Mathematical Creativity Among (Preservice) Teachers: An Assessment Approach

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Many nations today include the goal of creativity in their curriculum. Goals on their own, however, are insufficient without support by teachers for students to become creative themselves. This paper describes the development of an assessment rubric, that includes three characteristics of creativity (*fluency*, *flexibility*, and *novelty*), to analyse mathematical creativity in written work. It then applies the rubric to responses gathered from 19 preservice secondary mathematics teachers (PSMTs) in the context of completing multiple solution problems. The overall distribution of ratings indicates that level of creativity among the solutions of PSMTs was generally modest to moderate.

Creativity in mathematics has long been recognised as an essential component of mathematical activity, not only for mathematicians, but also for students and teachers of mathematics across the entirety of educational levels (e.g., Sriraman, 2005). Earlier works conceive creativity as an innate quality (which one is born with), but according to most current mathematics education works, creative abilities can be gained through one's effort, commitment (Bicer, 2021a) and are supported by instruction and curriculum materials (Hadar & Tirosh, 2019). While most research has focused on students, there is a growing recognition that teachers' own mathematical creativity is a crucial part of the puzzle (Bicer et al., 2023; Hadar & Tirosh, 2019). Given that there is a current international focus in educational policy and curricula to develop creative thinking skills in students (see Wilkie, 2024), this component is particularly important as teachers design and select tasks, anticipate and respond to students' solution strategies, and model what counts as creative mathematical work (Bicer et al., 2023; Bolden et al., 2010). Against this background, understanding and assessing the mathematical creativity of teachers (both practicing and preservice) becomes a key part of preparing them to support creativity in their students. To date, research on teachers' mathematical creativity is scarce in the literature, especially at secondary school levels (Suherman & Vidákovich, 2022). In this paper, we describe the development of an assessment rubric to analyse mathematical creativity among preservice secondary mathematic teachers (hereafter PSMTs). We apply this rubric to PSMTs' written responses to a division related multiple solution (MS) problem consisting of the approximation of a quotient of two fixed, co-prime positive integers.

Mathematical Creativity and Its Assessment

There are different definitions for mathematical creativity in the mathematics education literature, and these largely come from the psychology and educational psychology literatures (Sriraman, 2005; Wilkie, 2024). At the school level in particular, mathematical creativity can be characterised as the ability to produce unusual or novel and insightful solutions to a given problem, even if not original in a historical context (Bicer, 2021a; Sriraman, 2005). The relevant literature largely has conceived mathematical creativity compose of the following main criteria: *flexibility*, *fluency*, and *novelty*. According to the systematic review conducted by Suherman and Vidákovich (2022) onto assessment of mathematical creative thinking, *fluency* is about if one can produce multiple solutions or ideas. *Flexibility* is associated with one's ability to chance thinking pathways or to generate different types of solutions or ideas. *Novelty* refers to one's

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ability to seek out a solution pathway that is particularly original, distinctive, or uncommon for one's existing or expected knowledge level. Wilkie (2024) argued that students' mathematical creativity, in terms of the identified characteristics, can be investigated through processes they use and products they create in the content of problem-solving or problem-posing.

Substantial work has been done on ways to assess mathematical creativity, particularly in regard to school students, using a variety of approaches (Suherman & Vidákovich, 2022). Early approaches adapted general creativity tests, such as the Torrance Tests of Creative Thinking to mathematical content, but more contemporary studies have favoured tasks with mathematics embedded in them, such as open-ended questions, MS tasks, problem-posing activities (Leikin, 2009), and tasks that give students opportunities to develop creative thinking types (Wilkie, 2024). More recently, multiple representations have been proposed as both a didactical and psychometric tool for assessing and boosting mathematical creativity. In a study with elementary preservice teachers (PSTs), Bicer (2021b) developed a rubric to score the *fluency*, *flexibility*, and *originality* of representations used in fraction multiplication problems, demonstrating the use of multiple representations as a creativity measure. Within PST education, assessment work is less developed. Some studies have focused on PSTs' conceptions of creativity (e.g., Bolden et al., 2010), while others report on the creativity score of PSTs on MS tasks or problem-posing tests. Bicer (2021b) in particular showed that a methods course emphasising multiple representations and creativity can increase PSTs' performance on creativity tasks. However, very few studies have developed and validated assessment approaches specifically targeting teachers' mathematical creativity as measured through their own solutions to tasks that align with prescribed curricula.

