Seeing the Invisible: Exploring the Use of Spatial Reasoning in Mathematics Problem-Solving Beyond Geometry at the Secondary School Level

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Spatial reasoning has been widely studied and strongly associated with students' mathematics achievement, particularly in geometry and early grades. This study aimed to explore the use of spatial reasoning in solving mathematics problems beyond geometry. An algebra test was administered to six eighth-grade students from three different schools in Depok, Indonesia, followed by semi-structured interviews. The findings showed that the students use spatial reasoning to understand the problem context, model the problem in representations, and plan problem-solving strategies. Spatial reasoning supported problem comprehension and strategic planning, but procedural skills remained crucial for accurate problem solving.

Spatial reasoning refers to the ability to use spatial skills, including understanding, representing, and manipulating spatial properties of objects and spatial relations (Lowrie et al., 2019; Bruce et al, 2017). In this study, spatial reasoning is evaluated through three constructs: spatial visualization, mental rotation, and spatial orientation (Ramful et al., 2017). This recent study focuses on the use of spatial reasoning in mathematical problem-solving beyond geometry, referring to tasks in the algebra domain. Spatial reasoning is a specific cognitive ability that also strongly appears in the area of mathematics education. The development emerged as most research showed that mathematics performance and spatial reasoning are highly correlated. (Mulligan & Woolcott, 2015). Moreover, numerous studies have shown that spatial reasoning interventions significantly impact students' mathematics performance, as shown in the meta-analysis study conducted by Uttal et al. (2013).

Several studies indicated that spatial reasoning has a robust and consistent relationship with solving geometric problems, evidenced by the high correlation between spatial ability and geometry problem-solving performances (Buckley et al., 2019) and the effect of spatial training on students' geometry and measurement performances (Adams et al., 2023). For instance, students who have good spatial abilities tend to be able to demonstrate solving open geometric problems (Wang & Chiew, 2010).

Since the relationship between spatial reasoning and geometry is well established, it is necessary to investigate its relationship with domains beyond geometry. The domains include numbers, algebra, data, and uncertainty, which are studied at the secondary level based on the Indonesian mathematics curriculum (Ministry of Education and Culture, 2018). The studies about the relationship between spatial reasoning and mathematics are dominated by the geometry domain, but the investigation of the relationship between spatial reasoning and domains beyond geometry is still under research. In line with Newcombe et al. (2019), who stated that several algebraic materials may be related to 3D spatial concepts, such as function graphs, but the relationship between them has not been tested.

In addition, several studies related to the relationship between spatial reasoning and problem-solving beyond geometry were mostly conducted at the primary and early school level. For example, spatial visualization and mental rotation ability can predict numeracy skills in preschool to elementary school children (Gunderson et al., 2012). Research has also found that mental rotation training can improve arithmetic ability, especially in math problems involving missing numbers (Cheng & Mix, 2014). In addition, spatial orientation also correlates positively with students' abilities on number lines in elementary school (LeFevre et al., 2013). The main

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focus of this study is to explore in what ways students use spatial reasoning in solving mathematics problems beyond geometry at the secondary mathematics level.

Spatial Reasoning and Its Constructs

Spatial reasoning is a combination of several different skills divided into several constructs (Hawes & Ansari, 2020). Although spatial reasoning is a complex collection of skills, the relationships within it are not clear (Lowrie & Logan, 2018). Despite the recent end, there have been attempts to classify spatial skills into different dimensions, such as static and dynamic or intrinsic and extrinsic. However, this classification is not sufficient to accommodate the broad range of applications of spatial skills (Mix et al., 2018). Furthermore, this study employs three spatial reasoning constructs, including spatial visualization, mental rotation, and spatial orientation, as used in a study by Ramful et al. (2017) at the middle level. Spatial visualization refers to the ability or skill to mentally manipulate or transform an object's spatial properties (Lowrie et al., 2017). Meanwhile, spatial orientation requires an egocentric transformation of imagining a change in one's own perspective (Hegarty & Waller, 2004). Then, mental rotation is a specific type of object-based transformation, often distinguished from measures of spatial visualization in factor analytic studies (Mix & Cheng, 2012).

