Teaching Experience and Teacher Competence for Multiplicative Thinking in the Early Years

Mayamiko Malola
Charles Sturt University
mmalola@csu.edu.au

Max Stephens
University of Melbourne
m.stephens@unimelb.edu.au

Early development of multiplicative concepts in students has been associated with success in mathematics in later years of schooling. This paper reports on a study that employed an embedded mixed methods approach to investigate the association between primary school teachers' teaching experience and pedagogical content knowledge for multiplicative thinking in the early years. Our focus was on teachers' capacity to support student transition from additive to multiplicative thinking. The study used a questionnaire with 62 primary school teachers in Australia at different levels of teaching experience from Pre-service teachers to Novice, Experienced, and Expert teachers. Longer teaching experience was important but not a condition for sound pedagogical content knowledge.

Introduction

The importance of multiplicative thinking in supporting students' learning of key topics and success in further mathematics is widely and clearly stated in mathematics education literature. It forms the basis for understanding proportions, patterns, fractions, measurement, rates, percentages, statistical thinking, the development of algebraic thinking and understanding the complex issues in society (Askew et al., 2019). However, there is evidence of low student performance in this area (Siemon, 2013). Fennema and Franke (1992) emphasise that teachers' pedagogical content knowledge (PCK) is critical in determining students' learning attainment. In relation to multiplicative thinking, Siemon and Breed (2006) suggested that the transition from additive thinking to multiplicative thinking, in contrast to the implication that most curricula provide, is a major conceptual leap for most students who continue to rely on repeated addition strategies for problems that require efficient multiplicative strategies. It requires expert pedagogical understanding by their teacher to shift their spontaneous conceptions to the conceptually correct "scientific conceptions". The study reported here interrogated the capacity of teachers (Pre-service, Novice, Experienced, and Expert) to support this transition.

Literature Review

Student transition from additive to multiplicative thinking

Research in mathematics education is clear that additive thinking forms the basis for multiplicative thinking (Askew et al., 2019; Malola et al., 2020). For instance, Askew et al., (2019) acknowledged that although multiplicative thinking can be developed through repeated addition in primary schools, the teaching of multiplicative thinking focusing on proportional reasoning supports students' learning of higher-level multiplicative thinking problems and success in mathematics subject. Siemon et al. (2006) suggested that the learner transition from additive thinking to multiplicative thinking, in contrast to the implication that most curricula provide, is a major conceptual leap and for most students requires expert pedagogical understanding by their teacher.

Accordingly, the focus of this research study is to examine how teachers support students' transition from additive to multiplicative thinking (the transitional teaching stage). Teachers therefore need to possess appropriate understanding of the conceptual field and diverse

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pedagogies to successfully move learners from additive to multiplicative thinking. Further to this argument, Jacob and Willis (2001) emphasised that teachers need to understand the difference between additive and multiplicative thinking to effectively support students' development and understanding of multiplicative thinking in their further schooling. Teachers' PCK of multiplicative thinking becomes important because the teachers are key to students' development of this concept.

Teaching experience and competence

The current decade is seeing a growing interest among mathematics educators inquiring into the association between teaching experience (number of years one has been teaching) and teacher competence (how teaching practices influence student learning outcomes). For instance, In Australia, Gore et al. (2023) worked with 512 primary school teachers to investigate the relationship between the number of years spent teaching (teaching experience) and teaching quality. These teachers had a wide range of teaching experience (< 1 to 24 years), and the lessons were conducted over a period of 4 years (2014–2015, then 2019–2012). They found no significant differences in teaching quality across the range of teaching experience.

In Germany, König et al. (2021) investigated the connection between teachers' mathematical competence and students' achievement. This large-scale study involved 89 teachers and 3,496 students in junior secondary school in the federal states of Hamburg and Thuringia. The researchers found a positive relationship between teachers' mathematical competency and students' achievement on national examinations. On the other hand, Irvine (2019) reviewed several studies on the relationship between teaching experience and teacher effectiveness. He concluded that this relationship "as measured by student learning gains is complex nuanced, and non-linear" (page 1). While several studies have examined the association between the total number of years of teaching and teacher competence, the studies are general with limited attention to the teaching of complex mathematical concepts such as multiplicative thinking in the early primary (K-4) school years. This study intended to contribute knowledge to the teaching profession and empirical evidence to mathematics education community on the relationship between teaching experience and teacher competence in developing foundational mathematical competencies such as multiplicative thinking in students. This has implications to school practices in allocating teaching classes to teachers.