The Current Study

The current study is an exploratory pilot study designed to identify preliminary patterns in mathematical creativity among secondary mathematics teachers. It is situated in a larger body of research focused on the knowledge and understandings of PSMTs in teaching algebra, wherein solving a set of MS problems were examined (Hatisaru et al., 2024). The first goal of this paper is to develop an assessment rubric for the analysis of mathematical creativity, and we ask: *What kinds of tools can be used in assessing mathematical creativity among mathematics teachers?* The second goal is the application of this rubric to the written responses of a sample of PSMTs to a MS problem centred on the approximation of a quotient of two fixed, co-prime positive integers. We ask: *When the rubric is applied, what does it reveal about PSMTs' mathematical creativity with respect to fluency, flexibility, and novelty?*

Methods

Data for the larger study was collected from 19 PSMTs enrolled in a mathematics pedagogy unit at a metropolitan university in Australia, completing a mathematics minor or major. These PSMTs would be expected to teach mathematics to students in Years 7-10 (ages 12-16) upon graduation. At the time of the study, all participants had prior experience supporting school students in mathematics, either as PSTs on placement or as private tutors. Within this unit, PSMTs engaged in problem-solving activities as part of their learning for five weeks during Semester 2, 2023, with a focus on Number and Algebra. Participation in the research was voluntary, and informed consent was obtained from all participants. At the beginning of each class, PSMTs completed a reflection form containing a purposefully selected MS problem. In total, five reflection forms (detailed below) were administered across the five weeks, each featuring a different problem. PSMTs were asked to complete each form within 20–25 minutes.

Research Instrument and Task

The reflection forms comprised two pages. The first page presented a MS problem and required PSMTs to solve the problem in as many different ways as they could (Item #1). This approach aligned with the broader study's aim of examining PSMTs' capability to generate multiple solutions and provided a suitable context for exploring variations in *fluency*, *flexibility*, and *novelty* in their written responses. The second page contained reflective items that extended Item #1 and situated the task within a teaching context; however, these items fall outside the scope of this paper. Among the five problems, we selected the Division problem stated below. This choice was due to the fact that data related to the other problems have been reported elsewhere (Hatisaru et al., 2024).

The Division Problem

> Divide 800 by 303 and round your answer to the nearest tenths, without using a calculator. Think and explain as many different possible solutions to the problem as you can.

The problem entails approximating, to the nearest tenth, the division of two co-prime, positive integers, namely 800 divided by 303. It is a typical type of problem in school algebra that is both simple to state, is of a form that is both familiar to the PSMTs and their future students, but one that is also complex enough that it provides multiple avenues of approach to solving. We envision that this is a suitable style of problem that could be used to demonstrate a teacher's own creativity, as well as reflecting the type of creativity they could potentially model for their own students.

The Meaning of Division

According to Siemon et al. (2020), there are two meanings for division: *partition* and *quotition*. They are distinguished by the meaning attached to the divisor. For instance, in the problem '128 divided by 8', 8 can be interpreted as the number of groups (*quotition*) or the number in each group (*partition*). A more generalised conception of division is *division as the inverse of multiplication*, and this conception (i.e., *multiplication and division as inverse operations*) supports work with fractions, algebraic texts, and more complex problems.

