Affect in Mathematics Education: Blurring the Boundaries

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In this paper I describe the status of affect as an educational outcome and argue for more meaningful integration of affective and cognitive aspects of mathematics learning and mathematics education research. I provide examples from projects with which I have been involved of directions that could be pursued towards this goal and present systems thinking as a perspective that offers considerable explanatory value and can foreground the whole of cognition and affect as more than its constituent parts allowing a more realistic approach to and view of the likely impacts of interventions.

The study of affect in mathematics education has been of interest to researchers for many decades with foci following developments in funding and access to research participants as well as conceptual progress. There is consensus that students should, in addition to learning mathematical content, develop positive attitudes, self-efficacy, and confidence in relation to the subject, and the ability to flexibly apply the mathematics that they know. To assist students in acquiring these capabilities, teachers need also to have them. There has, consequently been considerable attention to the affect of teachers, particularly primary school teachers, often preservice, many of whom have negative views of mathematics and of their own ability to learn and teach it (Itter & Meyers, 2017). Mathematics anxiety has been of particular interest in relation to these teachers as well as students.

The field has been characterised by a proliferation of terms to describe the aspects of affect that are being considered, along with pleas, only partly successful, for researchers to define precisely what they mean by whichever terminology they use. Attempts to systematise the study of affect have been built on recognition that affect and cognition are not neatly separable. They have included continua ranging from constructs considered most affective (i.e., emotions) to those considered to have a considerable cognitive component (i.e., beliefs) (McLeod, 1992), and models that take account of other dimensions of affective constructs. Hannula (2012), for example, proposed categorising affective constructs according to three dimensions: cognition, motivation, and emotion, each of which can be considered as states or traits and can be examined from physiological, psychological or social perspectives.

Much attention has been devoted to examining the beliefs of teachers of mathematics and how they might be influenced in ways considered desirable by researchers and, although the literature is replete with accounts of changed practices resulting from specific projects some of which have included a focus on affect, the teaching of mathematics at scale has changed little (Beswick et al., 2023). In keeping with the theme of this conference, Beyond Boundaries: Strengthening Futures in Mathematics Education, there have also been calls to integrate affect into content focussed mathematics education research which have resulted in aspects of affect being considered in addition to mathematical attainment. These attempts have, however, often fallen short of genuine integration. It is worth bearing in mind that, in research, analytic approaches whereby broad concepts are broken into separately examinable components to allow detailed investigation are useful, but the work of re-constituting the pieces is arguably as important yet receives less attention.

In my research I have considered what teachers believe about the nature of mathematics and the logical implications of these beliefs for their beliefs about how mathematics is best taught and learned (e.g., Beswick, 2012). There is explanatory value in considering beliefs as constituting systems. In these systems teachers' beliefs about themselves in relation to

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (pp. 2–14). MERGA.

mathematics and mathematics teaching and about their students' capacities to learn the subject are especially consequential for their practice (Beswick, 2018). Although teachers' knowledge and their beliefs can and usually are considered separately (even within single studies) I have argued that distinguishing these constructs, while empirically useful, disguises the practical influence of beliefs relative to knowledge. Beswick (2011) provided a detailed argument for this stance.

Affect as an educational outcome

Although from a theoretical standpoint the boundaries between affect and cognition are not clear cut there is considerable and consequential divergence in the ways in which they are treated and valued in practice. Affective constructs feature, for example, in high-level statements about the aims of education such as the Alice Springs (Mparntwe) Education Declaration (Council of Australian Governments Education Council, 2019) which includes in the second of its two goals for education that all young Australians become "confident and creative learners" (p. 4). This is unpacked in terms of a range of affective constructs including self-worth, self-awareness, self-identity, emotional wellbeing, personal values such as honesty, loyalty, responsibility and respect, resilience, adaptability, optimism, initiative, creativity, and imagination. Other aspects of the goals refer to motivation, confidence, moral and ethical integrity, empathy, and civic values.

