From Beliefs to Practice: Secondary Mathematics Teachers' Conceptions and the Teaching of Mathematical Proof in Singapore Classrooms

Weng Kin Ho
Nanyang Technological University
National Institute of Education
wengkin.ho@nie.edu.sg

Berinderjeet Kaur
Nanyang Technological University
berinderjeet.kaur@nie.edu.sg

Navinesh Thanabalasingam
Singapore Polytechnic
Navinesh_thanabalasingam@sp.edu.sg

This study examines secondary mathematics teachers' conceptions, beliefs, and instructional experiences related to the teaching of proof in the Singapore curriculum. Data from written proof tasks and interviews with two teachers indicate that proof is associated with reasoning and developed mathematical thinking shaped by prior experiences. Teachers also reported tensions between curricular expectations and classroom constraints, while adopting strategies to support students' justification. The findings highlight how teachers' conceptions interact with curricular contexts in shaping proof instruction.

Introduction

Mathematical proof plays a central role in the discipline of mathematics, serving not only to establish the validity of mathematical statements but also to explain why mathematical relationships hold. In mathematics education, proof has been widely recognised as an important means of developing students' mathematical reasoning and conceptual understanding (Stylianides, 2007). However, the role of proof in school mathematics differs from its role in professional mathematics. Within school contexts, proof is often shaped by curriculum expectations, students' developing mathematical knowledge, and constraints of classroom practice. In the Singapore secondary mathematics curriculum, proof is not always presented as formal deductive argument in the Euclidean sense, but often emerges through reasoning, justification, and explanation of mathematical relationships. Students are expected to articulate logical arguments, explain results, and justify procedures, even when fully formal proofs are not explicitly required (MOE, 2023, p. 3). In this study, we therefore conceptualise *mathematical proof* broadly as forms of reasoning and justification that establish the validity of mathematical claims, ranging from informal explanatory arguments to more structured deductive reasoning, while distinguishing these from purely empirical verification based on examples or patterns.

Research in mathematics education has shown that teachers' conceptions of proof vary considerably and influence how proof is interpreted and enacted in classroom practice. For example, Harel and Sowder (1998) identified different proof schemes, demonstrating that individuals may rely on external authority, empirical evidence, or deductive reasoning when justifying mathematical claims. Subsequent studies have shown that teachers, like students, may draw on multiple forms of reasoning, sometimes privileging example-based or procedural explanations over formal deductive argumentation (Stylianides, 2007; Weber, 2001). In the Singapore context, Thanabalasingam (2024) found that teachers often associate proof with advanced mathematical thinking, and may perceive formal proof as more aligned with higher-level mathematics rather than everyday classroom practice.

While these studies have documented teachers' conceptions of proof and their beliefs about its role in mathematics learning, less attention has been given to how these conceptions interact

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with teachers' instructional experiences and the challenges they encounter in classroom contexts. In particular, there remains a need to better understand how teachers negotiate tensions between their beliefs about the value of proof and the practical constraints of curriculum, assessment, and student readiness.

This study builds on earlier work examining secondary mathematics teachers' conceptions and beliefs about proof in the Singapore secondary mathematics curriculum (Thanabalasingam et al., 2025). Extending this line of inquiry, the present study investigates how teachers' conceptions, beliefs, and instructional experiences collectively shape their approaches to teaching proof. By examining teachers' engagement with proof-related tasks alongside their reflections on classroom practice, this study aims to provide a more integrated account of how proof is interpreted and enacted in the secondary mathematics classroom.

The study addresses the following research questions:

1. What conceptions of mathematical proof do secondary mathematics teachers demonstrate when engaging with proof-related tasks?
2. What beliefs do teachers hold regarding the role and purpose of proof in teaching and learning in mathematics?
3. What challenges do teachers encounter when teaching proof in the secondary mathematics classroom?

By addressing these questions, this study seeks to contribute towards a deeper understanding of the interplay between teachers' conceptions of proof, their beliefs about teaching, and the contextual factors that shape proof instruction in school mathematics.