The Division problem can be approached in a variety of ways depending on one's conception of division, as well as one's knowledge of the field of mathematics. Traditional, algorithmic approaches to the problem include less novel methods such as long/short division, which is a partitional approach to the problem. More novel approaches include: (i) estimation approaches wherein a simpler, but related problem is created and solved, to infer a solution to the original problem, (ii) considering division as the inverse of multiplication, which can be used to help refine bounds established in estimation methods, (iii) establishing connections to other areas of mathematics, which involves the use of more complex mathematical topics, such as calculus, or number theory, or (iv) a combination of these approaches. On this basis, the problem offered a rich and authentic context for examining mathematical creativity. The multiple solution nature of the problem (Leikin, 2014), grounded in the varied conceptions of division encourages PSMTs to move beyond routine procedures and engage in diverse strategies. That has made the problem particularly suitable for understanding mathematical creativity in this study.

The Development of the Assessment Rubric

We first undertook an inductive review of all 19 responses to the problem in order to identify the solution methods produced by PSMTs. We started with open coding to identify all solution methods and then grouped them into standard/conventional versus novel, which supports openness to unexpected creativity. Our approach was primarily data-driven. That is, we examined each response to determine the methods used, considered how these methods could be grouped, and explored similarities and distinctions among them. This process revealed three most common solution methods, referred to here as Methods A, B, and C. While these methods are distinct, they are not entirely independent; in particular, Methods B and C share notable interconnections.

Method A: Algorithmic division (long or short). An iterative process of determining how many times 303 can be repeatedly subtracted from the remainder, and then subtracting 303 that many times to get the next remainder. This is a standard/conventional approach to division, used in many mathematics classrooms.

Long division

$$\begin{array}{r} 2.\overline{6402} \\ 303\overline{)800.00000} \\ -\underline{606.00000} \\ 194.00000 \\ -\underline{181.80000} \\ 12.20000 \\ -\underline{12.12000} \\ 0.08000 \end{array}$$

The decimal is recurring as we are back to diving 800 by 303.

Short division

$$\begin{array}{r} 2.\overline{6402} \\ 303\overline{)800.^{194}0^{122}0^{8}000} \end{array}$$

The decimal is recurring as we are back to diving 800 by 303.

Method B: Estimation. Estimating the numerator or denominator in a way that makes the division easier to do (e.g. via inspection). In its most basic form, it does not answer the question fully. To arrive at more accurate solutions would typically involve multiple estimations (e.g. upper bound/lower bound) and/or refinement using division as the inverse of multiplication (see below).

Given that $303 \approx 300$ it follows that

$$\frac{800}{303} \approx \frac{800}{300} = \frac{8}{3} = 2.\overline{6} \approx 2.7$$

Furthermore, since $303 > 300$ it follows that

$$\frac{800}{303} < \frac{800}{300} = 2.\overline{6}$$

However, further investigation would be necessary to confirm whether this would result in an answer of 2.6 (or less) to the nearest tenth. A further refinement would be to proceed as follows

$$\begin{aligned} \frac{800}{303} &= \frac{606 + 194}{303} \\ &= \frac{606}{303} + \frac{194}{303} \\ &= 2 + \frac{194}{303} \\ &\approx 2 + \frac{194}{300} \\ &= 2 + \frac{194/3}{100} \\ &= 2 + \frac{64.\overline{6}}{100} \\ &= 2 + 0.64\overline{6} \\ &= 2.64\overline{6} \end{aligned}$$

In a similar way to the previous estimation, we can conclude that

$$\frac{800}{303} < 2.64\overline{6}$$

which in turn means that the answer to the nearest tenth will be less than or equal to 2.6. Obtaining a lower bound is less neat but could be achieved (for example) by estimating 303 as 320. This would yield

$$\frac{800}{303} \approx \frac{800}{320} = \frac{8 \times 100}{8 \times 40} = \frac{10}{4} = 2.5$$

Given that $303 < 320$ it follows that

$$\frac{800}{303} > \frac{800}{320} = 2.5$$

Hence, we now know that to the nearest tenth the answer is either 2.5 or 2.6. Further refinement of the lower bound and/or the use of a multiplication as the inverse of division argument could lead to the correct answer (below).

Method C: Division as the inverse of multiplication. This approach is in some ways a 'guess and check' approach. An initial guess is required which can then be refined. An estimation approach is a good starting point.