Curriculum documents that have been created since the Declaration have mentioned many of these. The Australian curriculum (Australian Curriculum and Reporting Authority, [ACARA], 2023) for example, makes explicit reference to the declaration. The most recent version, version 9, comprises three dimensions, namely learning areas or subjects, general capabilities, and cross-curriculum priorities. General capabilities include critical and creative thinking which is purported to support, among other things, "Dispositions such as inquisitiveness, reasonableness, intellectual flexibility, open- and fair-mindedness, and a readiness to try new ways of doing things", ethical understanding that includes an understanding of the ways in which their values impact others, and personal and social capability, elements of which are self-awareness encompassing personal awareness including of their feelings, and emotional awareness. Social awareness includes empathy and both relational and community awareness (ACARA, 2023).

Numeracy is another of the general capabilities included in the Australian Curriculum. Its definition includes knowledge and skills along with the ability and confidence to apply mathematics in everyday life (ACARA, 2023). This formulation resonates with the description of numeracy of Geiger et al. (2015) who additionally emphasised the importance of a critical orientation that supports questioning, and critique as an overarching aspect of numeracy. Numeracy thus defined includes both cognitive and affective aspects.

Despite the prominence of affective outcomes in key documents intended to guide the teaching of mathematics, such outcomes appear to receive little attention in practice compared with knowledge and skills. At least part of the reason for this is likely the relative difficulty of assessing affective outcomes in ways analogous to the regular and familiar approaches to assessing cognitive mathematical outcomes. Although there is a plethora of instruments used to measure various aspects of affect in psychological studies, assessing students' affect in everyday classrooms is necessarily subjective and unreliable compared to assessments of cognitive outcomes (although this is not as clear cut as might be assumed). As a result, it is difficult to imagine how affective outcomes might be taken as seriously as cognitive outcomes (i.e., achievement). Nevertheless, high achievement with negative affect does not suggest a student likely to exhibit numerate behaviour or to continue studying mathematics beyond when it is compulsory or of other utilitarian value, yet we know that this describes a great many students at the end of secondary schooling. One wonders whether such an outcome is (or should be) more or less acceptable than its converse: relatively low achievement with positive affect. Of course we want both. One possible direction in which an answer to this conundrum might be found is in the integration of affective and cognitive outcomes such that achievement is construed and assessed as entailing relevant affect. In addition, there are pedagogical approaches and organisational choices in schools that can be part of a commitment to taking student affect seriously.

Approaches to integrating cognition and affect

In this section I describe in chronological order five examples from projects with which I have been involved to illustrate attempts at incorporating affect and cognition to demonstrate ways in which the integration of cognition and affect might be achieved. Some examples provide evidence of the utility of basing research on frameworks that include both cognitive and affective constructs. In many, if not all, one can reasonably infer that beliefs about the self (i.e., aspects of identity) underpin observable phenomena. Each offers an invitation for further research using or adapting the tools described or further developing or challenging the theoretical ideas and frameworks.

Empirical evidence for the integration of affect and cognition

As part of the Providing the Mathematical Foundation for an Innovative Australia within Reform-Based Learning Environments (MARBLE) project, Beswick et al. (2012) conceptualised teacher knowledge as including the seven types of knowledge identified by Shulman (1987), the knowledge categories identified by Ball (Ball et al., 2008) along with confidence to teach various aspects of the mathematics curriculum (e.g., fractions, mental computation, chance and data), and beliefs about such things as numeracy in the classroom (e.g., Teachers of mathematics should be fascinated with how children think and be intrigued by alternative ideas), and numeracy in everyday life (e.g., I often use mathematics to make decisions and choices in everyday life). The profile instrument comprised open response and Likert type items and was completed by 62 Year 5-8 teachers at the start of their involvement in a professional learning program. The use of Rasch measurement techniques showed that the profile was a measuring a single underlying construct thereby demonstrating that teacher knowledge can be considered a rich uni-dimensional construct that incorporates many elements.

Considering the many facets of knowledge on a single scale allowed us to see that confidence, which is often regarded as a dimension of attitude (Beswick et al., 2006), was evident at levels of knowledge that demanded little by way of content or pedagogical content knowledge suggesting that pursuing this aspect of positive affect for pre-service teachers independent of attention to cognitive aspects of knowledge may not be wise and certainly not sufficient (Beswick et al., 2012). The authors noted that there is, "value in pursuing holistic conceptions and measures of mathematics teachers' knowledge as well as ongoing detailed analyses of categories". There is scope for the use of this of similar instruments in investigations of teacher knowledge, broadly defined including to test whether the findings apply to teachers in other contexts and with differing mathematical backgrounds.

