Mathematics Education as Mathematical and Educational

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We bring two schools of research thought into a conversation. One engages with mathematics teaching, learning, and instructional design built on Freudenthal's views of *mathematics* as a human activity, where learning emerges in participation. The other is Biesta's philosophy-based perspective, where *education* is existential work in which students encounter the world and come to terms with it. We clarify when activities in classrooms are (or are not) *mathematical*, when they are (or are not) *educational*, and show that these pursuits align. This opens possibilities for clarifying what is involved in designing school mathematics that is both mathematical and educational.

Engaging with the work of Gert Biesta (2022) introduces one to arguments and language for considering education from a perspective where a person's place in, and their relationship to, the world are of key concern. Biesta's ideas resonate strongly with the well-established design research tradition in mathematics education (Cobb et al., 2008; Freudenthal, 1973) that have guided our research journey (e.g., Cortina et al., 2014; Visnovska et al., 2025). We introduce and discuss some of those resonances and elaborate on their significance to the mathematics education research community in grounding decisions about what teaching and learning mathematics should (and should not) entail, and why the difference matters. We illustrate how thinking with Biesta provides orientation for making instructional design decisions and how it can inform thinking about school mathematics at the curricular scale.

Drawing on Biesta's work broadly and on the *World-Centred Education* book specifically, we first introduce the key notions. We attend to how educational work is existential; outline the three domains of purpose in education; clarify the distinction of the student as being object or subject of their education. Our aim is to foreground educational considerations for mathematics education and clarify both the mathematical and educational value of mathematics in school.

Learning from World-Centred Education

Biesta's work is grounded in the position that the work of education is existential, rather than conceptual. For researchers in mathematics education—a field that is, and has been, so prominently about conceptual change and understanding—this perspective on education might be far from obvious. Biesta (2017) maintains that education should not merely be a means for equipping students for their understanding and making sense of the world, but it must be a place for encountering the world and coming to terms with it.

> The real educational work … is about … bringing children and young people *into dialogue* with the world. It is about turning them towards the world and about arousing their desire for wanting to be *in* the world and *with* the world, and not just with themselves. (p.37, italics in the original)

Biesta draws on Meirieu as he elaborates that education is about exploring what it might mean to exist in the world in a grown-up way, that is, "in the world without occupying the centre of the world" (Meirieu, 2007, p. 96). As Biesta elaborates in *World Centred Education*,

> educational questions are fundamentally existential questions, that is, questions about how we try to exist as human beings, how we try to live our life in and with a world that is not of our making and that is under no obligation to give us what we want from it or expect from it. (2022, p. 9)

In other words, as the world is not of our making and does not play by our rules, it is often confronting to encounter and to understand it. And it then matters what we choose to do with what we came to understand, and that we come to own the impacts our choices have on the world and on those around us.

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In mathematics education, Hans Freudenthal (1973) has imagined school as a place where students can encounter mathematics, come to terms with it, and be brought into dialogue with it. He viewed mathematics as a human activity and wished that the students encounter mathematics as such—as an activity of organising the world aimed at being in the world and acting within it. Freudenthal sought and proposed ways for supporting students to progressively become capable in organising the world—and subsequently mathematics—mathematically. We are all aware that mathematics created through such organising is not docile. It is governed by internal necessity and is self-constraining. Once an organised system of relations is established, it "talks back" to its organiser, revealing inconsistencies, and prompting the organiser to reconsider what has been pondered, concluded, or built so far. The *world of mathematics* can be confronting too. And while mathematics education might choose to focus on the conceptual work of how students are gaining mathematical understandings in school, it cannot escape the bind of how much it matters what those students choose to do with their understandings, starting with what they choose to do in the classroom, and how their choices impact those around them.

As mathematics educators, we aim to turn students towards mathematics and arouse their desire for wanting to be in and with mathematics—even though mathematics is under no obligation to give the students what they want or what they might, initially, expect from it. Mathematics education, too, faces questions that are existential. Biesta's view illuminates the dangers of staying purely in the conceptual when we organise mathematics education. It also indicates how much more can be accomplished in conceptual development when the existential work of education is taken seriously in mathematics classrooms.

Purposes of Mathematics Education

Framing educational work as existential, Biesta (2022) illustrates that education always serves three parallel functions, suggesting three domains of educational purpose, namely, qualification, socialisation, and subjectification. He argues that each of these domains is shaped through schooling, whether this happens by design or incidentally. As we introduce the three notions, we contextualise them for mathematics education, linking them to relevant research.