From the first estimation above we know that the answer is at most 2.7 to the nearest tenth. We can calculate $2.7 \times 303 = 818.1$ (confirming that 2.7 is an over-estimate). Next, we try $2.6 \times 303 = 787.8$, from which we conclude that 2.6 is a lower bound. We could evaluate $2.65 \times 303 = 802.95$, from which we conclude that 2.65 is an overestimate; hence 2.6 is the correct answer to the nearest tenth. That is, the answer is bound by 2.6 and 2.65 and this entire interval would round to 2.6

Unconventional and less common, and therefore somewhat more novel methods, involving slightly more sophisticated or rigorous estimation were as follows. *Method D*: Establishing an initial estimate and then refining using multiplication (e.g., initial estimate 2.7, so try 2.6 × 303 to see if it is closer). *Method E*: Applying the estimate of 303 ≈ 300 only to the remainder part of the division to improve overall accuracy. *Method F*: Using the Maclaurin series of $\frac{1}{1+x}$ to first order.

Next, we drafted the rubric for assessing mathematical creativity presented in Table 1. This process was partly deductive as we used the three characteristics of mathematical creativity defined in the existing literature. And we established a rating for each characteristic along with its description, making interpretation of each rating straightforward and reproducible. The procedure was transparent about its inductive approach to identifying solution methods followed by a deductive approach. This blended reasoning strengthens how accurately the rubric measures the actual mathematical creativity of the participants.

Table 1

Three Characteristic of Mathematical Creativity and Their Ratings

Characteristic	Rating and description
Fluency Number of different non-repeating solution method	0: No correct solution method 1: A correct solution method containing at least one error and/or lacking in rigor 2: A well explained solution method with rigorous justification 3: At least one well explained and rigorous method, plus a method lacking rigor or containing an error/s 4: At least two well explained and rigorous methods
Flexibility Variety of solution methods or shifts in methods based on distinct / different meanings of division	0: No correct solution method 1: One correct solution method 2: Two correct solution methods based on distinct / different meanings of division 3: Three correct solution methods based on distinct / different meanings of division
Novelty Aspect of unconventionality in a solution method	0: No correct solution method 1: A correct solution method containing at least one error and/or lacking in rigor, or the only solution method being standard/conventional algorithmic division 2: The most basic estimation (303 ≈ 300) without any systematic / justified refinement 3: A solution based on one of the least common correct approaches (Methods D, E, or F in this case)

Application of the Rubric and Findings

The first two authors of this paper applied the rubric to the responses of 19 participating PSMTs independently. Here, a response refers to a reflection form (19 in total); in a response, there can be more than one solution method. Across 57 rating decisions (19 responses evaluated against three creativity characteristics), the agreement was 45, which corresponds to approximately 80% inter-rater reliability. Figure 1 illustrates the distribution of consolidated creativity ratings for *fluency*, *flexibility*, and *novelty*.

The Ratings of Mathematical Creativity for Each Characteristic

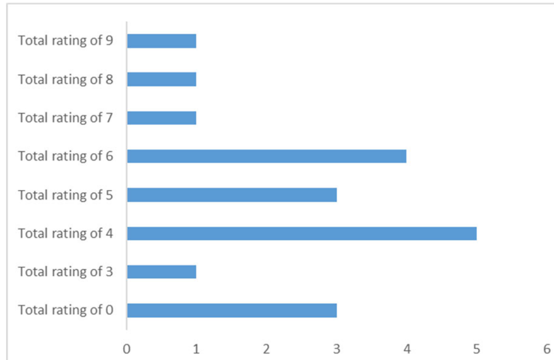
Fluency was observed across all five ratings. Three responses were rated 0, ten were rated 1, three rated 2, two rated 3, and one rated 4. This pattern suggests that while most responses demonstrated limited fluency (ratings 0 and 1), a small number showed moderate fluency (ratings 2 and 3), and only one response reached the higher rating. *Flexibility* was grouped at the limited to moderate end of the rating. Three responses were rated at 0 and eight at 1 (limited flexibility), with the rest were rated at 2 (moderate flexibility). This indicates that the majority of responses relied on one or two methods, and none exhibited the level of variation expected at the higher rating for this characteristic (producing more than two solution methods). *Novelty* spanned the four ratings. Three responses were rated at 0, two at 1, eight at 2, and six at 3. This distribution suggests that while six responses incorporated elements of novelty beyond the most standard/conventional methods, relatively original methods (Methods D, E, or F) were seen less.