Extending the conceptualisation of mathematical pedagogical content knowledge

Beswick and Chick (2020) presented a framework for teachers' pedagogical content knowledge that built on prior work by Chick and colleagues (e.g., Chick et al., 2006; Chick and Beswick, 2017; and Dyment et al., 2018) and that incorporated aspects of teachers' beliefs. Beliefs related to each of the three categories of PCK: first, aspects of PCK in which content and pedagogy are indistinguishable (Clearly PCK), second, aspects of PCK in which content knowledge appears to be prominent but deployed in a pedagogical context, and third, the converse in which pedagogical knowledge is used in a content context. Table 1 provides excerpts from the framework that concern beliefs.

Table 1

Aspects of PCK concerning teachers' beliefs in Beswick and Chick (2020)

	PCK for School Mathematic Teachers (SMTPCK)	
PCK category	*Evident when the teacher …*	*Example*
Clearly PCK		
Beliefs about The Nature of Teaching and Learning	Discusses or uses personal or established philosophies and approaches to teaching the discipline	Discusses the use of pedagogies consistent with a constructivist view of learning as a key facet of teaching mathematics; requires students to memorise times tables because of a belief that students should have fast recall of key number facts
Student Affect (in relation to content)	Discusses or addresses students' affective responses to particular mathematics content	Recognises that adolescent students may have negative emotional reactions to the prospect of learning algebra
Content Knowledge in a Pedagogical Context		
(Beliefs about) The Nature of Content	Expresses an appreciation of the nature of mathematics that goes beyond the school curriculum and aligns with mathematicians' views of the discipline	Compares the aesthetic qualities of two solution methods
Pedagogical Knowledge in a Content Context		
Student affect (general)	Describes how student affect influences pedagogical approach	Knows a particular student will respond negatively to being asked for an answer in a large group session

The framework was used as a lens to consider the PCK of experienced mathematics teachers comprising two focus groups in which they discussed, among other things a student's non-standard but mathematically correct response to a fraction division task as part of the Powerful Knowledge project led by Rosemary Callingham (Beswick & Chick, 2016). The inclusion of beliefs in the framework allowed their interaction with knowledge (traditionally defined) to be seen more clearly than had been the case with earlier versions. In addition to the belief categories incorporated in the framework, beliefs about themselves and their efficacy as teachers of mathematics and beliefs about the capabilities of their students were prominent in the teachers' conversations. Beliefs about themselves and particularly how difficult they found the task, appeared to influence the teachers' judgements of the cognitive demand of the task. Beliefs about student affect were evident in such things as their attribution of motives (e.g., laziness, enjoyment) to students' behaviours. Beliefs in these two areas are arguably among the most important in shaping teachers' practice (Beswick, 2018) and Beswick and Chick (2020) conceded there is a case for their inclusion in the framework. Further research testing the value and comprehensiveness of the framework and modifications thereof is warranted along with its use to examine PCK for mathematics teaching in highly granular and nuanced ways that can get more precisely than other measures to what it is that teachers of mathematics know and use in their teaching.

Conceptualising Science, Technology, Engineering and Mathematics capability

Geiger et al. (2023) developed a framework for principals' Science, Technology, Engineering and Mathematics (STEM) capability that drew on earlier work on numeracy (e.g., Goos et al., 2014). The framework, which underpinned the Principals as STEM Leaders (PASL) project, comprises four aspects: STEM discipline and interdisciplinary knowledge and practices, Task, Context, and Dispositions, all embedded in a Critical Orientation. Each of these aspects were unpacked as a series of elements that recognised the role and responsibilities of the principal. For example, the knowledge aspect was framed in terms of awareness rather than as an expectation that a principal should have knowledge of STEM disciplines in the same way that a teacher of a STEM discipline would need. The elements related to dispositions are shown in Table 2.

