. This, in essence, is learning—ontological transformation by being “stirred into” mathematical practices through participation (Kemmis, Edwards-Groves, & Grootenboer, 2025). This view is grounded in the seminal work of Lave and Wenger, where they showed that identity, practice and learning are inter-related. In 1998 Wenger said, “there is a profound connection between identity and practice” (p. 149), and

The experience of identity in practice is a way of being in the world. It is not equivalent to self-image . . . Who we are lies in the way we live day-to-day, not just in what we think and say about ourselves, though this is of course part (but only part) of the way we live. (p. 151)

Currently, mathematics is generally seen as being experienced at school in classrooms and textbooks. These classrooms and lessons, and the teaching practices that are employed, create the conditions for the formation of mathematical identities, and so for many years students have developed mathematical identities that would appear to be debilitating and even inconsistent with the nature of mathematics. If mathematics education is not so much about *learning something*, per se, but rather an initiation into practices through “legitimate peripheral participation” (Lave & Wenger, 1991), then the social participation in communal activities, where the individuals create and recreate their personal and shared identities through their common practices (i.e., in communities of practice) will be crucial. In this session we will discuss: If mathematical identities are formed through the participation in mathematical practices in classrooms:

- How can these practices be ‘authentically’ mathematical? And how can the sites where they are ‘learned’ be reflective of the ‘mathematics community of practice’?
- Is the purpose of mathematics education to provide conditions and arrangements which enable learners to participate in substantive mathematical practices, and so foster robust mathematical identities—identities that would see them as knowledgeable and confident in learning and doing mathematics?

References

- Kemmis, S., Edwards-Groves, C., & Grootenboer, P. (2025). Reframing learning: Changing practices, sites, histories, lives. Routledge. DOI: 10.4324/9781003581710
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (p. 469). MERGA.

Mapping the Ecosystem of Out-of-Field Teaching in Mathematics

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Teacher shortages in Australia and internationally have prompted systems and schools to rely increasingly on teachers working outside their area of qualification or teaching out-of-field (OOF) (Hobbs, 2013). This practice has significant implications for workforce planning (AITSL, 2021), with estimates that up to 40% of mathematics teachers may be teaching OOF. Research links OOF teaching to reduced student achievement (Van Overschelde, 2022), higher teacher attrition (Sharplin, 2014), and diminished teacher confidence (Du Plessis et al., 2015). The aim of our ARC-funded project, *Shifting the Culture of Out-of-field Professional Education for Teachers (SCOPE-T)*, is to investigate the broader interplay among policy settings, school priorities, stakeholder expectations, and systemic incentives that shape teachers' engagement with professional education (PE). Drawing on Bronfenbrenner's (1994) ecological systems theory, these influences can be understood as interacting across multiple layers of the education system, from macro-level policy discourses to school-level practices. In mapping the current ecosystem, our research team has identified three domains that operate across all levels of the ecosystem and provide pathways for OOF teachers to become more in-field: the *workplace*, *initial teacher education*, and *professional education*. We propose that such pathways can elevate the cultural and policy recognition of teacher re-specialisation, but may need flexibility to represent the needs of the heterogeneous group. This roundtable begins with a presentation of our draft mapping of the current mathematics OOF ecosystem (10 min.). Participants will then break into three groups, one for each potential pathway to becoming in-field, with each group led by a roundtable presenter. Each group will be provided with a poster-sized map of the relevant section of the ecosystem (workplace, initial teacher education, professional education) and invited to annotate the map to create a record of their discussion. The purpose of the group discussions is to interrogate the related domain-specific mapping, identify local drivers and responses, and consider what motivates or constrains teachers' engagement with PE (20 min.). The roundtable will conclude with a plenary discussion and participants will share insights from each domain of the current ecosystem and suggest features of an ideal ecosystem that values diverse pathways towards teachers becoming more in-field (10 min.).

References

- AITSL (2021). *Australian teacher workforce data: National teacher workforce characteristics report*. <https://www.aitsl.edu.au/research/australian-teacher-workforce-data/atwdreports>.
- Bronfenbrenner, U. (1994). Ecological models of human development. In T. Husén & T. N. Postlethwaite (Eds.) *International encyclopedia of education* (2nd ed., Vol. 3, pp. 1643–1647). Elsevier.
- Du Plessis, A.E., Carroll, A. & Gillies, R.M. (2015). Understanding the lived experiences of novice out-of-field teachers in relation to school leadership practices. *Asia-Pacific Journal of Teacher Education*, 43(1), 4–21. <https://doi.org/10.1080/1359866X.2014.937393>
- Hobbs, L. (2013). Teaching 'out-of-field' as a boundary-crossing event: Factors shaping teacher identity. *International Journal of Science and Mathematics Education*, 11(2), 271–297.
- Sharplin, E.D. (2014). Reconceptualising out-of-field teaching: experiences of rural teachers in Western Australia. *Educational Research*, 56(1), 97–110.
- Van Overschelde, J. (2022). Value-lost: The hidden cost of teacher misassignment. In L. Hobbs & R. Porsch (Eds.), *Out-of-field teaching across teaching disciplines and contexts* (pp. 71–97). Springer.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 470). MERGA.

Integrating Data Science, Computational Thinking, and AI into Mathematics Education: An Australian Challenge

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Overview

The Australian Curriculum Version 9.0 provides a context for integrating Data Science, Computational Thinking, and Artificial Intelligence into mathematics education. Data Science is emerging as a standalone subject, offering a pathway into statistical reasoning, modelling, and real-world problem solving in mathematics; however, awareness among mathematics teachers remains low and uptake is modest. Computational thinking is embedded across the curriculum as a core component of mathematical thinking, supporting abstraction, algorithmic reasoning, and investigation processes central to mathematics education. Artificial intelligence is rapidly evolving and potentially disruptive, creating new contexts in which mathematical ideas are applied, interpreted, and scrutinised. Together, these areas highlight how digitalisation is reshaping mathematics education. Increasing reports of AI disruption strengthen the case for critical data and AI literacy within mathematics, preparing students for adaptability, systemic understanding, and informed citizenship, and challenging educators to distinguish between foundational and genuinely new mathematics (NASEM, 2025).

Key Issues

- Visibility: Data Science exists as a subject, but how do we ensure students, teachers, and school leaders know about it and understand its value?
- Consistency: States and territories will implement these changes differently. What mechanisms support cross-jurisdictional learning without imposing uniformity?
- Teacher readiness: Most teachers of mathematics have had limited exposure to data science and AI. What professional learning is working, and what more is needed?
- Curriculum and assessment: Curriculum structures have not kept pace with these changes and risk falling short of educating young people for the world ahead of them.

Questions for Discussion

1. What is making computational thinking difficult to consistently implement?
2. What are structural barriers, beyond awareness, limiting uptake of Data Science?
3. How can curriculum and assessment practices respond to AI?
4. How can teachers design tasks where AI opens inquiry and support mathematical thinking?
5. How should MERGA respond, from a research and leadership perspective?

Australia has strong curriculum foundations to build from - an opportunity to examine what is working, and why AI requires rethinking about the teaching and learning of mathematics.

Reference

National Academies of Sciences, Engineering, and Medicine. [NASEM] (2025). Artificial intelligence and education. In *Artificial intelligence and the future of work* (pp. 133–146). The National Academies Press. <https://doi.org/10.17226/27644>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 471). MERGA.

Positioning, Purposes, and Policy: Examining the Purposes of Middle Leading of Mathematics in Primary Schools

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Research on mathematics leadership in schools increasingly positions it as a form of middle leading practice situated at the intersection of policy implementation, research translation, and classroom practice (Sexton et al., 2025). In this Round Table, we attend to the purposes of middle leading of mathematics as part of an ongoing scoping review examining how mathematics leadership is represented in MERGA-affiliated literature. Despite growing interest in the field, the purposes attributed to mathematics leadership remain diffuse and, in many cases, underspecified across the literature. While a dominant framing positions the purpose of mathematics leadership as improving student learning outcomes through school-based professional learning leadership (Copping et al., 2024), our review reveals several nuanced and varied purposes. This raises questions about how conceptualisations of mathematics leadership shape its purposes in schools, and what potential influence these might have on policy and practice. This Round Table is organised around three interrelated dimensions: positioning (how mathematics leaders are situated within organisational and relational structures), purposes (the intentions attributed to the work), and policy (how research might inform policy discussions). Together, these dimensions provide a lens for examining not just what mathematics leaders do, but why and for whom.

Participants are invited to contribute their responses and perspectives about the emerging themes generated through preliminary analysis of our scoping review. Round Table dialogue will be mediated by the following questions:

- How do the emerging purposes identified in our scoping review resonate with the work of mathematics leaders in practice?
- What purposes of middle leading of mathematics remain silent in research literature?
- What might be the purpose of research into mathematics leadership beyond exploratory descriptions of practice?
- How might MERGA engage policymakers with mathematics leadership research?

The aim of the Round Table is to generate insights from dialogue about how MERGA can extend mathematics leadership research that strengthens connections between practice, research, and policy. The session welcomes mathematics leaders, including aspiring and emerging leaders, and researchers with an interest in the field.

References

- Copping, K., Ziebell, N., Sexton, M., Downton, A., Pearce, B., O'Connor, A., Gould, L., & Grootenboer, P. (2024). From tensions to opportunities: Evidencing mathematics leadership. In J. Višňovská, E. Ross, & S. Getenet (Eds.), *Surfing the waves of mathematics education: Proceedings of the 46th annual conference of the Mathematics Education Research Group of Australasia* (pp. 35–47). MERGA.
- Sexton, M., Copping, K., Driscoll, K., & Downton, A. (2025). *Mathematics leadership during educational reform: Navigating pedagogical tensions with MERGA support* [Unpublished round table abstract]. MERGA.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (p. 472). MERGA.

Number as Quantity in the Australian Curriculum

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We analysed Version 9 of the Australian Curriculum: Mathematics, Foundation through Year 6 (Australian Curriculum, Assessment and Reporting Agency [ACARA], n.d.), by searching for presence and role of continuous and discrete quantities in how the notion of number is developed. The analysis is a case highlighting that some mathematical ideas that are central to teaching, and may seem obvious or common-sensical (Freudenthal, 1979, 1981), might, in fact, not be present in national curricula. This finding is of significance and requires research attention given that a curricular absence of a mathematical idea, shaping the form and content of teaching resources, is likely to carry into classrooms (Rezat et al., 2021).

In this round table, we justify the focus for this analysis by drawing on number conceptualisations that allow students to encounter fractions and whole numbers as the same kind of number (Bass, 2018; Cortina et al., 2014), and highlight the importance of quantity as a basis for the meaning of number in those conceptualisations (Chambris & Višňovská, 2022). We introduce the coding rubric we developed for this purpose and engage participants in the analysis, demonstrating how, in the analysed curriculum, attention to continuous quantities is highly restricted. Elsewhere (Višňovská & Cortina, 2025), we explained that this curricular stance has a historical connotation, suggesting that the phenomenon is likely to be found in many other countries. We also argued that the perpetuation of the largely forgotten historical rationale prevents such curricula from providing more adequate support for teaching and learning number.

We view this round table as an opportunity to debate the ideas related to the place of quantity in teaching and learning number with the participants. We hope to collate research literature, anecdotal episodes, and diverse classroom experiences to push our collective thinking about this curricular phenomenon.

References

- Australian Curriculum Assessment and Reporting Authority. (n.d.). *Australian curriculum: Mathematics (Version 9), Years F-6*. https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-1_foundation-year_year-2_year-3_year-4_year-5_year-6/
- Bass, H. (2018). Quantities, numbers, number names and the real number line. In M. G. Bartolini Bussi & X. H. Sun (Eds.), *Building the foundation: Whole numbers in the primary grades* (pp. 465–475). https://doi.org/10.1007/978-3-319-63555-2_19
- Chambris, C., & Višňovská, J. (2022). On the history of units in French elementary school arithmetic: The case of proportionality. *Historia Mathematica*, 59, 99–118. <https://doi.org/10.1016/j.hm.2021.04.003>
- Cortina, J. L., Visnovska, J., & Zúñiga, C. (2014). Unit fractions in the context of proportionality: Supporting students' reasoning about the inverse order relationship. *Mathematics Education Research Journal*, 26, 79–99. <https://doi.org/10.1007/s13394-013-0112-5>
- Freudenthal, H. (1979). New math or new education? *Prospects*, 9(3), 321–331.
- Freudenthal, H. (1981). Major problems of mathematics education. *Educational Studies in Mathematics*, 12(2), 133–150.
- Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM Mathematics Education*, 53, 1189–1206. <https://doi.org/10.1007/s11858-021-01309-3>
- Višňovská, J., & Cortina, J. L. (2025). Curriculum and mathematical coherence: Exploring the tensions in teaching and learning number. *ZDM Mathematics Education*, 57, 1051–1063. <https://doi.org/10.1007/s11858-025-01706-y>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 473). MERGA.

From Instructor to Facilitator of Learning: Shifting the Practices of Secondary Mathematics Teachers

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For students to thrive in a rapidly evolving, data and technology-driven world, student-centred learning must become an essential component of a teacher's repertoire of practice. Policy, curriculum and industry demands support the need for fostering STEM and 21st century capabilities in students, however, traditional, teacher-centred practices continue to dominate as the most regularly used practice in secondary mathematics classrooms.

Mathematics education is a dynamic, complex and multi-dimensional system, shaped by internal and external pressures, including political, cultural and societal. Despite consensus that mathematics education must support student proficiency, the emphasis on procedural knowledge acquisition remains common place in many mathematics classrooms. Contemporary numeracy demands require computational thinking and critical AI literacy (Whitney-Smith, 2023). To address declining dispositions and mirror processes used beyond the classroom, mathematics teaching needs to shift from instructor models, often associated with a heavy reliance on textbooks, independent practice and routine tasks (Schmidt et al., 2022), to facilitator models that incorporate student-centred, collaborative, integrated and technology-enhanced practices (Hanifah et al., 2025).

In this round table, I will present findings from a study exploring the extent to which essential STEM and 21st-century capabilities are supported by school mathematics in Years 7-10, and invite members of the audience to contribute to a discussion on how to instigate system-level change, sharing any insights, research or ideas on ways to support this necessary system-wide shift in practice. I am particularly interested in gaining insight into how we can best support secondary mathematics teachers to become facilitators of learning. The focus questions for this session include:

- What are the leveraging points to instigate a change?
- What is required to support secondary teachers to confidently act as a facilitator?
- How do assessment and reporting practices need to change?