Literature Review

The Role and Functions of Mathematical Proof in Mathematics Education

Mathematical proof is widely regarded as a central component of mathematics and an important means of developing students' mathematical reasoning and conceptual understanding (Stylianides, 2007). In mathematics education, proof not only verifies mathematical statements but also supports explanation and conceptual understanding. Research has identified several functions of proof in mathematical activity, including verification, explanation, and systematisation (de Villiers, 1990; Hanna & de Villiers, 2012). Despite its recognised importance, many students struggle to transition from empirical reasoning based on examples or patterns to deductive argumentation (Harel & Sowder, 1998; Weber, 2001). In response to these challenges, researchers have argued that students should be given opportunities to engage in reasoning and justification from the early stages of mathematical learning. Such experiences can help students gradually develop an understanding of proof as a meaningful mathematical activity rather than a purely formal exercise (Stylianides, 2007). Consequently, the way in which proof is introduced in classrooms plays a crucial role in shaping students' understanding of mathematical reasoning.

Teachers' Conceptions of Mathematical Proof

Teachers' conceptions of proof have been widely studied in mathematics education research (Harel & Sowder, 1998; Stylianides, 2007; Thanabalasingam, 2024), as these conceptions influence how teachers interpret and implement proof-related activities in their classrooms. Teachers' views about what constitutes a valid proof and the purposes of proof in mathematics may shape the types of explanations they provide, the tasks they select, and the expectations they set for students.

One influential framework for analysing individuals' reasoning in proof-related contexts is the Proof Scheme Framework proposed by Harel and Sowder (1998). In this framework, proof schemes refer to the types of arguments that individuals use to justify mathematical claims, and thus reflect their underlying conceptions of what constitutes a valid proof. These schemes are generally categorised into three broad types: external conviction schemes, empirical schemes, and analytical schemes. External conviction schemes rely on authority or ritual procedures, whereas empirical schemes validate claims using examples or patterns. Analytical schemes correspond to formal deductive reasoning (Harel & Sowder, 1998). These categories provide a useful lens for examining the nature and purpose of proof, particularly in distinguishing between reasoning based on verification through examples and reasoning based on general deductive argumentation.

Research has shown that both students and teachers may exhibit a range of proof schemes when reasoning about mathematical statements. In some cases, teachers themselves rely on empirical or perceptual reasoning when explaining mathematical ideas, particularly in classroom contexts where visual or example-based explanations are emphasised. Understanding teachers' underlying proof schemes can therefore provide insight into how they interpret mathematical arguments and how they guide students' reasoning in proof-related activities.

Teachers' Beliefs and Teaching of Mathematical Proof

In addition to their conceptions of proof, teachers' beliefs about mathematics and mathematics teaching can influence how proof is taught in classrooms. Teachers' beliefs encompass their views about the nature of mathematics, the role of proof in mathematics education, and the ways in which students should engage with mathematical reasoning.

Cabassut et al. (2012) examined teachers' beliefs related to proofs and identified several dimensions that influence instructional practices. These include beliefs about the role of proof in mathematics, beliefs about teachers' own mathematical competence, and beliefs about appropriate pedagogical strategies for teaching proof. These beliefs can shape teachers' instructional priorities and the degree to which proof is emphasised in classroom activities.

Previous studies have also shown that teachers may experience various challenges when teaching proof (Stylianides, 2007; Weber, 2001; Thanabalasingam, 2024). For instance, teachers may feel that the curriculum does not provide sufficient guidance or emphasis on proof-related activities. In addition, the absence of proof-related items in high-stakes assessments may discourage teachers from allocating substantial time to proof instruction. Teachers may also encounter difficulties in supporting students' reasoning processes, particularly when students struggle to articulate logical arguments or to understand abstract mathematical concepts.

Understanding teachers' beliefs and challenges related to proof instruction is therefore important for improving the teaching and learning of proof in school mathematics. By examining how teachers' conceptions of proof interact with their beliefs about teaching, researchers can gain deeper insight into the factors that shape the enactment of proof in classroom practice. Such understanding may inform the development of teacher education programmes, professional development initiatives, and curriculum resources aimed at strengthening the teaching of mathematical reasoning and proof.