Table 2

Elements of STEM capability for principals related to Dispositions

Dispositions	
	Belief that all students and teachers can develop/enhance their STEM capability
	Openness to the implementation of innovative STEM teaching practices
	Confidence that innovative STEM programmes can be initiated, managed and brought to fruition
	Flexible and adaptable change leadership practices so that all feel supported and encouraged in their engagement with the STEM project, regardless of differences in levels of knowledge, skill and confidence
	Willingness to be personally and actively involved in establishing and sustaining the school STEM teaching and learning programmes
	Flexible and adaptive thinking as their institution continues to innovate (e.g., openness to changes in school curriculum delivery structures and practices)

As can be seen from Table 2, Dispositions include beliefs and confidence along with qualities such as openness, flexibility, willingness and adaptability. These latter qualities are likely underpinned by principals' beliefs about themselves, their role and their efficacy in that role especially as it relates to fostering a positive STEM culture in their school.

In relation to dispositions Fraser et al., (2021) reported that principals in the PASL project were committed to equity through developing the STEM capability of all students, not only high achievers in mathematics and science, and that teachers agreed that principal dispositions were key to progressing STEM education in their schools. Among the recommendations in the final report of the project was that:

> Greater attention should be paid to the development of positive dispositions towards STEM among principals, teachers, students, and school communities; to build parental awareness of the importance of STEM learning as relevant for all; and to acknowledge its various demands in principals' professional learning programs.

Inclusive mathematics teaching as site for difficult questions about affect

Inclusive education is founded on a series of rights declarations (UNESCO 1994, 2020; United Nations 1989, 2006) and subsequent legal requirements (e.g., in Australia the Disability Discrimination Act (1992), Disability Standards for Education (2005)). The Disability Discrimination Act includes three requirements that constitute legal obligations for Australian schools and teachers. These are:

1. The right to education on the same basis as other students.

2. Reasonable adjustments are made to ensure that students with disability have opportunities to participate in education and training.
3. Consultation between the school, parents/carers and student takes place prior to adjustments being made.

In addition, teachers feel a moral imperative to meet the learning needs of all their students (Beswick et al., 2025).

Inclusion appears to be a context in which the obligations and demands upon, and the aspirations of teachers to do the best for their students meet in ways that are emotionally charged as a result of the perceived difficulty of the task and inadequate resourcing, and conflicting beliefs about how teachers perceive their role: is it to teach those able to learn from standard approaches or to endlessly differentiate for individuals? Nilsson (2026) used affect to describe emotions or feelings which she considered dynamically constructed through interactions within classrooms, schools and the broader society to analyse what she described as a negative mood surrounding inclusive education in Denmark. Central to her analysis was the “inclusion-child”, a construct to which teachers’ feelings of frustration, guilt and fear adhere construing inclusion as an impossibly burdensome task. Conversely, Nilsson (2026) also noted that some teachers expressed pride and satisfaction when they believed they had achieved something positive in relation to inclusion suggesting that a positive reframing is at least possible. Nilsson’s framing of emotions as attaching to objects and carrying meaning is consistent with the view that emotions, like beliefs, constitute systems (DeBellis & Goldin, 2006). Such systems can be considered to exist separately from or as part of belief systems.

The Learning Year Level Appropriate Curriculum (LYLAC) project, led by Rhonda Faragher, and its antecedent projects have been sites in which we have confronted and considered sometimes uncomfortable issues about inclusion. In terms of affect we have wondered whether affective outcomes take on greater importance for students with intellectual of development difficulties compared with their higher attaining peers because positive affect is implicitly seen as a consolation prize rather than the main objective of schooling. This highlights our earlier questioning of the status of affective relative to cognitive outcomes of education. Faragher et al. (2019) reported findings from a smaller study involving students with Down syndrome that focussed on the positive affect these students experienced in inclusive settings. In each case the students were also able to engage with the same lesson as their year level peers with minimal adjustments or none at all. We have wondered also whether low attainers are uniquely scrutinised as to the extent of their ‘understanding’ of mathematical concepts: We know that many students successfully complete mathematical tasks with minimal conceptual understanding, yet the veracity of their attainment is rarely questioned.

Conceptualising critical mathematical thinking

Geiger et al. (2023) provided a conceptualisation of critical mathematical thinking (CMT) as the basis for critical citizenship. That is, CMT is a quality that empowers individuals to deliberate upon and make decisions about mathematical solutions to real world problems taking account not only of the mathematics but also of the social and environmental implications of various courses of action. They have argued that responses to social and ecological disruptions that increasingly characterise human experience require more than purely cognitive analyses (Geiger et al., 2025). Their framework comprises five dimensions: Mathematical capability, Critical capability, Evaluating, Reasoning, and Dispositions with the latter in its initial form defined as “Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, and displaying perseverance”. This aspect is key to students’ engagement with socio-ecological issues in ethical and responsible ways (Geiger et al., 2024).

