

# Looking Backwards, Looking Forwards, and Looking for Direction

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This personal and reflective story presents some incidents from my professional life that have (or have not) influenced me as a mathematics educator. Looking backwards I see some mistakes I made, opportunities I missed, and lessons I learnt. Looking forwards I see areas within our discipline that seem worthy of exploration, boundaries that may need to be crossed, and initiatives that need to be taken. From these perspectives a sense of direction is emerging for me and conferring with colleagues will help me see this more clearly.

## Introduction

Why tell you my story about where I have been, where I am going, and why—not so others might learn from my mistakes, nor because of the conference theme ‘beyond 2000’. The reason is to get my work into perspective and review my direction. This is consistent with my views of conferences and research. I see the benefits of conferences coming from collegial discussions and from the preparation rather than the presentation of papers. In research I see us often looking outside ourselves for causes and solutions to problems rather than seeing ourselves as significant and inseparable from our problems.

One aspect of story-telling that intrigues me is how incidents are chosen as important. When researching myself, how do I reconstruct my reality, which incidents lie forgotten, and why do I choose or reject things to report on when life is complex and all incidents are intertwined. In acknowledging this I use the word story rather than autobiography.

A researcher always makes the research instruments explicit. This is my story, I am the research instrument, so I must expose myself. One influence stands out. Descartes contributed much to Western thought, but Damasio (1994) and others have questioned these accepted dichotomies, and I think this is important. These dichotomies include the separation of self from others, mind from body, and personal from professional and social. We seem to have extended these to include mathematics from other subjects, learning from teaching, schools from life, schooling from education, past from present, and work from play (apart from at conferences). Many of us who are males, also separate cognitive from non-cognitive knowing. This contrasts with Asian thinking where contradictory elements are juxtaposed and seen as complementary—I believe we need to consider incorporating such thinking into our work.

This paper looks backwards and forwards. The first is historical, the second involves my concerns within mathematics education, but the two directions are mutually dependent.

## Part 1: Looking Backwards

### *Primary School*

In 1945 I began school in a small country town. I was quite able and have happy memories of my time there. I remember three incidents about mathematics (or arithmetic as it was called). Firstly, age six, walking home and wondering why every second number was omitted from consecutive letter boxes, and later finding them on the other side of the road. Secondly, chanting tables, multiplication to  $12 \times 12$ , and addition, subtraction, and division tables. Thirdly, getting  $99\frac{1}{2}\%$  and being told by my teacher that he didn't think anyone

deserved 100% (but more on assessment later). My best memory of learning at this school was not in mathematics, it was in social studies (although it involved scale drawings, graphs and statistics). Each student selected a project topic and did their research. I was very satisfied when I handed in an exercise book full of work on Canada.

During these years I enjoyed family, I swam and I played games like monopoly and cards that reinforced my competitiveness, number skills, and capitalist upbringing.

### High School

My memories of high school as a boarder at a single-sex, city school are mixed and generally pleasant, but I did resolve not to send my children to boarding school.

The first incident I remember was in year nine geometry. Each period started with a test on the previous theorem, then a new one was taught, and we did riders (examples) based on it. I remember one theorem about isosceles triangles, see Figure 1.

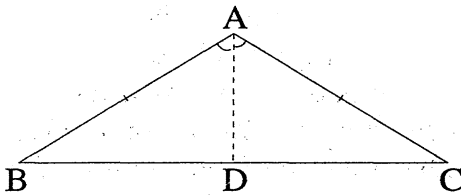
Theorem: If two sides of a triangle are equal, the angles opposite those sides are equal.	
	
Given	Let ABC be any triangle, in which $AB = AC$
Required	To prove $\angle B = \angle C$
Construction	Draw AD to bisect $\angle A$ , with D on BC
Proof	In triangles BAD and CAD Because $AB = AC$ (given) and $AD = AD$ (common) and $\angle BAD = \angle CAD$ (construction) therefore $\triangle BAD \cong \triangle CAD$ (SAS) thus $\angle B = \angle C$ <span style="float: right;">Q.E.D</span>

Figure 1. Isosceles triangle theorem.

For the test we knew that the diagram would be positioned and labelled differently to confuse pupils who had learnt the theorem by rote, see Figure 2.

