Finding Common Ground: Perspectives on Spatial Reasoning and Patterning in Mathematics Education

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Patterning and spatial skills are central to mathematics achievement and are precursors to generalising in mathematics. This paper draws together three complementary lines of research on the role of pattern and structure, number sense, and STEM in early learning to consider the interrelations between patterning and spatial skills, and their influence on mathematical achievement. An overview of the 'Pattern and Structure' project, studies on patterning and number sense, and the Early Years STEM Australia (ELSA) project highlight commonalities in approaches and findings. An integrated approach aims to develop an improved understanding on how and why patterning, spatial skills and number sense are linked and are 'transferred' to mathematical achievement.

There is an increasing interest in identifying key skills that underpin early mathematics learning (Resnick et al., 2026). One line of work focuses on the role of patterning and structure, highlighting how the ability to detect, represent and generalise regularities contributes to the development of early algebraic and numerical concepts (Mulligan & Mitchelmore, 2009; Rittle-Johnson et al., 2019). A second, largely parallel line of research has emphasised spatial thinking skills, finding that skills such as mental rotation and perspective taking are robust predictors of mathematics performance from preschool onwards (e.g., Gilligan et al., 2020; Mix, 2019).

Although these bodies of work have developed with different emphases and methodological traditions, more recent research suggests patterning and spatial skills reflect interdependent foundations of mathematical reasoning (Lowrie & Logan, 2023; Mulligan et al., 2020a; Rittle-Johnson et al., 2019). Patterning tasks often require children to utilise spatial processes (e.g., structuring, transformation, and visualisation) with their developing understanding of numerical or algebraic relations. At the same time, spatial skills may support children's capacity to perceive and abstract structure within patterns. Emerging studies suggest that these domains mutually reinforce one another, providing complementary pathways into the development of mathematical generalisation, number sense, and problem solving (Junker et al., 2025; Lowrie & Logan, 2023; Mulligan et al., 2020a; Rittle-Johnson et al., 2019). Indeed, developmental progressions (trajectories) and learning pathways on patterning, have involved spatial skills (Baroody et al., 2021; Mulligan et al., 2020a).

This convergence signals a need for a more integrated theoretical treatment of patterning and spatial skills in mathematics learning. Accordingly, the aim of this paper is to synthesise these complementary lines of research and suggest a more integrated framework of how patterning and spatial skills jointly contribute to early mathematical learning. We provide a summary of our studies focused on the interrelated roles of patterning and spatial skills in: mathematical structure, number sense, and STEM learning.

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The role of patterning in early mathematical development

There is strong evidence that patterning is central to mathematics achievement and is a precursor to generalising in mathematics (e.g., Acosta et al., 2026; Larkin et al., 2024; Mulligan & Mitchelmore, 2009; Zippert et al., 2020). Repeating patterning ability remains a unique predictor of success, even after accounting for a range of demographic factors such as receptive mathematics language, geometry, counting and cardinality, gender, age, ethnicity, basic number skills, or prior mathematics knowledge and spatial skills (Gilligan et al., 2020; Hawes et al., (2022), Rittle-Johnson et al., 2019; Wijns et al., 2019). Repeating patterning skills at the end of PreK have been found to predict mathematics achievement in first and fifth grade (Rittle-Johnson et al., 2013), and other studies have found pattern understanding a predictor of early reading and arithmetic skills (e.g., Burgoyne et al., 2019).

There are large individual differences among four-year olds' repeating pattern knowledge, with 77% of the four-year-olds able to copy a simple pattern, approximately 50% able to extend patterns, 33% capable of translating (abstracting) a repeating pattern, and only 10% being able to identify the 'unit of repeat' (Rittle-Johnson et al., 2019). A large scale, naturalistic Australia-wide study confirms existing laboratory-based research, whilst also indicating that some preschool children were more capable at creating three and four element patterns than previously thought (Larkin et al., 2024). In a suite of studies with four-to-eight year-olds Mulligan and colleagues found that the recognition of the 'unit of repeat' was more advanced than simple repetition, and that the unit of repeat was critical to abstraction and generalisation (mathematical structure), and multiplicative reasoning (e.g., Mulligan & Mitchelmore, 2009; Papic et al., 2011). In a mixed-method study of three-to-seven year-olds Lüken and Sauzet (2021) found a clear developmental progression in patterning skills: simple repetitions as alternate items, to those with a more advanced composite structure of the 'unit of repeat', and to the extension of growing patterns. These findings were consistent with Papic et al. (2011).

