Noticing Affordances of Tasks for Use with Low-Progress Learners in Mathematics: The Case of Mr Faizad

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Teaching through problem solving (TTPS) is an instructional approach that introduces new concepts by giving students opportunities to solve carefully crafted mathematics problems. However, many teachers struggle to design and implement TTPS lessons, especially when working with low-progress or emerging learners. In this paper, we compare how a teacher designed and implemented tasks to teach through problem solving for a Grade 3 class of 15 emerging learners, drawing data from two lessons to examine what he noticed about the task affordances.

In mathematics classrooms, a step-by-step approach to problem solving is often modelled by teachers, particularly for low-progress or emerging learners—those who require additional support or scaffolding. This is also the case for some Singapore teachers, who choose to first teach strategies or methods and then have these emerging learners apply them to solve less challenging questions (Kaur et al., 2019; Wang et al., 2014). Although such an "ends" approach (Lester, 2013, p. 246) provides a clear guideline for emerging learners to follow without requiring them to grapple with the problem, offering clearly articulated steps may hinder students' development of problem-solving skills and dispositions (Goulet-Lyle et al., 2019). This approach stands in contrast to a "means" approach, through which "mathematical concepts, processes, and procedures are learnt" through problem solving (Lester, 2013, p. 246). Takahashi's (2021) notion of teaching through problem solving (TTPS) is one such approach, in which problem solving goes beyond an "add-on" activity in a lesson and instead directs the flow of the lesson (Takahashi et al., 2013, p. 239).

A TTPS lesson begins with a carefully chosen problem, which is not only "low floor" (Resnick, 2008, p. 1) but also affords multiple solution pathways. Students solve the problem and present the different solutions to the whole class, which then sets the stage for the teacher to facilitate the comparison and discussion of these solutions. Such an approach provides opportunities for the teacher to revisit the solutions and connect these responses to the goal of deriving the intended facts, concepts, and procedures (Takahashi, 2021). Therefore, TTPS goes beyond show-and-tell, building on students' ideas to explore and learn new mathematical content (Smith & Stein, 2011). The TTPS approach aligns well with the recent Singapore mathematics curriculum push towards "learning mathematics as a tool to solve problems and learning mathematics as a discipline" (Yeo & Choy, 2023, p. 125).

However, while many teachers believe that students should learn mathematics through problem solving, most of them may not make problem solving and discussions around problems part of their everyday mathematics lessons because they struggle with the idea of TTPS (Takahashi, 2021). These tensions are further exacerbated when teaching emerging learners (Kaur et al., 2019; Wang et al., 2014), as teachers may feel compelled to teach basic skills, even though these learners prefer lessons that engage them in thinking (Kaur & Ghani, 2012). A key lever for supporting teachers lies in helping them notice and harness the affordances of tasks (Choy & Dindyal, 2021) and students' thinking during lessons (Goodwin, 1994; van Es, 2011).

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Given that a well-designed task is the centrepiece of TTPS, it is important for us to understand what teachers perceive as the tasks' affordances to better support teachers in enacting TTPS.

This paper adds to ongoing conversations about supporting teachers in enacting mathematically productive teaching approaches, such as TTPS, for emerging learners. Here, we focus on Mr Faizad, a mathematics teacher who taught a Grade 3 class of 15 emerging learners at Zane Hall Primary School (all names are pseudonyms) in Singapore. We are interested in examining changes in his teaching as he learned to teach his emerging learners through the TTPS approach. The following question frames this paper: What did Mr Faizad notice about the affordances of tasks, and how did he harness them as he learned to teach through problem solving for his emerging learners?

Noticing Affordances of Tasks

In the context of mathematics tasks for use in TTPS lessons, we refer to their affordances in enabling teachers to orchestrate discussions with and between students to bring out the lesson's mathematical point. We follow Choy and Dindyal's (2021) argument that "the way tasks are designed and used" (p. 197) determines "the range of possibilities for human activity" (Brown, 2009, p. 20). Put in another way, this range of possibilities can be conceived as the *affordances* of the tasks. According to Gibson (1986), "the affordances of the environment are what it offers the animal, what it provides or furnishes, either for good or ill" (p. 127). Greeno (1994) explains affordance as "whatever it is about the environment that contributes to the kind of interactions that occur" (p. 338). While task affordances exist relative to teachers' actions and capabilities, their existence is independent of teachers' ability to perceive them, and these affordances do not change in response to changes in teachers' instructional goals (Choy & Dindyal, 2021). Therefore, for teachers to use tasks productively for TTPS, they will need to recognise the tasks' affordances and decide how to use them to craft instructional episodes aligned with the lesson's goals (Brown, 2009). The main challenge is for teachers to *notice* the task's hidden affordances to unlock its full potential when teaching by focusing on their students' thinking.