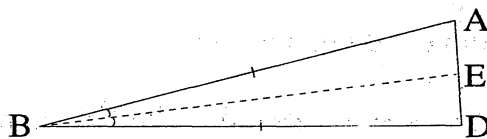


Figure 2. Alternative positioning and labelling.

A friend and I thought a shorter proof with no construction was possible. We thought (using Figure 1) that:  $AB = AC$  (given),  $AC = AB$  (given),  $BC = CB$  (common), thus  $\triangle BAC \cong \triangle CAB$  (SSS), hence  $\angle B = \angle C$ . I used this in the test but my friend avoided the risk of an after-school detention. The 'correct' work was handed back, then the ones that earned a detention, then mine. The teacher made me squirm then made me feel great by saying that one person in the class was thinking and demonstrating my proof to the class.

The same teacher said we needed automatic recall with basic facts so we could do difficult problems without our logic being disrupted. Basic facts were extended to 20 x 20 multiplication, squares to 30 squared, and cubes to 12 cubed. At the end of year 9 he gave us

most of the year 11 external examination and marked it. He assured us we could pass it, and told us not to think about year 11 but to set our sights on scholarship in year 13.

He thought mathematics was the most important subject. His mark schemes ensured that the top marks were spread over a broad range. This meant that when subject marks were aggregated mathematics results determined the top of the class. This was fine if one was good at mathematics, but if one was average one only attained 10%!

I learnt about assessment in other subjects too. I learnt to hate languages—translations were marked by deducting half a mark for each error from the 20% allowed for each translation in the test. Typically I had -3.5 and -4 added to the few marks I gained in the other sections. Meanwhile in science, with questions such as ‘list the properties of oxygen’ for 10 marks, I could list 24 properties at a half mark each and get 12 from a possible 10.

In five years at high school in pure and applied mathematics we were never given notes nor asked to write summaries. Teachers thought we learnt by doing mathematics, not by writing about it. When inspectors arrived in year 13 we were told to bring that years ‘scribblers’ to school, and I took 26 exercise books from my two mathematics subjects.

Our exercise books had always been called scribblers—not quad paper, not ruled, just plain paper. There were no marks for borders, crossed out work was acceptable, graphs were sketched not plotted, and figures represented, as far as possible, general triangles, circles, or whatever. We were expected to be legible, organised (equal signs under equal signs), use correct symbols ( $=$ ,  $\cong$ ,  $\therefore$ , etc), and justify geometric statements we made.

### *University*

University was different. Lecturers made little attempt to get to know students. Notes were dictated on material in our textbooks and we copied them and did not think. We had examples set and tutorials for help. It was fine for students from small schools who had learnt to work independently, and for those who had prepared for scholarship as no new topics were introduced. For many, disaster struck after year one because we did not know how to learn. Most scraped through year two, many failed at year three. I recall being able to follow lectures from line to line, but having little idea about the subject matter. I did not blame lecturers, I assumed I was not as bright as the students who passed with As or Bs.

There were lighter moments—one professor warned us about rigour, rigo(u)r, rigor mortis. There were exceptions too in terms of getting-to-know students—one lecturer learnt our names from enrolment photos, greeted us by name outside class, and asked individuals questions in lectures.

### *Teachers’ College*

I had a year at teachers’ college preparing to teach and was introduced to ‘new math’ by a lecturer who spent many hours apologising for it. We ‘did’ education, but it had little connection with teaching or learning. Lesson planning and discussing teaching and management strategies were what we found useful. Our planning followed the emerging behaviourism and school ‘sections’ provided opportunities to try what we planned. The direct instruction that prevailed was hardly different from behaviourism and our teaching was generally acceptable. When pressured I reverted to how I had been taught, and coming from a school with an emphasis on ‘doing’ academic mathematics, this was not successful.

### *Teaching*

My first teaching position was in the school that I had attended. I knew the staff, the routines, and the trouble spots. I taught mathematics and science and ran extra-curricular activities (swimming, lifesaving, gymnastics, table tennis and computing), and I learnt that respect earned from extra-curricular activities was of more benefit than that earned in class.