Recent studies on patterning have extended to the investigation of growing and spatial patterns (e.g., Wijns et al., 2021) and how these may be related to concepts such as counting, multiplication, number sense, transformations and dimensionality (Junker et al., 2025; Mulligan, 2025; Rittle-Johnson et al., 2019). Lüken (2025) investigated pre-schooler's self-initiated patterns, with most comprising transformations, constructed as both repeating and growing patterns. However, the role of different types and modes of patterning has not been sufficient to establish causal relations between patterning and spatial skills, and numerical competencies (Mulligan, 2025; Resnick & Lowrie, 2023).

The role of spatial reasoning in early mathematical development

The relation between spatial skills and mathematics achievement across development is consistent, predictive, and strengthens over time (Resnick et al., 2019). Higher levels of spatial skills are correlated with higher performance in mathematics (Mix et al., 2019). This relation begins in preschool (Verdine et al., 2014) and is increasingly associated with mathematics achievement over the intermediatory through adult years (Hawes et al., 2022). For example, spatial skills at age three accounts for 27% of the variance in mathematics performance at age four, even after controlling for SES, gender, fine-motor skills, vocabulary, and executive function (Verdine et al., 2017).

Preschool children demonstrate a broad range of spatial skills, even before formal schooling (for review, see Davis & Spatial Reasoning Study Group, 2015). By age three, many children can mentally represent and manipulate simple spatial configurations, identify shapes, navigate familiar environments using landmarks, and complete basic spatial visualisation tasks such as assembling puzzles or block constructions. Across the preschool years, spatial thinking becomes more flexible and efficient, which reflects both increasing experience with spatially-rich activities and a growing capacity for relational reasoning.

Patterning and spatial skills in developing mathematical structure

Awareness of Mathematical Pattern and Structure (AMPS) has been observed, described, and measured as a general construct that underpins structural thinking and its ties to emergent generalisation in early years' mathematics (Mulligan & Mitchelmore, 2009). AMPS comprises two interdependent components: one cognitive, a knowledge of structure, and one meta-cognitive, a tendency to seek and analyse patterns.

Over the past decades the Pattern and Structure Project conducted a series of related studies on AMPS with four-to-six year-olds (including task-based interviews, Rasch-modelled scales of AMPS, longitudinal interventions, and developmental design studies). A Pattern and Structure assessment interview (PASA) and program (PASMAP) was developed and evaluated, comprising an interrelated network of mathematical concepts and relationships, implemented through a pedagogical framework: modelling, visualising, representing, generalising and sustaining.

Students demonstrating highly-advanced AMPS spontaneously searched for mathematical patterns, noticed and represented common structural features, and formed emergent generalisations. In contrast, children showing pre-structural AMPS used idiosyncratic ideas and did not recognise structural features. The majority of children were able to spontaneously construct simple repeating patterns, while others applied a 'unit of repeat' and constructed growing patterns. 'Spatial structuring' and visualisation skills were found integral to AMPS and related mathematical knowledge. For example, structuring 'equal groups' required spatial skills such as co-linearity of arrays and grids in two and three dimensions, and transformation and orientation skills were used in extending patterns systematically.

Secondary qualitative analyses examined how different pattern types, including growing patterns, accounted for wide variation in children's AMPS, illustrating how spatial and patterning skills are interrelated (Mulligan, 2025). These analyses described how multiplicative thinking, transformation, navigation (directionality) and perspective-taking skills influenced the child's perception of both repetitions and growing patterns, and how the use of dynamic spatial visualisation determined whether the structure of a pattern could be noticed and generalised.