This places teacher noticing at the heart of the work of supporting teachers in enacting productive teaching practices (Choy, 2016; Mason, 2002; van Es, 2011). Broadly, noticing can be understood as the process of attending to and making sense of instructional moments to decide on instructional approaches (van Es, 2011), which can be seen as a form of "professional vision—consisting of socially organised ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (Goodwin, 1994, p. 606). Mason (2002) sees noticing as a discipline, highlighting it as "the heart of all practice" and offering it as a means by which teachers can "do something about it [teaching] in a practical and disciplined manner" (p. 1). Choy (2016) provides three focal points—mathematics, student thinking, and teaching actions—to direct teachers' noticing in ways that support their enactment of productive teaching practices, such as those needed in TTPS. Moreover, a teacher's ability to notice task affordances before lessons is critical (Choy & Dindyal, 2021). In this study, we focus on understanding how a teacher's noticing of task affordances changed as he learned to use TTPS tasks with emerging learners.

Methods

Data for this paper were collected as part of a project aimed at developing a pedagogical approach based on the notion of TTPS for emerging learners. The project involved three schools (Two secondary and one primary) and a total of 79 teachers. For each school, we conducted two professional development sessions, one at the beginning of the study and the other in the middle of the study (2 hours per session), to introduce TTPS ideas, practices related to orchestrating discussions (Smith & Stein, 2011), building thinking classrooms (Liljedahl, 2020), and universal design for learning (UDL) principles (Rose & Meyer, 2002). We then engaged extensively with one or two teachers from each school to develop and trial mathematics tasks for use with emerging learners in their TTPS lessons over another four sessions (1 to 2 hours per session). Mr Faizad is one of the teachers we worked with at Zane Hall Primary School, a typical public school whose student profile spans from emerging to talented. As an experienced teacher, Mr Faizad was assigned to teach a Grade 3 class of 15 emerging learners, who were specially pulled out as part of the school's Learning Support for Mathematics programme. Many of them struggled with English, the language of instruction, and had issues with factual fluency, resulting in very low achievement scores. Mr Faizad started out with a 'teach first, then apply' approach to problem solving, but his practices changed significantly towards the end of the project. The changes in his practices are of interest in this paper. For this paper, we will focus on the first and third TTPS lessons to examine what he noticed about the affordances and how he harnessed them.

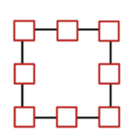
We collected his instructional materials and sampled students' work related to the lesson observed. We also conducted a post-lesson discussion with Mr Faizad, which was voice-recorded, to understand his thinking and instructional decisions after the lesson. To understand Mr Faizad's noticing of task affordances (Choy & Dindyal, 2021), we used Choy's (2016) three focal points to create snapshots of his noticing. First, we segment the lesson according to the stages described by Takahashi (2021)—introduction, posing of problem task, students working on the problem, whole-class discussion, and conclusion. Next, we analysed the task in terms of its mathematical point and possible students' responses (both correct and incorrect) to develop what we perceived to be the task's affordances. Third, we examined his teaching actions and his students' responses at different points in the lesson. We paid attention to how Mr Faizad used the tasks with his students, focusing on the kinds of talk moves (Van Zoest et al., 2023). Insights from these snapshots were developed by comparing them against Mr Faizad's responses during post-lesson interviews. More specifically, we want to highlight the changes in Mr Faizad's design and implementation of the problem task used in his TTPS lesson, focusing on what he noticed and how he harnessed the tasks' affordances.

Snapshots of Mr Faizad's First TTPS Lesson

The first TTPS lesson was centred on the problem of arranging a given number of cubes into a square while ensuring the same number of cubes on each side. Mr Faizad started with 8 cubes and asked his students to determine how many cubes were on each side of the square. See Figure 1 for the introductory problem and a visual representation of the solution.

Figure 1

Introductory Problem and its Visual Representation

Problem	Visual Representation
David arranges 8 cubes and ensures an equal number of cubes on each side. How many cubes did he arrange on each side of the square?	

