Defining Mathematical Thinking for Classroom Practice

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The term mathematical thinking is frequently used in research and policy writing as a key construct for mathematics education. Despite a clear shift from content-focused skills toward broader, holistic processes, a review of the use of the term revealed persistent ambiguity and tensions in how mathematical thinking was defined and conceptualised. This paper presents an overview of the evolution of mathematical thinking and its components, in order to propose a clear and contemporary working definition of mathematical thinking to support teachers' reflections on their pedagogical decision-making.

As a secondary mathematics teacher, the first author noticed students who could explain their thinking were often more engaged, persistent, and generally achieved higher academic outcomes. Whereas students who struggled to think of ways to approach problems or lacked confidence to discuss strategies disengaged quickly, avoided difficult questions, and had poorer long-term outcomes. These observations provided the impetus to better understand how to support students' in developing their mathematical thinking for future success.

Understanding mathematics is essential for secondary students, as it underpins many trades and professions and influences future employment opportunities. Limited mathematical proficiency has been found to restrict social privileges, employment, and economic security, while strong mathematics ability is associated with up to 28% higher wages (OECD, 2024). Mathematical thinking is widely recognised as an important goal of mathematics education (Stacey, 2006) to prepare students to participate in an increasingly technological and globalised society (Geiger et al., 2023). Although mathematical thinking has been identified as essential for future Science, Technology, Engineering and Mathematics (STEM) workers (Geiger et al., 2023), a powerful way of learning mathematics (Goos & Kaya, 2019; Isoda & Katagiri, 2012; Stacey, 2006), and involves the development of interpretative and sense-making dispositions (Goos, 1997), a review of contemporary research offered limited clarity regarding a contemporary definition of mathematical thinking.

In accepting the importance of helping students to develop their mathematical thinking, we sought a clear and contemporary definition. In this paper, we present an overview of the evolution of mathematical thinking and its components to support future research on teachers' pedagogical reasoning/decision-making.

Evolution of Mathematical Thinking

Historically, mathematical content was positioned separately from mathematical thinking in problem-solving. Burton (1984) conceptualised mathematical thinking as "operations, processes and dynamics" (p. 35), proposing "entry, attack and review" (p.41) affective phases of thinking. Students would experience curiosity and productive tension to initiate persistence during entry, confidence sustained problem-solving during attack, and review focused on checking and refining solutions. Building on Polya's work, this model was later extended to include heuristic strategies (e.g., drawing diagrams, simplification, working backwards) positioned as skills to be practiced and routinised for fluency (Schoenfeld, 2016).

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Mathematical thinking was also explicitly framed as a set of skills supporting procedural fluency (Schoenfeld, 1985; Stacey, 2006). Grounded in Schoenfeld's (1985) problem-solving framework, early research examined productive behaviours and explicitly taught heuristics to support efficient solution development (Kilpatrick et al., 2001). Instruction emphasised guided support, self-belief, and the application of practised strategies to support knowledge comprehension, and application (Stacey, 2006), which are lower-order cognitive processes in Bloom's taxonomy (Bloom et al., 1956; Schoenfeld, 2016). From this perspective, mathematical thinking was conceptualised as identifiable skills, including numerical fluency and conceptual understanding across content areas, such as measurement, algebra, geometry and operations (Burton, 1984).

Leaning more into processes rather than static skills, Goos (2014) described mathematical thinking as "the act of sense-making" (p. 196), grounded in the processes of specialising, generalising, conjecturing, and convincing. This emphasised that processes could be strengthened through classroom activity and social interaction. Although Stacey et al. (1985) initially positioned generalising and specialising as skills of problem-solving, mathematical thinking was later reframed as a dynamic process encompassing "deep mathematical knowledge, general reasoning abilities and knowledge of heuristic strategies" (Stacey, 2006, p. 41).