References

- Hanifah, U., Budayasa, I. K., & Sulaiman, R. (2025). Technology, pedagogy, and content knowledge in mathematics education: A systematic literature review. *Journal of Education and Learning*, 19(1), 579–586.
- Schmidt, W., Houang, T., Sullivan, W., & Cogan, L. (2022). *When practice meets policy in mathematics education: A 19 country/jurisdiction case study*. OECD Education working papers, No. 268. OECD Publishing.
- Whitney-Smith, R. (2023). The emergence of computational thinking in national mathematics curricula: An Australian example. *Journal of Pedagogical Research*, 7(2), 41-55

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 474). MERGA.

Special Working Groups

Effective Pedagogies in Mathematics Education: Implications for Initial Teacher Education

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Currently we find ourselves in turbulent waters within the context of mathematics education research and in debates about current policy, curriculum, and practice reform. In particular, there are ongoing tensions in Australia in the field of Initial Teacher Education due to the mandated inclusion of core content in ITE programs. Through a plenary panel including Greg Oates, Margaret Marshman and Emily Ross we seek to explore the future implications for mathematics education posed by this mandated core content for ITE programs.

The Teacher Education Expert Panel (TEEP) discussion paper made recommendations for reforms to Initial Teacher Education (ITE) Programs (Department of Education, 2023). One recommendation related to mandating core content to be taught in ITE programs, with one area of core content being effective pedagogical practice.

MERGA published a response to the TEEP Discussion Paper outlining a number of concerns related to mathematics education for ITE programs. Concerns included privileging a narrow teaching approach, and drawing upon a narrow evidence base, with a heavy reliance on a single area of research, that of cognitive science. Deuchar (2025) echoed the concern that the mandated core content will result in a narrowing of pedagogical possibilities available to the teachers of the future.

Hubbard, Russo, Livy, and Sullivan (2026) suggest that in addition to student performance, effective mathematics pedagogy needs to consider affective and dispositional elements of mathematics learning; they highlight the value of pedagogical approaches that consider student sense-making and focus on the learning process rather than only student output.

The MERGA Special Working Group for Effective Pedagogies in Mathematics invites you to this SWG meeting where we will have a plenary panel, including Emily Ross, Margaret Marshman and Greg Oates to discuss:

- What tensions are present in ITE programs in relation to effective mathematics pedagogies and mandated core content required for accreditation?
- What implications does the mandated core content pose for future mathematics teaching and learning?

References

- Department of Education. (2023). *Teacher Education Expert Panel Discussion Paper*. Retrieved from <https://www.education.gov.au/quality-initial-teacher-education-review/resources/teacher-education-expert-panel-discussion-paper>
- Deuchar, A. (2025). Policy makers as experts and teachers in need of reform: a critique of 'evidence-based' reforms of initial teacher education in Australia. *Australian educational researcher*, 52(2), 1163-1180. doi:10.1007/s13384-024-00757-6
- Hubbard, J., Russo, J., Livy, S., & Sullivan, P. (2026). Optimizing learning conditions for self-determination in primary mathematics education: A theoretical perspective. *Mathematics education research journal*. doi:10.1007/s13394-025-00555-y

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 476). MERGA.

Spotlight on Out-of-Field Teaching in Mathematics: Fast-Paced Research Highlights to Build Community Connections

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Australia's long-term teacher shortage has reached a crisis point, with mathematics among the most significantly impacted disciplines. Current estimates indicate that as many as 40% of secondary mathematics teachers are teaching out-of-field (OOF) in Australian schools (Australian Institute for Teaching and School Leadership (AITSL), 2021), highlighting the systemic nature of the problem. Out-of-field teaching in mathematics (OOFTM) is unevenly felt across the system, further exacerbating equity issues (Hobbs et al., 2022). This situation poses challenges to the integrity, quality and continuity of mathematics education, as schools increasingly struggle to recruit and retain knowledgeable and skilled teachers (Goos & Marchant, 2025). As concerns about OOFTM have intensified over the past decade, MERGA researchers have increasingly sought to deepen their understanding of this nuanced phenomenon and identify strategies to educate and support teachers, school leaders, policymakers, and the public. The growing prominence of this issue within the MERGA community is evident, with conference papers and presentations focused on OOFTM being a stable feature of the MERGA conference program since MERGA42 in 2019. This growing body of work reflects a sustained, collective effort to study and develop interventions to address one of the most pressing and complex challenges currently facing mathematics education. This roundtable marks the beginning of a community conversation. Short five-minute lightning talks from MERGA members working on OOFTM projects will offer a snapshot of the field, showcase the breadth of expertise within the community, surface emerging themes and tensions, and spark conversations about future directions for research and collaboration. By bringing together researchers at different stages of their OOFTM inquiries, the roundtable aims to build a clearer picture of the landscape and strengthen the foundations for an ongoing Special Working Group (SWG). Participants will be invited to reflect on the collective insights that emerge: What patterns or gaps become visible when we view OOFTM research side by side? How might these insights inform the development of a coordinated research agenda? And, in what ways can a MERGA SWG foster deeper connections, shared resources, and sustained collaboration across the community? Through fast-paced sharing and open discussion, this session seeks to energise and connect the growing network of OOFTM researchers within MERGA.

References

- Australian Institute for Teaching and School Leadership (AITSL) (2021). *Australian teacher workforce data: National teacher workforce characteristics report*. <https://www.aitsl.edu.au/research/australian-teacher-workforce-data/atwdreports>
- Goos, M. & Marchant, T. (2025). *Relieving out of field teaching in Australian secondary mathematics: Upskilling program design elements*. Australian Mathematical Sciences Institute (AMSI).
- Hobbs, L., Du Plessis, A.E., Oates, G., Caldis, S., McKnight, L., Vale, C., O'Connor, M., Rochette, E., Watt, H., Weldon, R., Richardson, P. & Bateup, C. (2022). *National Summit on Teaching Out-of-field: Synthesis and Recommendations for Policy, Practice and Research*. <https://oofas-collective.squarespace.com/s/TOOF-National-Summit-Report.doc>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 477). MERGA.

Short Communications

Using Eye-Tracking Software to Examine the Relation Between Spatial Reasoning and Mathematical Problem-Solving Behaviour

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Spatial reasoning is the ability to visualise and manipulate spatial images in the mind and is a strong predictor of mathematical achievement. Research has demonstrated that spatial reasoning skills can be improved through practice, and that improvements in spatial skills lead to corresponding improvements in mathematics (see Hawes et al., 2022 for review). Although this relation has been well established, transfer does not occur in every instance, with different types of mathematical problems benefitting more from improvements in spatial skills and others not demonstrating any transfer (e.g., Adams et al., 2023). Although this is unsurprising given the multifaceted nature of both spatial skills and mathematics, current research methods do not allow for explicit identification of how spatial skills support performance on mathematics tasks (Resnick & Adams, 2024). For this reason, new methodologies are needed to better explore this complex relation. The use of eye-tracking software is a recent, but increasingly popular approach in mathematics education to examine individuals' strategic behaviour when solving problems (e.g., Boels, 2025). This technology has the potential to shed light on the specific ways in which individuals might apply spatial skills when solving mathematics problems.

We examined undergraduate students' (n=43) strategic behaviour using eye-tracking software to capture real-time visual-attentional patterns when solving different spatial and mathematics tasks, and asked them to report the strategic approach they used to arrive at their answer. Spatial tasks included mental rotation and perspective-taking. Mathematics tasks included word problems, missing term problems, and questions involving the interpretation of pictorial representations. Individuals also completed measures of visuospatial working memory and executive function skills to explore how these related to specific strategic behaviors.

This presentation will explore these connections, such as how differences in spatial skills influence time spent interpreting written spatial information, and how visuospatial working memory aligns with efficient symbol scanning in missing-term problems. We will also discuss the implications for how to better leverage spatial skills to support students' success in mathematics.

References

- Adams, J., Resnick, I., & Lowrie, T. (2023). Supporting senior high-school students' measurement and geometry performance: Does spatial training transfer to mathematics achievement? *Mathematics Education Research Journal*. <https://doi.org/10.1007/s13394-022-00416-y>
- Boels, L., Bakker, A., Van Dooren, W., & Drijvers, P. (2025). Secondary school students' strategies when interpreting histograms and case-value plots: An eye-tracking study. *Educational Studies in Mathematics*, 118(3), 479–503. <https://doi.org/10.1007/s10649-024-10351-3>
- Hawes, Z., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology*, 58(1), 112–137. (2022-25661-002). <https://doi.org/10.1037/dev0001281>
- Resnick, R., & Adams, J. (2024). Spatial Reasoning Interventions and Transfer to Geometry: What we Know about Mechanism. In A. G. Rodrigues, T. Lowrie, & F. Emprin (Eds.). *Proceedings of the Twenty-Sixth ICMI Study: Advances in Geometry Education*. IREM de Reims.
- (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 479). MERGA.

Pre-service Teacher Experience Framing Generative Artificial Intelligence as a Thought Partner in Learning Mathematics

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Generative Artificial Intelligence (GenAI) is transforming K-12 mathematics learning, offering individualised support and changing student interactions (Walkington, 2025). Pre-service teachers (PSTs) must be prepared to teach in a world where over-reliance on GenAI is a concern (Aga et al. 2024; Biton & Segal, 2025; Engelbrecht & Borba, 2024). Caution is warranted because students may use it merely for a quick solution, offloading critical thinking and productive struggle essential for developing deep mathematical understanding (Biton & Segal, 2025; Engelbrecht & Borba, 2024). PSTs need experiences in their teacher education programs to become aware of the opportunities and concerns regarding the use of GenAI in teaching and learning of mathematics (Aga et al. 2024).

This presentation will share findings from an action research cycle within a mathematics methods course. Participants were four mathematics PSTs in their final year of a teacher education program. The intervention required PSTs to develop an original mathematics task and implement it at least once with a friend or a family member. The goal of the task was to engage the participant in learning mathematics using GenAI as a thought partner. Data sources were tasks, participant work, GenAI outputs, and PST reflection post implementation.

PSTs found that GenAI use was dependent upon the user's existing knowledge. When implementing the task, participants who were confident in their mathematical content knowledge were less reliant on GenAI and more critical of GenAI output. PSTs found GenAI as a useful tool that must always be accompanied by resources (e.g., textbooks, Desmos, etc.) for students to check for accuracy of GenAI output. PSTs also stressed that teachers should teach GenAI pedagogical actions that can mediate student interaction and include guidance for students themselves to help them engage with GenAI. PSTs found GenAI to be lacking in the answers provided without student engagement in any thought-provoking dialogue.

Analysis of PSTs' activities and their reflections on this experience will be shared. In addition, recommendations on the use of GenAI as a collaborator for teachers and as a thought partner for students when learning mathematics, will be highlighted.

References

- Aga, Z. G., Sawyer, A. G., & Wolfe, M. (2024). More confidence more critical: Investigating secondary mathematics preservice teachers' use of an Artificial Intelligence chatbot as a curriculum development tool. *American Journal of Creative Education*, 7(2), 63–78. <https://doi.org/10.55284/ajce.v7i2.1278>
- Biton, Y., & Segal, R. (2025). Learning to craft and critically evaluate prompts: The role of generative AI (ChatGPT) in enhancing pre-service mathematics teachers' TPACK and problem-posing skills. *International Journal of Education in Mathematics, Science, and Technology (IJEMST)*, 13(1), 202-223. <https://doi.org/10.46328/ijemst.4654>
- Engelbrecht, J., & Borba, M.C. (2024) Recent developments in using digital technology in mathematics education. *ZDM Mathematics Education*, 56, 281–292. <https://doi.org/10.1007/s11858-023-01530-2>
- Walkington, C. (2025). The implications of generative artificial intelligence for mathematics education. *School Science and Mathematics*, 1–10. <https://doi.org/10.1111/ssm.18356>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 480). MERGA.

Artificial Intelligence in Secondary Mathematics Classrooms: Insights from a Scoping Review

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Despite numerous advancements in educational methods, mathematics remains a challenging subject for many students worldwide (Li & Schoenfeld, 2019). It is typically perceived as difficult and accessible only by those who are deemed “smart” (Boaler, 2022). Traditional teaching approaches that emphasise memorising procedures over developing a deep conceptual understanding of mathematics do not accommodate diverse learners, resulting in significant achievement gaps among students (Mohamed et al., 2022). In addition, traditional mathematics education, defined by the disconnect between school mathematics and real-world applications, does not support the learning of all students (Banerjee et al., 2025). In recent years, artificial Intelligence (AI) has rapidly gained attention in the educational sector. In mathematics education particularly, the potential of AI tools offers new affordances for students, from personalising learning to adaptive assessment (Opesemowo & Ndlovu, 2024), real-time feedback (Opesemowo & Ndlovu, 2024), to building problem-solving capacity (Thanasi & Mema, 2024), and providing interactive learning environments (Opesemowo & Ndlovu, 2024). However, the integration of AI into mathematics education remains understudied. There is limited research which synthesises the outcomes and impacts of AI use within secondary mathematics classrooms. To address this gap, a scoping review has been conducted to identify AI use in secondary mathematics classrooms by examining teacher practices and AI impact on student outcomes. This presentation will report the findings of the scoping review, addressing the research question: *How has AI been used in secondary mathematics settings?*

References

- Banerjee, A. V., Bhattacharjee, S., Chattopadhyay, R., Duflo, E., Ganimian, A. J., Rajah, K., & Spelke, E. S. (2025). Children’s arithmetic skills do not transfer between applied and academic mathematics. *Nature*. <https://doi.org/10.1038/s41586-024-08502-w>
- Boaler, J. (2022). *The elephant in the classroom: Helping children learn and love maths* (Revised and updated UK edition). Souvenir Press.
- Li, Y., & Schoenfeld, A. H. (2019). Problematizing teaching and learning mathematics as “given” in STEM education. *International Journal of STEM Education*, 6(1), 44, s40594-019-0197-0199. <https://doi.org/10.1186/s40594-019-0197-9>
- Mohamed, M. Z. B., Hidayat, R., Suhaizi, N. N. B., Sabri, N. B. M., Mahmud, M. K. H. B., & Baharuddin, S. N. B. (2022). Artificial intelligence in mathematics education: A systematic literature review. *International Electronic Journal of Mathematics Education*, 17(3), em0694. <https://doi.org/10.29333/iejme/12132>
- Opesemowo, O. A. G., & Ndlovu, M. (2024). Artificial intelligence in mathematics education: The good, the bad, and the ugly. *Journal of Pedagogical Research*, 3. <https://doi.org/10.33902/JPR.202426428>
- Thanasi, T., & Mema, O. (2024, May 17–18). Enhancing mathematics education with artificial intelligence. *EIRP Proceedings: 19th International Conference on European Integration – Realities and Perspectives*, Galați, Romania. <https://dp.univ-danubius.ro/index.php/EIRP/article/view/509>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 481). MERGA.