This introductory problem served as a launchpad for the generalised problem of finding the number of cubes on each side, given the total number of cubes. An algebraic solution is not required at Grade 3. To support his emerging learners in the problem-solving process, Mr Faizad provided counting blocks for them to work out the answers to the cases of 12 cubes and 16 cubes, respectively. It appears that Mr Faizad wanted his learners to figure out the general method for obtaining the answers to any given number of cubes, as he tried to convince them of the need for a more general approach, beyond using the counting cubes:

Mr Faizad: During exams, did you get cubes? Then how?

Students: Draw!

Mr Faizad: But what if they say, David uses 144 cubes? Are you going to draw 144 cubes? So, what do we need? We need to transform what you put on the squares into a set of mathematical steps.

Although Mr Faizad circulated around the classroom and observed how his students worked on the problem, he did not call on them to share their answers, methods, or thinking processes. Instead, he was focused on sharing his approach to solving the problem:

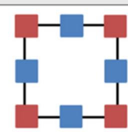
1. Subtract 4 from the total number of cubes (take away the four cubes at each corner of the square)
2. Divide the remaining cubes by 4 (because there are four sides in a square)
3. Add 2 to the quotient (to account for the two cubes at the corners on each side of the square)

Algebraically, we interpreted this solution as $\frac{x-4}{4} + 2$, where x is the total number of cubes given (Figure 2). Note that the algebraic expression is not needed for this lesson.

Figure 2

Visualisation of Mr Faizad's solution.

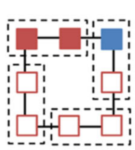
Note: The algebraic interpretation was ours and was not shared with the students.

Visual Image	Generalised Steps
 The students should exclude the red cubes first, before distributing the blue cubes equally to the 4 sides. Then add the 2 corner cubes to the answer to obtain the number of cubes on each side.	Minus 4: e.g., $8 - 4 = 4$ Divide by 4: e.g., $4 \div 4 = 1$ Plus 2: e.g., $1 + 2 = 3$ Number of cubes on each side = 3
Algebraic Interpretation	
Let the total number of cubes be x, Number of cubes on each side = $\frac{x-4}{4} + 2$	

When asked about his thinking behind his decision to focus solely on one solution, Mr Faizad reasoned, "I find that students need repetitive action to understand thoroughly, that same concept with the changed variable. I wanted students to build their muscle memory". Mr Faizad's approach to supporting his low-progress learners reflected the common belief that these emerging learners need a systematic, step-by-step approach to problem-solving (Goulet-Lyle et al., 2019; Kaur et al., 2019).

Figure 3

Other Possible Solutions to the Same Problem.

Method 2	
Visual Image	**Algebraic Interpretation**
	Let the total number of cubes be x, $\frac{x}{4} + 1$

Observing a Pattern by Listing

Total Number of Cubes	Number of Cubes on each side
8	3
12	4
16	5
20	6
…	…

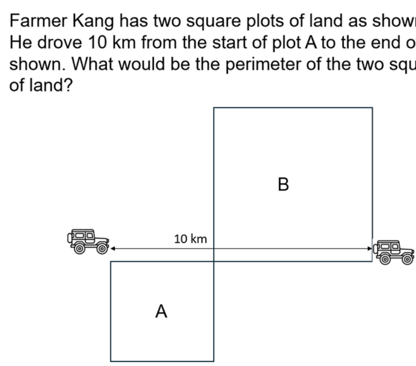
However, focusing on a single solution pathway means that these learners would not have the opportunity to see other solutions and discuss the learning points afforded by the different solutions—elements that are critical to the TTPS approach (Smith & Stein, 2011; Takahashi, 2021). For example, students could have opportunities to discuss how Method 2 (Figure 3 left) is similar to or different from Mr Faizad's solution (Figure 2) or highlight how systematic listing might lead students to see a pattern (Figure 3 right), which can provide another way of thinking about the problem. Here, we see Mr Faizad's goal to "build their muscle memory" may have hindered him from perceiving the affordances of the task he used. Furthermore, it was evident that he did not fully harness the task's potential in eliciting different generalisation strategies from his students.

Snapshots from His Third TTPS Lesson

Towards the end of the project, Mr Faizad's design and implementation of his TTPS task reflected a fundamental shift in how he identified and harnessed the task's affordances. For this lesson, Mr Faizad designed a sequence of tasks leading to a challenging perimeter problem for his Grade 3 pupils, as shown in Figure 4. In Grade 3, perimeter is introduced as "distance around a figure" (Danielson, 2005, p. 31), and students are required to find the perimeter of rectilinear figures and rectangles given their side lengths. The problem is considered challenging even for some Grade 4 students because none of the squares' dimensions is given.