The school taught traditional mathematics but I was encouraged to try new maths and taught it for a year to an able year-ten class from a book by Papy (1963) written in French. I had a few language problems; I thought *cercle* meant circle, not circumference then found *disque* meant circle. This experience influenced my use of diagrams, in particular Papy-grams or arrow-graphs as they are generally known, and made me realize that the context for teaching mathematical concepts can be pure mathematics.

I taught applied mathematics (mechanics) and thought it useful in terms of 'modelling'. I particularly enjoyed this with a class who did five years of work in four years. I was humbled each term when marking their examination as I checked my mark scheme against the best student's script and saw how he did things much more efficiently than I did. After six years at this school I met the 'country service bar', so I looked for another position.

At this time in-service education was rarely offered by tertiary institutions. New mathematics was introduced by government advisers, by mathematics associations, by teachers reading overseas journals and textbooks, and by participation at teacher-refresher-courses that were organized by teachers, held in vacation time, and funded by government.

I continued part-time university study while teaching. I thought I had reached my limit with mathematics so started with education, then transferred to commerce. This reinforced my capitalist upbringing, and extended the contexts I used in class from the male 2Ms to 3Ms—machines, military, and money. An incident ten minutes after my final economics lecture is memorable; two of us were in the tavern talking. My friend asked if I enjoyed the lecture, I replied that I had and that many topics from the course seemed to click together. He looked at me quizzically. I asked him how was it for him? His response made me aware of our differences, he said the course had been great, but the final lecture had thrown many doors wide open! (He is now an economics professor in a prestigious US university.)

### *Teaching Again*

To 'jump' the country service bar I successfully applied for the head of mathematics position in a Presbyterian boys school where I was to introduce modern mathematics. In this school the 'ethic of caring' that Noddings (1992) writes about was evident. The Principal knew all 600 boys in the school and their parents, and he taught for half the week. I was new, I was expected to make changes, the department was small, we worked well together, and in my four years I felt I was a colleague rather than an agent for change.

During this time our teacher union published a delightful book (Munro, 1969) that included a set of aims for education. These influenced my thinking, although possibly not my practice for some time. The aims were "the highest value is placed on the urge to enquire, self-respect, and a concern for others". These reflect the academic, personal, and social purposes of school. In hindsight I interpreted these too narrowly. I saw enquiry as cognitive enquiry about mathematics and not non-cognitive enquiry, or values, attitudes, beliefs, or emotions. I accepted self-respect but had not thought about self-knowledge. I interpreted a concern for others at a person-to-person level, but did not extend it to include a concern for culture or environment. I focussed on teaching and was concerned with self-respect and concern for others because they influenced conduct within my classroom.

### *Teaching Afresh*

A new state coeducational high school was to open in 1973 beginning with only year 9 students. Having heard who the Principal was, I applied to be head of mathematics. Here I 'started' to learn about education, and to teach 'students' rather than 'mathematics'. A month before our first staff meeting we were told to prepare for it by reading "Teaching as a subversive activity" (Postman and Weingartner, 1971). Students had no uniforms—they were

expected to develop responsibility for dress. Classes were not 'streamed'. Picking up rubbish was not a punishment as if we had pride in our school and environs then we would naturally keep our school tidy. Everyday at lunchtime the Principal delighted in walking around, chatting to students, and picking up rubbish with them. Each year many students had their names put into the 'honours' book but prizes were not part of the school's emerging traditions, all students were growing, and external rewards were irrelevant.

A regular task I had was collating cyclostyled chapters of a draft text for the students and I used this as a detention task for my unruly form class. One young woman always seemed to be in this detention. After a while I spoke to her about this. She said, "it's a game, if I behave badly you put me in detention, then I stay and help sort this paper. That's more pleasant than going home". When I invited her to stay and help without being in detention her behaviour improved. Later the Principal said that school detention was not working as it was always the same students in detention.

Teaching seven year-9 classes in one year nearly drove me crazy. After two months I moved different classes to different topics to keep sane. I had never had unstreamed classes and to cope I designed an individualised programme. I did not know how important social interaction was and only later restructured the course so groups progressed together.