Building Adaptive Expertise by Making Expert Assessment for Learning in Mathematics Visible

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Assessment for Learning (AfL) is a high-leverage approach for improving student learning because it foregrounds in-the-moment evidence of learning during instruction (Black & Wiliam, 1998). In mathematics, effective AfL relies on teachers' capacity to attend to students' mathematical thinking, interpret partial or emergent understandings, and respond in ways that advance learning (Jacobs et al., 2010). Developing adaptive expertise - the ability to draw on sources of knowledge and apply it flexibly to specific problems - remains a goal of, and persistent challenge in, initial teacher education (ITE) (Anthony et al., 2015; Timperley et al., 2017). While expert teachers routinely draw on such expertise to interpret evidence of learning and make instructional decisions, these judgements are often enacted with such fluency that they remain largely invisible to novices. As a result, preservice teachers (PSTs) may struggle to recognise what matters in moments of AfL and why. Video examples afford PSTs opportunities to slow down and decompose classroom practice, focusing attention on significant moments and expanding the time and space for developing professional noticing (Jacobs et al., 2010; Sherin & van Es, 2009). This study uses video models of expert practice to address the question: What aspects of expert mathematics AfL are noticed by PSTs compared with in-service teachers, and what becomes newly visible when expert teacher reasoning is made explicit? The study explores how expert AfL practice in mathematics becomes visible to teachers. Participants describe what they notice about teacher actions and assessment opportunities by first viewing raw classroom footage, then re-viewing the same footage accompanied by the expert teacher's reflective commentary. The protocol is implemented with both in-service teachers and PSTs, enabling comparison across cohorts. The study aims to identify patterns of *expert ease* in mathematics AfL - those fluent, often tacit judgements that underpin responsive teaching - to inform the design of ITE approaches that develop adaptive assessment expertise.

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University of Queensland ethics approval [8754] and all participants gave informed consent.

References

- Anthony, G., Hunter, J., & Hunter, R. (2015). Prospective teachers' development of adaptive expertise. *Teaching and Teacher Education*, 49, 108-117.
- Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education. Principles, Policy and Practice*, 5(1), 7-74. <https://doi.org/10.1080/0969595980050102>
- Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169-202.
- Sherin, M., & van Es, E. A. (2009). Effects of Video Club Participation on Teachers' Professional Vision. *Journal of Teacher Education*, 60(1), 20-37.
- Timperley, H., Ell, F., & Le Fevre, D. (2017). Developing adaptive expertise through professional learning communities. In M. Jones, J. B. Huffman, & A. Harris (Eds.), *Teachers Leading Educational Reform* (pp. 175-189). Routledge.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 482). MERGA.

Investigating Teacher Noticing in Technology-Enhanced Mathematics Classrooms

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It is widely recognized that the quality of mathematics instruction plays a crucial role in improving learning in mathematics classrooms (Hiebert & Douglas, 2007; Yang & Kaiser, 2022). A key aspect of high-quality instruction is centred around how teachers orchestrate their interactions with students around mathematics tasks (Smith & Stein, 2018). However, teachers face multiple simultaneous events, ranging from students writing down solutions to their peer discussions, when orchestrating these interactions in classroom practice. This makes it challenging for teachers to attend to and interpret critical classroom events for the purpose of responding to students' thinking. Teacher noticing has been widely studied as an important component of teaching expertise (Bastian et al., 2022; Sherin et al., 2011; van Es & Sherin, 2021), needed for such responsive teaching practices. While teacher noticing has been extensively studied in standard mathematics classroom contexts, less is understood about how teacher noticing unfolds in technology-enhanced classrooms, against the backdrop of ensuring high-quality instruction in the age of digital transformation. In this study, I investigate what and how in-service mathematics teachers notice in technology-enhanced classrooms. Drawing on the framework of Pedagogical Technology Knowledge (Thomas & Hong, 2013), I focus on how teachers' mathematical knowledge for teaching, personal orientations, goals and technology instrumental genesis influence their noticing. In this presentation, I will present a case study of a secondary mathematics teacher drawn from my doctoral study. Data sources include lesson preparation artifacts, classroom observations, and a series of teacher interviews. The findings of this study may provide insights into high-level teacher noticing in technology-enhanced classrooms and inform the development of teachers' noticing expertise.

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Nanyang Technological University granted ethics [IRB-2025-174], and all participants gave informed consent.

References

- Bastian, A., Kaiser, G., Meyer, D., Schwarz, B., & König, J. (2022). Teacher noticing and its growth toward expertise: An expert–novice comparison with pre-service and in-service secondary mathematics teachers. *Educational Studies in Mathematics*, 110(2), 205–232.
- Hiebert, J., & Douglas, A. G. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 371–404). Information Age.
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 3–13). Routledge.
- Smith, M. S., & Stein, M. K. (2018). *Five practices for orchestrating productive mathematics discussion* (2nd ed.). National Council of Teachers of Mathematics.
- Thomas, M. O., & Hong, Y. Y. (2013). Teacher integration of technology into mathematics learning. *International Journal for Technology in Mathematics Education*, 20(2), 69–84.
- van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM – Mathematics Education*, 53(1), 17–27. <https://doi.org/10.1007/s11858-020-01211-4>
- Yang, X., & Kaiser, G. (2022). The impact of mathematics teachers' professional competence on instructional quality and students' mathematics learning outcomes. *Current Opinion in Behavioral Sciences*, 48, 101225.
- (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 483). MERGA.

Insights into Very Young Children's Mathematical Thinking from Psychological Research

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Very young and preverbal children can engage in mathematical thinking. Often their opportunities to use, show, or develop mathematical thinking is dependent on their environment—and this is impacted by the attitudes and beliefs of early childhood educators who work with them (MacDonald & McGrath, 2022). Early childhood educators also need to be able to recognise the mathematical understandings that the children have and then use what they have observed in creating additional opportunities for further developing the children's mathematical understandings (Cohrssen & Tayler, 2016). Research from the field of psychology could help early childhood educators refine their beliefs about very young and preverbal children's capacity to engage with mathematics. Due to the work with very young and preverbal children not being able to ask the children for their thinking, psychological research often uses the violation-of-expectation method to identify the thinking of these children—as Stahl and Kibbe (2022, p. 1) state, violation-of-expectation provides “insights into the knowledge and cognitive capacities of infants who otherwise would not be able to verbally report those abilities”. The violation-of-expectation method involves familiarising or habitualising to a specific situation or image by showing this to the child; the novel situation or image is then shown and the looking time is measured (Turk-Browne et al., 2008).

As can be seen in the description of psychological research methodology above, the context in psychological research is very different from that in an early learning setting. However, sharing findings from psychological research still provides insights into the mathematical thinking that very young and preverbal children can engage with, and early childhood educators could benefit from being aware of this mathematical thinking when working with these very young and preverbal children. Although a range of research investigating the mathematical thinking of children up to 12 months of age from the field of psychology is presented, this is neither a systematic review nor a scoping review, rather, a snowball process that grew as research was identified; as a result, it is not exhaustive. This presentation aims to demonstrate how extensive very young and preverbal children's mathematical thinking is through mapping the findings from psychological research to broad mathematics curriculum areas of number and algebra (for example, quantity, operations, and patterns), measurement (such as size, time, and speed), geometry (including identification of attributes to create categories, comparisons of attributes, and spatial concepts), and prediction.

References

- Cohrssen, C. & Tayler, C. (2016). Early childhood mathematics: A pilot study in preservice teacher education. *Journal of Early Childhood Teacher Education*, 37(1), 25-40. <https://doi.org/10.1080/10901027.2015.1131208>
- MacDonald, A., & McGrath, S. (2022). Early childhood educators' beliefs about mathematics education for children under three years of age. *International Journal of Early Years Education*, 30(4), 847-862. <https://doi.org/10.1080/09669760.2022.2107493>
- Stahl, A. E., & Kibbe, M. M. (2022). Great expectations: The construct validity of the violation-of-expectation method for studying infant cognition. *Infant and Child Development*, 31(6), e2359. <https://doi.org/10.1002/icd.2359>
- Turk-Browne, N. B., Scholl, B. J., & Chun, M. M. (2008). Babies and brains: habituation in infant cognition and functional neuroimaging. *Frontiers in human neuroscience*, 2, 333. <https://doi.org/10.3389/neuro.09.016.2008>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 484). MERGA.

MERGA Reports of Primary Mathematics Middle Leadership Practices: A Scoping Review

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Mathematics education research has increasingly recognised the importance of school mathematics leaders (SMLs) in influencing student outcomes, teacher development, and teaching and learning (Driscoll, 2022). Mathematics leadership is often conceptualised as a form of middle leadership, positioned between senior leadership and teachers, involving practices, knowledge, and dispositions that support planning, coaching, and leading professional learning (Copping, 2022). In primary schools, where generalist teachers are responsible for mathematics instruction, SMLs play an important role in supporting pedagogical and content knowledge development as they lead change and navigating challenges in highly contextualised school environments (Sexton & Downton, 2014). Despite growing interest in this area, there is limited understanding of how knowledge in this field is created, represented, and shared in research communities.

MERGA is central to advancing mathematics education research across Australasia and represents a significant source of knowledge creation where evolving understandings of mathematical leadership, particularly in primary school contexts, are reported. However, there is limited synthesis of how this research conceptualises and reports mathematics leadership in primary schools as a form of middle leadership. A scoping review was chosen to explore the nature of this focus, diversity of study designs, theoretical perspectives, and reporting practices from MERGA publications. This scoping review addresses the question: *How has the MERGA community reported knowledge creation concerning mathematics leadership in primary schools as a form of middle leadership?* Through the systematic identification and analysis of relevant MERGA publications, the review examines how mathematics leadership is conceptualised, the forms of knowledge produced, and the methodological and theoretical approaches which underpin this work. In doing so, it offers insight into how mathematics education leadership is understood and how research communities construct and share knowledge. This short communication presents preliminary findings from the scoping review.

References

- Copping, K. (2022). Perceptions of the role of primary mathematics leaders. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys. Proceedings of the 44th annual conference of the Mathematics Education Group of Australasia*, (pp. 146–153). MERGA.
- Driscoll, K. (2022). Primary school mathematics leaders' actions that facilitate effective mathematics planning and support teachers' professional learning. In N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys. Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 178–185). MERGA.
- Sexton, M., & Downton, A. (2014). School mathematics leaders' perceptions of successes and challenges of their leadership role within a mathematics improvement project. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice. Proceedings of the 37th annual conference of the Mathematics Education Group of Australasia* (pp. 581–588). MERGA.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (p. 485). MERGA.

Can Young Children Divide Using Ideas of Rate?

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Earlier research reported the importance of the rate variable in understanding multiplicative reasoning (e.g., Lamon, 1993). A lack of experience with various semantic structures particularly rate and multiplicative comparison, can cause difficulties for students in upper primary and lower secondary school when faced with situations that require proportional reasoning (e.g., Lamon, 1993; Siemon, 2008).

Allocation/ rate is an alternative conceptualisation of the Equal Groups structure. In this context the multiplier is a rate variable that defines the relationship between a target object and its multiple counterparts (e.g., four cookies per child). The multiplicand specifies the number of target objects and hence the number of iterations of that relationship (e.g., 3 children, so 3 iterations of the 4 cookies) (Downton & Sullivan, 2017). Previous studies have explored rate problems in the context of multiplication (e.g., Cheeseman et al., 2023; Lu & Richardson, 2018), or both multiplication and division with older children (e.g., Askew, 2018; Downton & Sullivan, 2017; Siemon, 2008).

In this study, the problem was presented to 23 children (5-6 years olds) by their teacher, both as a poster and read aloud: “Fluffy can have four dog biscuits each day. Look into your packet of dog biscuits. How many days can you give Fluffy dog biscuits?” Each child was given a packet of 12 “biscuits” to use to solve the problem and illustrate their solution. To solve this problem children had to interpret the word “each” and consider how many biscuits per day and the number of days; this interpretation can be quite challenging for 5-6 year olds.

Analysis of video recordings, children’s drawings and their thinking during the lesson revealed that all 23 children could arrange the “dog biscuits” into groups of four on their whiteboards. Sixteen of these children were able to interpret the rate concept in the context of division, as evidenced by their drawings. These children recorded the number of days the dog would receive biscuits and some also included the days of the week. The remaining seven children struggled to translate the concrete representation of the problem into a recorded solution on paper. In this short communication we will examine the children’s representations and thinking in more detail more, inviting discussion on how simple rate problems can be explored with young children, and the implications for teaching and learning in Years 3 and 4.

References

- Askew, A. (2018) Multiplicative reasoning: Teaching primary pupils in ways that focus on functional relations, *The Curriculum Journal*, 29(3), 406-423, doi: <https://doi.org/10.1080/09585176.2018.1433545>
- Cheeseman, J., Downton, A. Roche, A. & Ferguson, S. (2023). Meeting multiplication through thought-provoking tasks. *Mathematics Education Research Journal*. <https://doi.org/10.1007/s13394-022-00413-1>
- Downton, A., & Cheeseman, J. (2025). Young children’s intuitive ideas of division: What do they think about “leftovers”? In *Elementary mathematics: Building strong foundations*. Charles University.
- Lamon, S. J. (1993). Ratio and proportions: Connecting content and children’s thinking. *Journal for Research in Mathematics Education*, 24(1), 41-61.
- Lu, L., & Richardson, K. (2018). Understanding children’s reasoning in multiplication problem solving. *Investigations in Mathematics Learning*, 10(4), 240-250.
- Siemon, D. (2008). From additive to multiplicative thinking – a major challenge in lower secondary mathematics education. *Paper presented at the 11th International Congress on Mathematics Education, Mexico*.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (p. 486). MERGA.