Figure 4

A Challenging Perimeter Problem for Grade 3 Students

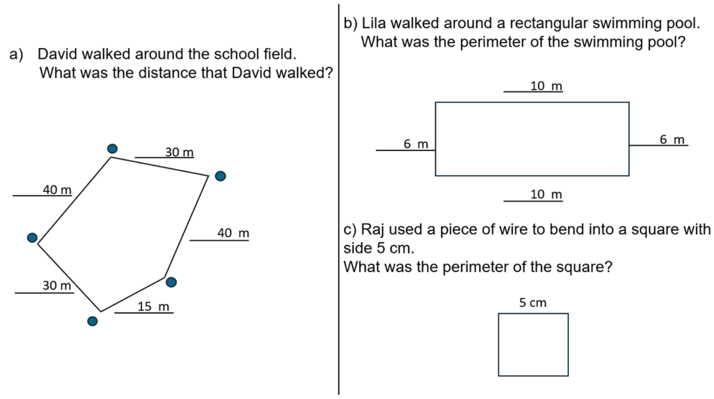


With the aim of supporting his low-progress students to understand perimeter, which is not always trivial (Danielson, 2005), Mr Faizad used contexts such as fences, walking around a field or a swimming pool, and bending wires to form rectilinear shapes (Figure 5). The sequence of tasks presented in Figure 5 was intentionally designed by Mr Faizad: Parts (a) and (b) shared the same context of "walking around", but part (b) used the context of a rectangular pool for students to realise that the perimeter could be determined if the length and breadth are given; part (c) builds on the idea of a special rectangle and uses a new context of bending wires to form shapes. When put together, these three tasks gave students opportunities to apply their understanding of perimeter and provided Mr Faizad with a context to highlight the properties of rectangles and squares, which are necessary for solving the main problem.

Furthermore, Mr Faizad created animations (using movable dots) to show his students how David might have moved around the school field in part (a). Similarly, he created an animation showing how Farmer Kang drove from one end of the field to the other. These animations were created to support his students' understanding of the problem by presenting information in a visual, dynamic way rather than in words alone (Rose & Meyer, 2002), so they can connect the concept of perimeter to the problem scenario (Danielson, 2005). Here, we see how Mr Faizad's sequence of selected tasks and his presentation of the main problem reflect what he productively noticed and how he intended to harness the tasks' affordances (Choy & Dindyal, 2021).

Figure 5

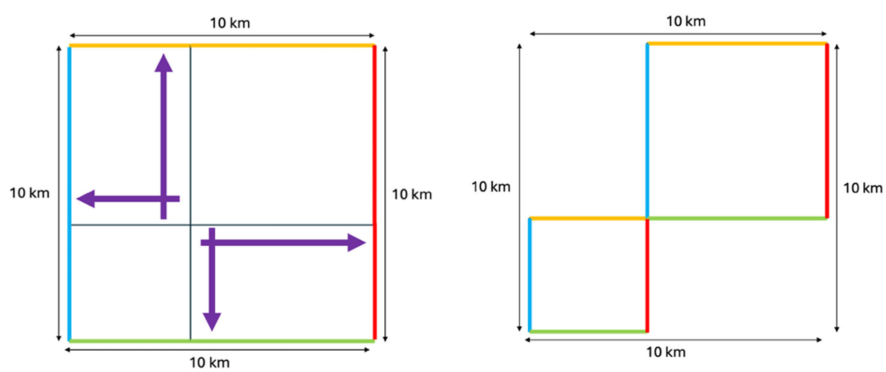
Sequence of Tasks Leading to the Perimeter Problem



Unlike the baseline study lesson, Mr Faizad asked his students to present their ideas and explain their thinking. In many ways, Mr Faizad's practices reflect the five practices for orchestrating productive discussions (Smith & Stein, 2011). While monitoring his students' work on the problem, Mr Faizad selected three students to present their solutions to the whole class in the following order. The first student "moved" selected lines parallelly to form a "big square of 10 km length" (see left of Figure 6), the second student coloured the lines to show that there are "four sets of 10 km" (see right of Figure 6), and the third student substituted 4 km and 6 km respectively for the sides of the smaller and larger squares respectively.