The framework was the basis for the development of the Critical Mathematical Thinking Noticing Instrument (CMTNI) which has been used with teachers involved in the project

(Geiger et al., 2025). The instrument comprises six 2–3-minute video vignettes of teacher-student and/or student-student interactions. The teachers can see each vignette just once and then respond to a series of questions related to their noticing of aspects of CMT with reference to the CMT framework. Initial use of the CMTNI has demonstrated differences in the breadth of aspects of CMT to which teachers refer in their responses.

The framework has also been used by teachers as they plan to implement the tasks used in the project and to define the components of CMT for which the teacher is looking as their students work on the tasks. Geiger et al. (2025) reported on the use of one of the project tasks with a Year 10 class. The task is shown in Figure 1. In each class in which the task was used there were some students who responded from a purely financial perspective, asserting the right of owners to do as they wish with their property. In the example described by Geiger et al. (2025) the teacher's prompts for students to consider ethical issues and the plight of those disadvantaged by the current housing market were insufficient to move all students beyond consideration of the financial impacts on AirBnB owners. Subsequent discussions with teachers resulted in one coining the term *empathy primer* to describe the use of an introductory discussion focussing on the importance of empathy and compassion in CMT before a specific problem context is presented. This approach has been broadly adopted across the project and appears to be successful in broadening students' thinking to include the implications of possible solutions. The disposition dimension of the CMT framework has consequently been amended to read, "Believing mathematics is relevant to a real-world problem, showing initiative, taking intellectual risks, displaying perseverance, openness to alternative solutions, *preparedness to empathise and show compassion*" (Geiger et al., 2025).

Figure 1

AirBnB and the rental crisis

Are short-stay properties like Airbnb the reason for the rental crisis?

The following video and media report discuss the apparent shortage of affordable rental accommodation because owners are making their properties available for short-term arrangements. This raises the concern that such properties are taken out of the pool of available housing for permanent residents.

Please discuss whether the Brisbane City Council (media report) made the right decision to impose a registration fee on properties that offer short-stay opportunities through online agents (e.g., Airbnb and Stayz) or are they overreacting? Justify your response using available information about the availability of affordable accommodation.

Investigate the rental prices and short stay accommodation in Gympie. Based on your findings suggest whether the Gympie Regional Council should implement a short stay registration fee as imposed in Brisbane. Who would be advantaged or disadvantaged? Suggest other possible avenues to alleviate the rental problems in Gympie. Justify all responses using available information.

It will be interesting as the project draws to a close to analyse our data which include considerable video footage for evidence of the range of aspects of affect that teachers and students display, and the ways in which the affective aspects of CMT interconnect with other dimensions of the construct.

Systems thinking

The examples presented in this paper relied with varying degrees of explicitness on an underlying assumption that beliefs and other aspects of affect exist not in isolation but as parts of systems. Systems thinking has been relatively underutilised in mathematics education research but has the potential to offer much to efforts to integrate cognition and affect. Indeed, it has much to offer the study of these domains separately as well. To avail ourselves of the opportunities system thinking affords will require ongoing innovation in our theorising and methodological approaches. Following is an overview of systems thinking and an outline of two forms of such thinking that have been useful and that show promise for progressing our understanding of affect in mathematics education research.

Systems thinking arose in the 20th century, principally in the biological sciences. It offers a way of holistically considering relationships between the interacting parts that comprise a system and the system as a whole. A system can exhibit properties that are not present in the parts, that is, the system has properties that are emergent from the interaction of its constituent parts. As a result, analytic approaches that break systems into parts can fail to discern the crucial features of the system as a whole.