Towards Levels of Reading Mathematical Expressions to Comprehend the Underlying Phenomenon

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Access to mathematical objects is mediated through representations that express their characteristics using symbols and expressions. In mathematics education research, reasoning that investigates mathematical objects through such symbolic representations is commonly referred to as *algebraic thinking* (e.g., Kaput, 2008). Algebraic thinking involves expressing mathematical objects symbolically, transforming these expressions, and reading the characteristics of the objects in the expressions. This study focuses on the reading of mathematical expressions and explores the following research question: What qualitative differences can be identified in how students interpret mathematical expressions when they observe and attempt to understand the properties of mathematical objects? To address this research question, we planned and conducted a design-based experiment. The lessons we designed were based on a problem from *Jinkoki*, a Japanese mathematics textbook from the Edo period. The problem was to determine the number of thieves and the number of bolts of cloth they had stolen from a conversation among the thieves about how the bolts of cloth should be distributed (see Enomoto & Tohei, 2025, for details). The solution described in *Jinkoki* was to determine the number of thieves by adding 7 and 8. This solution reveals a curious phenomenon: the number of thieves equals the sum of the number of bolts of cloth left over and the number lacking when the cloth is distributed. To investigate students' interpretations of this phenomenon, we designed two tasks: explaining why the solution in *Jinkoki* works and examining whether it holds in all cases. In this study, students' thinking during the lessons were analyzed using an analytical framework developed by Tondorf and Prediger (2022). The analysis revealed qualitative differences in how students interpreted the expressions in this task. Some students focused solely on solving the equation from an operational perspective. In contrast, other students interpreted the equation from a relational perspective by connecting it with other representations, thereby revealing the structure of the solution in *Jinkoki*. Students who read the expressions in relation to the context and other representations tended to represent the structure of the solution in *Jinkoki* by intentionally retaining the operations rather than treating the solution simply as a numerical result. In addition, they considered the situation by treating constants as variables.

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References

- Enomoto, S., & Tohei, S. (2025). Students' Algebraic Thinking as Reading of Expressions to Comprehend the Reality of the Phenomenon. *Proceedings of the 47th Annual Conference of the Mathematics Education Research Group of Australasia* (p. 537).
- Kaput, J. J. (2008). What is algebra? What is algebraic reasoning? In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp.5–17). Lawrence Erlbaum.
- Tondorf, A., & Prediger, S. (2022). Connecting characterizations of equivalence of expressions: design research in Grade 5 by bridging graphical and symbolic representations. *Educational Studies in Mathematics*, 111, 399–422. <https://doi.org/10.1007/s10649-022-10158-0>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (p. 487). MERGA.

Development of Teacher and Student Perception Surveys of Mathematical Resilience

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Mathematics education aims to support students developing the capacity to reason, problem solve, communicate mathematically, and make connections across mathematical ideas (Niss & Højgaard, 2019; Phonapichat et al., 2014). Achieving these forms of mathematical proficiency, however, requires students to engage with tasks that are both cognitively and emotionally demanding. A substantial body of research shows that many students experience mathematics as challenging and anxiety-provoking, which can limit their persistence and willingness to engage with complex problems (Quintero et al., 2022).

In response to these concerns, researchers have increasingly emphasised the importance of mathematical resilience, defined as students’ capacity to persist, adapt and remain engaged when encountering difficulty in mathematics (Lee & Johnston-Wilder, 2017). Resilient learners are able to respond constructively to challenge and sustain effort when developing new mathematical understanding (Ariyanto et al., 2017). While the concept has gained attention, there are few validated instruments for measuring how teachers and students understand it as a multidimensional construct.

To address this gap, the lead author’s doctoral study developed perception surveys to investigate teachers’ (n=82) and students’ (n=781) understandings of mathematical resilience as a multidimensional construct. Practitioner insights and a targeted literature review informed survey development, which was framed by two overarching theoretical frameworks and operationalised through five constructs, emotion regulation, meaningful conceptual understanding, motivational engagement, self-regulation strategies and supportive learning environment, translated into aligned survey items. This talk outlines the methodological approach used to develop and operationalise the surveys.

References

- Ariyanto, L., Herman, T., Sumarmo, U., & Suryadi, D. (2017). Developing mathematical resilience of prospective math teachers. *Journal of Physics: Conference Series*, 895, 12062. <https://doi.org/10.1088/1742-6596/895/1/012062>
- Lee, C., & Johnston-Wilder, S. (2017). The construct of mathematical resilience. In (pp. 269-291). <https://doi.org/10.1016/B978-0-12-802218-4.00010-8>
- Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, 102(1), 9-28. <https://doi.org/10.1007/s10649-019-09903-9>
- Phonapichat, P., Wongwanich, S., & Sujiva, S. (2014). An analysis of elementary school students’ difficulties in mathematical problem solving. *Procedia – Social and Behavioral Sciences*, 116, 3169-3174. <https://doi.org/10.1016/j.sbspro.2014.01.728>
- Quintero, M., Hasty, L., Li, T., Song, S., & Wang, Z. (2022). A multidimensional examination of math anxiety and engagement on math achievement. *British Journal of Educational Psychology*, 92(3), 955-973. <https://doi.org/10.1111/bjep.12482>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilshire (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 488). MERGA.

Beyond Mathematical Thinking: Teachers' Noticing of Engagement and Dispositions in Primary Mathematics

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Research on teacher noticing in mathematics education has traditionally foregrounded teachers' attention to students' mathematical thinking, particularly their use of strategies, representations, and reasoning (Jacobs et al., 2010; Van Es & Sherin, 2002). While this focus has been foundational to understanding responsive teaching and formative assessment, teachers' classroom experiences suggest that noticing may extend beyond students' mathematical thinking alone. This short communication reports a finding from a qualitative study that explored primary teachers' reflections on noticing as an informal formative assessment practice across online and face-to-face mathematics teaching contexts.

The study involved semi-structured interviews with eight primary teachers, in Melbourne government schools, conducted on two occasions in 2020: once during Emergency Remote Teaching (ERT) and once after teachers and students returned to the physical classroom. Although the study was initially designed to investigate teachers' noticing of students' mathematical thinking, inductive analysis revealed that teachers frequently attended to aspects of learning beyond mathematical content. In particular, teachers described noticing students' engagement, dispositions, and approaches to learning as meaningful indicators that informed instructional decision-making.

Across both contexts, teachers used these relational and dispositional insights to interpret students' thinking, adapt tasks, provide feedback, and plan subsequent instruction. During ERT, reduced access to students' mathematical work and interactions heightened teachers' attention to individual learning behaviours and engagement patterns. However, such noticing was not limited to online teaching; upon returning to face-to-face classrooms, teachers continued to value and draw on insights about students' dispositions and engagement when responding to learning needs. These findings suggest that engagement and dispositions may function as important forms of evidence within teachers' informal formative assessment practices in mathematics classrooms.

This short communication contributes to research on teacher noticing by highlighting how teachers make sense of students as learners of mathematics, not solely through their demonstrated mathematical thinking but through their participation, affective responses, and learning behaviours. These findings extend current discussions in teacher noticing, formative assessment, and teacher education by highlighting the value of a more holistic conceptualisation of what counts as meaningful evidence of mathematics learning.

References

- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202. <https://doi.org/10.5951/jresmetheduc.41.2.0169>
- Van Es, E. A., & Sherin, M. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, 10(4), 571–596.

Rationales Behind Problem-Solving Implementation: Perspectives From Primary Mathematics Teachers

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The Australian Curriculum (ACARA, 2022) and international documents (National Council of Teachers of Mathematics (NCTM), n.d.; Organisation for Economic Co-operation and Development (OECD), 2024) position problem-solving as a key proficiency, prompting teachers' rationales for its implementation. This qualitative case study examined the rationales given by two primary school teachers for implementing mathematical problem-solving and explored how these rationales shaped their instructional practices.

This study was conducted as part of a larger research project in a Transition-to-Year-12 non-government school in Greater Western Sydney and involved two primary teachers of a Year 5–6 class and a Year 4 class. Data were collected through semi-structured interviews conducted at mid- and end-year points. Interview transcripts were analysed using a combination of inductive thematic analysis (Braun & Clarke, 2006) and deductive analysis.

Both teachers framed their implementation of problem-solving around three key rationales. Firstly, they pointed to future preparedness viewing problem-solving as essential for success in high school and beyond, highlighting the importance of real-world application and noting that problem-solving tasks support students to see the connection between mathematical concepts and authentic contexts. Secondly, problem-solving was seen as creating opportunities to address strategies explicitly, using modelling and discussion embedded within the context of mathematical problems. Thirdly, teacher interview data associated problem-solving with student confidence and engagement, suggesting that implementing such tasks promotes positive attitudes toward mathematics and encourages persistence.

By examining these rationales, the study provides understanding into the belief-practice relationship and highlights the need for professional learning that supports teacher mathematics problem-solving values and attitudes. Specifically, it demonstrates how teachers' rationales reveal underlying beliefs that shape instructional choices, thereby offering insight into areas where targeted professional learning may foster more consistent alignment between beliefs and practice.

References

- Australian Curriculum, Assessment and Reporting Authority. (2022). *Australian curriculum: Mathematics (Version 9.0)*. Australian Curriculum. <https://www.australiancurriculum.edu.au/>
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp0630a>
- National Council of Teachers of Mathematics. (n.d.). *Problem solving* [Research brief]. National Council of Teachers of Mathematics. <https://www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Problem-Solving/>
- Organisation for Economic Co-operation and Development. (2024, December 18). *An evolution of mathematics curriculum: Where it was, where it stands and where it is going* (Future of Education and Skills 2030). OECD Publishing. <https://doi.org/10.1787/0ffd89d0-en>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 490). MERGA.

Micro-Credentials as a Response to Out-of-Field Mathematics Teaching: Emerging Evidence

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Australia continues to face a persistent shortage of qualified secondary mathematics teachers, resulting in significant levels of out-of-field teaching where teachers without formal mathematics qualifications deliver mathematics instruction. Research and national workforce reports indicate that out-of-field teaching can limit curriculum depth, constrain pedagogical choices, and reduce teacher confidence and self-efficacy in mathematics instruction (Australian Mathematical Sciences Institute, 2024). These issues contribute to disparities in student learning opportunities and highlight an urgent need for targeted, high-quality professional learning for out-of-field mathematics teachers.

Micro-credentials and other structured upskilling programs have emerged as flexible ways to strengthen teachers' content and pedagogical knowledge. Evidence indicates that well-designed professional learning featuring active learning, strong content focus, classroom-embedded practice, and sustained support can positively influence teacher beliefs, confidence, and instructional practices (Australian Council for Educational Research, 2021; Goos et al., 2023). However, empirical evidence remains limited regarding the ways mathematics-focussed micro-credentials affect the experiences and instructional approaches of out-of-field teachers in Australia.

This project examines how mathematics micro-credentials and related professional learning influence out-of-field teachers' confidence, beliefs, and classroom practice. Using semi-structured interviews with teachers across Australia who have completed mathematics upskilling programs, the study explores perceived changes in content and pedagogical knowledge, shifts in instructional approaches, ongoing challenges, and the supports needed to sustain improvement. The aim is to generate evidence that informs future program design, strengthens support for out-of-field mathematics teachers, and contributes to national efforts to address workforce capacity. The presentation will share initial findings, including early themes related to teacher confidence, self-efficacy, and pedagogical change following engagement in mathematics micro-credentials. It will also highlight emerging insights into the strengths and limitations of current upskilling models and their implications for long-term professional growth.

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Ethics approval [H26004] was granted by Charles Sturt University and participants gave informed consent.

References

- Australian Council for Educational Research. (2021). *Professional learning modes: Literature review*. <https://doi.org/10.37517/978-1-74286-695-6>
- Australian Mathematical Sciences Institute. (2024). *Relieving out-of-field teaching in Australian secondary mathematics*. <https://amsi.org.au/?publications=position-paper-relieving-out-of-field-teaching-in-australian-secondary-mathematics>
- Goos, M., Ni Ríordáin, M., Faulkner, F., & Lane, C. (2023). Impact of a national professional development programme for out-of-field teachers of mathematics in Ireland. *Irish Educational Studies*, 42(3), 401–421. <https://doi.org/10.1080/03323315.2021.1964569>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 491). MERGA.

Emotions as Signals: A Methodological Framework Combining Achievement Emotion Theory and Mathematical Wellbeing

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Emotions are central to students' mathematical experiences, shaping how they engage, persist, and ultimately flourish in mathematics. Pekrun's (2006) Control-Value Theory (CVT) provides a well-established framework for understanding how achievement emotions arise through students' appraisals of control and value in learning contexts. The mathematical wellbeing (MWB) framework complements CVT by specifying the content of those value appraisals in mathematics through seven empirically validated ultimate values: accomplishments, cognitions, engagement, meaning, perseverance, positive emotions, and relationships, which, when fulfilled, foster students' MWB (Hill & Hunter, 2024). Importantly, positive emotions are consistently the value students are least likely to fulfil yet most strongly associated with overall MWB (Hill & Hunter, 2024), underscoring the need to better understand students' emotional experiences in mathematics. Together, CVT and MWB offer a theoretically coherent pairing in which achievement emotions function as signals of value fulfilment or conflict, and the MWB framework provides the broader context for understanding those emotions as an appraisal process, revealing which values are at stake.

To explore this combined framework (CVT+MWB), we applied it to existing datasets of students' open-ended written responses about their mathematical experiences, testing whether achievement emotions could serve as diagnostic signals and the MWB framework as the analytic tool for interpreting underlying values. For example, one student wrote: "When I finally get a question right that I've been struggling with for a while, it makes me feel really happy". Here, happiness functions as the emotional signal, while the MWB values of perseverance and accomplishments provide the interpretive context: the student's positive emotion is contingent on sustained effort through difficulty and the sense of achievement that follows. This example, like many others in our dataset, illustrates how emotions rarely appear in isolation from values; rather, each emotional experience points toward specific conditions of value (un)fulfilment that characterise a student's mathematical experience.

Building on these findings, we next apply this combined framework to analyse primary students' emotional responses during mathematics learning, captured through emotion diaries. In doing so, we offer an innovative approach to understand not only how students feel in mathematics, but why.

References

- Hill, J. L., & Hunter, J. (2024). Student mathematical wellbeing across schools in New Zealand. *New Zealand Journal of Educational Studies*, 59, 211–234.
- Pekrun, R. (2006). The control-value theory of achievement emotions. *Educational Psychology Review*, 18(4), 315–341.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 492). MERGA.

Out-of-Field Teachers: Who is Teaching Senior Secondary Mathematics in New Zealand?