Mathematical pedagogical content knowledge

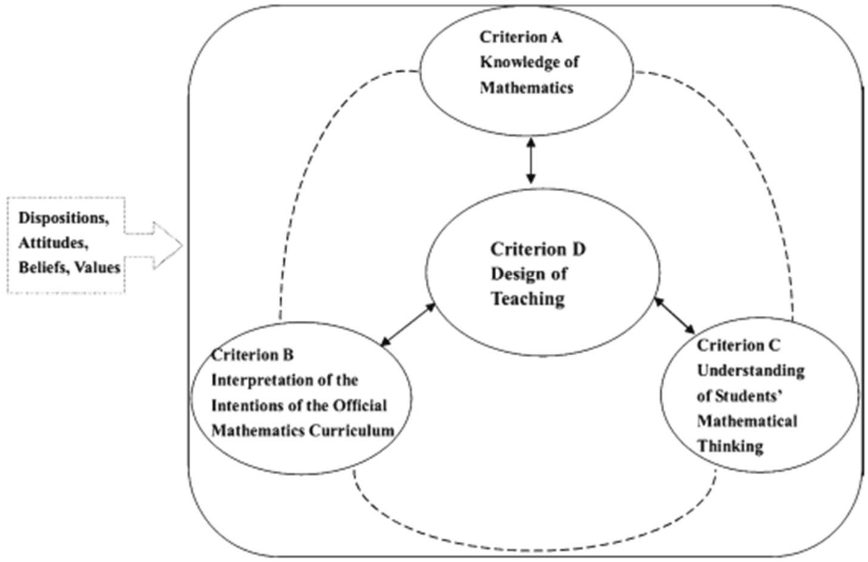
Informed by Shulman's (1986) seminal work on PCK, various scholars (Baker & Chick, 2006; Ball et al., 2008; Fennema & Franke, 1992; Zhang & Stephens, 2013) have developed models for mathematical knowledge for teaching (MKT). These models have provided useful lenses through which teachers' knowledge for teaching mathematics can be examined. Ball et al. (2008) and all the above authors underscore the claim that MKT is necessary to effectively teach complex concepts such as multiplicative thinking which was the focus of our research. This study used the model of Teacher Capacity (Zhang & Stephens, 2013) as the lens through which teachers' pedagogical content knowledge for developing foundational concepts of multiplicative thinking was understood.

The teacher capacity model

In developing a construct of teacher capacity, Zhang and Stephens (2013) worked with 120 teachers from Australia and China. Figure 1 presents the four elements of teacher capacity model to help investigate teachers' capacity for teaching mathematics: knowledge of mathematics, knowledge of curriculum, knowledge of students' thinking, and design of instruction (p. 489). Collectively, these elements influence teachers' PCK for Mathematics

Figure 5

Zhang and Stephens Model of Teacher Capacity



For this study, these four elements were understood in terms of teacher capacity for multiplicative thinking.

Summary of components of the Teacher Capacity model

- **Mathematical knowledge:** Mathematical language, multiplicative ideas, the effectiveness of solution strategies, student correct and incorrect responses, variety of teaching/solution strategies, connection to key topics, knowledge of possible student future challenges with the use of inefficient strategies.
- **Knowledge of students:** Knowledge of where students "should be" supporting individual students, knowing when to move students forward, knowledge of potential challenges and successes, identifying and correcting errors.
- **Curriculum knowledge:** Identifying multiplicative thinking in curriculum, knowing where multiplicative thinking begins and aspects of multiplicative thinking suitable for each year level, knowing where multiplicative thinking knowledge is applicable in curriculum, and knowing how multiplicative thinking is connected to other key topics in mathematics.
- **Design of instruction:** Lesson design, addressing misconceptions, adherence to curriculum standards, variety of resources solution strategies, and choice of teaching.

Research Design

The following research question guided this study: What is the association between teaching experience and teachers' pedagogical competence for supporting students' development of multiplicative thinking in the lower primary (K-4) school years? This study employed an embedded mixed methods approach and used the Zhang & Stephens (2013) model of Teacher Capacity as the framework for teacher questionnaire design and data analysis where Mathematical Knowledge, Knowledge of Students' Thinking, Curriculum Knowledge, and Design of Instruction are all seen as essential components of teachers' PCK for mathematics.

Participants

The participants in this study were 62 primary school preservice and in-service teachers in Australia. For this study, Preservice teacher participants were Master of Teaching (MTeach) students from one institution of higher learning in the State of Victoria. These students already held a first degree in other fields of study at the time they enrolled for the MTeach degree. Table 2 shows the distribution of teacher participants according to teaching experience.