Qualification

The first of the domains, the educational pursuit of mathematical *qualification*, is the most familiar. It is observable in how curricula list the content to be learned, understandings to be derived, and skills to be developed. Most clearly perhaps, pursuit of qualification is observable in the assessment instruments and practices used to determine who learned mathematics, or how well did they learn it. This purpose of education is rarely challenged, as it aligns with the present broad focus on learning (Biesta, 2015), predominantly viewed as acquisition of knowledge, skills, or proficiencies (Sfard & Cobb, 2014). Qualification purposes determine how mathematics is organised for teaching, and how it is approached in schools today.

One of the consequences of singling out the qualification purposes as the focus of education is the reinforcement of framing education as a cognitive commodity. The students should be personally motivated to acquire such commodity (of facts, skills, and ideas) because those who succeed in its acquisition gain a competitive advantage over those who do not (Sfard, 1998). Under these assumptions, organising the mathematics curriculum as a list of facts, skills, and ideas that need to be handed over to conceivably willing and eager recipients sounds like a straightforward job. The years of earnest efforts at doing so show that many recipients do not tend to be quite so willing, and that even some of the willing ones tend to face considerable difficulties in acquiring what is handed over to them. We return to qualification as the source of these difficulties when we discuss how this purpose interacts with the remaining domains.

Socialisation

Whether by design or not, education inevitably plays a role in "providing the new generation with orientation into [some of] the traditions, cultures, and practices of past and present" (Biesta, 2022, p. 7-8). Biesta considers this role as constitutive of the purposes of the domain of *socialisation*. Which practices students are enculturated into depends greatly on whether the domain of socialisation receives serious consideration in curricular and instructional design.

Biesta's examples describe the students' socialisation into practices of the society broadly, highlighting issues like democratic participation and citizenship. Following Freudenthal's vision of school mathematics, Yackel and Cobb (1996) sought *classroom practices* that made all students' participation in mathematical discussions viable, revealing the value of socialisation as a means of supporting students' mathematical learning. Aligned with Biesta's proposition, they brought attention to the fact that there always are social practices that students learn to enact in their classroom. The fact that students know who is expected to speak and when, or what are the acceptable ways to contribute to a conversation, are some of the indications of practices that have become normative in their classroom. Cobb and colleagues (e.g., 1997) documented that what those classroom social practices are matters both for the teacher's success in shaping students' mathematical learning and for the students themselves. By distinguishing between social and social-mathematical norms as constitutive of classroom practices, Cobb and colleagues brought attention to a *domain-specific layer* of socialisation that takes place in classrooms—the layer responsible for what students come to see as mathematics, and which activities, actions, and contributions they come to see as mathematical.

The classroom practices into which students are socialised are consequential to whether students identify with, comply with, or resist, the forms of participation that are expected of them (Cobb et al., 2009). In far too many mathematics classrooms, students are socialised into the traditions and practices of mathematics schooling, which are not only different from those of mathematics, but are incompatible with them. Students may come to learn that being correct and being fast in calculations are the most highly valued attributes. They may come to learn that mathematical correctness, just like correctness in spelling, is arbitrated by external sources—the textbook, the test answer key, or their teacher's final say—and that to be good in mathematics one must rely on their memory (cf. Boaler, 2000). These student observations are then confirmed in how they are assessed.

Enculturation of this kind is *mathematically* problematic: Accomplished mathematicians do not recognise the classroom practices focused on speed and correctness as being central to mathematics (e.g., Boaler, 2016). Speed and correctness may emerge as side products of doing mathematics with curiosity and dedication for extended time, but chasing these qualities in their own right does not provide a viable path to becoming good at, or interested in, mathematics (Boaler, 2000). The reliance on external arbiters to determine the correctness of a mathematical argument is outright un-mathematical (cf. Bass, 2017). Making inferences and calculations without logical or arithmetic errors is certainly helpful to a mathematician. But the mathematical value of correctness is rooted in the possibility to determine whether something is correct or incorrect by appealing to the authority of mathematics itself, not to its external arbiters. The removal of the possibility (and indeed the necessity) to determine the correctness of one's mathematical actions through one's own intellect effectively removes mathematics from activities in mathematics classrooms, and renders those activities un-mathematical.

The images students form of what mathematics is and what it requires of them, as they are socialised into practices of mathematics in their classroom, shape whether they will be willing to take this kind of mathematics on. Only a few students can be the fastest in one classroom. When a competitive speed is a requirement for "doing well in school mathematics", this systemically convinces large numbers of students that there is something about themselves as a person that prevents them from ever being good at mathematics, in other words, that they are not "mathematical". This kind of enculturation embeds what Rancière (1987/1991) refers to as a *stultification* of students: a process of bringing students to view themselves as stupid in the face of what is being required of them in school. In addition to bringing too many students to see themselves as not mathematical, mathematics schooling of this kind also does injustice to those who identify with it and succeed. Sooner or later, these students will be confronted with the mismatch between what they are good at and what is genuinely mathematical.