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Confident, experienced, well-qualified, and well-supported mathematics teachers are more likely to enjoy and be satisfied with their teaching, to use pedagogical practices known to be effective in mathematics, and to provide equitable, high-quality learning experiences (Berg et al., 2025; See & Gorard, 2020). A steady supply of competent and confident secondary mathematics teachers with strong mathematical content and pedagogical knowledge, who remain in the profession, is therefore essential. Despite this, shortages of mathematics teachers remain an enduring problem internationally (Chauke et al., 2025) and in New Zealand (Steering Committee, 2024).

In 2025, our research team surveyed teachers of secondary mathematics in New Zealand to investigate who is teaching these classes and their experiences of doing so. We were particularly interested in the proportion of out-of-field mathematics teachers. For the purposes of this study, these were defined as teachers who had not completed a qualification that included at least one 300-level university mathematics paper (3rd year of a bachelor's degree), or equivalent, and/or who were not qualified secondary teachers. We also explored teachers' experiences of teaching mathematics in these circumstances.

In this short communication, we present a subset of preliminary findings focusing on the most senior level of mathematics taught by respondents in relation to gender, qualifications, experience and experiences of out-of-field teachers. Teaching senior mathematics is particularly sensitive to teachers' subject expertise (Vale et al., 2021), as it demands conceptual clarity, deep mathematical understanding, and less reliance on procedural approaches. Topics such as calculus, proof, and advanced algebra require specialised content knowledge and pedagogical content knowledge that are unlikely to develop without substantial disciplinary preparation.

References

- Berg, D. A., Ingram, N., Asil, M., Ward, J., & Smith, J. K. (2025). Self-efficacy in teaching mathematics and the use of effective pedagogical practices in New Zealand primary schools. *Journal of Mathematics Teacher Education*, 28(1), 129-149.
- Chauke, C., Mosia, M., Egara, F. (2025). On teacher shortages: A bibliometric analysis of the supply and demand of mathematics teachers from 2019 to 2024, *AJSTME*, 11(1), 102-119
- See, B. H., & Gorard, S. (2020). Why don't we have enough teachers? A reconsideration of the available evidence. *Research Papers in Education*, 35(4), 416-442.
- Steering Committee, 2024, Report on the Teacher Supply Summit Welling September 2-3, 2024 and the Education Sector Groups <https://www.ppta.org.nz/about-ppta/publication-library/document/2536>
- Vale, C., Campbell, C., & White, P. (2021). Beliefs and practices of secondary teachers crossing subject boundaries to teach mathematics out-of-field. *Mathematics Education Research Journal*, 33(3), 589-612.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 493). MERGA.

Learners' Negotiation on Mathematical Task Choices

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This study explores how groups of students select and justify their choice of which mathematical tasks to work on. Four groups of three students enrolled in a university bridging mathematics course were presented multiple tasks and were asked to choose one task at a time to work on together. The tasks varied in terms of familiarity, openness, context and representation. Following Yeo (2017), both open and closed tasks were included. Contexts included word problems, modelling-oriented situations and critical mathematics contexts. They were presented in symbolic, visual and digital forms of representation. Each group participated in three sessions, making a total of 12 sessions, and students participated in focus group interviews afterwards. Sessions were conducted outside regular classroom hours to fit their timetable.

Data was collected through work samples, video-recorded observations, and focus group interviews. Their written work, including task selection and working sheets, were collected as work samples. Each session, lasting approximately 90 minutes, was video recorded using two cameras, one capturing students' interaction and the other their mathematical work. Following each task choice activity, semi-structured focus group interviews of around 25 to 30 minutes were conducted to explore students' justifications behind task choices, their experience of working on the tasks, and their feelings towards mathematics.

Positioning Theory (Herbel-Eisenmann et al., 2015) is used as an analytical framework to examine students' interactional processes during task choice at the episodic level, with interview data used to understand students' reasoning for their task choices and their learner identity construction.

Preliminary findings suggest that mathematical task choice is an interactional accomplishment, shaped by both mathematical and non-mathematical justifications. Tasks perceived as familiar were selected most frequently; however, familiarity did not operate as a simple dichotomy between familiar and unfamiliar tasks. Instead, familiarity emerged as a negotiated resource within group discussions. Initial analysis also indicates that students' positioning within the group influenced how task preferences gained acceptance. Dominant participants were more likely to shape decisions using more non-mathematical arguments, whereas flexible collaborators used mathematical arguments in the decision-making process, and followers often adapted their preferences during discussion.

During the presentation I will share these preliminary findings on how students make task choices and invite discussion on how such interactional processes may relate to the mathematical learner identity construction.

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Ethics approval UAHPEC28826 was granted by University of Auckland, and all the participants gave informed consent.

References

- Herbel-Eisenmann, B., Wagner, D., Johnson, K., Suh, H. & Figueras, H. (2015). Positioning in mathematics education: Revelations on an imported theory. *Educational Studies in Mathematics*, 89(2), 185-204.
- Yeo, J. B. W. (2017). Development of a Framework to Characterise the Openness of Mathematical Tasks. *International Journal of Science and Mathematics Education*, 15(1), 175-191. <https://doi.org/10.1007/s10763-015-9675-9>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 494). MERGA.

How Prospective Mathematics Teachers Elicit, Explore, & Apply Classification Models in Unplugged Image Classification Tasks

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The advent of big data and AI has heightened the necessity of developing instructional approaches and professional learning to integrate core topics of data science (DS) and AI within mathematics learning at the primary and secondary levels (Leavy et al., 2026). Both practising and teaching DS are required as part of initial teacher education (ibid.). In this context, unplugged DS tasks and activities that do not rely on computational tools and programmes are effective for teachers who are not familiar with data science to experience fundamental DS practices (e.g., Biehler et al., 2025; Hilger & Büscher, 2025; Lindner et al., 2019). This study explores how prospective mathematics teachers (aged 20–21 years) elicit, explore, and apply classification models in two unplugged image classification tasks based on *Models and Modelling Perspectives* (MMP; Lesh & Doerr, 2003). MMP focuses on real-world problem-solving and conceptual development through the modelling process (Doerr et al., 2017). Consequently, MMP aligns with DS practices centred on modelling, involving the development of their own models and key ideas related to DS. Based on the MMP, this study developed two DS units (total nine 90-min lessons). In the first unit (six lessons), the prospective teachers collaboratively elicit classification models in an unplugged image classification task, explore and revise them using new data, and apply the models they developed in the first unit to another image classification task in the second unit (three lessons). This study analysed the group's model development through the units. Preliminary results indicate that they constructed decision tree models, and one group of them developed informal random forest models to ensure the simplicity, shareability, and generalisability of the models. In addition, they creatively developed informal classification models, including point system models that incorporate weights for each attribute and classification success rates. These results highlight the potential of a modelling approach to achieve open-ended and creative DS learning in initial teacher education.

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References

- Biehler, R., Kawakami, T., Lampen, E., Weiland, T., & Zapata-Cardona, L. (2025). Statistics and data science education as a vehicle for empowering citizens – short summary of a survey. *European Mathematical Society Magazine*, 136, 49–52. <https://doi.org/10.4171/MAG/257>
- Doerr, H., delMas, R., & Makar, K. (2017). A modeling approach to the development of students' informal inferential reasoning. *Statistics Education Research Journal*, 16(2), 86–115.
- Hilger, S., & Büscher, C. (2025). Richness of reflective processes on algorithmic decision-making systems by prospective teachers. In M. Bosch et al., (Eds.), *Proceedings of CERME14* (pp. 875–882). ERME.
- Leavy, A., Kazak, S., Podworny, S., & Frischemeier, D. (2026). Designing data science learning in initial teacher education: The EDUCATE conceptual framework. *Education Sciences*, 16(2), 307.
- Lesh, R., & Doerr, H. (2003). *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Routledge.
- Lindner, A., Seegerer, S., & Romeike, R. (2019). Unplugged activities in the context of AI. In S. Pozdniakov & V. Dagienė (Eds.), *Informatics in schools: New ideas in school informatics* (pp. 123–135). Springer.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 495). MERGA.

Mathematical Affective Factors Experienced by Year 7 Students During Transition to Secondary School

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The transition for students from primary to secondary school is one of great change. This can have a large impact on students' progress in mathematics. This may not only be reflected in academic achievement but also a decline in interest in and motivation to learn mathematics (Kaur et al., 2022). This can then lead to the development of negative attitudes towards mathematics (e.g., Rončević Zubković et al., 2023) and a potential increase in mathematics anxiety (e.g., Deieso & Fraser, 2019). However, a longitudinal study by Klee and Miller (2019) showed that there were a variety of different mathematics anxiety trajectories that students experience across the transition period.

The current study looked at what students identify as factors that influence feelings about mathematics during transition from primary to secondary school including attitudes towards mathematics and mathematics anxiety. Year 7 students ($n = 286$) were measured for these aspects of mathematical affect at the end of Term 1 and the end of Term 3 and were also asked to write a metaphor about what mathematics was like for them at both time points.

It was found that students' attitudes towards mathematics significantly deteriorated. The metaphor analysis was able to illustrate a richer understanding of students' attitudes (e.g., "math is like a big, scary bear"; "math is like tasty food"; "maths is like 1967 television") and how they changed over time. This presentation will explore the complex relations between students' metaphors and their overall feelings about mathematics.

A purposive sample of students ($n = 16$) were also interviewed to clarify their attitudes and what factors influenced these. These participants were selected to represent three groups based on changes in attitudes towards mathematics: those whose attitudes improved, those whose attitudes remained unchanged, and those whose attitudes deteriorated. It was found that for those students whose attitudes towards mathematics deteriorated, this was related to negative experiences of secondary school mathematics. The factors driving this were harder content, content moving at a faster pace and the high use of textbooks in secondary school. When this group is compared to the improved attitude group, they reported a lower rate of lack of self-confidence, and higher stress and anxiety.

References

- Deieso, D., & Fraser, B. J. (2019). Learning environment, attitudes and anxiety across the transition from primary to secondary school mathematics. *Learning Environments Research*, 22(1), 133–152. <https://doi.org/10.1007/s10984-018-9261-5>
- Kaur, T., McLoughlin, E., & Grimes, P. (2022). Mathematics and science across the transition from primary to secondary school: A systematic literature review. *International Journal of STEM Education*, 9(1), 13. <https://doi.org/10.1186/s40594-022-00328-0>
- Klee, H. L., & Miller, A. D. (2019). Moving Up! Or Down? Mathematics Anxiety in the Transition From Elementary School to Junior High. *The Journal of Early Adolescence*, 39(9), 1311–1336. <https://doi.org/10.1177/0272431618825358>
- Rončević Zubković, B., Pahljina-Reinić, R., & Kolić-Vehovec, S. (2023). Age and gender differences in mathematics learning during school transition. *International Journal of School & Educational Psychology*, 11(1), 20–33. <https://doi.org/10.1080/21683603.2021.193420>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 496). MERGA.

Spatial Training Improves Mathematics Performance Independent of Mathematics Anxiety in Early Adolescence

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The training of spatial skills such as mental rotation and perspective taking has been shown to improve mathematics performance, yet the mechanisms underlying this relation remain poorly specified (Resnick & Adams, 2024). Most accounts emphasize shared cognitive resources or representational overlap between spatial and mathematical thinking, but comparatively little attention has been given to the role of learners' affective experiences in mathematics. Mathematics performance depends not only on competence, but also on one's attitudes towards mathematics and other affective responses such as mathematics anxiety, which can constrain working memory, reduce persistence, and impair performance (e.g., Barroso et al., 2021).

Training spatial skills may alter these affective factors by increasing perceived control, fluency, and intelligibility when engaging with mathematically relevant representations. As a result, improvements in mathematics following spatial training may be mediated by changes in mathematics-related beliefs and anxiety. The current study explored this mediation effect, by examining whether the relation between spatial training and mathematics performance is explained by changes in mathematics affect. By evaluating affective pathways alongside performance outcomes, this work aims to clarify how and why spatial training transfers to mathematics, and to integrate cognitive and affective perspectives on learning and transfer.

Drawn from a larger classroom-based digital spatial intervention study, which include pre/post measures on spatial skills and mathematics performance (Harris et al., under review), 320 Year 7 and 8 students also completed pre/post measures on mathematics anxiety and attitudes towards mathematics. Qualitative data, including interviews and metaphor analysis, were used to elaborate on and contextualise quantitative findings. It was found that spatial training produced a significant improvement in mathematics performance irrespective of the level of mathematics anxiety experienced by the student, highlighting the impact of spatial training.

Drivers of mathematics anxiety and attitudes towards mathematics in Years 7 and 8 appear more complex. The study found that there was no significant reduction in mathematics anxiety due to spatial training. In contrast, attitudes towards mathematics declined across the entire cohort, independent of participation in the intervention. Initial analysis of the qualitative data highlights the complexity of factors that are present in these year groups which may be obscuring the potential benefits of spatial training on mathematics affect.

References

- Barroso, C., Ganley, C. M., McGraw, A. L., Geer, E. A., Hart, S. A., & Daucourt, M. C. (2021). A Meta-analysis of the Relation Between Math Anxiety and Math Achievement. *Psychological Bulletin*, 147(2), 134–168. <https://doi.org/10.1037/bul0000307>
- Harris, D., Resnick, I., Kovacs, N., Younes, F., Patahuddin, S. & Lowrie, T. (2026). *Comparing Spatial Training Pathways: Mental Rotation, Perspective Taking, and Their Links to Mathematics*, PsyArXiv, https://doi.org/10.31234/osf.io/nkcdv_v1
- Resnick, I., & Adams, J. (2024). Spatial reasoning interventions and transfer to geometry: What we know about mechanism. *Proceedings of the 26th ICMI Study Conference (Advances in Geometry Education)*, (134), 117–124. <https://doi.org/10.4171/mag/208>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 497). MERGA.

Design Principles for Teacher Discourse: Developing Adaptive Reasoning in Mathematics Classrooms

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Supporting students' capacity to reason mathematically is widely recognised as an important focus of mathematics education, yet research continues to show that students often struggle to explain, justify, and reflect on their mathematical work (e.g., Putri & Warmi, 2024). These capabilities are captured in the construct of adaptive reasoning which is a component of mathematical proficiencies (Kilpatrick et al., 2001). They defined adaptive reasoning as the capacity for logical thought, reflection, explanation, and justification in mathematical activity. Reasoning becomes adaptive when learners revise their thinking, justify their ideas, or adjust their strategies in response to new problems, feedback, or alternative perspectives (Jeannotte & Kieran, 2017). In this sense, adaptiveness lies not merely in the presence of reasoning processes, but in how these processes are flexibly mobilised and revised across mathematical situations, especially in the classroom.