Conceptual Framework

This study adopts an integrated conceptual framework combining Harel's Proof Scheme Framework (Harel, 2007) and Cabassut et al.'s (2012) Belief System Framework. Harel's framework focuses on the types of reasoning individuals use when justifying mathematical claims, distinguishing between external conviction, empirical, and analytical proof schemes. External conviction schemes rely on authority or ritualised procedures, empirical schemes validate claims using examples or patterns, while analytical schemes correspond to deductive reasoning. In this study, this framework is used to analyse teachers' reasoning when engaging with proof-related tasks, and thus their conceptions of what constitutes a valid proof. Cabassut et al.'s (2012) Belief System Framework complements this perspective by examining teachers' beliefs about the nature and role of proof, their own mathematical competence, and appropriate pedagogical approaches to teaching proof. In particular, it provides a lens for understanding how teachers view the purpose of proof in the curriculum and how these beliefs shape their instructional decisions.

In this study, the two frameworks are integrated to examine both the reasoning and belief dimensions of teachers' engagement with proof. The Proof Scheme Framework is used to analyse how teachers justify mathematical claims (i.e., the forms of reasoning they employ), while the Belief System Framework is used to examine teachers' beliefs about proof and its role in teaching and learning. By bringing these perspectives together, the study investigates how teachers' conceptions of proof and their beliefs about teaching interact to shape their instructional approaches in the secondary mathematics classroom.

Methodology

Research Design

A qualitative multiple-case study design was adopted to examine teachers' conceptions of proof and their instructional experiences. In this study, two in-service secondary school mathematics teachers were examined as individual cases, allowing a detailed exploration of their conceptions of proof, their beliefs about proof instruction, and the challenges they encounter in teaching proof.

Participants

Two secondary school mathematics teachers were purposively selected. Both held degrees in pure mathematics and postgraduate teaching qualifications and taught Elementary Mathematics and Additional Mathematics at upper secondary level.

Data Collection

Data were collected using a conceptions questionnaire consisting of nine proof tasks and semi-structured interviews. Teachers' written responses to these tasks were analysed to identify the types of reasoning they used when justifying mathematical statements. The conceptions questionnaire consisted of proof-related tasks drawn from secondary school topics (e.g., geometry and algebra), requiring teachers to justify mathematical statements, evaluate the validity of given arguments, and explain their reasoning. These tasks were designed to elicit the types of arguments teachers use when engaging with proof, including empirical and deductive forms of reasoning. In addition, a beliefs questionnaire was administered to capture teachers' views about the nature and role of proof in mathematics, their perceptions of students' ability to engage with proof, and their approaches to teaching proof in the classroom. The questionnaire included open-ended items that invited teachers to elaborate on their perspectives.

To explore Research Question 3, which concerns the challenges teachers encounter when teaching proof, semi-structured interviews were conducted with each participant. The interviews provided opportunities for teachers to elaborate on their questionnaire responses, describe their experiences in teaching proof, and discuss the difficulties they face when supporting students' reasoning in proof-related tasks.

Data Analysis

Teachers' responses to the conceptions questionnaire were analysed using Harel's (2007) Proof Scheme Framework, which categorises reasoning into external conviction, empirical, and analytical proof schemes. This analysis enabled the identification of teachers' underlying conceptions of proof and addressed Research Question 1.

Responses to the beliefs questionnaire were analysed using categories derived from Cabassut et al. (2012) Belief System Framework, examining teachers' beliefs about the nature of proof, its role in the curriculum, and appropriate instructional strategies for teaching proof. The analysis provided insight into how teachers' conceptions relate to their instructional approaches, addressing Research Question 2.

Interview transcripts were analysed thematically to identify recurring patterns in teachers' reported experiences and challenges in teaching proof. The thematic analysis was conducted inductively, allowing themes to emerge from the data while being interpreted in relation to the study's conceptual framework. These themes were used to address Research Question 3, and to provide a deeper understanding of the factors shaping teachers' instructional practices.

Triangulation across three data sources – questionnaire responses, task performance and interview data – was used to enhance the credibility of the findings and to provide a more comprehensive understanding of teachers' instruction of mathematical proof.

In addition to identifying teachers' conceptions and beliefs, the analysis examined how these elements were reflected in teachers' reported instructional approaches and experience when teaching proof.

Findings

The analyses of the data revealed that teachers' conceptions of mathematical proof were closely related to ways in which they approached the teaching of proof in their classrooms. Three themes emerged: (1) alignment between teachers' conceptions and instructional goals, (2) tensions between beliefs about proof and classroom realities, and (3) pedagogical adaptations in teaching proof.