Lack of proactive attention to what students are being socialised into in schools and in mathematics classrooms has a significant (and far too familiar) moral as well as mathematical cost. On the positive side, the work of Cobb and colleagues, and similar research conducted within the design research tradition in mathematics education, shows how this cost can be avoided. These studies document productive social and social-mathematical norms, how these are proactively negotiated in classrooms, and how—once established—they support both students' willingness to engage in mathematical activities (cf. Cobb et al., 2009) and their learning. We add with Biesta that socialisation into well-chosen classroom practices has an educational value in its own right, beyond being a means of supporting mathematical learning.

Subjectification

The third, and the most debated of Biesta's (2022) domains of educational purpose, is *subjectification*. This centres on "bringing the subject-ness of the child 'into play'" (p. 7) in their school experiences. Biesta evokes this subject-ness by highlighting the *difference* between activities that invite a student's "arrival of the 'I' in the world as *subject* of its own life" and those which construe the student's 'I' "as *object* of forces or desires from 'elsewhere'" (p. vii, italics added). He refers only to the former type of activities as being *educational*.

For Biesta, "the very appearance of the 'I' in the world" (p. vii) is at stake in education, leading him to suggest that subjectification should be its primary focus. Biesta's suggestion is based on the observation that if, during the years of schooling, students are treated as objects that comply with what others desire of them, then this gives them a wrong indication about how to be in the world and with the world, to the detriment of us all. What students do not get to experience when education objectivises them is the difficult freedom of making their own decisions regarding their acting in the world and experiencing the consequences of those actions as the world speaks back to them. Biesta (2022) suggests that organising education for students' subject-ness is a major challenge for education and education research.

If we accept Biesta's challenge, the educational value of mathematics would lie in its potential to help students recognise themselves, first, as subjects of their mathematical activity, rather than as objects of what someone else desires them to do, be, and know. Findings from design research studies that pursue Freudenthal's vision (e.g., Cobb et al., 1997; Cortina et al., 2014) show that when mathematics is thoughtfully organised for teaching, students can make independent mathematical decisions, can compare, debate, defend, and change them, and in the process experience (mathematical) consequences of their decisions. In Biesta's terms, students get to enact their subject-ness. We suggest that coming to experience themselves as subjects in relation to mathematics is one layer in an arrival of one's 'I' in the world as a subject—rather than an object—of one's life. In this way, mathematics in school can provide a path for calling students to attend to the world, to observe it, to make sense of it, and to recognize themselves as subjects who live, act, and remain in continuous relationship with it.

In the discourse of schooling, however, mathematics has been framed as content, techniques, skills, and more recently proficiencies *to be acquired* and *possessed*. Students are envisioned as the ones who comply and engage in these acquisitions. The acquisitions are justified to them almost exclusively in terms of the possible (yet elusive) qualification-related benefits to their distant future selves. Operating under this acquisitionist paradigm (Sfard & Cobb, 2014), school mathematics often asks students to refrain from exercising their subject-ness, and to agree—even enthusiastically—to become objects of forces and desires from elsewhere for the sake of their and their society's future. We know too well that when driven to succeed, students comply with such objectivisation processes by suppressing their selves, along with their curiosity and criticality, temporarily or permanently.

Biesta (2022) is deeply concerned about the dangers of equating education with its qualification purposes, primarily when students' subject-ness is too readily sacrificed in the process of accomplishing their qualification. To make this point, he introduces the Parks-Eichmann paradox, a harrowing illustration of how something that "appears as educational success from one perspective, is actually quite problematic when viewed differently" (p. 26). Rosa Parks's and Adolf Eichmann's actions left significant marks on the 20th century. Rosa Parks, fully aware of the possible consequences in 1955 Alabama, refused to obey the bus driver's order and did not give up her seat in a "coloured" section of the bus to a white passenger. It was through individual actions of Parks and others that a bigger change in the societal order was taking shape. Biesta contrasts Parks's decision to act in a particular way with the actions of Adolf Eichmann, the high-ranking Nazi SS officer, who successfully completed considerably more schooling. Eichmann, when convicted on crimes against humanity during World War II, admitted arranging the mass deportation of Jews and others to extermination camps. At the same time, he denied responsibility for the consequences of his actions, stating that he was only following orders. Biesta uses this contrast to highlight that it was not the differences in qualification between Parks and Eichmann that mattered the most to the ways in which they acted in the world. It mattered more how they related to their 'I' as either a subject of their own life or as an object of (or a tool for enacting) someone else's desires. Remaining an object allowed Eichmann to not face the difficult decisions in the world as being his own.

