Towards Levels of Reading Mathematical Expressions to Comprehend the Underlying Phenomenon

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Access to mathematical objects is mediated through representations that express their characteristics using symbols and expressions. In mathematics education research, reasoning that investigates mathematical objects through such symbolic representations is commonly referred to as *algebraic thinking* (e.g., Kaput, 2008). Algebraic thinking involves expressing mathematical objects symbolically, transforming these expressions, and reading the characteristics of the objects in the expressions. This study focuses on the reading of mathematical expressions and explores the following research question: What qualitative differences can be identified in how students interpret mathematical expressions when they observe and attempt to understand the properties of mathematical objects? To address this research question, we planned and conducted a design-based experiment. The lessons we designed were based on a problem from *Jinkoki*, a Japanese mathematics textbook from the Edo period. The problem was to determine the number of thieves and the number of bolts of cloth they had stolen from a conversation among the thieves about how the bolts of cloth should be distributed (see Enomoto & Tohei, 2025, for details). The solution described in *Jinkoki* was to determine the number of thieves by adding 7 and 8. This solution reveals a curious phenomenon: the number of thieves equals the sum of the number of bolts of cloth left over and the number lacking when the cloth is distributed. To investigate students' interpretations of this phenomenon, we designed two tasks: explaining why the solution in *Jinkoki* works and examining whether it holds in all cases. In this study, students' thinking during the lessons were analyzed using an analytical framework developed by Tondorf and Prediger (2022). The analysis revealed qualitative differences in how students interpreted the expressions in this task. Some students focused solely on solving the equation from an operational perspective. In contrast, other students interpreted the equation from a relational perspective by connecting it with other representations, thereby revealing the structure of the solution in *Jinkoki*. Students who read the expressions in relation to the context and other representations tended to represent the structure of the solution in *Jinkoki* by intentionally retaining the operations rather than treating the solution simply as a numerical result. In addition, they considered the situation by treating constants as variables.

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