The Relationship between Spatial Reasoning and Mathematics Problem-Solving Beyond Geometry

In the case of secondary level, the relationship between spatial reasoning and mathematics varies and is not consistent across mathematics domains. Research by Bremigan, (2005) investigated the association between students' spatial ability performance and advanced mathematics. The results showed that spatial ability was significantly correlated with performance in advanced mathematics. However, the latest research by Adams et al. (2023) stated that spatial training can support achievement in certain contexts, such as measurement and geometry, but not in numbers and algebra.

Spatial reasoning plays a significant role in mathematics problem-solving beyond geometry. Although these domains are not directly related to the concept of space, as in geometry. However, it allows individuals to visualize and connect elements in an imaginary space. For example, algebra materials often involve lines and curves in their concepts (Newcombe et al., 2019).

The mechanism of how spatial reasoning is used in problem-solving processes beyond geometry has been widely investigated at the early levels, and it is still unclear to what extent it is used at the secondary school level. A study by Empson and Turner (2006) stated that multiplicative thinking requires an understanding of how parts relate to the whole to recognize patterns at an early level. In addition, some component skills in algebra (and likely other facets of algebra that are yet untested), 3-D spatial visualization may mediate the relation between working memory and mathematics achievement (Tolar et al., 2009). In this study, the role of spatial reasoning in the process of solving mathematics problems beyond geometry at the secondary mathematics level still needs to be explored further.

Methodology

The qualitative study uses a case study design to elaborate on how and to what extent spatial reasoning is used in solving mathematics problems beyond geometry (Creswell, 2015). This design emphasizes an in-depth analysis of the case within the context of the use of spatial reasoning by students in mathematics problem solving. The participants were six Grade 8th students from three secondary schools in Depok, Indonesia. After completing the 30-item Spatial Reasoning Instrument (SRI) developed by Ramful et al. (2017), the students were classified into several categories: two students were categorized at the high level (≥75% correct answers), two students were at the intermediate level (50%–74% correct answers), and two students were classified at the low-intermediate level (25%–49% correct answers). The selection of these levels ensured that the students had sufficient spatial reasoning performance to be used in solving problems beyond geometry. In addition, the variation in spatial reasoning levels and school districts was intended to capture diverse perspectives on students' problem-solving approaches.

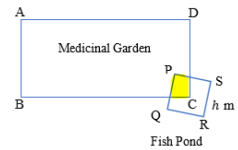
In developing the instrument, this study identified two aspects that serve as the main criteria for determining mathematics problems that elicit spatial reasoning, as they reflect the cognitive demands involved in solving problems. First, the problems require students to construct mental representations to encourage spatial reasoning before performing calculations (Marikyan, 2023) and support in modelling the structure of the problem accurately and transforming objects within the problem (Sorby et al., 2022). Second, the problems are non-routine or contextual, requiring students to mentally interpret relationships among information. In this process, spatial reasoning becomes important for working with number lines, graphs, and internal visual models (Harris et al., 2021).

This study selected algebra problems because that domain is newly introduced at the secondary level, making it a relevant domain for representing problem solving beyond geometry in secondary mathematics. After the tasks were designed, the instrument was validated by two expert validators. The revision process focused on refining the spatial reasoning components and the structure of the problem-solving tasks in the instrument. Therefore, the instrument developed in this study is presented in Table 1.