In the light of the stark illustration, Biesta maintains that the role of education is not realised if it produces people who are able and willing to follow orders with excellence but without willingness or ability to recognise that they indeed have the difficult freedom to make their own decisions about the orders they are asked to execute.

As Biesta clarifies, teaching to pursue students' subject-ness is not an easy path for students or for teachers. Refusing to treat students as objects of their schooling may be a good start, but a suggestion of this kind does not clarify what specifically could or should be done in classrooms instead. What is the teacher to do to 'allow' the student's 'I' to 'arrive' to the world, even if just to the world of mathematics in their classroom? Part of the difficulty lies in the magnitude and severity of the examples like Parks-Eichmann paradox, which illustrate the need to act educationally but do not give any indication of what could or should be done in a classroom on a day-to-day basis. Allowing for such arrival does not mean that the teacher can be passive, but it is also very clear that nothing here can be given to students to acquire.

As hinted earlier, the classroom design studies built from Freudenthal's line of thought offer practical insights on how to design, and teach, for subject-ness. These researchers did not conceptualise, or pursue, students' subject-ness—they pursued good mathematics teaching and learning. Is it then true that students' subject-ness emerges in those classrooms, and if so, how?

Designing for Participation in Classroom Mathematical Practices

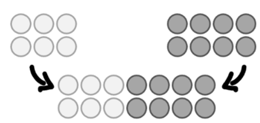
Hans Freudenthal is widely credited for presenting a view of what mathematics is, and what it should be about in classrooms, characterising mathematics as a particular type of human activity, in which one increasingly gets to take part (e.g., Freudenthal, 1973). Freudenthal suggested that to teach mathematics well, we must approach it—and organise it for teaching—with this view in mind. We would like to highlight that this approach assumes a perspective of learning which is different from that of an acquisition. Learning here is characterised by participation and measured by documenting changes in forms of participation (Cortina et al., 2014). To Freudenthal, mathematics is a practice, something in which humans engage and participate together. He clarified that mathematics entails—first and foremost—the act of *mathematising*. For our present purposes, let us define mathematising as an act of gradually generating more mathematics by noticing, exploring, and making clearer the patterns of, and structures within, how things are or how they behave. The 'things' here are both the 'real' things we encounter in the world, and the mathematical things that came to be of interest through prior mathematising. In this sense, research mathematicians mathematise to develop new insights and theories; and young children mathematise to organise patterns observed around them.

As an example of an activity of mathematising, consider adding different numbers. One may notice that a sum of even numbers brings an even result. But would this always be so? Is this an inevitability, or is this a chance occurrence only true for the numbers we have tried so far? How could we know for sure? Could we check with all the numbers? By seeking answers to these and related questions, the numbers that students already understood previously would come to be understood in a new way (see Figure 1): The classroom may first notice that an even number of cookies can be organised into a double row of cookies. Aligning two such double rows of cookies gives a longer double row, thus again an even number.

The informal patterns of adding even numbers, and the reasons for these patterns, are not just pieces of mathematical knowledge to add to one's memory. In the classroom, these would become part of what numbers are and what can be done with them or explored about them. The activities may extend into exploring parity patterns in subtraction, and with odd numbers. The habit of concocting and pursuing questions about one's mathematical activity is being established as itself being deeply mathematical: As we seek answers to our question, we also wonder what else is interesting to ask. In addition, the processes by which one can ascertain or refute that mathematical relations hold are becoming part of students' mathematical repertoires.

Figure 1

An argument to show that adding two even numbers must yield an even number. Specific amounts illustrate a generalised geometric principle, which can later be formalised using algebraic notation.



If students are to come to associate mathematics with their activity of mathematising, and thus socialise into mathematics in a broad sense, the activity of mathematising must be present in their classrooms. Instead of a ready-made body of knowledge that must—as Biesta says—be brought "into someone" (the student; Biesta, 2017, p. 75), Freudenthal viewed the mathematical ideas and processes as having to be created and re-created anew by each group that takes part in this type of human activity. In this view, students are brought to mathematics (cf. Biesta, 2017) and taught to experience it, and face it, so to speak, on their terms.