Problem-solving became a clear focus in the 1980's, which saw a pedagogical shift from rote learning of discrete skills to viewing mathematical thinking as integrating content knowledge and techniques within broader problem-solving processes. Although closely linked to problem-solving, mathematical thinking was described using varied terminology, including specialising, generalising, conjecturing and convincing (Stacey et al., 1985); formulating, representing, and solving mathematical problems (Goos & Kaya, 2019; Schoenfeld, 1982); and abstraction, generalisation, symbolisation, and logical reasoning (Stacey, 2006). These processes were conceptualised as actions through which students explored complex ideas and extended mathematical understanding (Schoenfeld, 2016; Stacey et al., 1985). However, substantial variation persisted in defining these processes. For example, Stacey (2006) emphasised specialising, generalising, conjecturing, and convincing, whereas Tall (2008) highlighted abstraction, synthesis, modelling, problem-solving and proof. More holistically, mathematical thinking was framed as both a mathematical perspective and use of mathematical tools such as abstraction and symbolic representation (Schoenfeld, 2016). More recent literature aligned mathematical thinking with higher-order thinking skills (HOTS), emphasising abstraction, reasoning, generalising, hypothesising, and proving (Brown, 2017; Stacey et al., 2015).

This conceptual shift carried important pedagogical implications. When mathematical thinking was framed as a set of discrete skills, instruction emphasised procedural fluency, strategy instruction, and guided practice (Kilpatrick et al., 2001; Stacey, 2006). By contrast, process-oriented perspectives positioned teachers as facilitators who design learning environments that promote exploration, reasoning, and sense-making (Goos, 2014). Students are required to generalise, conjecture, and justify through collaborative discourse (Stacey, 2006), requiring practices beyond repetitive exercises to foster flexible thinking (Isoda & Katagiri, 2012; Tall, 2008). Supporting mathematical thinking requires higher-order pedagogical strategies, including critical questioning and exploratory practices that structure knowledge, deepen conceptual connections (Van Zoest et al., 2017), and enable real-world application (Schoenfeld, 2016; Stacey, 2006). This pedagogical shift reflect movement from developing skills towards processes cultivating inquiry, dialogue, and the construction of mathematical meaning (Schoenfeld, 2016).

Despite this apparent shift to processes, some contemporary authors continued to define these actions as skills. For example, Delima et al. (2018) framed specialising, generalising, conjecturing, and convincing as skills of procedural competency. Similarly, Tong et al. (2020) emphasised the skills of "predicting results, assessing knowledge, analysing and generalising" (p. 484). These examples illustrate the continued lack of consensus regarding whether mathematical thinking is best conceptualised as a set of skills or as a set of processes.

Not all definitions emphasised problem-solving in mathematical thinking. Tall (2008) conceptualised mathematical thinking as a developmental progression through conceptual-embodied (perception and action), proceptual-symbolic (calculation and manipulation), and axiomatic-formal (definitions and proof) worlds, where mathematical processes are compressed into procepts (thinkable concepts) enabling the construction of formal axiomatic theories that can link back to more sophisticated forms of embodiment and symbolism (Tall, 2008). Conversely, mathematical thinking was positioned as a multidimensional construct of mathematical attitudes (mindset), methods (abstraction, generalising, specialising and symbolising), and content, where students are nurtured to be independent learners (Isoda & Katagiri, 2012). Devlin (2012) argued mathematical thinking reflected how mathematicians *think* when problem-solving, developing ideas, and constructing arguments for real-world contexts. In addition to logical and analytical thinking, and quantitative reasoning, this framework suggests *doing* mathematics through collaboration will support engagement and understanding (Devlin, 2012).

Mathematical Thinking as a Multifaceted Construct

Positioning problem solving, reasoning, and communication as central to conceptual understanding (National Council of Teachers of Mathematics, 1989) broadened the focus of mathematics beyond procedural skills to include; the articulation of reasoning, recognition of the affective domain encompassing students' beliefs, feelings, and dispositions toward mathematics (Grootenboer & Marshman, 2016). Productive disposition was introduced as the fifth strand of mathematical proficiency, alongside curriculum aspects of conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning (Kilpatrick et al., 2001). Productive disposition has a learner focus, and the "tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001). From this perspective, teacher beliefs, modelling of mathematical value, and pedagogical choices were thought to cultivate students' productive dispositions through sustained engagement and opportunity for productive struggle (Marshman & Goos, 2018). Consequently, the development of productive disposition appears closely intertwined with mathematical thinking, as students who perceived mathematics as meaningful were more likely to engage in the reasoning, problem-solving and reflection that underpin deep understanding (Kilpatrick et al., 2001).