Belief systems (Green, 1971)

Beliefs (and other aspects of affect) have been acknowledged to constitute systems. It is unreasonable and likely misleading, for example, to expect that a particular belief will correspond with a particular behaviour even when that belief appears to be the logical antecedent of the behaviour. Instead, behaviours are based on dynamically interacting groups of beliefs whose composition and relative importance to the behaviour of interest vary with context. Failure fully to appreciate this has been a reason for which observations such as that a teacher professes to value group work but is not observed using it have been characterised as contradictions between espoused and enacted beliefs. Green's (1971) description of belief systems has been a powerful and widely used for many years (e.g., Grootenboer & Marshman, 2016; Haser & Doğan, 2012; Mewborn, 2002) as a means of understanding such phenomena in terms that honour the integrity of teachers.

According to Green, beliefs vary in terms of the strength with which they are held with more strongly held beliefs considered more central. Centrality is also a function of the extent to which a belief is connected to others in the system, a fact that makes beliefs acquired earlier in life more likely to be central and that contributes to central beliefs being resistant to change. It has been argued that the beliefs an individual has about themself are among their most central (e.g., Beswick, 2006; 2018). Belief systems are structured such that some beliefs are the consequences of others, that is they are derivative of other beliefs, chains of which can be traced by questioning the basis of successive beliefs offered as reasons for another until the chain ends with a belief that is primary in that it is not derived from another (Green, 1971). Green also suggested that beliefs can be held in clusters, that is groups of beliefs that are held in isolation from others. This is most likely to occur when beliefs arise in different contexts and means that an individual can unknowingly hold conflicting beliefs. There is also evidence that clusters might be created, probably unconsciously, to avoid the need to reconcile conflicting beliefs. Schuck (1999), for example, suggested that pre-service primary teachers who believed that learning mathematics should be enjoyable, but who had negative beliefs about their ability to teach the subject constructed the belief that knowing mathematics is not necessary for teaching it effectively. This allowed them to preserve a belief in themselves as potentially effective teachers. Beswick (2006) observed, based on Schuck's work, that clusters of beliefs, for example about mathematics and mathematics teaching, may form, or emerge, from the system, in order to avoid confronting conflicts. Finally, Green (1971) pointed to beliefs being either evidentially or non-evidentially held, with the latter impervious to contradictory evidence that since they are held on the basis of the authority of the source of the belief or because they support other, likely central, beliefs.

In contrast to beliefs which are largely cognitive, emotions are "feelings or states of consciousness, distinguished from cognition" (Philipp, 2007, p. 259) and include a physiological response (Hannula, 2002). They have been described as the energy that drives cognition (Evans, 1999) and as influencing cognition by biasing attention and memory (Hannula, 2002). Beswick (2011) argued that beliefs and emotions, although possibly constituting separate systems (DeBellis & Goldin, 2006) are intimately connected. Beliefs may be associated with emotions and emotions with beliefs but both beliefs and emotions may also encode information independently, that is without such associations. Beliefs may also evoke emotions and emotions may evoke beliefs with the order often difficult to discern. For example, when confronted with an apparently difficult mathematics problem a student may be reminded that they have been unsuccessful with similar problems in the past, evoking both the belief that they are unlikely to be successful and the emotion of anxiety. This reaction may surface a further belief that despite the discomfort they have sometimes been successful with difficult problems when they have remained calm and made an attempt, leading to a moderation of their emotional state and, should success ensue, emotions of joy and pride and strengthened beliefs in their capacity to solve difficult problems.

An individual's most central beliefs such as those about themselves are likely to be strongly connected to emotions with the linkages with emotions adding to the sum of their connections and hence their centrality (Beswick, 2011). Challenge of centrally held beliefs is likely to give rise to negative emotions manifesting as resistance and adding to the upheaval that is both cognitive and affective (Beswick, 2011). Emotions seem also to be related to memory formation with strong emotional responses at the time of an event becoming part of the memory (McClay et al., 2023).

Complex systems

Complexity science offers an enriched understanding of Green's description of beliefs systems that adds to its explanatory power. Complex systems typically involve collections of living, but not necessarily conscious, agents that are at some level autonomous. Interactions among agents give rise to characteristics that are not attributable to any particular individuals in the system but rather are emergent properties of the system. Complex systems are commonly nested within other complex systems forming hierarchical structures. For example, cells, organs, individuals, social groups and societies can all be regarded as complex systems with each emerging from the interactions of agents comprising the previous level (Davis & Simmt, 2003). In addition, complex systems are self-organising in that they form patterns in the absence of central coordinator (e.g., flocks of birds) assisted by multiple feedback loops and non-linear interactions among agents. These features contribute to the ability of complex systems to embody their histories and adapt or learn such that the reaction of the system to an influence upon it may differ on different occasions (Miller et al., 2001).

