



Making Multiplication Structure Visible: Using Spatial Artefacts to Develop Distributive Reasoning

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Young students often encounter multiplication as repeated addition or as a collection of facts to remember. An important next step is helping them recognise multiplication as a structure that can be composed and decomposed. For example, 5×12 can be understood as $5 \times 9 + 5 \times 3$. Such decomposition supports flexible calculation and provides an important foundation for distributive reasoning. Making these relationships visible can help students move beyond recalling facts toward understanding how multiplication works.

Our study explored how spatial artefacts supported 24 Year 3 students through three sequenced tasks using multiplication bars and a bar model. The tasks progressively moved students from noticing relationships between products, to composing products, and finally to independently decomposing and justifying multiplication.

In the first task, students used multiplication bars to compare rectangles. Initially, many relied on perceptual descriptions such as “wider” or “taller”. By selecting, rotating, and combining the bars, they began to notice factor relationships more explicitly. For example, students recognised that 3×8 could be composed from 3×5 and 3×3 , making visible that 3 stayed the same while $8 = 5 + 3$.

The second task shifted students from recognising these relationships to deliberately constructing them. Students combined bars with a shared dimension to create new rectangles. For example, 5×4 and 5×3 were combined to produce 5×7 . This helped students see that $5 \times 4 + 5 \times 3 = 5 \times 7$: one factor stays the same while the other is composed as $4 + 3 = 7$.

In the final task, the physical bars were removed and students solved 5×12 by partitioning a bar model. They generated decompositions such as $5 \times 12 = 5 \times 9 + 5 \times 3$ and $5 \times 12 = 5 \times 5 + 5 \times 4 + 5 \times 3$. Students checked that the partitions recombined to make 12 and that the partial products gave the original product. The bar model had therefore become a tool for independently representing and justifying multiplication decomposition.

For teachers, the key message is that spatial representations are most powerful when used as tools for reasoning rather than simply as illustrations. The progression from noticing relationships, to composing products, to decomposing and justifying products helped students see how one factor can stay the same while another is composed or partitioned. Questions such as “*What stays the same?*”, “*How has this factor been split or combined?*”, and “*How do the partial products justify the whole product?*” can direct attention to these relationships. These prompts can also encourage students to explain their reasoning rather than focus only on obtaining an answer. With carefully sequenced tasks and purposeful questioning, spatial artefacts can help students see, construct, and explain multiplication structure before encountering it as a formal symbolic rule.

For more information, please refer to the following paper presented at the 48th Annual Conference of MERGA in July 2026.
Putrawangsa, S., Patahuddin, S.M., & Lowrie, T. (2026). making multiplication structure visible: Spatial artefacts, instrumental genesis, and emerging distributive reasoning. In N. Berger, M. Thompson, J. Ley & C. Wilmore (Eds.), *Beyond boundaries: Strengthening futures in mathematics education*. Proceedings of the 48th annual conference of the Mathematics Education Research Group of Australasia, Sydney (pp. 329–336). MERGA.