Table 1

Algebra Problem-Solving Administered to 8th Grade Students

Description	Problems
Narrative	**Family Medicinal Garden** Mr. Dimas is a retired doctor who spends his days tending a garden planted with various medicinal plants such as ginger, turmeric, and curcuma. He has a fairly large, rectangular family medicinal garden behind his house.  Sources: id.pinterest.com Initially, Mr. Dimas only knew that the length of his garden was 6 meters longer than its width (as shown in the picture) because he hadn't measured it directly. Over time, as Mr. Dimas's garden yields increased, he wanted to expand his garden by adding 2 meters to each of its length and width to grow more plants and help more people in need. Therefore, Mr. Dimas wanted to increase the size of his garden while maintaining its shape and planned to make several modifications to it.
Part a (Focus: SV)	Mr. Dimas tries to make a new sketch of his garden's dimensions. He knows that the perimeter of the new garden is 60 m, measured using a measuring tape that only shows the final result. Determine the area of Mr. Dimas's garden with the new dimensions!
Part b (Focus: SO)	Then, Mr. Dimas plans to create a T-shaped path (from his perspective), with the front on one of the wide sides of the garden, as follows: • One path runs vertically from the front to the back of the garden, right in the middle. • Another path runs horizontally, 1 m wide, behind the garden, along the wide side. If the area of the walkway is 20.5 m^2, determine the width of the vertical pathway!
Part c (Focus: MR)	Later, Mr. Dimas's medicinal garden experienced a drought due to the medicinal plants being sensitive to changes in humidity. Therefore, Mr. Dimas planned to build a rectangular fish pond to help maintain soil moisture in the medicinal garden. Mr. Dimas sketched a rectangle to represent the medicinal garden, as shown in the following figure.

	The pool is designed to extend slightly into the garden area, while most of it remains outside the garden. It is known that each vertex of the pool is at the same distance from the corner point on the edge of the garden (point C). To achieve this design, Mr. Dimas directly measured the area of the fish pond that lies within the garden, obtaining 16 m² (as shown in the yellow shaded region). Determine the length of the pond's side (h)!	

Note: SV = Spatial Visualization; MR = Mental Rotation; SO = Spatial Orientation

Findings

This section presents the findings on the extent to which six eighth-grade secondary school students used spatial reasoning in solving mathematics problems beyond geometry, as illustrated in Table 2. The findings show that spatial reasoning (spatial visualization, spatial orientation, and mental rotation) helped students understand the context, model the problem, and plan solution strategies; however, the accuracy of their answers was still strongly determined by the accuracy of algebraic modelling and procedural skills.

Table 2

Findings on Students' Use of Spatial Reasoning in Algebra Problem Solving

SR constructs (function to PS)	Level of SR	The example of excerpts	Outcomes of PS (S)	Description of PS
SV (Mentally modelling the problem).	Developed	"The shape of the garden? It's a rectangle, but its length and width aren't the same." (S1)	Correct (S1, S3)	Found the correct area of the garden using an algebraic equation.
	Developed	"In my opinion, it's still a rectangle but a bit larger in size." (S5)	Incorrect (S2, S4, S5, S6)	Made an error in the algebraic modelling.
SO (Understanding the problem context)	Developed	"So, I imagined that the front sides of the pathway were on the width side. I drew the square so that the width side would be in the front." (S1)	Correct (S1)	Determined the width of the pathway by using algebraic modelling and applying algebraic operations.
	Developed	"For question b, it's like I positioned myself in the middle as if I were standing on the path... the horizontal part would be behind me." (S3)	Incorrect (S2, S3, S4, S5, S6)	Made an error in algebraic modelling and in applying algebraic procedures.
MR (Planning a problem-solving strategy)	Developed	"If the distance from the corner point to point C is the same, then it represents the diagonal of a square. Therefore, the square was rotated to make it perpendicular to the garden. Consequently, the shaded region is a square because its area is 16 m². Thus, the side length of the small square is 4 m" (S1)	Correct (S1, S2)	Performed a transformation to determine the targeted construction, which supports planning the strategy.
	Not Developed	"Not really, if I were to rotate it" (didn't think that far)" (S4)	Incorrect (S3, S4, S5, S6)	Not identifying the appropriate construction led to an incorrect problem-solving strategy.

