Investigating Primary Students' Images of a Whole in Fractions

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This paper aims to investigate what images of a whole that Grade 4 students have when solving problems involving unit, proper, and improper fractions. Data were collected through fraction tasks, post-test on fraction schemes, and interviews. Coding of the data reveals that the students had visual and symbolic images of the whole. However, they have not had a flexible and connected image of the whole across the fractions.

A whole is one of the essential concepts for fraction understanding. Students' ability to determine a whole in, for example, 2/5 as two parts *out of* or *within* five equal parts of a chocolate bar, gives meaning to the fraction as a part-whole relation (Hackenberg, 2013). Yoshida and Sawano (2002) documented that the concept has a significant impact on students' understanding of fraction equivalence and magnitude. For example, when comparing 2/7 and 5/7, students who are aware of the whole could likely answer that 5/7 is greater than 2/7. Moreover, the whole is essential to the understanding of fraction arithmetic, such as multiplication (Behr et al., 1992). A problem like 1/3×1/4 could be perceived as finding one-third of one-fourth *of a whole* chocolate bar. Another critical role of understanding the whole is when encountering improper fractions. In a proper fraction, such as 2/5, students might observe that the whole is the greater number in the fraction. This observation is misleading when working with improper fractions, such as 7/5, in which the whole is not the greater number.

Despite its critical role, determining a whole in fraction-related problems is challenging even for teachers (Bednarz & Proulx, 2014). This challenge is magnified by the fact that the concept is not explicitly taught in the mathematics classroom (Yoshida & Sawano, 2002). Furthermore, there is a lack of research on what students perceive as a whole in fractions, especially students' images of a whole in different fractions, for example, from unit/proper fractions to improper fractions. Such research is pivotal to improving fraction teaching and learning. Therefore, this paper aims to investigate the images of a whole that Grade 4 students have when solving problems involving different fractions. The findings of this paper are expected to extend our insights into students' understanding of the concept and, more importantly, to contribute to improving fraction instruction.

Images of a Whole in Fractions

This paper is part of a classroom intervention study that aimed to support Grade 4 students in Indonesia in learning fractions through a spatial intervention. The study design was underpinned by spatial reasoning, growth of mathematical understanding (Pirie & Kieren, 1994), and fraction schemes (Steffe & Olive, 2010). This paper focuses on the images students have about a whole in fraction-scheme-related problems. This section reviews a whole in problems involving different fraction schemes and *images having* (Pirie & Kieren, 1994), one of the stages in the growth of mathematical understanding. The conceptualisation of the whole and the images having underpin this paper.

Scheme theory is one of the theoretical lenses for interpreting students' reasoning about fractions. In this lens, a fraction scheme is a model of students' recurring ways of operating in situations involving fractions (Steffe & Olive, 2010). The scheme consists of three parts: an experiential situation, an activity associated with the situation, and the result of the activity (von Glasersfeld, 1980, cited in Steffe & Olive, 2010, p. 20). For example, when asked to represent 2/5 in a chocolate bar (a *situation* relating to the *part-whole scheme* involving unit or proper

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fraction), a student may partition (*the activity,* also called an *operation*, Hackenberg, 2013) the bar into five equal parts and then shade (*the activity*) two parts of the whole partitioned parts. The term *operation* is used in this paper. The *result* of the activity is the student's constructed representation of 2/5 or two shaded parts within the chocolate bar. The intervention in the present study provided Grade 4 students with spatial-thinking situations, such as paper folding, that provoked the construction of fraction operations, such as partition.

To come to the result, the student might have identified the whole, which is the five parts, by partitioning the chocolate bar. Determining the whole in other fraction schemes: *partitive* (unit fraction, proper fraction), *reversible partitive* (proper fraction)*, and iterative* (improper fraction), requires not only partition but more complex operations, such as iterating and splitting (Hackenberg, 2007; Norton & Wilkins, 2009). Table 1 summarises how students operate to determine the whole. For example, splitting is required to determine the whole in problems involving reversible partitive and iterative fraction schemes. It is the way students partition and iterate simultaneously. An example of a splitting problem is "This is your friend's choco bar, which is three times as long as yours. Draw your choco bar." The problem uses an iterative language, but students need to partition the "your friend's choco" into three equal parts, while, at the same time, iterate the one part to fit the whole choco. Consider Test 10 (Reversible partitive scheme, Table 1), the ability to split is foundational to solving the test; the students partition the given rope into two equal parts (2/6) and, at the same time, iterate one part (1/6) to make the whole six parts (6/6).