Davis and colleagues (Davis, 2004; Davis & Simmt, 2003; Davis & Sumara, 2005) have proposed five necessary but insufficient conditions for the emergence of complexity from the interactions of a collective. These are *diversity* which refers to differences among agents in the system that allow the possibility of new responses. It is balanced by *redundancy*, the common ground among agents that allows meaningful interactions. *Enabling constraints* limit the activity of the collective in ways that provide focus while still allowing diversity to be expressed. *Decentralised control* emphasises the autonomy of agents from whose actions complexity emerges in ways that cannot be planned or predicted. *Neighbour interactions* occur between agents and are facilitated by redundancy. In educational contexts they most importantly refer to interactions between ideas.

Beswick (2006; 2018) described in detail how belief systems, understood according to Green's (1971) description can be considered complex. She argued that they are characterised by self-organisation, emergence, nestedness, and include networks of connections facilitating feedback loops and non-linear interactions among the agents which are in this case individual beliefs. If we accept that belief systems are complex, it makes sense that the five necessary but insufficient conditions for complex emergence identified by Davis and colleagues may also apply. Belief systems are characterised by diversity both in terms of the subjects of beliefs and the range of beliefs that can be held about specific objects. Redundancy arises from the multiplicity of connections among beliefs in a system. Beswick (2018) provided the example of teacher who believes that students should do regular homework perhaps because they also believe that establishing regular study habits is important and that homework is an important way to consolidate new learning. Each of these beliefs may also be derivative in that they follow from the teacher's own experiences as a learner, though not necessarily the same ones. Derived beliefs may not, therefore, necessarily be undermined if one or maybe many of the related beliefs was no longer held and in that sense some beliefs are redundant. Redundancy thus seems connected to the centrality of beliefs because more centrally held beliefs have more connections to other beliefs in the systems and hence are in a sense protected by the redundancy in the system that the connections afford.

Much of the activity of belief systems appears to happen unconsciously rather than being controlled by a central authority and therefore are characterised by decentralised control. Nevertheless, the conscious mind is able to direct an individual's attention casting it in the role of providing enabling constraints analogously to the way that Davis (2005) described the teacher in the complex system of a classroom as the consciousness of the collective, making decisions about the focus and direction of collective activity. Neighbour interactions exist in the connections between beliefs.

Complexity also reminds us that despite our efforts to direct learning in both classrooms and professional learning contexts, the learning (new knowledge and affect) that emerges may be surprising. Our best efforts can set conditions that make the emergence of intended outcomes more likely but cannot guarantee their achievement.

Conclusion

Although positive affective outcomes are universally endorsed as valuable their value in practice remains limited by difficulties in assessing them. This difficulty is particularly evident in mathematics where traditional pen and paper tests that focus on the accurate and timely performance of algorithmic solutions continue to be the norm. I have argued that removing or blurring the boundaries between cognitive and affective outcomes offers possibilities for taking affective outcomes more seriously. If knowing mathematics is understood as not only being able to solve mathematical problems but also to have productive beliefs about the subject, positive emotions in relation to it, well-founded confidence in one's ability to use mathematical tools in unfamiliar as well as routine tasks, and a critical orientation to the use of mathematics, then supporting the development of students' positive affect becomes not just a nice additional outcome but central to what it means to teach mathematics.

The explicit inclusion of affective constructs in frameworks such as those described in the examples provided in this paper offer opportunities to explore interconnections among aspects of teachers' and students' affect, for example between their beliefs and emotions (compassion and empathy) related to CMT, or between aspects of affect and cognition, such as between knowledge, beliefs and confidence in the MARBLE project. A systems perspective makes this an entirely natural and comprehensible thing to do.

Acknowledgments

The MARBLE project was funded by Australian Research Council (ARC) grant LP0560543.

Powerful Knowledge was funded by ARC grant DP130103144 with input into the framework reported here from ARC grant FT140101351.

The PASL project was conducted with funding support from the Australian Government Department of Education, Skills and Employment.

LYLAC was funded by ARC grant LP200300740.

CMT was funded by ARC grant DP220101015.

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