Table 2

Distribution of participants according to teaching experience

Experience	Frequency	Percent	Valid percent	Cumulative percent
Preservice	11	17.7	17.7	17.7
Novice	5	8.1	8.1	25.8
Experienced	13	21.0	21.0	46.8
Expert	33	53.2	53.2	100.0
Total	62	100	100.0	

The study initially aimed at obtaining 25 participants from each group of teachers. These groups based on a classification used by DEECD (2004) are: Graduate, Novice (1-3 years), Experienced (4-7 years), and Expert (> 8 years). However, this was not achieved due to challenges encountered in the data collection phase of this study in a Covid-19 environment. Nevertheless, the performed statistical procedures controlled the possible limitations to study results and conclusions associated with the uneven sample sizes.

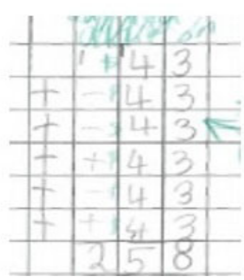
Data collection

A questionnaire comprised of four extended responses items that underwent expert validation and piloting was used to collect both quantitative and qualitative data. A task suitable for Year 3 & 4 students was used and informed the development of questionnaire items that interrogated teachers' capacity for supporting students' transition from additive to multiplicative thinking.

The task was "A new theme park has opened in Geelong, and you are going there for your birthday party. You and five friends are going to the party. Tickets cost $43 each. How much will it cost altogether?" Figure 3.2 is a response from Student C.

Figure 3.2

Solution Strategy Used by Student C.



Student C

This problem was used to develop four questionnaire sub-items addressing the four components of the Model of Teacher Capacity (Zhang & Stephens, 2016). Table 3 is a matrix of questionnaire items (sample of first four items out of seven items) and components of Teacher Capacity model which they address.

Table 3

Questionnaire item versus element(s) of Teacher Capacity Model.

Items	MK	CK	KS	DI
Item 1.1. What Year level is this question suitable for?		✓		
Item 1.2. Can you identify other additive strategies for solving the above problem which are more efficient than used by student C?	✓		✓	✓
Item 1.3. What are the present and future challenges that students are likely to encounter if they continue to rely on additive strategies to solve this problem?	✓		✓	
Item 1.4. How will you support students moving forward from relying on additive strategies to using more efficient (multiplicative) strategies? Please provide as many as possible.	✓			✓

Data analysis

The teachers were assigned pseudonyms (Teacher 1 to Teacher 62). Their responses to the questionnaire were scored using a rubric and were recorded on a spreadsheet ready for analysis. Again, the rubric underwent multiple stages of expert evaluation and validation before its final use in this study. The total possible score for this questionnaire was 22 points spread across the four elements of the Teacher Capacity model. The data were analysed quantitatively using SPSS and also qualitatively providing further understanding of quantitative analysis.

Results and Discussion

Table 4

Estimated Marginal Mean (EMM) scores for the four groups of teachers.

Experience	Mean	Standard error	95% confidence interval Lower bound	Upper bound
Preservice	15.8	1.0	13.7	17.8
Novice	14.4	1.3	11.8	16.9
Experienced	16.1	1.0	14.1	18.2
Expert	17.1	.57	16.0	18.3

Table 4 shows scores for groups of teachers ordered based on the EMMs from the highest to lowest. The Expert teachers scored the highest (17.1), followed by Experienced teachers (16.1), then Preservice (15.8), then Novice teachers (14.4) who scored the lowest. A thematic analysis of the teachers' responses to the questionnaire was conducted based on the quality and depth of responses originating from teachers at different levels of teaching experience and helped make connections to the quantitative data.

In relation to Item 1, teachers from preservice to expert demonstrated high PCK by including skip counting, partitioning, doubling, halving, area model, using known multiplication facts, times tables, multiplying by place value, and grouping in their responses as efficient multiplicative strategies for solving the problem. On the other hand, limited PCK was evident when respondents wrote “multiplication” (Teacher 37, Experienced) and “extending multiplicative strategies” (Teacher 56, Preservice). In relation to Item 1.3, teachers demonstrated high PCK by including responses such as using arrays, teaching times tables, splitting larger numbers, teaching ratios, place value, skip counting, and doubling and halving. However, limited PCK was evident when teachers wrote responses, for example, “showing them tricks” (Teacher 51, Novice) and “using concrete pictorial abstracts” (Teacher 33, Expert). Likewise, for Item 1.4, teachers demonstrated high PCK by including responses such as “place value” (Teacher 37, Experienced), “basic money concepts” (Teacher 8, Expert), “skip counting” (Teacher 55, Preservice), “partitioning” (Teacher 52, Preservice), “arrays” (Teacher 33, Expert), and “doubling and halving” (Teacher 4, Expert). On the other hand, low PCK was evident when teachers wrote responses such as “crossing the decade when adding” (Teacher 44, Experienced), and “self-efficacy around multiplication” (Teacher 51, Novice).

Two exceptional Expert teachers: What did they know and show differently?

