Teachers' Awareness of Students' Difficulties with Algebra Word Problems

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This paper reports findings from a diagnostic questionnaire completed by eight junior high school mathematics teachers in Ghana before participating in a Community of Practice. Analysis of the questionnaire explored how the teachers' responses revealed aspects of their pedagogical content knowledge for teaching algebra word problems, particularly their awareness of task complexity, student reasoning, and potential misconceptions. Findings suggest that while the teachers demonstrated awareness of common student difficulties, their pedagogical content knowledge was uneven, especially in relation to explanations, representations, and curriculum-task use.

Introduction

Algebra word problems continue to present substantial challenges for students in school mathematics. Unlike routine symbolic exercises, they require students to interpret language, identify quantitative relationships, represent those relationships algebraically, and reason with the resulting expressions or equations. The demands of algebra word problems therefore extend across linguistic, conceptual, and representational domains, making them an important site for examining student thinking and understanding (Kieran, 2020; Verschaffel et al., 2020). For this reason, they also provide a valuable context for examining the teaching practices needed to support students' learning.

Difficulties with algebra word problems have been widely documented. Students often struggle with the meaning of variables, the structure of multiplicative relationships, and the translation of verbal statements into algebraic form. Reversal errors are especially common when students attempt to represent comparison statements symbolically, and such errors often reflect deeper difficulties in coordinating language with algebraic structure rather than mere carelessness (Clement, 1982; Kieran et al., 2016). These challenges matter because success in algebra depends not only on symbolic fluency but also on the ability to reason about relationships, generality, and representation.

In this context, teachers' pedagogical content knowledge (PCK) is particularly important. Effective teaching of algebra word problems requires more than knowing the correct answer or procedure. It also requires teachers to anticipate likely student difficulties, interpret incorrect responses, explain relationships clearly, and select tasks or representations that support understanding (Shulman, 1987; Ball et al., 2008; Chick & Beswick, 2018). Teachers' awareness of students' difficulties is therefore an important aspect of PCK, especially in topics such as algebra where misconceptions can be conceptually persistent and instructionally consequential.

The Ghanaian context provides a strong rationale for examining these issues. Concerns about students' achievement in mathematics have persisted in national reports and related research, with algebra and solving worded problems continuing to present difficulties for many learners (Abreh et al., 2018; Osei & Agyei, 2023; West African Examinations Council [WAEC], 2023). Yet, while there is substantial discussion of students' performance, much less attention has been given to what teachers themselves know about students' difficulties with

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algebra word problems and to what their responses reveal about their PCK before any targeted professional learning begins.

This paper addresses that gap through findings from a diagnostic questionnaire completed by eight junior high school mathematics teachers in Ghana prior to their participation in a Community of Practice (CoP) focused on teaching algebra word problems. The questionnaire generated two connected sets of findings. First, teachers ranked a set of algebra items according to how difficult they believed those items would be for their students and explained the reasons for their judgements. Second, teachers responded to hypothetical student answers and algebra tasks designed to reveal aspects of their PCK. Bringing these two strands together makes it possible to examine not only whether teachers were aware of likely student difficulties, but also whether that awareness was accompanied by pedagogically robust ways of responding to those difficulties. This paper is guided by two research questions:

1. What does the diagnostic questionnaire reveal about teachers' awareness of students' difficulties with algebra word problems?
2. What does the diagnostic questionnaire reveal about the pedagogical content knowledge teachers draw on in responding to those difficulties?

Theoretical Framing

This paper is framed by the concept of PCK, understood as the specialised knowledge that enables teachers to transform disciplinary content into forms that are accessible and meaningful for learners (Shulman, 1987). In mathematics education, this includes knowledge of content, knowledge of students' thinking, and the capacity to make pedagogically sound instructional decisions (Ball et al., 2008; Chick & Beswick, 2018). In algebra word problems, this is especially important because students must coordinate language, quantitative relationships, and symbolic representation, and often struggle with variables, relational statements, and translation from words to equations (Kieran, 2020; Verschaffel et al., 2020).