A Spatial Reasoning Mathematics Program (SRMP) extended the Pattern and Structure studies, developed through a 2-year intervention, and a two-phase design study in Grades 3 and 4 (Mulligan et al., 2020b). The SRMP involved complex repetitive and growing patterns including spirals; area/perimeter/shape relations; directionality, and perspective taking. In summary, the intervention study found significant gains by the experimental group on two interview-based assessments (comprising rotation, reflection, equipartitioning, directionality, perspective-taking tasks) and marked growth in patterning and spatial structuring skills, for example, visualising and structuring arrays and grids and extending rotational growing patterns. The SRMP also enabled the integrated structural development of related numerical and measurement concepts. Further studies aim to describe how and why the SRMP concepts are inextricably linked to improved number sense, mathematical achievement, and problem solving.

Patterning and spatial skills in number sense learning

Although there has been an unprecedented number of studies on early patterning in recent decades (e.g., Björklund & Pramling, 2014; Mulligan, 2025; Pitta-Pantazi et al., 2025; Wijns et al., 2019), there remains limited understanding of how patterning is associated with specific aspects of number sense, particularly for students with high or low mathematical ability. Junker (2024) theorised that AMPS is essential for the development of early number sense and formulated a new study to investigate the interrelations between pattern and number sense.

A cross-sectional mixed-method doctoral study of 75 Norwegian 6-year-olds analysed counting, repeating and sequencing patterning, number-sense performance, and strategy use through a digital assessment, and task-based interviews (Junker et al., 2025). Fifty-two embedded case studies differentiated high (HNS) and low number sense (LNS) groups. A digital pre-assessment of number sense included 69 items: eight interdependent skills drawn from Andrews and Sayers' (2015) Foundational Number Sense framework: *Numeral Identification, Systematic Counting, Number* and *Quantity, Estimation, Quantity Discrimination, Arithmetic Competence, Number Patterns*, and *Subitising*. Number patterns included *Sequencing patterning* focused on systematic counting tasks and repeated patterning involving duplication, extension, transfer, and unit isolation.

For students with LNS, success in extending repeating patterns was linked to quantity discrimination, while success in sequencing N±1 and N−2 patterns was linked to number and quantity and additive concepts—areas in which these students struggled. For HNS students, success in extending repeating patterns was associated with arithmetic, and success in sequencing N ± 1 patterns was associated with estimation. In-depth multimodal analysis of the LNS and HNS students' strategies found a statistically significant group difference for extending repeating patterns. Both groups used recursive strategies but only students with high number sense used relational strategies. LNS students used more variations of recursive patterning strategies, such as omitting, adding or mirroring the order of pattern elements. In contrast to HNS students, LNS students attended to superficial pattern features, which likely hindered identification of a pattern's underlying structure. For example, LNS students constructed patterns based on colour and shape rather than attending to the underlying structure that the colours and the shapes represented. By contrast, those with a high number sense exhibited (de)composition, a strategy indicative of relational understanding. Notably, repeating and sequencing patterning activities requiring unit identification significantly distinguished the student groups and predicted success.

Follow-up case studies of HNS and LNS students were selected on the basis of assessment data and their use of finger and gestural strategies. The study primarily aimed to video students' spontaneous hand and finger movements and analyse these in relation to number sense. *Systematic Counting* utilised traditional tasks: verbal counting, forward/backward sequences 0–20, counting on/back from 7 and 14, and counting by twos from 0, 10, and 20. Manipulatives were available, though rarely used. For the LNS case, counting occurred as rote verbal sequences with no evidence of decomposing quantities or their relations, unlike the HNS case. Video analysis revealed two themes: (1) decomposition of individual quantities, which may or may not indicate structural awareness, and (2) more advanced decomposition of whole or partial counting sequences into unequal or equal parts (composite units), where unequal parts may be associative and equal parts multiplicative (e.g., Mulligan & Mitchelmore, 2009).