Figure 1

Distribution of Ratings Across Three Characteristics of Mathematical Creativity

Figure 2

Distribution of Total Mathematical Creativity Ratings



The Overall Ratings for Mathematical Creativity

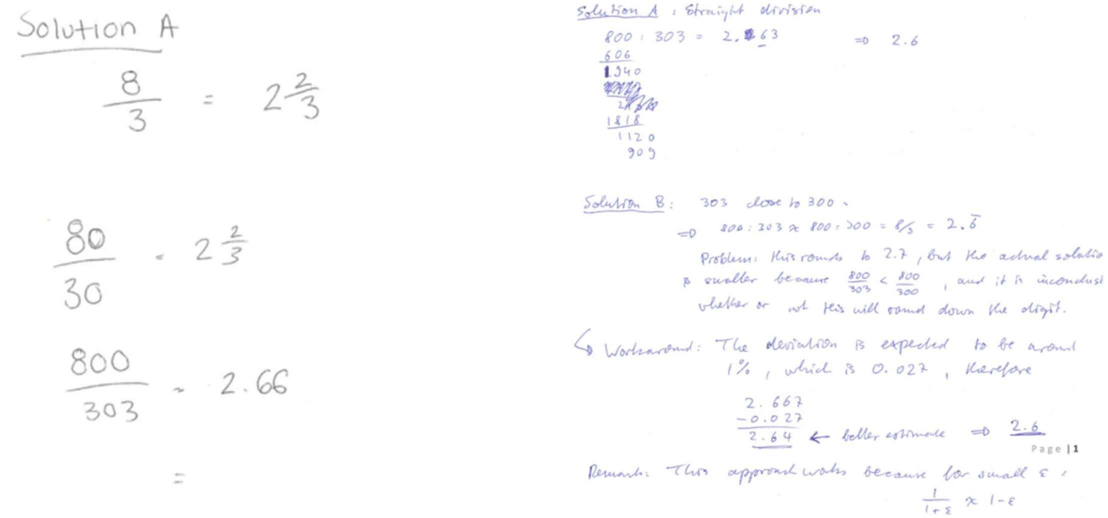
Figure 2 presents the total creativity ratings for the 19 responses, where each total is the sum across three characteristics (*Fluency*: from 0 to 4; *Flexibility*: from 0 to 3; *Novelty*: from 0 to 3), yielding a possible range from 0 to 10 or above. In the current sample, observed totals span 0 to 9. The distribution across observed totals is as follows: one response rated 3; five rated 4; three rated 5; four rated 6; one rated 7; one rated 8; and one rated 9. All counts together account for the 19 responses. Three responses rated 0, as there was any correct solution in these responses. Overall, the ratings are grouped in the lower-to-mid range (3 to 6), with fewer responses attaining totals at the higher end (7 to 9). This pattern suggests that, among these PSMTs, and for the Division problem, many responses combined the three characteristics to achieve moderate totals, while only a small number achieved higher totals indicative of stronger creativity across multiple characteristics simultaneously.

Example Responses Illustrating Mathematical Creativity

To illustrate what mathematical creativity looks like in the responses to the Division problem, we now present two representative examples from the two ends of the creativity rating: one response with low total creativity rating (PSMT #18; Figure 3, left) and another with a higher rating (PSMT #14, right). These examples provide concrete insight into how PSMTs approached the problem and demonstrate the variation in creativity observed in this study.