Despite its importance, students' adaptive reasoning remains underdeveloped, particularly among average- and low-achieving learners. Currently, instructional approaches rarely focus explicitly on cultivating this aspect of mathematical proficiency. At the same time, a growing body of research highlights the potential of discourse-rich mathematics classrooms to support students' reasoning. Through classroom discourse, students articulate their thinking, justify strategies, critique arguments, and revise their ideas. Because adaptive reasoning involves explaining, justifying, and reflecting, it is often made visible during classroom discourse as learners respond to and build on one another's reasoning. However, while prior studies highlight the role of discourse, less is known about the specific teacher discourse strategies that actively elicit students' adaptive reasoning during mathematical activity.

In the presentation, we will describe and exemplify strategies for eliciting adaptive reasoning, informed by both the Teacher Moves for Supporting Student Reasoning framework (Ellis et al., 2019) and our initial empirical analysis of four lesson transcripts from previous professional work with teachers. The design principles for teacher-orchestrated discourse that emerge from this synthesis will be used to craft intervention modules intended specifically for average- and low-achieving learners in secondary schools in the Philippines. Ultimately, these modules will inform the design and implementation of a teaching experiment, which is the next stage of our work.

References

- Ellis, A., Özgür, Z., & Reiten, L. (2019). Teacher moves for supporting student reasoning. *Math Ed Res J*, (31), 107–132. <https://doi.org/10.1007/s13394-018-0246-6>
- Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educ Stud Math*, (96), 1–16. <https://doi.org/10.1007/s10649-017-9761-8>
- Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics. Washington, DC: The National Academies Press. <https://doi.org/10.17226/9822>
- Putri, D. A., & Warmi, A. (2024). An analysis of adaptive reasoning ability in middle school students mathematics learning. *Jurnal Pendidikan Matematika*, 10(2), 117–125. <https://doi.org/10.33474/jpm.v10i2.21981>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 498). MERGA.

Do Paradigms Matter in Mathematics Education Research?

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Education as an interdisciplinary field draws on and adapts methodologies and findings from within educational research as well as a broad range of related fields. Mathematics education research values outcomes from different research perspectives and paradigms as they provide opportunities for debate and to enrich our understanding of findings within mathematics education research. There is recent pressure, however, to privilege research findings about teaching and learning in school mathematics that have been conducted in other fields; as a source of theoretical and practical guidance, these findings often neglect or dispute research conducted in mathematics education (cf. MERGA, 2023; 2025). This misalignment of findings has created tensions when policymakers have accepted findings about mathematics education conducted by those outside the field, in preference to findings within mathematics education research. This presentation discusses a small personal exploratory study with an aim to better understand how findings about the teaching and learning of mathematics—promoted in a discipline outside of education—could come to conclusions that so strongly contrasted with findings within mathematics education research.

The presentation will first discuss a pilot exercise that observed the methodology and subsequent claims made in an article published in a high-ranking journal in psychology about whether it is best to teach content before problem-solving in mathematics or vice versa. A subsequent exploratory analysis was carried out based on this pilot exercise to respond to the research question, *What kinds of methodologies and evidence are used in psychology to substantiate claims about teaching and learning in mathematics education?* Observations from the pilot exercise were used to create criteria to select a set of articles published in 2025 about the teaching of mathematics obtained from the APA Psych Info database. The analysis examined the methodological choices, evidence and claims made in these articles about mathematics education.

The findings were inconclusive and on their own were not meant to be generalisable, but insights from the analysis provided a reminder about the difference paradigms and disciplines can make in mathematics education research. That is, along with the findings, one must understand the paradigms, methodologies and communities within which research is situated to be wise in how research can be adopted or adapted to mathematics classrooms. The conclusion discusses implications for the mathematics education community and why attending to paradigms is so critical at the time we are in right now.

References

- Mathematics Education Research Group of Australasia (MERGA) (2023). *Response from the Mathematics Education Research Group of Australasia (MERGA) to the Teacher Education Expert Panel Discussion Paper*. Author. https://www.education.gov.au/system/files/documents/submission-file/2023-07/TEEP_Mathematics%20Education%20Research%20Group%20of%20Australasia.pdf
- Mathematics Education Research Group of Australasia (MERGA) (2025). *MERGA Position Statement on Effective Mathematics Teaching*. Author. <https://merga.net.au/positionstatement/>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 499). MERGA.

Analysis of Unitizing and Norming Abilities in the Process of Reasoning Up and Down in Fraction Learning in Year 6

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The importance of learning fractions has been emphasized because of its potential to enhance mathematical understanding (Lamon, 2020). However, many studies report that children struggle to develop fraction concepts (e.g., Shimomura et al., 2022; Singh et al., 2018). In response, Lamon (2020) highlights the importance of “reasoning up and down,” a process that supports useful and flexible thinking strategies from the earliest stages of fraction learning. She explains: “When units are defined implicitly, we have an excellent opportunity to use a process that I refer to as reasoning up and down, because it often entails reasoning up from some fraction to the unit, then back down from the unit to another fraction” (Lamon, 2020, p. 107). This explanation suggests that children solve problems by comparing the magnitude of a fraction with the unit value of 1. Since fractions are numbers derived from dividing 1, comparing the magnitudes of 1 and fractions are essential for understanding fractional relationships. Furthermore, in the process of reasoning up and down, “unitizing”—treating an object as a fixed unit—and “norming”—restructuring quantitative relationships based on that unit—come into play (cf. Masutani, 2025). Examining these abilities may therefore provide important insights for fraction instruction. However, to the best of our knowledge, no studies have examined the characteristics or developmental stages of unitizing and norming abilities in this process. Therefore, this study aims to identify the developmental stages of unitizing and norming observed in reasoning up and down. We conducted a study with 21 students in a Year 6 Japanese elementary school and analyzed their problem-solving processes using workbook entries and classroom utterances within Lamon’s framework of unitizing, norming, and partitioning. This presentation reports the developmental stages of unitizing and norming abilities demonstrated in children’s reasoning up and down and discusses their implications for fraction instruction.

Acknowledgments

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References

- Lamon, S. J. (2020). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instruction Strategies for Teachers* (4th ed.). Routledge, UK: Newgen Publishing.
- Masutani, Y. (2025). Analysis of reasoning up and down in Grade 2 fraction learning: A case study based on Lamon’s theoretical framework. *Proceedings of the 58th Autumn Research Conference of the Japan Society of Mathematical Education*, 281–288. [in Japanese]
- Shimomura, T., Okabe, Y., Shimomura, S., & Saito, H. (2022). Characteristics of mathematical negotiation in the process of building a fractional concept: Based on an analysis of small group learning in quantity fraction. *Journal of Science Education in Japan*, 46(4), 283–298. [in Japanese]
- Singh, P., Hoon, T. S., Nasir, N. A. M., Han, C. T., Rasid, S. M., & Hoong, J. B. Z. (2021). Obstacles faced by students in making sense of fractions. *The European Journal of Social & Behavioural Sciences*, 30(1), 34–51.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 500). MERGA.

Engaging Pre-Service Teachers in Asynchronous Learning: Evidence from a Systematic Review

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University teaching has progressively moved from traditional lectures to blended and online models in which core content is delivered asynchronously through curated digital resources (Talbert, 2017). This trend is also evident in Initial Teacher Education (ITE), where online and asynchronous learning experiences are now commonplace (Sheehan et al., 2025). Although effective lecture practices are well established (e.g., Harrington & Zakrajsek, 2017), evidence regarding effective practices for asynchronous learning in ITE teaching remains fragmented (Dyment & Downing, 2020). To inform the design of mathematics and science ITE subjects and enhance pre-service teacher learning, the authors (experienced teacher educators in a faculty transitioning to curated online learning) conducted a systematic literature review to identify evidence-based practices that support engagement in the asynchronous components of blended ITE programs. Guided by a structured search strategy and the PRISMA framework (Page et al., 2021), the review progressed through four stages: identification, abstract and full-text screening, and inclusion. The review identified practices and task characteristics that supported cognitive, affective, and behavioural engagement. Cognitive engagement refers to pre-service teachers' investment in learning through reflection, critical analysis, and problem solving; affective engagement encompasses emotional responses such as confidence, satisfaction, and belonging; and behavioural engagement is reflected in active participation and deliberate contributions to the learning environment. Three interrelated themes emerged: engagement was strengthened by intellectually demanding, carefully scaffolded tasks; many practices supported multiple dimensions of engagement simultaneously; and effective designs consistently promoted autonomy and personalisation through learner choice, self-pacing, and flexible pathways.

References

- Dyment, J. E., & Downing, J. J. (2020). Online initial teacher education: A systematic review of the literature. *Asia-Pacific Journal of Teacher Education*, 3(48), 316-333.
- Harrington, C., & Zakrajsek, T. D. (2017). *Dynamic lecturing: Research-based strategies to enhance lecture effectiveness*. Routledge.
- Page, M. J., Moher, D., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., Brennan, S. E., Chou, R., Glanville, J., Grimshaw, J. M., Hróbjartsson, A., Lalu, M. M., Li, T., Loder, E. W., Mayo-Wilson, E., McDonald, S., ... McKenzie, J. E. (2021). PRISMA 2020 explanation and elaboration: updated guidance and exemplars for reporting systematic reviews. *BMJ (Online)*, 372, n160. <https://doi.org/10.1136/bmj.n160>
- Sheehan, H., Cain, M., & Taouk, S. (2025). Preservice teachers' experiences of relationality in asynchronous online learning: "There's not really an opportunity to have those networks". *Asia-Pacific Journal of Teacher Education*, 53(4), 449-463. <https://doi.org/10.1080/1359866X.2025.2539248>
- Talbert, R. (2017). *Flipped learning: A guide for higher education faculty*. Stylus.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 501). MERGA.

Curiosity as Circuit Breaker: Deconstructing Teacher Educator Deficit Perspectives of Pre-Service Teacher Resistance

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Teacher educator work with primary PSTs related to mathematics pedagogies is important to support the opening of more equitable possibilities (Boylan & Woolsey, 2015). In exploring more socially just approaches to mathematics teaching with PSTs, Nolan and Keazer (2021) and Boylan and Woolsey (2015) discuss the potential for PSTs to respond with resistance. However, Guzmán (2019) critiques this narrative, noting that this perspective is a deficit perspective which frames PSTs as problematic and teacher educators as the ones bringing the solutions to those problems. Graham and Slee (2008) use the concept of normalisation and normalising discourses as a contrast to deficit discourses. They say that normalising discourses “affirm or negate particular ways of being” (p. 282). This means normalising discourses communicate what is the desirable subject, seeing those who do not meet the criteria of a desirable subject as deficient. Past experiences and beliefs systems shape the practice and assumptions of teacher educators, meaning that teacher educators need to deconstruct their discourse to open up their teaching practices (Leibowitz & Bozalek, 2016). This presentation shares some reflection and unlearning I undertook as a teacher educator, specifically exploring my deficit assumptions towards the primary PSTs with whom I work. It proposes that a pedagogy of curiosity may provide space to examine and reframe deficit assumptions towards PSTs. Undertaking a process of self-study led to an awareness that I regularly respond to PSTs with frustration and deficit assumptions when met with responses I frame as resistance. At times I respond with blame and frustration towards PSTs for their unwillingness to be open to the new possibilities for learning and teaching mathematics that I seek to introduce. The process of self-study helped me to see that my ways of thinking about and relating with PSTs need to change. I propose that a response of curiosity may act as a circuit breaker (Graham & Slee, 2008) to a deficit perspective, bringing opportunities to reshape the deficit perspective and opening up opportunities to respond to PSTs in ways that foster relational encounters between myself and the PSTs with whom I work.

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References

- Boylan, M., & Woolsey, I. (2015). Teacher education for social justice: Mapping identity spaces. *Teaching and Teacher Education*, 46, 62-71.
- Graham, L. J., & Slee, R. (2008). An illusory interiority: Interrogating the discourse/s of inclusion. *Educational Philosophy and Theory*, 40(2), 277-293.
- Guzmán, L. D. (2019). Complex and contradictory conversations: Prospective teachers interrogating dominant narratives within mathematics education discourse. *Mathematics Teacher Educator*, 8(1), 7-20.
- Leibowitz, B., & Bozalek, V. (2016). The scholarship of teaching and learning from a social justice perspective. *Teaching in Higher Education*, 21(2), 109-122.
- Nolan, K., & Keazer, L. (2021). Mathematics teacher educators learn from dilemmas and tensions in teaching about/through culturally relevant pedagogy. In M. Goos & K. Beswick (Eds.), *The learning and development of mathematics teacher educators: International perspectives and challenges* (pp. 301-320). Cham, Switzerland: Springer.
- (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 502). MERGA.

From Outcomes to Opportunities: Broadening the Conception of School Data for Reflective Mathematics Teaching

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An accountability-driven view of data has led to several problems in mathematics teaching, including curriculum narrowing, rigid grouping practices, and deficit views of students (Datnow & Park, 2018). Insufficient attention has been given to how routinely produced school documents might provide insight into intended mathematics opportunity to learn (OTL) within schools. To address this gap, selected findings from the intended mathematics OTL data derived from existing school documents are reported.

Two government primary schools, referred to as School A and School B, participated in the study. Year 3 lesson plans and daily timetables from the first eleven weeks of the school year, along with mathematics scope and sequence documents, were analysed from each school. Documents were coded into quantitative values and analysed using descriptive statistics, including counts, percentages, and measures of central tendency. Although the broader analysis examined several aspects of intended OTL, this presentation reports on the time allocated to mathematics learning, the distribution of planned lessons across strands and proficiencies, and students' grouping practices.

Analysis of school data identified both similarities and differences in the intended OTL across the two schools. Time allocated to mathematics learning was consistent across the four Year 3 classes in School B (Mean: 5.58 hours per week), whereas School A showed greater variation (Mean: 3.42 to 4.23 hours per week). In both schools, Fluency was the most frequently addressed proficiency (School A: 85%; School B: 84%), followed by Understanding (School A: 61%; School B: 53%). Problem Solving appeared more frequently in School B (20%) than School A (10%), while Reasoning was minimally represented (5%) across both contexts. Both schools prioritised the Number and Algebra strand (School A: 60%; School B: 75%), followed by Measurement and Geometry (School A: 29%; School B: 13%) and Statistics and Probability (School A: 10%; School B: 9%). Grouping practices also varied by strand. Teachers in both schools predominantly planned homogeneous groups for Number lessons, informed by test outcomes, whereas Statistics and Probability lessons exclusively involved heterogeneous grouping. In Measurement and Geometry, heterogeneous grouping was common, occurring in approximately 80% of lessons in School A and 70% in School B.