The analysis was guided by an adapted framework informed by Chick and Beswick (2018) and Charalambous (2016), focusing on five aspects of PCK: algebra content knowledge, selecting and using representations, providing and evaluating explanations, analysing students' errors and misconceptions, and selecting and using curriculum tasks. These aspects were selected because they are central to effective mathematics teaching and highly relevant to the instructional demands of algebra word problems (Charalambous, 2016; National Council of Teachers of Mathematics [NCTM], 2014).

Within this framing, teacher awareness is understood as more than knowing whether students succeed or fail. It includes judging likely task difficulty, interpreting incorrect responses, identifying possible sources of misconceptions, and proposing pedagogically appropriate support. At the same time, awareness alone is not sufficient. A teacher may recognise that students are likely to misinterpret variables or reverse a multiplicative relationship yet still lack the pedagogical means to address those difficulties effectively. This framing makes it possible to examine not only what the teachers noticed about students' difficulties, but also the pedagogical resources they drew on in responding to those difficulties.

Methodology

Participants and Context

This paper draws on the diagnostic phase of a broader qualitative study that investigated how a CoP supported junior high school mathematics teachers' PCK for teaching algebra word problems in Ghana. The paper focuses exclusively on the questionnaire administered before the CoP began, because these data provided baseline evidence of what the teachers knew about students' difficulties and the pedagogical resources they drew on in responding to those

difficulties. Eight teachers were purposively selected from public junior high schools in the study area using criteria that included professional teaching qualification, willingness to participate, and availability for CoP meetings. The teachers were drawn from both average- and below-average-performing schools. This number was suitable for collaborative professional learning and detailed qualitative analysis.

Instrument

A diagnostic assessment questionnaire was used to gather initial data on teachers' PCK in relation to algebra word problems. The questionnaire focused on five aspects of PCK: algebra content knowledge, selecting and using representations, providing and evaluating explanations, analysing students' errors and misconceptions, and selecting and using curriculum tasks. The questionnaire comprised three sections. Section A collected demographic information indicating variation in the teachers' ages, qualifications, and years of teaching experience. Section B required teachers to rank seven algebra items according to the level of difficulty they believed those items would present for their students and to justify those rankings. Section C presented hypothetical student answers to algebra word problems and asked teachers to judge whether the responses were correct, explain their reasoning, identify likely student mistakes, describe possible sources of those mistakes, and propose strategies for addressing them. These hypothetical answers were informed by prior literature on common student difficulties in algebra word problems (Clement, 1982; Ladele, 2013). The questionnaire was peer reviewed, piloted with five teachers outside the final sample, and revised for clarity. A copy of the questionnaire is presented in Kasimu (2024).

Data Analysis

A hybrid analytic approach was used. First, teachers' rankings of the seven algebra items in Section B were analysed descriptively using mean rankings (see Table 1). This descriptive analysis was used alongside the qualitative explanations teachers gave for their rankings. Second, teachers' responses to the hypothetical student answers to algebra word problems were analysed deductively using an adapted PCK framework informed by Chick and Beswick (2018). The analysis focused on the teachers' knowledge of algebra word problems, selecting and using representations, providing and evaluating explanations, analysing students' errors and misconceptions, and their judgements in selecting and using appropriate curriculum tasks, using representations, providing and evaluating explanations, and analysing students' errors and misconceptions. Responses were coded as satisfactory, unsatisfactory, or not evident. A response was coded as satisfactory when it showed mathematically and pedagogically appropriate understanding of the relevant task, unsatisfactory when it reflected misconceptions or significant errors in reasoning, and not evident when the relevant aspect was absent from the response. To strengthen consistency in coding, the categorisation of responses was reviewed by the co-authors, and any differences in coding were discussed and resolved through consensus. Third, teachers' open-ended responses about likely student errors, possible sources of those mistakes, and proposed strategies were analysed using thematic analysis (Braun & Clarke, 2019). The combination of descriptive, deductive, and inductive analysis enabled a more nuanced account of teachers' awareness of students' difficulties with algebra word problems and the PCK evident in their responses.