Figure 3

Examples from the Two Ends of the Mathematical Creativity Rating



PSMT #18; *fluency* rated at 1, *flexibility* at 1, and *novelty* at 2. Total rating of 4.

PSMT #14; *fluency* rated at 3, *flexibility* at 2, and *novelty* at 3. Total rating of 8.

PSMT #18 rated low on *fluency* and *flexibility* as they only presented a single estimation method, and there was no explanation provided to explain their solution. Furthermore, the solution is not complete as they do not attempt to establish the correct answer to the nearest tenth, as stated in the problem. The *novelty* rating is in the mid-range as the use of an estimation method (estimating 303 as 300) was considered more unconventional than a routine algorithmic method (e.g., long division). PSMT #14 rated well on *fluency* as they presented two reasonably complete solution methods. The long division method (Solution A) unfortunately contained an error, so did not allow for a rate of 4 on *fluency*. That said, the classification of Solution B as complete was generous in the sense that the PSMT did not confirm that 2.6 was correct to the nearest tenth, with the generosity justified by the unique sophistication of using the first two terms of the Maclaurin series of $\frac{1}{1+X}$ to estimate the solution. The rating for *flexibility* was 2; that is, two correct methods provided, while the rating for *novelty* was 3 in acknowledgement of the sophistication of the Maclaurin series method.

Discussion

The application of the rubric to the analysis of preservice secondary mathematics teachers' (PSMTs') solutions has shown that the rubric effectively captures the mathematical creativity among PSMTs with respect to the three characteristics of creativity. *Fluency* showed the widest variation, with a few responses reaching higher rates, whereas *flexibility* was consistently limited to one or two solution methods. This may be a reflection of the time students were allocated to spend on the task (25 minutes), with higher scores in *fluency* requiring more detailed and rigorous explanations, this may have been a barrier to providing more than two solutions. Student familiarity with different areas of mathematics would also be a determining factor here, and it may be of interest for future studies to examine the outcome when certain methods, such as the less novel approaches, are prohibited, or provided to students, to understand the full depth of their creativity. These results are consistent with existing literature wherein in general those assessed score higher on *fluency* than *flexibility* (e.g., Bicer, 2021b). *Novelty* was present in some responses, while standard/conventional methods were more common. This is possibly a result of the previous experience of the PSMTs themselves and the opportunities provided for the development of their own creative abilities through instruction wherein more traditional, algorithmic approaches would be more familiar and widely taught.

The overall distribution of ratings indicates that level of creativity was generally modest to moderate, with the majority of responses received a total rating of 4, 5, or 6, which broadly aligns with a previous study as to the total creativity score of elementary PSTs (Bicer, 2021b).

In this study, we sought to develop an assessment approach and a rubric for evaluating mathematical creativity, based on empirical data from a PSMT sample. The rubric should be applied to similar other mathematical problems and contexts, offering a pathway for future research. It is also important to extend the analysis into the classroom to understand how teachers' mathematical creativity, with respect to its identified characteristics, influence their instructional decisions, and their students' mathematical creativity. The rubric used in this study shows that teachers may display varying levels of mathematical creativity, some strong, others moderate or limited. Mathematics teacher educators can use this rubric to decide what to emphasise in initial teacher education and ongoing professional development. For researchers, the rubric can be used in studies of the impact of teacher education or teacher professional development on mathematical creativity. For teachers, it offers a tool to understand students' mathematical creativity to support them adequately in the development of their own creative ability. We argue that more detailed information on the levels of mathematical creativity among mathematics teachers is needed (Suherman & Vidákovich, 2022), which opens the door for future research.

Acknowledgments, Author Contribution

Ethics approval 2022-03252-HATISARU was granted by Edith Cowan University, and all participants gave informed consent. All authors conceived of the work and contributed to writing the paper. Investigation: VH and SR. Conceptualisation and Methods: VH. Literature review: AM and VH. Data analysis: SR and VH. Findings: VH and SR. Discussion and Conclusions: VH and AM. Critical review of the paper: AM.

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