The discussion in this section focuses on Item 1.2 on the teacher questionnaire. Two Expert teachers, Teacher 4 and Teacher 8, whose excerpts of responses to the survey items at the transitional teaching stage are presented in Figures 2 and 3, all scored twenty-two points, representing scores of 100%. Teacher 4 was a leading teacher, while Teacher 8 was an assistant principal, and they both worked in different States and territories of Australia.

These two teachers mentioned that additive strategies will not be helpful in the learning and solving of problems involving fractions. They both highlight that those students who rely on additive strategies will encounter challenges working with decimals, for example, 1.33 × 2.41. Teacher 8 went further to mention that with continued reliance on additive strategies, students’ learning of the measurement topic will be impacted. This demonstrates that these two teachers understand the limitation of additive strategies and therefore the need to support students with more efficient multiplicative strategies. They also understand the connection between multiplicative thinking and other topics as impacting students’ success in mathematics.

Figure 2

An Exceptional Expert Teacher (Teacher 4)

1.1. Can you suggest other additive and/or multiplicative strategies for solving the above problem which are more efficient than used by student C?

40x6+3x6 =258 OR doubling 86+86+86=80+80+80+6+6+6=258 or algorithm

1.2. What are the present and future challenges that students are likely to encounter, if they continue to rely on additive strategies to solve this problem?

When the size of number gets bigger, students will experience work of overload. Repeated addition will not work for decimals or fractions such as 1.33 x 2.41.

1.3. How will you support students moving forward from relying on additive strategies to using more efficient(multiplicative)strategies? Please provide as many as possible.

Ensure students can recognise the structure of equal groups, develop an understanding of many to one counting, encourage using the doubling and halving strategies.

1.4. Can you suggest any prior mathematical knowledge and skills that students may need to solve the above problem? List as many as possible.

Understanding of place value, repeated addition, doubling, partitioning, multiplication facts

Figure 3

An Exceptional Expert Teacher (Teacher 8)

1.1. Can you suggest other additive and/or multiplicative strategies for solving the above problem which are more efficient than used by student C?

1. Partitioning to multiply 40 and 3 utilising place value to multiply by 6. 2. Multiplying 43 by 10, halving it and adding 43. 3. Using the area model to multiply 43 x 6 4. Providing models with arrays.

1.2. What are the present and future challenges that students are likely to encounter, if they continue to rely on additive strategies to solve this problem?

In K-6 additive strategies restrict mental computation, fluency, and flexibility. It restricts thinking in looking at fractions as division, decimals, limit abilities in measurement. So many challenges but mostly using inefficient strategies impacts negatively on a student's self-perception as a mathematician.

1.3. How will you support students moving forward from relying on additive strategies to using more efficient(multiplicative)strategies? Please provide as many as possible.

Share models and different representations. Build a strong foundation of multiplicative place value. Let children investigate with concepts like commutativity and inverse operations and show them the how these concepts work. Implement a relational pedagogy and utilise different pedagogies in teaching math's; explicit teaching, investigation, problem solving. Manage the cognitive load of students.

1.4. Can you suggest any prior mathematical knowledge and skills that students may need to solve the above problem? List as many as possible.

Unpacking word problems. Place value, addition but I'd prefer to see multiplication, mathematical language, basic money concepts.

It would possible also to point to high MKT amongst several Preservice teachers in the study.

Conclusion

This study showed that teaching experience tends to support effective teaching of key ideas associated with multiplicative thinking. The complexity of transitioning students from additive to multiplicative thinking and its pedagogical challenges are well emphasised in mathematics education literature (Clark & Kamii, 1996; Malola et al., 2020). While high level responses were demonstrated by Pre-service, Novice and Experienced teachers, Expert teachers tended to demonstrate higher pedagogical competence in supporting students through this process by clearly stating limitations with additive strategies suggesting the need for more efficient multiplicative strategies. Expert teachers also demonstrated understanding of the connection between multiplicative thinking and the learning of key topics in mathematics, a process that is well supported by developing efficient multiplicative strategies in students. Their capacity to support students' transition from additive to multiplicative thinking can be attributed to enhanced professional growth and development opportunities, engagement with peers and curriculum resources, and several opportunities for self-reflection on the lessons taught.

While there is a growing body of research demonstrating a weak association between teaching experience and pedagogical competence, this study has indicated that both MKT and experience are important for effective teaching of foundational mathematics concepts such as multiplicative thinking. This cautions us against discrediting the role of teaching experience. School leaders should consider allocating more experienced teachers to teach mathematics in the early years to effectively support students' development of foundational concepts (e.g., multiplicative thinking). The relatively small sample sizes used in this study point to the need to further research among the four groups of teachers on PCK for developing students' multiplicative thinking, especially in identifying which if any components of the Teacher Capacity model are improved as a result of teaching experience.

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