We know from Dewey (1916/2018) that children do not spontaneously explore the ideas that, with hindsight, would be of educational value to them (e.g., parity). How, then, can a teacher bring students to mathematics without objectifying them in the process? The design tools that oriented the classroom studies, specifically the instructional design theory of Realistic Mathematics Education (RME; Freudenthal, 1973; Treffers, 1987), offer an insight. The three tenets of RME envision a mathematics education in which students are to encounter and face mathematics. They also provide heuristics for planning such mathematics education.

To Freudenthal (1971), mathematics is primarily "an activity of organizing a subject matter" (p. 414), and to do such organising, a student needs to be placed in situations where organising would occur to them as a useful thing to do. The first of the three central tenets of RME design theory deals with making students' genuine engagement with mathematics possible in the classroom. It suggests that for students to have a chance of (re)creating a mathematical idea, they need to encounter this idea in activities in which they could and would want to be active, so that they can notice, interrogate, and eventually establish the mathematical idea or insight. Care is taken in design and enactment of activities to ensure that students develop a reason, and have a mathematical way, to engage in what is brought to their attention. This demand placed on RME-based design and teaching is rarely afforded elsewhere, but it functions to maintain the freedom for each student to engage (or not). It is a responsibility first of design, and then of teaching, not to exclude anyone for lacking a reason or mathematics needed to engage.

The second tenet of RME reminds that the mathematics in the classroom must be capable of gradually evolving into the mathematics of mathematicians, the disciplinary mathematics. It states that the starting point of students' activity must be justifiable in terms of the potential end goal of the mathematical sequence. The starting point needs to allow for mathematics to grow towards this end goal *in a manner that upholds the internal coherence of mathematics*, while gradually transitioning from the initial informal ways of speaking, symbolising, and reasoning towards the sophisticated and increasingly standardised ways (Treffers, 1987). The activities must target those ways of participation that are organically expandable over time into the advanced mathematical practices of mathematicians. At the very least, the earlier ways of engaging in the classroom (including meanings and definitions of the elements of mathematical work) should not create obstacles to how students will need to engage with the discipline as they progress. This functions to maximise the freedom for students to engage over the long run.

The third tenet of RME ensures that the evolution of students' classroom activities towards greater mathematical sophistication *always proceeds by mathematical means*. This demands to sequence mathematical ideas for teaching in a way that would allow students to conjecture, glean, approach the next idea via the mathematical tools that they already have at their disposal. This ensures that the students—like mathematicians—experience how they can progressively mathematise the phenomena and situations at hand, rather than merely observe how the teacher or somebody else does so.

In Conclusion

Thinking with Biesta helps us to name the key strength of RME instructional design theory: RME is not a guide on how to bring mathematics to students. Instead, it is best viewed as a theory that engages with how to bring students to mathematics. The RME tenets overtly guard against objectifying students who engage in the designed activities. The need for students to experience their own mathematising shows that the very arrival of their 'I' in the world of mathematics as a *subject* of their own mathematical activity is inherent to RME. Guiding the classroom activities to be recognisably mathematical (generative and reflective), the activities are well suited to socialise the students into what *mathematical* means and feels like. Using RME in instructional design means designing for all three domains of educational purpose.

Classroom design studies in mathematics education (e.g., Cobb et al., 1997) tend to position socialisation- and subjectification-related parts of the design as the *means of supporting* the specific forms of mathematical reasoning in the classroom (i.e., the qualification purposes). While these studies show that profound classroom mathematical learning is possible, the means that supported this learning have often been critiqued as being unrealistic under current school conditions. In highlighting socialisation and subjectification as essential legitimate purposes of education, Biesta removes the possibility to treat their pursuit as "unrealistic" and encourages us to turn the question around. How can we organise the qualification requirements in mathematics education so that adequate mathematical socialisation and subjectification will become part of teaching and learning in mainstream mathematics classrooms?

We discuss elsewhere (Višňovská & Cortina, 2025) that mathematics content is often organised for teaching so that students experience mathematics as lacking coherence.

Disruptions of mathematical coherence mean that new mathematical ideas at times cannot be developed from previous ideas present in the classroom and must instead be given to the students. We now view curricular sequencing of this kind as omitting the reasoning resources essential to teaching for students' subject-ness. While such curricula may appear adequate when judged in terms of the mathematics content they bring to students, they do not adequately support teachers in the difficult task of bringing students to that mathematics. This limitation is significant as it cannot be overcome by good teaching without changing the curricular sequence.

The mathematics education research community has long been driven by the goal of making students' experiences of school mathematics more meaningful and more mathematical. Our discussion suggests that this goal could be advanced by examining the organisation of mathematics curricula and activities for their capacity to be, in Biesta's terms, educational.

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