The findings demonstrated that LNS students can demonstrate unitary and skip counting, cardinality, ordering, number-line skills, and basic facts to 10 without displaying structural awareness, or using decomposition or additive structure. The study has important implications for pedagogy and curricula that should emphasise the relationships between embodied counting and structural awareness. Another key insight was the students' use of dynamic embodied spatial skills, including use of finger patterning, shape, symmetry, and directional finger movements to track the count and visualise decomposed groups.

Patterning and spatial reasoning in early STEM learning

Patterning and spatial skills are critical in STEM (science, technology, engineering, and mathematics) learning (Resnick & Lowrie, 2023). Thus, the Early Learning STEM Australia (ELSA) programs emphasise patterning and spatial skill development as core components of the programs, which span preschool to Year 2. For example, the ELSA F-2 program includes topics such as *sorting objects based on spatial and numerical attributes; aligning numbers to spatial patterns; giving and following directions using landmarks* (Foundation); *measuring and estimating distance with informal units*; *decoding representations using maps; sorting and ordering visual data* (Year 1); *graphing using and X- and Y- axis*; *Fraction Families – Wholes, halves, quarters and eights,* and *transformations and rotations* (Year 2). Notably, this program begins by developing the requisite spatial skills, then spatial patterning, building towards numerical patterning understanding.

A randomised control trial (RCT) of the ELSA preschool program (Resnick & Lowrie, 2023), found that engagement with a play-based spatial program (which included playing patterning and spatial games) led to better overall spatial reasoning and transferred to better numeracy understanding for students compared with a business-as-usual control. The results underscore the importance of embedding spatial learning within strong pedagogy and authentic learning contexts, supported by the use of engaging digital games that can be played independently by the students with their learning automatically reported back to teachers, via a digital dashboard, at individual and cohort levels.

A study involving a nationally representative sample of 697 preschool children and 1011 kindergarten children (n=1708) found that patterning and spatial skills are separate but correlated, with each providing unique variance in predicting numeracy (Resnick et al., under review). Specific pathways remained stable over the transition from play-based (preschool) to more formal (kindergarten) learning environments, highlighting that the relations between patterning, spatial skills, and numeracy appear consistent. The interdependence of spatial skills and patterning is also evident in work adopting a strength-based perspective on children's spatial competencies. Harris et al. (2025) demonstrated that preschool children's spatial language and patterning skills are strongly related and that their expression is largely context dependent. In game-based assessment environments, children from diverse socio-economic and geographic backgrounds demonstrated robust spatial language and patterning skills that were not always apparent in formal test-based measures. This finding reinforces the idea that spatial patterning and spatial language develop together through engagement with rich, embodied, and contextualised experiences—experiences that are central to authentic STEM learning.

Further research directions and implications for pedagogy and curricula

Taken together these studies have not yet established *how* and *why* patterning, spatial skills, and number sense are interrelated, or which particular spatial skills are critical to improved and broader mathematics achievement. Hence our approach is to adopt a complementary perspective to leverage existing research and find 'common ground' in order to collaboratively develop a more coherent body of research on the role of spatial reasoning in mathematics learning. Based on our joint research findings, some development and integration of our theoretical and pedagogical frameworks may be proposed. Further research may design and implement interdisciplinary, 'mixed methods' studies, some at scale, that focus on more complex multi-dimensional patterning and spatial skills (beyond simple repetitions and rotation). This approach aims to find strong associations between patterning spatial reasoning, and number sense and advance understanding of the mechanisms of 'transfer' to mathematics achievement.

A collaborative, complementary approach may provide more convincing and cohesive evidence for impactful reform in mathematics pedagogy and curricula that currently limits the place of pattern and structural development, and spatial reasoning as core aspects of number sense. Our broad aim is to positively impact teacher practice: pedagogy, curricula, and assessment, that articulates and prioritises spatial reasoning, and embeds spatially-oriented programs within learning. More conceptually-advanced and explicit descriptors of spatial/mathematical interrelations are needed for us to achieve this aim.

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