Findings

The diagnostic questionnaire revealed an uneven pattern in the teachers' awareness and PCK. The ranking data indicated that the teachers generally perceived algebra word problems as more difficult for students than symbolic equations, particularly when tasks required interpreting variables and translating verbal statements into algebraic expressions. However, analysis of the teachers' responses to the hypothetical student answers to algebra word problems

suggested that this awareness was not consistently supported by strong PCK. Although the teachers often identified likely student misconceptions, there was less evidence of clear explanations, effective representational strategies, or curriculum-oriented task reasoning in their responses.

Teachers' Awareness of Students' Difficulty with Algebra Word Problems

As shown in Table 1, the teachers' rankings of the seven algebra items showed a clear pattern. Symbolic items were consistently perceived as easier, whereas worded items were ranked as more difficult. The symbolic items received mean rankings of 1.00, 2.38, and 2.75, while the worded items received higher mean rankings of 4.63, 5.13, 5.75, and 6.38. These rankings suggest that the teachers recognised the greater demands involved in translating words into algebraic form compared with direct symbolic manipulation.

Table 1

Teachers' Ranking of Questions from 1 (Extremely Easy) to 7 (Extremely Difficult)

Nature of question type	Code	Ranked questions	Mean ranking
Symbolic: *Questions involving direct mathematical symbols and equations*	Q6	Solve for x: 5x = 50	1.00
	Q4	Solve for x: 6/4 = 30/x	2.38
	Q2	Solve for x in terms of a: 9a = 10x	2.75
Worded: *Questions requiring translation of a word problem into a mathematical equation or concept*	Q6	Solve for x: 5x = 50	1.00
	Q3	There are 900 people at a parade. There are 40 more men than women. There are twice as many children as there are men. How many children are there?	4.63
	Q5	Ahmad has four times as much money as Betty. After Ahmad spent GHC160 and Betty spent GHC 40, they each had equal amounts of money. How much money did Ahmad have at first?	5.13
	Q1	Write an equation using the variables S and P to represent the following statements: "There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors."	5.75
	Q7	Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there are five people who ordered strudel." Let C represent the number of cheesecakes and S represent the number of strudels ordered.	6.38

Among the worded items, a further distinction was also evident. Problems that combined known quantities and variables were perceived as less difficult than those expressed mainly in terms of variables. This suggests that the teachers recognised the added challenge students face when reasoning about relationships between abstract quantities without the support of concrete numerical values. This pattern aligns with earlier research showing that verbally presented algebra problems are often more difficult than symbolic equations because they require interpretation as well as manipulation (Clement, 1982; Nathan & Koedinger, 2000).

Thematic analysis of the teachers' reasons for their rankings generated two main themes: *conceptual understanding of variables and equations*, and *difficulty translating words into mathematical expressions*. Under *the first theme*, the teachers indicated that students often struggled to understand that variables represent quantities and that letters in algebra stand for numbers rather than fixed objects. Their comments suggested that students found abstract representations difficult, especially when these had to be interpreted within a problem context. *The second theme* concerned students' difficulty in converting verbal statements into algebraic expressions. The teachers referred to students' struggles with interpreting written statements, identifying relationships, and expressing them mathematically. Several suggested that students found symbolic equations easier when these were provided but had difficulty generating them independently from written scenarios. Together, these themes indicate that the teachers were aware of two central barriers in students' algebra learning: the abstraction involved in working with variables and the complexity of translating natural language into algebraic notation.

Teachers' Responses to the Hypothetical Student Answers and Algebra Tasks

The deductive analysis of the teachers' responses to the hypothetical student answers revealed an uneven pattern across the teaching practices associated with PCK. Although the teachers often identified likely student misconceptions, this was not consistently matched by strong explanations, representations, or curriculum-task strategies.