These findings illustrate how routinely produced school documents can be used as a source of intended mathematics OTL data to support reflection on planning and instructional priorities. By focusing on planned learning opportunities rather than student characteristics alone, such data may help teachers and school leaders examine structural factors influencing mathematics outcomes. For researchers and policymakers, analysing intended OTL from school documents offers a complementary approach to survey-based studies. The findings are intended to invite discussion about how school-generated data might be used to support reflective mathematics teaching and planning practices. Limitations associated with using school documents as indicators of intended, rather than enacted, OTL are acknowledged.

References

Datnow, A., & Park, V. (2018). Opening or closing doors for students? Equity and data use in schools. *Journal of Educational Change*, 19(2), 131–152.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 503). MERGA.

Emerging Dimensions in Early Childhood Mathematics Education Research: Perspectives from the 15th International Congress on Mathematical Education (ICME-15)

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We provide an overview of the research contributions to the 15th International Congress on Mathematical Education (ICME-15), Topic Study Group (TSG) 3.1: Mathematics Education at Early Childhood and Primary Level. The forthcoming monograph, *Emerging Research Dimensions of Early Mathematics Education*, draws together the main themes and provides collaborative perspectives with over 90 authors (Mulligan et al., in press). This overview highlights some shifts in research priorities and emphases over the past decade, particularly from ICME-13 and ICME-14 (Björklund et al., 2020; Elia et al., 2023; van den Heuvel-Panhuizen, 2021).

The contributions represent a diverse range of cultural contexts drawn from 20 countries, comprising 35 papers and six posters. There were a large number of studies on early conceptual structures, patterning, and spatial and statistical reasoning, engagement in learning, and new technologies. Related themes focused on learning trajectories (Clements et al., 2023) and early childhood professionals' practice.

References

- Björklund, C., van den Heuvel-Panhuizen, M., & Kullberg, A. (2020). Research on early childhood mathematics teaching and learning. *ZDM– Mathematics Education*, 52, 607–619.
- Clements, D., Lizcano, R., & Sarama, J. (2023). Research and pedagogies for early math. *Education Sciences*, 13(8), 839; <https://doi.org/10.3390/educsci13080839>
- Elia, I., Baccaglioni-Frank, A., Levenson, E., Matsuo, N., Feza, N., & Lisarelli, G. (2023). Early childhood mathematics education research: Overview of latest developments and looking ahead. *Contemporary Research and Perspectives*, 28, 75–129. <https://doi.org/10.4000/adsc.3113>
- Mulligan, J. T., Björklund, C., Olfos, R., Clements, D., & Sarama, J. (2026, in press). *Emerging Research Dimensions of Early Mathematics Education: A Review of the ICME-15 Topic Study Group (TSG): Mathematics Education at the Early Childhood and Primary Level*. Springer.
- van den Heuvel-Panhuizen & A. Kullberg (Eds.) (2021). Proceedings of the 14th International Congress on Mathematical Education (ICME14), Topic Study Group 1 Mathematics education at preschool level. Shanghai, China, 11–18 July 2021.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 504). MERGA.

Qualitative Changes in Primary Pupils' Geometric Reasoning Through Engaging with Diagrams in Generative Mode of Interaction

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It has been suggested that the difficulties pupils encounter with proofs at the secondary school level stem from a discrepancy between their primary and secondary school learning experiences. In particular, they lack experience in explaining their reasoning at the primary level (Stylianides & Stylianides, 2017). This study examines what kinds of interactions between pupils and diagrams in geometry lessons might contribute to qualitative changes in their geometric reasoning. To theorise such interactions, we use Herbst's (2004) generative mode of interaction. This involves the actor (e.g. a learner) applying hypothetical operations to a diagram as an object and reinterpreting it as a semiotic resource, thereby discovering new meanings or mathematical structures within it and contributing to changes in the learner's geometric reasoning. To elicit this generative mode of interaction, we designed and implemented a lesson based on Wittmann's (2021) operative principle. The research question was: In primary school geometry lessons based on the operative principle, how does the generative mode of interaction emerge, and what effect does it have on pupils' geometric reasoning? We conducted a classroom-based study with two lessons in which Year 5 Japanese pupils (aged 10) were asked to explain why the areas of triangles created using GeoGebra were always equal. Following the operative principle, the pupils were allowed to interact with diagrams in both digital and non-digital formats. These lessons were recorded. We also collected the pupils' written responses and reasoning from their worksheets. In these lessons, pupils manipulated diagrams via GeoGebra and used paper to explain the equality of triangle areas based on their findings. Through operations such as adding auxiliary lines, the pupils transformed the diagram in various ways and discovered the underlying geometric properties. Because the pupils actively engaged with the geometric objects and generated new ways of seeing the figures, they were interacting with the diagrams in a generative mode. Furthermore, video analysis showed that task design and teacher questioning based on the operative principle worked in complementary ways, leading pupils to reinterpret shapes from mere visual objects as semiotic resources for reasoning. Observing the evolution of the pupils' explanations revealed a process in which their descriptions developed from intuitive visual imagery to logical explanations grounded in the constituent elements and geometric properties of the shapes. These findings suggest that constructing a learning environment that fosters generative interactions is an effective means for primary school pupils to develop geometrical reasoning based on geometric properties.

References

- Herbst, P. (2004). Interaction with diagrams and the making of reasoned conjectures in geometry. *Zentralblatt für Didaktik der Mathematik*, 36(5), 129–139. <https://doi.org/10.1007/BF02655665>
- Stylianides, G. J., & Stylianides, A. J. (2017). Research-based interventions in the area of proof: The past, the present, and the future. *Educational Studies in Mathematics*, 96(2), 119–127.
- Wittmann, E. Ch. (2021). *Connecting mathematics and mathematics education: Collected papers on mathematics education as a design science*. Springer. <https://doi.org/10.1007/978-3-030-61570-3>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 505). MERGA.

Can Research on Conspiracy Theory Inoculation Motivate Mathematics Instruction?

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Unwarranted beliefs—those not based on valid reasoning or credible data and which persist despite countervailing evidence—are a societal problem (Mihailidis & Viotty, 2017). Literacy and social studies are actively working on this problem, but mathematics, the purported site of careful reasoning, could do more to reduce susceptibility to unwarranted beliefs. Unfortunately, mathematical reasoning often fails to extend beyond school (Herbst & Brach, 2006). How might we teach mathematical reasoning in a way that *does* travel with students beyond the classroom? One possible answer to this question draws upon (A) boundary crossing theory and (B) social psychology research on inoculations against unwarranted belief. Boundary crossing (Akkerman & Bakker, 2011) is a way to build knowledge and skills that extend beyond the classroom, more bidirectional and accepting of the diversity of different domains than “transfer”. Boundaries are lines of separation between domains of action and interaction, such as between interactions at school and at home, but boundaries also entail proximity, such as the same people being present in the two domains. Boundaries of interest are those between school mathematics situations and social situations in which arguments are made about political or civics-related events. One central notion in boundary crossing theory is a boundary object, which is a tool or artifact that can carry meaning or spur interaction in both domains. In the present work, boundary objects can be inspired by research on conspiracy theory inoculation (e.g., Dyer & Hall, 2019), such as the inoculating power of pausing to ask the questions “How do you/I know this claim is true?” and “Could this claim possibly be false?” This *set of critical questions* could be a boundary object that spurs mathematical justifications but also builds a habit of careful thinking outside school (Otten et al., 2023). Another boundary object could be *diagramming an argument* (Krummheuer, 1995) to better ponder and diagnose the backing (or lack thereof) behind claims and inferences. A third could be a *confidence rating*, which makes explicit how confident someone is about a claim (PDPD, 2024). Exposing even small levels of doubt about unwarranted beliefs can lead to productive discourse, in class and beyond alike. The aim is to consider whether they can carry meaning in the mathematics classroom and also help students develop skills and identities that make them less susceptible to unwarranted beliefs in society.

References

- Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research*, 81, 132–169. <https://doi.org/10.3102/0034654311404435>
- Dyer, K. D., & Hall, R. E. (2019). Effect of critical thinking education on epistemically unwarranted beliefs in college students. *Research in Higher Education*, 60, 293–314.
- Herbst, P., & Brach, C. (2006). Proving and doing proofs in high school geometry classes: What is it that is going on for students? *Cognition and Instruction*, 24, 73–122. https://doi.org/10.1207/s1532690xci2401_2
- Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures* (pp. 229–269). Lawrence Erlbaum.
- Mihailidis, P., & Viotty, S. (2017). Spreadable spectacle in digital culture: Civic expression, fake news, and the role of media literacies in “post-fact” society. *American Behavioral Scientist*, 61(4), 441–454.
- Otten, S., Bemke, J., & Webb, J. (2023). Exploring the viral spread of disease and disinformation. In S. R. Harper & D. C. Cox (Eds.), *Modern Math Tasks to Provoke Transformational Thinking: Grades 9-12* (pp. 52–78). National Council of Teachers of Mathematics.
- Practice-Driven Professional Development (PDPD). (2024). Confidence Meter [webpage]. <https://practicedrivenpd.com/instructional-nudges/my-messy-idea/>
- (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 506). MERGA.

Reframing Reluctance as a Turning Point for Students' Critical Mathematical Thinking

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Students' reluctance in mathematics classrooms is commonly associated with disengagement or lack of understanding. Research on affect and productive struggle suggests that reluctance may reflect cognitive–affective tension when learners encounter conflicting ideas (Warshauer, 2015). In socially contextualised mathematical tasks, these tensions may extend beyond procedural difficulty, as students interpret quantitative results, evaluate consequences, justify decisions, and consider real-world implications. Such forms of engagement correspond to key dimensions of Critical Mathematical Thinking (CMT), defined as the capacity to use mathematics to address real-world problems while also considering the consequences of proposed solutions and associated actions. CMT integrates mathematical, evaluative, reasoning, and critical capabilities, together with dispositions to reconsider assumptions (Geiger et al., 2025). However, little is known about how reluctance relates to CMT and may be interactionally reframed as a turning point for its emergence. This study examined how reluctance, conceptualised as tension between students' mathematical results and their willingness to act on them, is interactionally constructed and reframed as a moment in which CMT can emerge. Drawing on a case study of a Year 7 classroom in Queensland, Australia, involving a socially contextualised fundraising task, the study captures how moments of reluctance emerged during mathematical problem-solving interactions between students and teachers. Data included video-recorded classroom interactions and a video-stimulated recall interview with the teacher. Using interactional analysis, episodes of reluctance were identified through observable indicators including hesitation, uncertainty, and resistance to revising solutions. Reframing was traced through shifts in classroom dialogue. Analysis showed that reluctance emerged when unexpected mathematical results conflicted with the feasibility of students' proposed solutions. Teacher contributions that redirected students' attention to task constraints were associated with shifts toward evaluative, reasoning, and consequence-oriented engagement aligned with dimensions of CMT. These findings offer insights into how classroom interactions can support the development of critical mathematical thinking during mathematical problem-solving.

References

- Geiger, V., Beswick, K., & Fielding, J. (2025). The role of compassion and empathy when engaging with tasks concerned with social and environmental justice. In S. M. Patahuddin, L. Gaunt, D. Harris, & K. Tripet (Eds.), *Unlocking minds in mathematics education. Proceedings of the 47th annual conference of the Mathematics Education Research Group of Australasia* (pp. 157–164). Canberra: MERGA.
- Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*, 18(4), 375–400. <https://doi.org/10.1007/s10857-014-9286-3>
- (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 507). MERGA.

An Investigation on the Impact of Screen Multi-Tasking on Secondary Students Behaviour and Wellbeing in Mathematics in NSW Secondary Schools

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Digital technology use has dramatically altered learning in secondary mathematics classrooms. Although the purpose of technology was to enhance engagement and streamline teaching (Yang et al., 2020), its impact can be contested, with concerns that devices may distract students and undermine learning (Kuş, 2025). Growing concerns about digital distraction have led to initiatives such as the NSW mobile phone ban, underlining the need to examine how laptops and multi-tasking behaviours affect students in mathematics. Adolescents engage in extensive daily screen use and frequent media multitasking (May & Elder, 2018), leading to divided attention, cognitive overload, and poorer learning outcomes (Saarinen et al., 2021). Excessive screen use is also associated with anxiety, sleep disruption, reduced self-control, and internet addiction (Nakshine et al., 2022). While purposeful technology integration can enhance conceptual understanding and engagement, unfocused multi-tasking may weaken working memory and hinder deep mathematical processing (Pellas, 2024). This study investigates the behavioural and socio-emotional impact of screen multitasking in secondary mathematics classrooms. It aims to provide evidence-based recommendations to support responsible technology use while maintaining focus and improving mathematics outcomes. This study is underpinned by Bandura's Social Cognitive Theory and Sweller's Cognitive Load Theory, which together provide a lens for understanding how screen use influences classroom dynamics, student engagement, behaviour, and wellbeing. Using a pragmatic and realist approach, data collected from a single Sydney secondary school include student survey, Flanders' Interaction Analysis observations, and semi-structured interviews of students and senior staff. Surveys capture students' self-reported task-switching; observations document classroom dynamics; and interviews explore perceptions of technology's influence on learning and wellbeing. Findings contribute to promoting healthier technology use, informing school practices, and guiding policy related to digital wellbeing.

References

- Kuş, M. (2025). A meta-analysis of the impact of technology-related factors on students' academic performance. *Frontiers in Psychology*, 16, 1524645. <https://doi.org/10.3389/fpsyg.2025.1524645>.
- May, K. E., & Elder, A. D. (2018). Efficient, helpful or distracting? A literature review of media multi-tasking in relation to academic performance. *International Journal of Educational Technology in Higher Education*, 15(1), 1-17.
- Nakshine, V. S., Thute, P., Khatib, M. N., & Sarkar, B. (2022). Increased screen time as a cause of declining physical, psychological health, and sleep patterns: A literature review. *Cureus*, 14(10).
- Pellas, N. (2024). Effects of Kahoot! on K-12 students' mathematics achievement and multi-screen addiction. *Multimodal Technologies and Interaction*, 8(9), 81.
- Saarinen, A. I. L., Lipsanen, J., Hintsanen, M., Huotilainen, M., & Keltikangas-Järvinen, L. (2021). The use of digital technologies at school and cognitive learning outcomes: A population-based study in Finland. *International Journal of Educational Psychology*, 10(10), 1-26.
- Yang, J. M., Sung, Y. T., & Chang, K. E. (2020). Use of meta-analysis to uncover the critical issues of mobile inquiry-based learning. *Journal of Educational Computing Research*, 58(4), 715-746.