Figure 6

Two of the Solutions Presented by Mr Faizad's Students



Although one might argue that Mr Faizad could have sequenced the presentation differently to strengthen the mathematical connections, the opportunity given for students to share and comment on the different solutions is a critical feature of TTPS (Takahashi, 2021). For instance, although the third solution is a special case, Mr Faizad built on the answer to show that any pair of numbers that add up to 10 could be substituted, and the answer would still be 40 km, alluding to how ideas about invariance could be used to solve the problem. After making connections between the first two solutions, Mr Faizad presented a prepared animation of those solutions to consolidate the ideas learned from solving the problem, highlighting that he had *anticipated* the solutions when planning the TTPS lesson.

Discussion

Taken together, the two snapshots provided insights into what Mr Faizad noticed about the task affordances and how he harnessed them during the lessons. It was evident that Mr Faizad had expanded his range of possibilities for how the task could be solved and used for TTPS. His teaching stance had begun to shift slowly from a "teach first, then apply" approach towards a TTPS approach. During the post-lesson discussion of the perimeter lesson, Mr Faizad said he was "personally surprised" by the solutions his emerging learners provided. We believe these "surprising" solutions from his students might be critical to Mr Faizad's incremental changes in his teaching stance over the course of the year. The findings presented in this paper also suggest that emerging learners could solve problems when given the opportunities to do so. As highlighted by Wang et al. (2014, p. 22):

> The right for LP [low progress] students to access high-quality teaching and education resources should not be deprived simply because of their current performance. To offer LP students an equal education opportunity, these students need to be encouraged and supported to engage with content knowledge and tasks that are above their current level of competencies or abilities.

As Mr Faizad worked with us through the project, it became evident that he became better at noticing and harnessing the affordances of tasks for TTPS. However, the challenge remains. How do teachers design and implement tasks that are not only accessible to emerging learners but also engaging and challenging at the same time? In this paper, snapshots from Mr Faizad's lessons offer two main insights into how we can support teachers to do this ambitious work, especially when working with low-progress students.

First, it is crucial for teachers to notice the affordances of the problems used (Choy & Dindyal, 2021) and presented the problem in a way, which makes it a "low floor (easy to get started with)", "high ceiling (opportunities for increasingly complex explorations over time)", and "wide walls" task—multiple pathways from floor to ceiling (Resnick, 2008, p. 1). As highlighted in this paper, although the task used in the first lesson could be seen as a low-floor, high-ceiling, wide-walls task, its presentation and implementation did not fully harness its affordances. In addition to designing for the mathematical quality of the task, it is equally important to consider how the UDL principles—providing multiple means of engagement, representation, action, and expression—can be applied in its design and implementation (Rose & Meyer, 2002). Mr Faizad's use of animations to present the problem and solution is a case in point, enabling his students to engage actively in problem solving.

Second, these task affordances can only be realised if teachers adopt a listening stance in their teaching: Mr Faizad, for instance, anticipated his students' responses, monitored their work on the task, and then purposefully selected and sequenced their work for presentation in the third lesson (Smith & Stein, 2011; Takahashi, 2021). Doing this involves using talk moves such as eliciting ideas from students, asking for clarification, and the teacher's revoicing of ideas to facilitate interactions (Van Zoest et al., 2023), while maintaining the accessibility of ideas for their students. These practices place great demands on a teacher's attention to listen to students and notice the mathematical aspects of students' ideas (Choy, 2016). This is critical because teachers will need to put in more effort to make sense of emerging learners' ideas.

How do we support teachers in this ambitious form of teaching, where all learners, regardless of their current levels of competence, have opportunities to engage in mathematics? While Mr Faizad's growth in his noticing of affordances is encouraging, the findings presented in this paper also suggest that more sustained professional development is needed. Perhaps it is important to consider ways for teachers to also engage in thinking and noticing task affordances as part of this professional development. As Mason (2002) has highlighted, noticing involves a shift in attention, which enables one to see new possibilities. For teachers, it is about adopting an open mind towards perceiving new possibilities in the tasks they use. For mathematics educators, it is critical to create opportunities for teachers to discuss and more carefully examine the hidden mathematical affordances embedded in these tasks (Choy & Dindyal, 2021). Focusing on enhancing teachers' ability to notice affordances may be a critical area for future research, as we learn to support teachers to do different forms of ambitious teaching.

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