Throughout this evolution, we see an unresolved tension between skill-based and process-oriented perspectives. Simultaneously, we have a broadening of conceptualisations, such as those that include reasoning, metacognition, discourse (Schoenfeld, 2016), the affective domain (Grootenboer & Marshman, 2016), productive disposition (Kilpatrick et. al., 2001), a deepening of understandings including axiomatic theories, embodiments, and symbols, with a persistent enticement for simpler notions of skills and processes. This unresolved tension between perspectives and underpinning components explains why mathematical thinking remains inconsistently defined in literature, difficult for teachers to operationalise, and only vaguely referenced in policy documents. The lack of definitional clarity has pedagogical implications, as skill-oriented approaches prioritise procedural fluency and accuracy, while process-orientated perspectives emphasise inquiry, reasoning, and communication (Goos, 2014; Schoenfeld, 2016). Therefore, a clear and contemporary definition of mathematical thinking that can support teachers to develop shared understanding and classroom applications is needed. In this paper we propose a clear, contemporary definition.

Components of Mathematical Thinking

Despite the ambiguity in definitions, several common components are identified across mathematical thinking literature. Across the literature, communication frequently emerged as crucial in enabling students to express and justify mathematical ideas, alongside problem-solving, reasoning, and metacognition as closely related components of mathematical thinking (Goos & Kaya, 2019; Schoenfeld, 2016). Within this framing, processes including specialising and generalising, conjecturing and convincing have been commonly discussed as key components of mathematical thinking (Burton, 1984; Stacey, 2006). It is noted that while these components have been commonly discussed, agreement on definitions has not always been achieved.

Communication

Communication was widely recognised as a fundamental component of mathematical thinking, functioning both as a skill (Bloom et al., 1956) and a process that underpins understanding and proficiency (Goos, 1997). It encompasses verbal, non-verbal, written, and symbolic forms through which students express, interpret, and refine mathematical ideas, translating between representations, employing precise language, and connecting ideas to support sense-making (Goos, 1997; Kilpatrick et al., 2001). Building on the work of both Polya and Vygotsky, communication involves dialogue, gestures, visual representations, diagrams, and concrete materials (Schoenfeld, 2016; Stacey, 2006). From a socio-cultural perspective, collaborative discourse between teacher and student, or among peers may mediate learning, make thinking visible, and support reasoning and mathematical understanding (Brown, 2017). Teacher-led discussions, modelling, and inquiry-based strategies further foster engagement, reasoning, and the development of mathematical thinking, highlighting the importance of effective pedagogy (Van Zoest et al., 2017).

Problem-Solving, Reasoning, and Metacognition

Problem-solving appears to be intertwined with mathematical thinking (Isoda & Katagiri, 2012; Schoenfeld, 1985), although the relationship is contested. Some research has suggested mathematical thinking is a fundamental mental activity that is activated when solving problems (Danoebroto et al., 2024), whereas other studies argued that problem-solving is a context for developing mathematical thinking (Burton, 1984; Isoda & Katagiri, 2012). Although Polya's (1975) four-phase model (understanding, devising, carrying out, and reflecting) remains influential for problem solving, contemporary frameworks extended this perspective to include affective domain elements of beliefs, attitudes, and metacognition (Schoenfeld, 2016). Schoenfeld (2016) argued student's affective domain beliefs shape their problem-solving behaviours. Problem-solving was also framed as iterative and socially mediated, with collaborative discourse supporting perseverance, reasoning, and strategic competence (Grootenboer & Marshman, 2016; Kilpatrick et al., 2001). In each of the Queensland senior mathematics syllabi, problem-solving and mathematical thinking are intertwined, stating "Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding" (Queensland Curriculum and Assessment Authority, 2024, p. 8). However, no definition of mathematical thinking is provided. This creates conceptual ambiguity about what constitutes mathematical thinking in classroom practice.

Mathematical reasoning is central to proficiency, enabling logical thought, explanation, justification, reflection, and application in real-world contexts (Kilpatrick et al., 2001; Stacey, 2006). The Australian Curriculum defines mathematical reasoning as:

> central to thinking and working mathematically . . . analysing, proving, experimenting, modelling, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, and prove that something is true or false. (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2025, Reasoning, para. 1)

According to this definition, students are required to communicate their mathematical thinking to show how they reached their conclusion. The curriculum further defines mathematical processes as "the thinking, reasoning, communicating, problem-solving and investigation *process skills* (emphasis added) involved in working mathematically" (ACARA, 2025, Mathematical Processes, para. 1). It is interesting to note the use of the term 'process skill' here furthering the lack of clarity of these terms. Mathematical reasoning is simultaneously mandated and underspecified, leading to uncertainty for teachers.

