Qualitative Changes in Primary Pupils' Geometric Reasoning Through Engaging with Diagrams in Generative Mode of Interaction

Shogo Obayashi
Saga University
obayashi@cc.saga-u.ac.jp

Taro Fujita
Exeter University
T.Fujita@exeter.ac.uk

Hidenori Kobayashi
Tennoji Primary School
kobahide40@gmail.com

It has been suggested that the difficulties pupils encounter with proofs at the secondary school level stem from a discrepancy between their primary and secondary school learning experiences. In particular, they lack experience in explaining their reasoning at the primary level (Stylianides & Stylianides, 2017). This study examines what kinds of interactions between pupils and diagrams in geometry lessons might contribute to qualitative changes in their geometric reasoning. To theorise such interactions, we use Herbst's (2004) generative mode of interaction. This involves the actor (e.g. a learner) applying hypothetical operations to a diagram as an object and reinterpreting it as a semiotic resource, thereby discovering new meanings or mathematical structures within it and contributing to changes in the learner's geometric reasoning. To elicit this generative mode of interaction, we designed and implemented a lesson based on Wittmann's (2021) operative principle. The research question was: In primary school geometry lessons based on the operative principle, how does the generative mode of interaction emerge, and what effect does it have on pupils' geometric reasoning? We conducted a classroom-based study with two lessons in which Year 5 Japanese pupils (aged 10) were asked to explain why the areas of triangles created using GeoGebra were always equal. Following the operative principle, the pupils were allowed to interact with diagrams in both digital and non-digital formats. These lessons were recorded. We also collected the pupils' written responses and reasoning from their worksheets. In these lessons, pupils manipulated diagrams via GeoGebra and used paper to explain the equality of triangle areas based on their findings. Through operations such as adding auxiliary lines, the pupils transformed the diagram in various ways and discovered the underlying geometric properties. Because the pupils actively engaged with the geometric objects and generated new ways of seeing the figures, they were interacting with the diagrams in a generative mode. Furthermore, video analysis showed that task design and teacher questioning based on the operative principle worked in complementary ways, leading pupils to reinterpret shapes from mere visual objects as semiotic resources for reasoning. Observing the evolution of the pupils' explanations revealed a process in which their descriptions developed from intuitive visual imagery to logical explanations grounded in the constituent elements and geometric properties of the shapes. These findings suggest that constructing a learning environment that fosters generative interactions is an effective means for primary school pupils to develop geometrical reasoning based on geometric properties.

References

Herbst, P. (2004). Interaction with diagrams and the making of reasoned conjectures in geometry. *Zentralblatt für Didaktik der Mathematik*, *36*(5), 129–139. https://doi.org/10.1007/BF02655665

Stylianides, G. J., & Stylianides, A. J. (2017). Research-based interventions in the area of proof: The past, the present, and the future. *Educational Studies in Mathematics, 96*(2), 119-127.

Wittmann, E. Ch. (2021). *Connecting mathematics and mathematics education: Collected papers on mathematics education as a design science*. Springer. https://doi.org/10.1007/978-3-030-61570-3

(2026). In N. Berger, M. Thompson, J. Ley & C. Wilsmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney* (p. 505). MERGA.