School organisation had three foci—curriculum, guidance and administration. I was involved with curriculum and administration and came to appreciate the complexity and the interconnectedness of these. I valued the way the Principal delegated tasks, he did so completely and with trust. After four years in this school a state alternative high school was opening and my Principal encouraged me to apply for the Director's position.

### *Alternative Teaching*

As Director/Principal of the alternative school I thought I had made it. Here was the opportunity and the expectation that one would be innovative. A school meeting, based on "Summerhill" (Neill, 1962), was established for decision-making. My role was to provide some input and ideas to meetings, and to implement their decisions (if they were legal). The staff including myself, students, and parents began to look at education more broadly, not just subjects. One result of this was that my awareness grew in terms of broader interpretations of the urge to enquire, concern for others, and self respect. The school had been conceived as a "school without walls", but most students wanted a place they could identify with and this was provided.

I had a supportive staff who shared the running of the school and I taught for 15 hours each week. Many students wanted traditional subjects, but in non-traditional ways, and we offered many non-academic learning activities. The school year was divided into six blocks. Students chose their courses each block and courses had to require as few pre-requisites as possible. As the maths teacher this meant ensuring that students could opt in and out, so I modified my individualised programme and encouraged students to work in groups. Assessment was not high stakes, we believed that students were responsible for their learning, so each six weeks they filled out their reports and staff countersigned them.

The school population was bimodal—some students and/or their families sought alternative education (whatever that meant), others were disenchanted with traditional schooling. Many parents expressed concern that their child might come to school but not attend class. I shared my feelings—I had two sons at the school, one did what parents hoped for, the other participated in numerous work-experience classes but avoided most formal classes in his three years at the school. I felt that formal academic classes were only one part of education. I saw schools as a safe and supportive space where students could learn by making mistakes and reflecting on them without fear of punishment.

There were no separate staff facilities and rules applied to both staff and students. The relationships between staff and students were close and we were all on first name terms. I heard of one ex-student who knew me as Andy and asked who Mr Begg was.

While the school was alternative, its value remained if it continued to experiment, push boundaries, and legitimise less radical innovation in other schools. Some other schools began doing some things we were doing (although not necessarily because we were doing these things), but others became more conservative and told parents ‘if that’s what you want, go to that other school down the road’. While I was at the school the politics of education had not swung to the right as it did later and when it did many of the ‘liberal’ activities were harder to achieve within the regulations. I enjoyed being at this school for nearly seven years, towards the end I felt we were not breaking new ground, I assumed my ‘marginal utility’ was decreasing and a new Director would bring in fresh ideas. Then, wondering about my future, I heeded a colleague’s advice “to stay in education you need to be decontaminated” and sought a position in the Government’s Curriculum Division.

### *Textbook Writing*

While teaching I was involved in resource development, I had been with the NZ Mathematics Magazine from 1964 and in 1965 began co-authoring textbooks. At the time most schools taught arithmetic, algebra and geometry from separate textbooks, decimal currency was coming, and a need was seen for a new arithmetic book. My colleagues at my first school and I wrote this and continued with a trigonometry book. My next writing started because of my interest in statistics. I was asked to write chapters for two year-12 new mathematics books, and we went on and co-authored text/work books for slower students. The third phase was with authors that I had brought together. We co-authored a second generation of New Zealand texts for new mathematics, and to keep out of mischief, I published them—this is known in the trade as ‘vanity press’.

At the time I believed that textbooks made a difference. I enjoyed the ‘guru’ status, the supplementary income, and the profile I gained from writing. I became aware of the awful ‘power’ of resources when students in class would not change a wrong answer in the back of the texts because ‘the text book must be right’, and when teachers wanted their planning done by the text rather than thinking through the issues for themselves.

### *Head Office*

My next job was as a public servant/bureaucrat. I stopped writing textbooks because of the conflict of interest, but there was writing and editorial work to be done with curriculum documents and teachers guides. In 1984, my first year in this job, ICME was held at Adelaide and this started my overseas conference addiction.