For the variable-equality word problems, four of the eight teachers provided satisfactory responses for algebra content knowledge, while the remaining four gave unsatisfactory responses. The same pattern was found for providing and evaluating explanations and for analysing students' thinking, errors, and misconceptions. This indicates that only half of the teachers were able to explain correctly why two different variables could represent equal quantities and to interpret the misconception embedded in the item appropriately. The unsatisfactory responses revealed uncertainty about the meaning of variables, with some teachers treating different letters as though they could not represent the same quantity.

For the verbal-to-symbolic problems, the pattern was similarly uneven, with some aspects revealing more limited content knowledge. For example, for the statement "*The number y is eight times the number z*", five teachers gave satisfactory responses, while three did not. However, for the students-and-professors statement (*There are six times as many students as professors at this university*), only two teachers produced the correct equation, while six gave unsatisfactory responses. Many of these unsatisfactory responses reflected reversal errors or incorrect operations, suggesting that some of the teachers themselves were not fully secure in representing multiplicative relationships algebraically. This finding is important because it suggests that some of the misconceptions teachers identified in students' work were also reflected in their own responses.

A further weakness was evident in the quality of the teachers' explanations. Although some teachers provided mathematically sound reasoning, others offered explanations that were incomplete, circular, or conceptually weak. This suggests that even when a correct answer was given, the pedagogical capacity to explain the underlying algebraic relationship clearly was not always secure. Such variability matters because explanation is central to helping students make sense of algebraic structure rather than simply reproduce a procedure.

A particularly significant result was that none of the teachers explicitly referred to the use of representations in response to the verbal-to-symbolic tasks, and none explicitly addressed the selection and use of curriculum tasks. These components were therefore coded as not evident. This absence is important because it suggests that, although several teachers could identify likely student difficulties, they did not draw on visual or concrete representations, nor did they propose task-based strategies that might help students build conceptual understanding progressively.

Teachers' Views of Student Errors and Support Strategies

The teachers' open-ended responses about the kinds of mistakes students were likely to make provide further support for this interpretation. They pointed to recurring errors such as misinterpretation of mathematical symbols or operations, confusion about the meaning of "is", incorrect use of addition in place of multiplication, interchange of variables, and reversal errors in forming equations. These responses suggest that the teachers had a reasonably strong awareness of the kinds of incorrect formulations students might produce when attempting to translate verbal statements into algebraic expressions.

When asked about possible sources of these mistakes, teachers' responses clustered around two main themes: language comprehension and lack of understanding of word problems. Some pointed to students' inability to read and understand the question properly, including difficulties with English language comprehension. Others suggested that students feared word problems or assumed that mathematics should mainly involve numbers and direct operations rather than verbal situations requiring interpretation. These responses indicate that the teachers understood students' difficulties as involving both linguistic and conceptual factors.

In proposing strategies to address students' difficulties, the teachers most commonly referred to providing explanation, engaging students in problem-solving, and student-centred learning. Their suggestions included explaining the statement to students, helping them understand the problem, solving more examples with them, and encouraging students to read and attempt problems independently before receiving corrective feedback. These strategies were pedagogically sensible, but they were also quite general. The teachers rarely moved beyond explanation and practice to more specialised pedagogical tools such as visual representations or deliberately sequenced tasks designed to build understanding.

Overall, the findings suggest that the teachers' awareness of student difficulty was stronger than the pedagogical repertoire they drew on in responding to that difficulty. They were relatively better at identifying likely errors and misconceptions than at providing strong explanations, selecting representations, or drawing on curriculum-task strategies that could support learning more effectively. This reinforces the broader finding that the diagnostic assessment revealed a distinction between recognising difficulty and having the pedagogical means to address it in mathematically powerful ways.

Discussion

This study examined what a diagnostic questionnaire revealed about teachers' awareness of students' difficulties with algebra word problems and the PCK they might draw on in responding to those difficulties. The findings show that the teachers in this study were aware of students' misconceptions. Their rankings of the seven algebra items, their explanations for those rankings, and their responses to the hypothetical student answers all indicate that they recognised algebra word problems as more demanding than symbolic equations and were aware of several common student difficulties, including poor understanding of variables, reversal errors, and challenges in translating verbal statements into algebraic expressions. This pattern is consistent with research showing that algebra word problems are often more difficult than symbolic equations because they require students to coordinate language, relationships, and symbolic representation (Clement, 1982; Kieran, 2020).