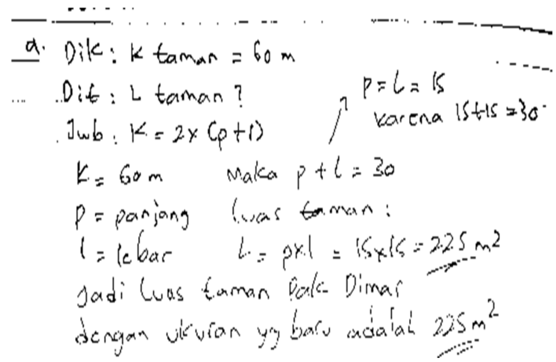
Note: SR=Spatial Reasoning; SV = Spatial Visualization; MR = Mental Rotation; SO = Spatial Orientation; PS = Problem-Solving; S = Student

In solving Problem a (spatial visualization), these excerpts indicated that the students perceived changes in the size of a shape as transformations that do not alter its initial form. The students stated that although the object undergoes widening or enlargement, its geometric characteristics are preserved. This suggests that the students effectively used spatial visualization to understand that the garden's dimensions would change while the form of the shape remained the same. In this case, spatial visualization supported the student's initial understanding and modelling of the problem situation.

However, there was a misunderstanding by the students in modelling this problem to algebraic variables, which led to errors in applying the algebraic procedure. As illustrated in one of the students' answers (see Figure 1), the student was able to recognize the relationship between the new length and width of the garden and the perimeter procedure. Figure 1 shows that the student began by writing the perimeter formula $K = 2(p + l)$ and correctly substituting the value $K = 60$. Nevertheless, the student did not fully understand the problem, assuming that the length and width had the same value, *"p + l = 30, therefore p = l = 15"*. This response suggests that although the student was able to visualize the change in the rectangle's dimensions, an accurate mathematical model was not constructed, resulting in incorrect length and width values and an inaccurate solution.

Figure 1

S5's Answer: Performing Algebra Equations in Problem a (the box on the right provides the English translation of the student's answer)

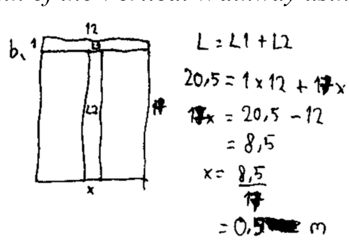


Translation:
Given: perimeter of the garden = 60 m
Asked: area of the garden?
Answer: $K = 2x(p + l)$
$K = 60$m
p =length
l =width
$p = l = 15$
because $15 + 15 = 30$
so $p + l = 30$
Area of the garden:
$A = p \times l = 15 \times 15 = 225$ m^2
Thus, the area of Mr. Dimas's garden with the new dimensions is 225 m^2

In solving Problem b (spatial orientation), these excerpts indicated that the student actively used spatial orientation to determine an object's position before representing it in the form of a drawing. By imagining the front side of the pathway as being on the width side, the student engaged in perspective taking to adjust the orientation of the shape to match the chosen viewpoint. This response shows that the student attempted to mentally reposition the pathways in order to understand their arrangement within the garden. This statement showed that the student was able to accurately recognize the positions of the pathways within the garden to understand the context of the problem.