The curriculum project I led was for years 11 to 13. It was planned to follow a 1–10 compulsory curriculum but to provide flexibility for alternative courses in the last three years of school. Over five years teachers thinking moved to a general acceptance of most of what the project team suggested, but a restructuring of the department, the closure of the curriculum development division, and the emergence of a new Ministry of Education with a different philosophy on curriculum overtook our initiative. Our curriculum was never implemented and some colleagues expected me to be disappointed—my feeling was that curriculum is a process not a product and that some people who would be involved in the next project had benefited from their participation in our one.

I was also involved with mathematics in the context of Maori education. This arose as part of the emphasis being put on Maori education and also my interest in ethnomathematics

from ICME. My main role was getting funding so groups could meet but I took part in some of the discussions and learnt a lot from this participation.

An incident occurred while I was representing New Zealand in Japan at a three-week workshop. The sessions were in English so that English conventions would prevail and younger people could question and disagree with older participants. Outside the conference I hardly heard any English. Over the three weeks I noticed that my perceptions changed, I saw things differently and was seeing more and was surprised by this altered perception.

Another incident I remember involved a teacher writing letters to influence policy matters although he had only made a minimal submission when given a chance to comment. Identical letters were sent to the Prime Minister, the Minister for Education, the Director General of Education, the Head of the Curriculum Development Division, and to me as Mathematics Curriculum Officer. Within a week the five letters were on my desk awaiting replies. I knew the teacher so I succumbed to temptation. I replied to the first letter on behalf of the Prime Minister, xeroxed four copies, then with a pen crossed out 'Prime Minister' and wrote the other titles. I stapled the five together, put them in one envelope and posted it. Later I met the teacher, he saw the funny side of it, and we discussed the incident and other ways of working.

In hindsight, a concern with the curriculum division was its structure. It ensured a continuation of a subject-based approach in schools. Some work was done on a curriculum framework but the Western partitioning of knowledge by subjects was never questioned. This may disadvantage students from other cultures, and reinforce the false dichotomies set up by Descartes that I am uncomfortable with. I found the curriculum division personnel were like high school heads of departments, and colleagues at university—concerned about their subject and status, reluctant to cross boundaries, cooperate fully, or question assumptions or bosses. (I wonder what this says about our education system.)

### *University Teaching*

In 1989 after the restructuring of our educational administration I was to be out of a job so I looked for a fresh challenge, and obtained my position at University. In this I am involved with graduate courses, research and supervision and not pre-service teacher education. I was appointed because of my background but had to upgrade my qualifications. I had perceived little take-up of innovations and curriculum during my time in head office so I chose professional development for my research topic. This made me aware of many misconceptions I had about teacher learning. Upgrading my qualifications also gave me an opportunity to design my first research project, and to become aware of how teachers feel when they return to study after some years.

Our centre was set up for education research in science, then expanded to include mathematics, technology and computing. Unfortunately there is little cross-fertilization between the disciplines and the centre is mainly an administrative unit with distinct subject foci. As I was teaching the first maths education masters papers in New Zealand it was important to network and I am thankful for the help I was given by colleagues from this association. This need also provided opportunities for me to satisfy my 'conference habit'.

Our research methods class and the approaches used in student research often remind me of the differences between my friend in year 3 economics and me. Many students think the course provides them with specific ways to do research, while I think it introduces methods that need modification for particular situations and it is intended to open doors for them. So far the ones that have been the least bound by the course are those from overseas who recognise cultural inadequacies in standard approaches.

An aspect of university study that concerns many students is that they think they are to learn what others think (and cite the sources of such ideas), but are not allowed to have their

own opinions. This is unhealthy and unproductive. To encourage people to enquire and to foster their self respect, we need to pay attention to what they think, and my courses need to change so this happens more. An example of this from a school arose with two teachers from Fiji. Their problem and discussion went something like this—a mother has four children, they are all girls, she is expecting another, what is the probability that it will be a girl. One teacher said that her class had discussed this in biology and were confused. The other said that she knew that the answer I wanted was  $1/2$  but that the sex of a child is decided by God, not by probability. This reinforced for me that there are many viewpoints, that mathematics should be tentative when it presents a model, and we have no right to undermine models from biological or religious education.