Table 1

Operations Involved in Determining a Whole in Different Fraction Schemes

Schemes	Sample problems	Ways of operating
Part-whole	*Task 4B*: The figure* shown below is a whole block. Draw 1/5 and 3/5 of the whole block.	*Partition* the given figure (the whole block) into five equal parts (5/5).
Partitive	*Task 4C*: The figure* shown below is 1/3 of a whole block. Draw the whole block.	Copy (*iterate*) the given figure (1/3) three times or *add* two similar figures to the right to make the whole block (three blocks, 3/3).
Reversible partitive	*Task 4D*: The figure* shown below is 2/6 of a whole block. Draw 6/6 of the block. *Test 10*: The rope** shown below is 2/6 as long as the whole rope. Draw the whole rope	*Splitting* the given figure/rope (2/6, two parts) to make 1/6 (one part) and the whole (6/6, six parts).
Iterative	*Task 4E*: The figure shown below is 9/7 of a whole block. Draw the whole block.	*Splitting* the given figure (9/7, nine parts) to make 1/7 (one part) and the whole (7/7, seven parts).
	Test 12: The given rope below is 7/4 as long as the whole rope. Draw the whole rope.	*Splitting* the given figure (7/4, seven parts) to make 1/4 (one part) and the whole (4/4, four parts).

*Notes: *The figure given in each Task is an unpartitioned block/fraction bar. **The rope given in the Test items is unpartitioned*

Students with a part-whole fraction scheme might be able to determine the whole in partitive scheme problems by treating them as part-whole relations. For example, a student is likely to consider Task 4C (Table 1) as making 1/3 within the given figure; therefore, partition it into three equal parts and then determine the whole as three equal parts, or 3/3. Norton and Wilkins (2010) point out this case, where students could use their part-whole fraction scheme to solve problems involving the partitive fraction scheme. Regarding this, it is likely that the reversible partitive problems (Task 4D, Test 10 in Table 1) will be treated as part-whole relations: partitioning the given figure/rope into six equal parts and selecting the six parts (6/6) as the whole.

The student's ability to determine the whole in various fraction-scheme problems, for example, three equal parts (Task 4C), might indicate that the student has an image of the fraction (1/3) and the whole within the fraction. Pirie and Kieren (1994) call this *image having* or having a mental model of 1/3, represented by the given figure, and the whole, which is three copies of the figure. The focus of this paper is on the images of a whole that Grade 4 students have when solving Tasks 4B-4E, Tests 10 and 12.

Martin and Pirie (2003) defined images as "any ideas the learner may have about the topic, any 'mental' representations, not just visuals or pictorial ones (p.173)." In this paper, images are perceived as students' mental representations of the whole in the problems involving different fraction schemes. The whole could be visual or pictorial in the form of partitioned parts of the fraction representation or symbolic fractions (n/n) representing the whole. The mental representations of the whole are observable in students' drawings when answering the problems and in their verbal explanations during the interviews. Based on the model of growth of mathematical understanding (Pirie & Kieren, 1994), the images students have in learning are the results of relevant *image making* activities, such as spatial activities in the present study. The images developed from the activities could be incomplete or incorrect (Pirie & Kieren, 1994), which may indicate the students' understanding. Presmeg (1986) identified different visual images that high school students had in solving math problems, for example, a student had a concrete image to represent trigonometric ratios. For primary students, a whole in fractions may be perceived differently, potentially leading to incomplete images as a result of their learning experiences, including the spatial intervention in the present study. For example, the students might think of the whole as 'always' the greater number in fractions because it is the longer parts if drawn in fraction bars. This image is incorrect because it does not apply to improper fractions. This incorrect image could shed light on (further) improvement of the fraction instruction.