Figure 2

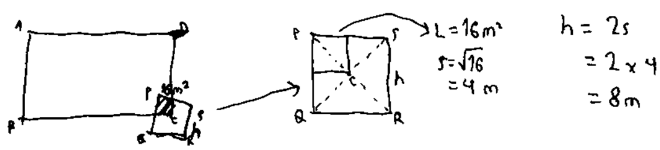
S1's Answer: Determining the Width of the Vertical Walkway using Algebraic Equation in Problem b



From this understanding, the students were able to visualize the shape of the pathway clearly. Some students even created drawings to help them remember it more easily and to identify the spatial relationships among the objects in the problem, as illustrated by Figure 2, realized that each pathway was also rectangular in shape. Consequently, they used the formula for the area of a rectangle to calculate the areas of the two pathways. They also recognized that there was an overlap between the two paths, so they proceeded to use the total area formula: $L=L_1+L_2$, so $20.5=1\times 12+17x$. After forming these equations, the student applied the correct algebraic calculations to obtain the value $x = 0.5$ m. Finally, in solving Problem c (mental rotation), the excerpts indicate that students understood rotation as a spatial transformation affecting orientation but not shape or size, enabling them to interpret the problem and plan an appropriate strategy to determine the intended geometric shape.

Figure 3

S1's Answer: Determining the Side Lengths of the Fish Pond in Problem c



The students with high problem-solving ability were able to think of the appropriate procedure to find the area of the shaded square. The students were able to solve the problem correctly because they used the appropriate strategy, namely applying the formula for the area of a square, as shown in Figure 3. The students easily used the square area formula (area of the fish pond), $L = 64\ m^2$, and therefore determined that the shaded area was $L = 16\ m^2$, which is one-fourth of the total area of the fish pond.

Discussion and Conclusion

The findings of this study suggest that spatial reasoning may assist the students to understand the problem situation by transforming verbal and symbolic information into relational structures that can be visualized and mentally manipulated. This is consistent with Lowrie and Logan (2018) who stated that to solve and understand mathematical problems, students must extract information from the problem and transform it into a form of representation referred to as information encoding. Furthermore, Zhu et al. (2024) added that when working on open-ended problems, students use spatial skills to process spatial information and employ analogies to understand spatial relationships.

In addition, utilizing spatial reasoning appears to support the students in constructing mental models of the problem situation to represent relationships among objects, positions, shapes, and orientations, so that the problem context is no longer understood as a collection of separate pieces of information, but as a meaningful whole (Duffy et al., 2024). This process enables students to organize complex information, identify relevant relationships, and bridge contextual situations with appropriate mathematical representations, thereby making their understanding of what is happening in the problem clearer and allowing important information to be comprehended effectively (Choi & Kim, 2021).

Once this spatial structure is established, students can identify relevant quantities (such as length, area, angle, or volume) and the relationships among them, and then represent these relationships in appropriate mathematical symbols, variables, and expressions (Maher & He, 2022). Thus, mathematical models may emerge as abstractions of spatial relationships that students have already understood making the translation from context to symbolic form more meaningful. This aligns with Hawes and Ansari (2020), who stated that spatial models help translate problems into symbolic form. Before students write equations, functions, or mathematical symbols, they first structure the problem visually, representing its components.

Other findings also suggest that spatial reasoning may help the students identify strategies for solving mathematics problems beyond geometry. Through mental transformations, the students may explore and manipulate mental models of the problem situation while performing preliminary reasoning before engaging in formal calculations (Lowrie et al., 2016). Spatial reasoning may allow students to anticipate possible subsequent solution steps and to foresee the impact of each step on the overall structure of the problem. This ability supports students in selecting and integrating visual and mathematical strategies, thereby making problem-solving more effective.

In conclusion, the findings suggest that students may use spatial reasoning to support three problem-solving processes: understanding the problem context, modelling the problem, and planning a solution strategy. Furthermore, students who were able to develop spatial reasoning tended to understand the problem, plan strategies, or both, more effectively. However, this does not necessarily guarantee correct mathematical solutions, because other cognitive abilities may also play an important role during the procedural stage. Lastly, these interpretations should consider that this study is exploratory and involved a small number of students; therefore, the findings should be viewed as preliminary insights highlighting the potential role of spatial reasoning in solving mathematical problems beyond geometry. Further studies need to integrate spatial reasoning with students' mathematical procedural skills in interventions to explore new approaches to mathematics problem solving beyond geometry.

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