An incident on sabbatical in Sweden at Luleå was important for me. I was researching science museums. I had been observing visitors at Teknikens Hus and was impressed by the quality of interaction with exhibits that was different from what I had seen at other interactive science and children's museums. I sensed engagement rather than the enjoyment and entertainment that I saw elsewhere. I was interviewing the Director and she told me that she had visited many other museums and had seen how they took a 'big idea' from science or mathematics and developed an exhibit to demonstrate it. She had not studied education but had heard the expression "start where the learner is". On this basis the exhibits at Luleå had been set up starting with familiar things that visitors knew, and then problematising the situations, and moving towards the underlying 'big ideas'.

At university we are getting more overseas/international students. The assumption seems to be made that if these people come to our universities then they want our degrees. However, fundamental to education is the notion that was reinforced for me at Luleå, one should start where the learner is. My concern is that we know very little about these students—their languages, beliefs cultures, attitudes and values, ways of thinking, and what they bring to education. Many are Asian and there are books that might help (for example Bond, 1996 and Watkins and Biggs, 1996), but how many of us have read them?

This increasing number of overseas students is partly due to an enticement policy to which the euphemism globalization is sometimes attached. Whether the real aim is international aid or whether it is pure greed is debatable but I wonder if it is not just a new form of colonization. I think that rather than working towards globalization we should celebrate cultural diversity. Diversity and differences seem to lead to new understandings, they make new syntheses possible, and they open the door for creative ways of seeing things. If we are to help people from other countries then we need to help them set up more tertiary institutions, or at least find their needs rather than satisfy our own.

After finishing my doctorate, I enrolled in an art class. At session one the tutor had us draw a boat, a house, and a person. He came round and gave us each a mark between 8 and 18. I scored 8. He then drew a series of pictures of boats, houses and people and labelled them all 8, 10, 12, ... 18 and told us that these were typical of drawings by children of these ages. He suggested that we would have been these ages, when we had been told we could not draw. Eight weeks later, with 2 hours each week, we were all drawing pictures of each other and of other objects. The tutor started by getting us to draw what we saw instead of what we thought we saw. He had high expectations of us and we met his expectations. Reflecting about this class has been valuable. It was based on non-cognitive rather than cognitive knowing with many techniques intended to make one focus on the unfamiliar. Secondly, art is the construction of representations (models, metaphors?) of reality, and this is similar to mathematics. Thirdly, we became totally involved in the activities and time in class just whizzed by—I wonder why does this not happen in the same way when I am teaching mathematics? I am sure that I have lots to learn from how people engage with and learn other



subjects and the separation of mathematics education from other forms of education may not be helping this learning process for me.

### *Missed Opportunities and Lessons Learnt*

From my experiences in these work situations I have become aware of some of the mistakes I have made, or put less negatively, some of the opportunities I have missed. Before looking forward, I want to briefly summarise these:

- My jobs (and aspirations) have actions implied by their names— teach, write, develop the curriculum, lecture, profess. I have done these things rather than facilitate learning.
- I do not remember being taught how to learn, nor was this a focus in my teaching.
- I have not often used student-chosen extended projects for study in mathematics. (Nor one problem per period that Stigler (1994) has discussed from Japanese classrooms.)
- My emphasis in teaching and resource production was the subject not the student.
- I have focussed on subject specific outcomes rather than on students' broader concerns, and have opened doors to specific pathways, instead of to many opportunities.
- My teaching focussed on learning objectives rather than on learning activities and I lacked an understanding of how learning occurs co-incidentally.
- My expectations of students have often not been high enough.
- I accepted the need for assessment when it was not contributing positively to learning.
- I assumed that punishment had a place in education.
- I have not always had an adequate idea of what my students already knew, what their interests were, or what might be suitable contexts in which to put my teaching.
- I have questioned the partitioning of knowledge by subjects but I did not seek ways around this problem at the alternative school or in the curriculum division.
- I have not adequately dealt with the complexity and interconnectedness of knowledge across subject boundaries or to the other aspects of the lives of students.
- I have concentrated on operationalising the aims of mathematics education at the expense of the more general aims of education.
- I assumed that analytical and logical thinking are the most important ways of knowing.