The ability to determine a whole is a fundamental part of fraction understanding. Such an ability depends on students' available operations, such as partition, iteration, or splitting, required to solve fraction scheme problems. The students might see the whole in the fraction problems differently; it could be right or incomplete. Therefore, this paper seeks to answer this question: *What images of a whole did Grade 4 students have when solving problems involving different fraction schemes?*

Methods

Research Context and Participants

The spatial intervention in the present study was delivered to 31 Grade 4 students over 11 lessons by a classroom teacher. The lessons provided students with spatial activities promoting fraction operations, such as partitioning through paper folding (lessons 1-5) and a number of fraction tasks (lessons 6-11). A fraction scheme test (Norton & Wilkins, 2009) was administered before and after the intervention. To solve the test, students need to have developed fraction operations, such as partition, allowing the present study to examine the impact of the intervention. This paper focuses on four tasks and two test items that asked the students to determine or draw a whole (Table 1). Six students (2 boys, 4 girls) with different levels of fraction schemes based on pre-test results were purposively selected. The students were interviewed after lessons containing the fraction tasks and after the pre- and post-test to further understand the impact of the intervention. The whole was not explicitly included in the spatial activities. It was only discussed in the lessons, where the tasks about the whole were given.

Data Collection and Analysis

Data sources for this paper were (1) the six students' answers to the four tasks (Table 1, Tasks 4B-4E, a total of 24 answers) and two post-test items (Table 1, Test 10, Test 12, 12 answers), and (2) their interviews on the tasks and test items (36 transcripts). Each student was interviewed individually, using a semi-structured approach, after the lesson containing the four tasks and the post-test. In the present study, the students' interviews chiefly aimed to investigate their growth of fraction understanding across the lessons and after the intervention, including the promoted fraction operations. For this, an interview protocol was developed following fraction schemes (Steffe & Olive, 2010) and growth of mathematical understanding (Pirie & Kieren, 1994) and used for the interviews. For example, in Task 4B, students were asked, "How did you divide the whole block?" Other sample questions are "Which one is the whole rope?" and "Why is it the whole rope?" for Test 10. These questions could show their images of the whole.

The term 'block' in the tasks and interview transcripts refers to the housing blocks, which served as the learning context. The blocks represent fraction bars. The ways in which the students determine or draw the whole are summarised in Table 1. Task 4C is the reciprocal of Task 4B: given 1/3 of a block, a whole block was to be drawn. Conversely, in Task 4 B, the students need to break the whole to make fractions, for example, 3/5. What makes Task 4B intriguing is that the students have to partition the unpartitioned block to make the whole, based on the fractions they are asked to draw. All the tasks reported here not only asked the students to draw the whole or the fraction representing the whole, but also other fractions, such as 4/6 (Task 4D), 2/3 (Task 4C), or 3/7 (Task 4E). The other fractions were excluded to focus on students' answers regarding the whole in the fractions.

In the context of the present study, the students' answers to the tasks/tests and interviews were coded referring to a combination of inductive and deductive approaches. The six students' test answers were coded inductively across two coding cycles: process and pattern coding (Saldaña, 2013). For example, the student's answer to Test 12 (Figure 1B) shows that she could partition the given rope into seven equal parts and make an additional four-part drawing representing her answer. The answer was coded as *drawing on the given representation with appropriate equal partition and additional drawing*. The student's interview transcript on the test item was coded deductively based on the conceptualisation of fraction schemes and growth of mathematical understanding. To capture the images the student has, the student's answer and interview on the item were combined. For instance, although her drawing seemed correct (Figure 1B), she identified the whole as 4 parts (4/4), but in the interview one part was identified as 1/7 instead of 1/4. This was coded as *no indication of three-level unit coordination* and *visual and symbolic images of the whole*.

Figure 1

Students' Answers

(A) Test 10 (B) Test 12

Findings

This part reports the emerging themes in students' images of a whole across different fraction scheme problems, along with supporting evidence. The students' names (Anna, Andi, Hana, Lili, Rian, Zane) reported here are pseudonymous. The main findings are two-fold. First, the six students have visual and symbolic images of the whole. Second, the six students have not had *a flexible and connected image* of the whole. Each finding is elaborated as follows.

Visual and Symbolic Images

The six students seemed to have visual and symbolic images of the whole within the fraction problems. Not only did they have visual representations of the whole, for example, the five partitioned parts within the representation in Task 4B, but they also translated the parts to a symbolic fraction, 5/5 (T1). Although the students could relate the whole parts within the representations to symbolic fractions, the images they had were not necessarily complete. This will be explained in the following section.