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 508). MERGA.

Consensus-Building in Mathematics Classroom Talk: A Speech-Act Perspective

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Discussion with others plays a central role in learning mathematics. Classroom talk not only promotes conceptual understanding (Mercer & Howe, 2012) but also supports broader intellectual and social capabilities (Lampert, 1990). However, the capabilities that should be intentionally fostered through classroom interaction are not always clearly specified. Without such clarity, it becomes difficult for teachers to design learning environments that systematically support students' participation in meaningful mathematical discussion. In this study, we focus on the concept of consensus-building and argue that the ability to reach agreement with others should be regarded as an important capability to be cultivated through mathematics lessons.

In contemporary society, characterised by rapid technological change and unpredictable global challenges such as the COVID-19 pandemic, citizens increasingly need to collaborate with others to address complex problems. The ability to build consensus is therefore an important civic capability. From a disciplinary perspective, mathematics itself has historically developed through processes of negotiation and agreement regarding the validity of reasoning and knowledge (Ernest, 1991). From this viewpoint, mathematics classrooms can be understood as spaces where students negotiate mathematical ideas with others and work towards shared agreement about the validity and appropriateness of mathematical reasoning.

Research question: How do students negotiate mathematical ideas when attempting to build consensus during classroom discussion?

This short communication proposes an instructional model designed to foster students' consensus-building in mathematics classrooms. The study analyses episodes of consensus-building observed in a Year 6 mathematics lesson conducted in Japan. To examine the processes through which students negotiate mathematical ideas, we draw on Searle's (1969) theory of speech acts and analyse the intentions embedded in students' utterances. By examining the sequence of speech acts within classroom dialogue, the analysis aims to clarify how consensus emerges in students' interactions and how instructional design can support the development of consensus-building through classroom talk.

Acknowledgements

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References

- Ernest, P. (1991). *The philosophy of mathematics education*. Falmer Press.
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 17, 29–64.
- Mercer, N., & Howe, C. (2012). Explaining the dialogic processes of teaching and learning: The value and potential of sociocultural theory. *Learning, Culture and Social Interaction*, 1(1), 12–21.
- Searle, J. R. (1969). *Speech acts: An essay in the philosophy of language*. Cambridge University Press.
- (2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 509). MERGA.

Exploring the Mathematics Experiences of Students with Down Syndrome

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It is often assumed that individuals with Down syndrome struggle and have difficulties learning mathematics (Bird & Buckley, 2001). Fortunately, contemporary perspectives highlight the need for encouraging further mathematics education research, aiming to find new ways to bring positive mathematics experiences to this population (Faragher & Gil Clemente, 2019). This need is grounded in the fact “that mathematics can contribute, like other disciplines (language, theatre, sports) to the holistic development of people with Down syndrome especially with respect to thinking skills and awareness of the world” (Faragher & Gil Clemente, 2019, p. 112).

This presentation shares findings from a larger study exploring the mathematics experiences of students with Down syndrome in Australian schools. Drawing on seven case studies across mainstream and segregated settings (mainstream setting $n = 4$ case studies; segregated setting $n = 3$ case studies), data includes semi-structured interviews with parents, teachers, and teacher’s aides, alongside two mathematics classroom observations per case study. Together, these data sources offer insight into the overall mathematics experiences of this student population.

Findings highlight the importance of authentically including students with Down syndrome in mathematics lessons, ensuring they do not develop an over-reliance on their teacher or teacher’s aide. Additionally, regular, learning-focused communication between teachers and parents emerged as a significant factor in supporting student success in mathematics, alongside the need for teachers and teacher’s aides to have dedicated opportunities to develop specialised knowledge in teaching and supporting this student population. Furthermore, the findings also point to the need to reconsider how the mathematical ability and knowledge of students with Down syndrome is understood and assessed, as shifting these perspectives is fundamental to creating meaningful opportunities for success in the subject.

These findings have significant implications for inclusive mathematics education practices as well as understanding how educational systems and student support structures can better prepare individuals with Down syndrome for mathematics success and independence.

Acknowledgements

Ethics approval was granted by Western Sydney University.

References

- Bird, G., & Buckley, S. (2001). *Number skills for individuals with Down syndrome– An overview*. Hampshire: The Down Syndrome Educational Trust.
- Faragher, R., & Gil Clemente, E. (2019). Emerging trends in mathematics education for people with Down syndrome: Current research and future directions. *International Journal of Disability, Development, and Education*, 66(2), 111–118. <https://doi.org/10.1080/1034912X.2019.1579891>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 510). MERGA.

Implementing Interactive Oral Examinations in Tertiary Mathematics Classes

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While traditional written mathematics examinations tend to focus on procedural understanding, interactive oral examinations can assess students' conceptual understanding and verbal communication skills (Theobold, 2021). This style of assessment provides an authentic check of understanding at a time of increasing artificial intelligence use in other assessment formats (Fenton, 2025). Despite this, the uptake of this assessment in tertiary mathematics has been slow due to a perception that this format will not scale to large class sizes (Videnovic, 2020). We present trial results for interactive oral assessments in a first-year tertiary mathematics class with initial enrolments of up to approximately 300 students. Each student was asked questions designed to explore conceptual understanding with ten minutes allocated per student, which has been found to be a sufficient length (Iannone & Simpson, 2012). Follow-up questions were used to prompt students to justify the answers provided. This assessment proved to be efficient and provided a better separation of final grades than seen from written examinations. Students were invited to complete pre- and post-assessment surveys, featuring the same seven Likert-scale questions and one text-based response, with 19 and 22 responses received, respectively. The results showed that students felt apprehensive about this new assessment type, with high levels of nervousness reported both before and after the oral exams, while also indicating that this assessment aided engagement and understanding. This trial revealed limitations in students' verbal communication skills and conceptual understanding, as reflected in students' perception of their own preparation, which are barriers to further study. A series of communication activities has since been embedded to address these issues and the design is presented here. Overall, this trial has demonstrated that oral assessment can be an effective method for assessing mathematics and can be run efficiently for large cohorts.

Acknowledgements

Ethics approval HREC 8122 was granted by Flinders University, and participants gave informed consent.

References

- Iannone, P., & Simpson, A. (2012). Oral assessment in mathematics: Implementation and outcomes. *Teaching Mathematics and Its Applications: International Journal of the IMA*, 31(4), 79–190. <https://doi.org/10.1093/teamat/hrs012>
- Fenton, A. (2025). Reconsidering the use of oral exams and assessments: An old way to move into a new future. *Educational Researcher*, 54(7), 430–436. <https://doi.org/10.3102/0013189X251333638>
- Theobold, A. S. (2021). Oral exams: A more meaningful assessment of students' understanding. *Journal of Statistics and Data Science Education*, 29(2), 156–159. <https://doi.org/10.1080/26939169.2021.1914527>
- Videnovic, M. (2020). Challenges with implementing oral exams in post-secondary mathematics courses. *Indonesian Journal of Mathematics Education*, 3(2), 44–51. <https://doi.org/10.31002/ijome.v3i2.3079>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 511). MERGA.

What Students Tell Us: Preferences for Learning Trigonometry Through Pen-and-Paper and Online Practice

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The integration of digital platforms has prompted debate about how students engage with mathematics practice environments. While technology-supported instruction enhances learning outcomes (Hillmayr et al., 2020), traditional handwriting practices remain essential for conceptual understanding (Dündar, 2015). This study addressed: What are Year 10 students' preferences for completing trigonometry practice in pen-and-paper and online environments, and what factors do they identify as supporting their learning?

This action research was conducted with a Year 10 mathematics class ($n = 24$) at an independent boys' school in New South Wales. As the classroom teacher and researcher, I designed a three-week trigonometry unit using both pen-and-paper textbook practice and online Mathspace practice. To mitigate bias, I prioritised student voice through anonymous surveys and class discussions. Inductive thematic analysis (Braun & Clarke, 2006) was employed, with codes emerging directly from student data.

Six themes emerged. *Immediate feedback* (31.2%) was dominant, with students valuing Mathspace's instant feedback. *Error identification* (23.4%) reflected appreciation for correcting mistakes in real time. *Blended approach* (20.8%) was most significant: 64.3% preferred combining both environments. One student noted, "Mathspace when we're learning initially, textbook for study when closer to assessment." *Distraction concerns* (14.3%) revealed some preferred pen-and-paper focus. *Memory and retention* (10.4%) reflected writing's value for consolidation. *Technical frustration* (3.9%) involved minor platform issues.

These findings reveal that students develop sophisticated metacognition when given experience with multiple practice environments. Rather than expressing universal preferences, students articulated context-dependent reasoning: digital environments for new concepts and traditional environments for consolidation. This suggests intentional pedagogical orchestration of both environments may better support student learning than reliance on either approach alone.

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References

- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. <https://doi.org/10.1191/1478088706qp063oa>
- Dündar, S. (2015). Does writing have any effect on mathematics success? *Journal of Education and Training Studies*, 4(1), 56–61. <https://doi.org/10.11114/jets.v4i1.989>
- Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. *Computers & Education*, 153, Article 103897. <https://doi.org/10.1016/j.compedu.2020.103897>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 512). MERGA.

Developing Critical, Adaptive, and Inventive Thinking through Mathematics: A Game-Based Approach

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Mathematically rich games can be effective in engaging students (Yeo, 2007), in exploring and applying key mathematical ideas, and in increasing motivation to learn mathematics (Heshmati et al., 2018). Existing literature has largely focused on identifying the key characteristics of such games and providing detailed illustrations of specific examples. However, there is comparatively limited literature detailing general guidelines for creating or modifying mathematically rich games, for implementing mathematical games in the classroom, as well as for teachers' facilitation of students' mathematical work during games (Heshmati et al., 2018) to support the development of students' critical, adaptive, and inventive thinking, a key component of the emerging 21st Century Competencies (Ministry of Education Singapore, n.d.). This study addresses this gap by reviewing current game-based approaches to distil a framework consisting of design principles for selecting, developing, and implementing mathematically rich games in classrooms. The key principles include the centrality of mathematics in gameplay, student engagement, a balance of skill and luck (Russo et al., 2018), and other factors such as accessibility and feasibility. Based on these design principles, I developed a rubric to assess the design and use of games for mathematics instruction and outlined key strategies for their effective implementation in the classroom (Klooger et al. 2025). In addition, the same framework can be used to evaluate the game's affordances after implementation.

In this presentation, I will illustrate the use of the framework with two specific examples of mathematically rich games—Battleship and Tower of Hanoi—and suggest how these games can be implemented in mathematics classrooms. Lastly, implications for practice and directions for future research will be discussed.

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References

- Heshmati, S., Kersting, N., & Sutton, T. (2018). Opportunities and challenges of implementing instructional games in mathematics classrooms: Examining the quality of teacher-student interactions during the cover-up and un-cover games. *International Journal of Science and Mathematics Education*, 16(4), 777–796. <https://doi.org/10.1007/s10763-016-9789-8>
- Klooger, M., Russo, J., & Kalogeropoulos, P. (2025). An instructional model to support planning and teaching with educationally rich mathematical games. *International Journal of Mathematical Education in Science and Technology*, 1–16. <https://doi.org/10.1080/0020739X.2025.2530968>
- Ministry of Education Singapore. (n.d.). 21st Century Competencies. <https://www.moe.gov.sg/education-in-sg/21st-century-competencies>
- Russo, J., Russo, T., & Bragg, L. A. (2018). Five principles of educationally rich mathematical games. *Australian Primary Mathematics Classroom*, 23(3), 30–34.
- Yeo, J. B. W. (2007). *Mathematically-Rich Games*. <https://doi.org/10.13140/RG.2.1.2648.1529>

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia*, Sydney (p. 513). MERGA.

A Comparative Analysis of Mathematics Textbooks between the 2022 South Korean Curriculum and the Australian Curriculum v9.0: Focusing on the Introduction to Functions in Secondary

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Introductory sections of textbooks play a significant role in learning mathematics. How mathematical concepts are introduced initially usually determines the direction of the whole picture. The Australian and Korean mathematics curriculums have common things, but they have their own purpose for education. This study aims to compare the introduction of functions to Year 7 students in both countries. Especially, this study focuses on analysing how the introductory section affects students building the concepts of functions with the aspect of inductive modelling and deductive reasoning. This study will conduct three types of comparison. First, the change of textbooks for Australian curriculum from v8.4 to v9.0. Second, the change of textbooks for Korean curriculum from 2015 to 2022. And finally, a comparison between the Australian and Korean mathematics curriculum.

According to the revised Australian Curriculum version 9.0 (ACARA, 2022), secondary students are required to describe the relationship between variables from authentic data. Version 8.4 also includes the contents about analysing graphs from authentic data. But we can find it gets more intentional in content descriptors as part of textbooks for v 9.0, where the textbooks emphasise the process of modelling real-world phenomena rather than the ability to transform a pattern to an equation. The textbooks don't refer to the word, function, but the relationship is related to function. Also, using digital tools has been combined more strongly with the content descriptors. Content descriptions describe what students should understand in the whole context of the concept and the way they would go.

The concept of Digital Literacy is one of the important factors in the South Korean 2022 revised Mathematics Curriculum. (Ministry of Education [MOE], 2022). Digital literacy means the ability to use digital tools to find information needed. In 2015 Curriculum, digital tools were tools reflecting data. In the 2022 Curriculum, it is a main field for mathematical thought by itself. The use of digital tools is a new capability for students to develop, which has been added in the revised 2022 curriculum that did not exist in the 2015 curriculum. 'Ability to Connect' stems from connecting mathematical concepts, principles, and rules, applying them to real-life problems and other subjects and recognising the usefulness of mathematics. The findings may support educators to be better able to design the next version of the curriculum that is more suitable for their students.

References

- Australian Curriculum, Assessment and Reporting Authority (ACARA). (2022). *Australian Curriculum:Mathematics (v9.0)*.
Ministry of Education. (2022). Mathematics curriculum (Proclamation Nop. 2022-33, Separate Volume 8).