Alignment Between Teachers' Conceptions of Proof and Instructional Goals

The interview data suggest that both teachers viewed proof as requiring reasoning beyond routine procedures, particularly in explaining and justifying mathematical relationships. For instance, Teacher 1 reflected that answering some of the proof-related questions in the conceptions questionnaire required "a higher level of mathematical maturity", noting that such tasks may be challenging for students with limited exposure to advanced reasoning.

Teachers also linked their own understanding of proof to their prior experiences studying mathematics at more advanced levels. Teacher 1 attributed his confidence in approaching proof tasks to his undergraduate mathematics training, explaining that his university education helped him develop the "mathematical maturity to understand how to write proofs, and also how to think of … the pathway into proving certain phenomena in mathematics." Similarly, Teacher 2 reported that her experiences studying mathematics in junior college (pre-university) and university had provided sufficient exposure to mathematical proof, although she noted that professional development opportunities specifically focused on the teaching of proof were relatively limited.

These reflections suggest that teachers' conceptions of proof were shaped by their prior learning experiences, with proof viewed primarily as a form of reasoning grounded in conceptual understanding. These views shaped teachers' instructional goals, emphasising students' ability to explain and justify mathematical relationships.

Evidence of teachers' conceptions of proof could also be observed in their written responses to proof-related tasks in the questionnaire. For example, in a geometry task, both teachers justified their solutions by transforming the problem into equivalent angle relationships and applying properties such as angle sums and cyclic quadrilaterals. This reflects a transformational proof scheme, in which claims are established through the application of known properties (Harel & Sowder, 1998).

At the same time, the written responses revealed subtle differences in how teachers articulated their reasoning. For instance, one teacher concluded that $\angle ABC + \angle ADC = 183° \neq 180°$ and therefore that the quadrilateral was not cyclic, without explicitly stating that the underlying geometric principle regarding opposite angles in cyclic quadrilaterals. In another instance, a teacher used the term *converse* when the intended logical reasoning corresponded more closely to a *contrapositive*. These examples suggest that while teachers were generally able to construct valid deductive arguments, the articulation of the underlying mathematical reasoning was sometimes implicit or imprecise.

Tensions Between Teachers' Beliefs About Proof and Classroom Realities

Although both teachers expressed positive views about the importance of proof in mathematics, the interview responses revealed several tensions between these beliefs and the realities of classroom practice. One challenge concerned the limited presence of proof-related tasks in the curriculum materials used in secondary school mathematics. Teacher 2 observed that while textbooks contain many challenging problems and investigative activities, "E-Math seldom have proving problems," noting that explicit proving tasks appear more frequently in Additional Mathematics topics such as trigonometric identities. This observation suggests that opportunities for students to engage with proof-related reasoning may be uneven across different strands of the curriculum.

Teachers also highlighted practical constraints that affect the extent to which proof-related activities can be implemented in classroom teaching. Teacher 1 acknowledged that although exploratory activities designed to support reasoning are available in textbooks, these were not always used in practice, explaining that he "did not use much of those exploratory activities … because of time shortage." Such comments point to the influence of curriculum pacing and time constraints on teachers' instructional decisions.

Student motivation was another factor shaping teachers' experiences of teaching proof. Teacher 2 explained that because the proof of formulas is often not explicitly required in the E-Math syllabus, many students are not keen to learn how such results are derived. She noted that students tend to focus on applying formulas rather than understanding the reasoning behind them.

Taken together, these responses illustrate how teachers' beliefs about the importance of proof can come into tension with the curricular structure, time constraints, and students' expectations of mathematics learning. While teachers recognised the value of proof in fostering mathematical understanding, these contextual factors shaped how proof-related reasoning was enacted in classroom practice.

Pedagogical Adaptations in Teaching Proofs

In response to the challenges associated with teaching proof in secondary mathematics classrooms, the interview data suggest that teachers adopt various pedagogical strategies to support students' engagement with mathematical reasoning. A recurring concern raised by teachers involved helping students make their reasoning explicit when constructing proofs. For example, Teacher 1 described difficulties in assessing whether students had correctly understood geometrical relationships when writing proofs. He explained that students often wrote brief statements such as "alternate angles" when justifying a step in a geometric argument, without clearly specifying which pair of angles they were referring to in diagrams that contained multiple pairs of parallel lines. As he explained, the challenge was "trying to get my students to write … to make their thinking visibly clear."