Metacognition is defined as a students' knowledge of their own cognitive processes, and monitoring and regulation of thoughts during mathematics tasks (Schoenfeld, 2016). These processes support cognitive regulation during complex tasks, with expert problem solvers demonstrating planning, checking, and strategy use to become autonomous learners (Goos & Kaya, 2019; Schoenfeld, 1982; Schoenfeld, 2016). Teachers who support the development of metacognition through pedagogical choices may be inherently fostering effective mathematical thinking.

Despite variations in emphasis, problem-solving, reasoning, and metacognition were consistently recognised as interdependent key components of mathematical thinking.

Specialising, Generalising, Conjecturing and Convincing

Although definitions of mathematical thinking varied, the interrelated processes of specialising, generalising, conjecturing, and convincing repeatedly appeared in the literature related to mathematical thinking despite shifts in conceptual emphasis. Specialising uses specific examples to explore and build understanding through inductive engagement with concrete representations, enabling students to recognise emerging patterns (Burton, 1984; Danoebroto et al., 2024). Building on this, generalising requires identifying regularities across examples and abstracting common elements that could be expressed symbolically or verbally, allowing learners generalise from specific examples (Stacey, 2006), through inductive and analogical reasoning (Isoda & Katagiri, 2012; Tall, 2008). Conjecturing occurs as students formulate and test hypotheses based on observed patterns, engaging in the cyclical process of hypothesising and revising (Goos & Kaya, 2019). Convincing involves validating conjectures through explanation, argumentation, or proof, reflecting the social and discursive nature of mathematics and supporting the development of adaptive reasoning (Kilpatrick et al., 2001).

Collectively, these processes were not enacted automatically by students but were shaped by teachers' beliefs about mathematical thinking and their pedagogical choices, including task selection, questioning, and the facilitation of discourse. Teachers who valued exploration, justification, and productive struggle were more likely to design learning environments that made these processes explicit, thereby supporting students' engagement in mathematical thinking as an enacted classroom practice rather than a fixed set of skills (Goos & Kaya, 2019; Schoenfeld, 2016).

Teachers' understanding of mathematical thinking

Teachers' beliefs about mathematics are generally regarded as internalised understandings about the world that an individual considers to be true, shaping their dispositions toward pedagogical practice (Marshman, 2021). Teachers' beliefs shape how mathematical thinking is interpreted and supported in their classroom. These beliefs influence instructional decisions, including task selection, questioning strategies, and tolerance for productive struggle, with students' views often mirroring those of their teachers (Marshman, 2021). When teachers value reasoning, problem-solving, and metacognition they are more likely to design learning environments that promote conceptual understanding and productive disposition, where mathematics is perceived as meaningful and attainable (Kilpatrick et al., 2001; Schoenfeld, 2016). For teachers to be able to support students to develop the essential construct of mathematical thinking, they need to know what constitutes mathematical thinking in their contemporary classrooms. This requires a clear and contemporary definition of the multifaceted nature of mathematical thinking.

Final Thoughts

This lack of clarity in the definition of mathematical thinking creates challenges for teachers seeking to intentionally foster this essential construct. A clear and contemporary definition is required to enable teachers to foster it in their classroom and enabling further research into mathematical thinking. The following definition is therefore proposed for mathematical thinking:

Mathematical thinking refers to the multidimensional processes through which students actively engage with, express, and develop mathematical ideas through reasoning, problem-solving, and reflective regulation of their thinking. It involves the coordinated processes of specialising, generalising, conjecturing, and convincing, and is enacted through language, representations, and often collaborative discourse to achieve idea refinement or consensus. Mathematical thinking integrates cognitive and affective dimensions, including metacognition, beliefs, and dispositions, enabling students to make connections, to move from concrete exploration toward abstraction and formal reasoning, justify conclusions, and apply mathematics beyond procedural execution into real-world contexts.

With a contemporary working definition of mathematical thinking for teachers, further research is needed to better understand teachers' perceptions about mathematical thinking and their reflections on their mathematical classroom pedagogies.

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