T1 (Transcript 1) R: Researcher Ai: Andi

1 R : what block is this? [Task 4B]
2 Ai: the whole block
3 R : how did you divide this? [pointing to the block]
4 Ai: I divided it into five parts […]
5 R : why did you make five?
6 Ai: to make 1/5
7 R : what is the whole for 1/5?
8 Ai: 5/5

Static, Inconsistent, and Flexible but Unconnected Images

In general, the students have not had flexible, connected images of the whole in the fraction problems. It is flexible if a student can see the change of the whole from unit/proper fractions (partitive, reversible partitive scheme) to improper fractions (iterative scheme); the (whole) partitioned parts or the greater number are only applicable to unit/proper fractions. A connected image of the whole is when the student can coordinate multiple units in improper fractions, for example, 1/4, 4/4, and 7/4. This allows the student to *ignore* the (whole) visible partitioned parts or the greater number in 7/4. This finding is characterised by three emerging categories of images that the students had, as follows.

First, *static images* of the whole. Two students were in this category. For Anna, the whole was *always longer*. For the tasks involving unit and proper fractions (Tasks 4B-4D), she referred the whole to the denominator. When asked how she solved Test 10 (Figure 1A), Anna said, "*it was asked to draw a whole rope this way (her hand goes beyond the given rope following her answer).*" This indicates that the whole for her is longer than the given rope. Similarly, she perceived the longer parts (the numerator) in improper fractions (T2, Test 12, Figure 2A) as the whole. Andi also had a static image of the whole, which was the (whole) visible parts within the representation he partitioned (Figure 2B, T3). Unlike Anna, he saw the whole as the greater number in all the fractions, which were drawn into all the partitioned parts.

Figure 2

Students' Answers

(A) Test 12 (B) Test 10 (C) Test 12

T2 R: Researcher A: Anna

1 R: what about the whole? [Task 4E, she divided the block into nine parts]
2 A: nine [nine parts in the block]
5 R: why nine?
6 A: because it is 9/7
[…]
7 R: how did you solve it? [Test 12, improper fraction 7/4]
8 A: It is asked to draw a whole rope so I drew it to make seven and then I divided it

T3 R: Researcher Ai: Andi

1 R: how did you solve it? [Test 10]
2 Ai: I divided it into six parts
3 R: why did you divide it into six parts?
4 Ai: to make 2/6 to make it the whole rope […]
5 R: what is the whole rope?
6 Ai: 6/6
[…]
7 R: what is the whole for 9/7? [Task 4E]
8 Ai: 9/9 […]
9 R: this block is 9/7 how did you divide it?
10 Ai: nine parts

Second, *inconsistent images* of the whole. Rian and Lili were included in this category. In Tasks 4B-4D, both students had the denominator as the whole (T4 lines 1-6). Rian was like Anna, who thought the whole was always longer in representations. This made him draw a longer rope (seven parts), extending the given rope in Test 12 as he had done in Test 10. However, he considered the whole to be the four parts (4/4) in the extended seven-part rope, representing the 7/4. In Task 4E, Lili had her nine partitioned parts as the whole (the numerator), but she answered four parts as the whole for Test 12 (T4 lines 7-12).

T4 R: Researcher Ri: Rian L: Lili

1 R: what block is this? [pointing to the given block, Task 4B]
2 Ri: a whole
3 R: what fraction is this whole block?
4 Ri: 5/5
5 R: why? […]
6 Ri: I can see from this [pointing to the denominator of the fractions to draw]
[…]
7 R: how did you solve it? [Test 12]
8 L: I divided this [the given rope] into seven parts
9 R: why?
10 L: because it is 7/4 and the whole is four […]
11 R: what fraction is the whole?
12 L: 4/4

Third, *flexible but unconnected images* of the whole. In this category, Hana and Zane flexibly perceived the whole as it was expected for the different fractions. The whole was the longer partitioned parts (the denominator) in the unit and proper fractions (Tasks 4B-4D). In the case of 9/7 (Task 4E), they considered seven parts (7/7) as the whole in the interviews, since they initially made seven parts for 9/7. However, they changed their answers when I had them refer to the problem statement: the figure shows 9/7 of a block. Hanna responded, "*ooh it is divided into nine and shade seven*." Despite this intended answer, they did not appear to have a solid image of 9/7 in the representation (e.g., Hana's response, "*ooh* ..."). Of the six students, only Hana and Zane seemed to have shifted their images of the whole when encountering improper fractions, 9/7 and 7/4. Moreover, they could argue why the whole was four parts in 7/4 (T5 lines 1-12). Thus, their images of the whole were likely flexible. Lili also had the same image of the whole in Test 12, 4/4 or four parts. She was excluded from this category because her image of the whole was unchanged from the unit/proper fractions, which was still reliant on the nine partitioned parts she made for 9/7. Although the two students flexibly perceived the whole across fractions, their images were *unconnected*. When asked what fraction represents one part within their seven-partitioned rope in Task 12, the answer was 1/7, instead of 1/4 (T5 lines 13-16). They might not have developed the ability to coordinate 1/4, 4/4, and 7/4, which is required for working with improper fractions.