Teachers also described using visual or illustrative approaches to support students' understanding of mathematical relationships before introducing more formal proofs. Teacher 2 mentioned using visual demonstrations (for instance, using dynamic pictorial proofs of the Pythagorean Theorem) when introducing certain results, explaining that such approaches could help students grasp the underlying relationships even though these demonstrations did not constitute formal mathematical proofs. At the same time, she emphasised that understanding the reasoning behind mathematical results remained important, commenting that it would be "ridiculous for students to apply the property but not knowing how it is derived."

These responses suggest that teachers attempt to balance the development of conceptual understanding with the formal requirements of mathematical proof. In particular, teachers appear to use explanatory discussion, visual representations, and guided reasoning as ways of helping students move toward clearer articulation of mathematical arguments. These practices support students in developing the ability to express and justify mathematical reasoning.

Discussion

The findings of this study provide insights into how teachers' conceptions of proof interact with classroom practice in the secondary mathematics curriculum. In particular, teachers associated proof with developed mathematical reasoning shaped by their prior learning experiences, aligning with Harel's (2007) notion of proof schemes, where individuals' justification of mathematical claims reflects their cognitive development and prior exposure to mathematical reasoning.

At the same time, the findings highlight tensions between teachers' conceptions of proof and the structural constraints of the curriculum. Opportunities for engaging students in proof-related reasoning appear uneven across topics, with explicit proving tasks more prevalent in areas such as trigonometric identities than in other parts of the curriculum. This reflects broader observations that the role of proof in school mathematics is shaped by curricular priorities and assessment structures (Stylianides, 2007).

The results also suggest that teachers respond to these tensions through pedagogical adaptations aimed at supporting students' reasoning. These include encouraging clearer articulation of arguments and using visual or explanatory approaches to bridge students' intuitive understanding and formal reasoning. Such practices indicate that proof instruction in school mathematics often involves supporting students' progression from empirical reasoning towards more structured deductive thinking.

Taken together, the findings highlight the interplay between teachers' conceptions of proof, their beliefs about students' learning, and the curricular contexts in which proof is taught, contributing to a more nuanced understanding of how proof is enacted in the secondary mathematics classroom.

Conclusion and implications

By integrating Harel's proof scheme framework with Cabassut et al.'s belief system framework, the study provides a lens for understanding how teachers' views about proof interact with the practical realities of classroom teaching. The findings suggest that teachers' conceptions of proof influence the instructional goals they prioritise in proof-related lessons, particularly in emphasising explanation and reasoning as important aspects of mathematical understanding. At the same time, the results reveal tensions between teachers' pedagogical beliefs about the value of proof and the constraints of curriculum pacing, examination demands, and students' readiness for abstract reasoning.

In navigating these tensions, teachers reported a range of pedagogical adaptations aimed at supporting students' engagement with proof-related ideas. These adaptations included the use of examples, exploratory reasoning, and explanatory discussion as ways of bridging students' intuitive understanding and more formal deductive reasoning. These practices suggest that proof instruction often involves a progression from empirical reasoning toward analytical proof.

The findings of this study have implications for teacher education and professional development. Within the scope of this study, which examined teachers' engagement with proof in specific mathematical topics (e.g., trigonometric identities), the results suggest that programmes designed to support the teaching of proofs may benefit from addressing not only teachers' mathematical knowledge of proof but also the pedagogical strategies needed to guide students' development of mathematical reasoning. In particular, opportunities for proof-related reasoning may be more readily embedded in some topics than others, and professional development programmes may therefore need to consider how proofs can be meaningfully integrated across different areas of the curriculum.

Supporting teachers in reflecting on their own conceptions of proof and their instructional practices may contribute to strengthening the role of reasoning and justification within school mathematics. However, given the small sample size and topic-specific nature of the study, these implications should be interpreted as exploratory rather than generalisable. Future research may examine how instructional approaches influence students' development of proof-related reasoning across a wider range of mathematical topics.

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