At one level, this is an encouraging finding. It suggests that the teachers demonstrated an important aspect of PCK, namely awareness of how students are likely to experience difficulty in algebra. This aligns with Shulman's (1987) argument that effective teaching depends not only on subject-matter knowledge, but also on knowledge of how learners engage with that content. It also supports research identifying knowledge of content and students as a central aspect of teaching expertise (Ball et al., 2008). In this study, the teachers were often able to

anticipate incorrect formulations students might produce and to identify plausible sources of those errors, particularly in relation to language comprehension and the difficulty of translating words into symbolic form.

However, the findings also reveal a more critical point: awareness of student difficulty was not consistently matched by strong pedagogical responses. Although several teachers could identify likely misconceptions, fewer demonstrated secure algebra content knowledge across the focal tasks, and even fewer showed strong explanatory power. Most notably, there was no explicit evidence of representations or curriculum-task use in their responses to the verbal-to-symbolic tasks. This matters because PCK involves not only recognising student misconceptions, but also being able to explain concepts clearly, choose useful representations, and select tasks that support learning meaningfully (Chick & Beswick, 2018).

The distinction between recognising difficulty and responding pedagogically is one of the main contributions of the paper. A teacher may recognise that students are likely to confuse variables, reverse a multiplicative relationship, or misread a verbal statement, yet still lack the representational, explanatory, or task-based resources needed to address those difficulties effectively. In this sense, the diagnostic questionnaire did not simply show how teachers interpreted students' difficulties. It also exposed the limits of the pedagogical tools they were drawing on when imagining how to support learners. This finding supports broader arguments that effective mathematics teaching requires more than identifying error; it requires transforming mathematical ideas into forms that are accessible and instructionally productive for students (Ball et al., 2008; Chick & Beswick, 2018).

The absence of representations is especially significant. Algebra word problems require students to move between language, quantities, and symbolic forms, and representations can help bridge these forms of reasoning. Visual and concrete representations can support conceptual understanding by making relationships more visible and less abstract (Kieran et al., 2016). Yet none of the teachers explicitly proposed such representations in response to the verbal-to-symbolic tasks. A similar concern applies to curriculum-task selection. Although some teachers suggested giving students more examples or practice, there was little evidence of deliberate attention to task sequencing or to the use of tasks that build understanding progressively. Together, these omissions point to an uneven pattern of PCK rather than a simple lack of awareness.

The findings therefore suggest that the teachers had a meaningful starting point for professional learning, but one that needed to be extended and deepened, especially in relation to explanation, representation, and task design. The diagnostic questionnaire was valuable not only as a baseline measure, but also as an analytic tool that revealed where subsequent professional learning needed to focus. In this sense, the study shows how diagnostic assessment can make visible both teachers' existing strengths and the pedagogical gaps that may constrain their responses to students' difficulties.

Conclusion

This paper examined what a diagnostic questionnaire revealed about teachers' awareness of students' difficulties with algebra word problems in Ghana. The findings showed that the teachers generally recognised that worded algebra tasks were more difficult for students than symbolic ones and were able to identify several important student difficulties, including weak understanding of variables, reversal errors, and challenges in translating verbal statements into algebraic expressions. These results suggest that the teachers possessed some awareness of the conceptual and linguistic demands that algebra word problems place on learners.

At the same time, the diagnostic questionnaire also revealed uneven PCK. Although several teachers were able to anticipate likely student misconceptions, fewer demonstrated secure algebra content knowledge across the focal tasks, and none explicitly referred to representations

or curriculum-task selection in response to the verbal-to-symbolic tasks. The findings therefore suggest that awareness of student difficulty, while important, was not consistently matched by strong pedagogical resources for responding to that difficulty. Diagnostic assessment can make visible both what teachers already notice and what pedagogical resources they still need, providing an evidence-informed basis for shaping targeted professional learning.

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