T5 Z: Zane H: Hana

1 R: why did you make this into four [four parts under the given rope, Figure 1B]?
2 Z: because it is asked for 7/4
3 R: it is asked for the whole rope is this the whole rope? [pointing to her drawn answer]
4 Z: yes
5 R: what fraction is this whole rope?
6 Z: 4/4
7 R: why?
8 Z: because it is the whole rope
[…]
9 R: why did you mark four? [the four parts, Figure 2C]
10 H: because of this [pointing to the denominator in 7/4 given in the test statement]
11 R: what is the whole rope?
12 H: four
13 R: what fraction is this? [pointing to the most left part of the rope in Figure 2C]
14 H: 1/7
15 R: why 1/7?
16 H: because taking one from seven parts

Discussion and Implications

The aim of this paper is to investigate Grade 4 students' images of a whole in different fractions scheme problems: unit and proper fractions (part-whole scheme, partitive scheme), and improper fractions (iterative scheme) after receiving a spatial intervention that promoted the construction of fraction operations, such as partition or iteration.

The data analysis reveals that students had visual and symbolic images of the whole across the different fraction problems. Translating the whole partitioned parts or fraction visual representation to symbolic fractions (*n*/*n*) in the fraction problems, for example, 5/5 in Task 4B (Transcript 1) requires the ability to perform partition and iteration. The students were only presented with an unpartitioned fraction bar and symbolic fractions to be drawn. They partitioned the bar into five equal parts (the whole) and likely iterated 1/5 to make 5/5 representing the whole. Prior publication of the results of the present intervention study documented that the six students were able to construct the symbolic fraction *n*/*n* from partitioned fraction bars (Wahyu, 2025). This finding indicates the potential of the spatial intervention delivered to students, in which relevant fraction operations, such as partition and iteration, were promoted through spatial activities.

Despite being able to relate the visual to symbolic images of the whole, most students have not developed a complete image of the whole throughout the different fractions. In problems involving unit and proper fractions, all six students perceived the denominator, the greater number in the fractions, or the whole visible parts within a partitioned representation as the whole. For example, three partitioned parts (3/3) are the whole for 1/3 (Task 4C). When encountering improper fractions, the whole for two students remained the same: the greater number or numerator, for example, nine partitioned parts or 9/9 for 9/7 (Task 4E). The other two students showed inconsistency: the whole for 9/7 was 9 partitioned parts, but for 7/4 it was four partitioned parts (4/4). The remaining two students were able to see the correct whole in improper fractions but were unable to determine that one of the seven partitioned parts representing 7/4 is 1/4, not 1/7. This finding confirms what Pirie and Kieren (1994) point out:

the images students have (*image having*) after *image making* activities, such as the intervention in the present study, may be incomplete.

One possible reason for the incomplete image is that fractions, in general, are demanding, especially improper fractions. Research on fraction schemes (Steffe & Olive, 2010) shows that different operations underpin fraction understanding (Table 1). For instance, students need to develop splitting (making partition and iteration simultaneously) and a three-level unit coordination (coordinating 1/4, 4/4, and 7/4) to work with improper fractions (Hackenberg, 2007). To draw the whole for 7/4 (Test 12), students need to simultaneously partition 7/4 into 1/4 (one part) and iterate 1/4 to make 4/4 (four parts) and 7/4 (seven parts). In addition, they need to coordinate 1/4, 4/4, and 7/4 or be aware that the seven parts are not 7/7. The students may not have developed the required operations, which are very complex, including the two students who identified one of the seven parts as 1/7. The absence of such complex operations may cause some students to rely on their part-whole understanding of fractions when determining the whole in improper fractions (Norton & Wilkins, 2010), for example, perceiving the whole as the greater number or numerator. The findings of this study imply that the underpinning fraction operations, such as partition, iteration, or unit coordination, need to be explicitly targeted in fraction interventions.

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