I would not want anyone to get the wrong impression. In spite of these many missed opportunities (and these are only the ones that I have time to recount), I do look back on my work to date with satisfaction and a feeling of achievement. I am also very aware of some of the lessons I learnt, these include:

- Teaching (and supervision) involves learning by all parties involved.
- Respect for students should be automatic; respect for teachers needs to be earned.
- Authoritarian schools and classrooms do not help students learn self-discipline.
- To gain power, the best thing to do is to give it away.
- Students of all ages want to learn but many have been taught not to.
- Teachers make a difference, success makes even more, and teaching needs to change so all students experience meaningful success.
- Teaching is neither a necessary nor a sufficient condition for learning.
- Assessment and evaluation of students and teachers do not contribute to learning.
- Students of all ages need to be trusted. Learning to trust and to be trusted sometimes involves making mistakes. We all need a second chance, a third one, a fourth one, ... .

- Students, parents, teachers, and bureaucrats all know what is best, but they can't agree. In this situation the views of the learner deserve to take priority, and the convictions of the teacher should take precedence over curriculum and/or regulations.

## Part 2: Looking Forwards

Changing direction? Looking forwards is not dichotomous from looking backwards. My past, my missed opportunities, and my lessons learnt influence the things that now interest and concern me. In linking these I will use four interrelated foci implied by our association's name—mathematics, education, research, and community.

### *Mathematics*

We often think of mathematics as hardly changing. In the six decades of my life there have been many changes in school mathematics, see Table 1.

Table 1

#### *Six Decades of Change in High School Mathematics in New Zealand*

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1940s	<ul style="list-style-type: none"> <li>- leaving age raised and some high school education compulsory</li> <li>- mathematics becomes part of compulsory core in education</li> <li>- 'core' mathematics introduced for non-academic students</li> </ul>
1950s	<ul style="list-style-type: none"> <li>- change from Euclidean to coordinate geometry in senior mathematics</li> <li>- calculus introduced into school mathematics</li> </ul>
1960s	<ul style="list-style-type: none"> <li>- growth of mathematical associations (and mathematics teacher journals)</li> <li>- disappearance of 'core' mathematics and growing acceptance of 'maths for all'</li> <li>- introduction of 'new maths' (with an emphasis on sets, logic and structure)</li> <li>- start of computer clubs in schools teaching simple programming</li> <li>- introduction of transformation and vector geometry</li> <li>- decimal currency introduced</li> <li>- applied mathematics (mechanics and statistics) replacing mechanics</li> <li>- experimental approach advocated for senior statistics</li> </ul>
1970s	<ul style="list-style-type: none"> <li>- metric system replaced imperial units</li> <li>- more emphasis on practical work at all levels</li> <li>- increasing emphasis on calculators and then on computers</li> <li>- mechanics returned to physics</li> <li>- increasing acknowledgment of needs of less-able senior students</li> </ul>
1980s	<ul style="list-style-type: none"> <li>- computer studies replaces computing taught within mathematics</li> <li>- increasing emphasis on mathematics rather than arithmetic in primary schools</li> <li>- swing back from the excesses of 'new maths'</li> <li>- project work emphasised (and used to supplement external assessment)</li> <li>- trend to internationalisation (Cockcroft Report, 1982; ICME Adelaide, 1984)</li> </ul>
1990s	<ul style="list-style-type: none"> <li>- mathematics positioned within a curriculum framework</li> <li>- statistics included at all levels of schooling</li> <li>- emphasis on ethnomathematics and Pangarau (mathematics for Maori in their language)</li> <li>- increasing emphasis on processes (doing) as well as on content (knowing)</li> </ul>

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In considering the future we need to ask some questions. What mathematics should be taught, is less mathematics required as society relies more on technology, if mathematics is useful in contexts, should it be taught in these contexts? In answering these questions, who

should have a say; whose mathematics is it; and should we listen to the voices of students, of our communities, or only of educators? Should we focus on Western mathematics? What is the future of mathematics in education and society, what sort of society do we want, and can we separate school from society? Groups such as Radical Statistics work for change in society, should we do the same? If we do not, will mathematics be the Latin of the 21st century, will it be important or impotent? The answers to these depend on our aims for teaching mathematics and our philosophies of education, but do we debate these enough? Scholarship is needed to move past the thinking of Descartes and others, to address basic issues about society and what we value, and to make our assumptions problematic.

### *Education*

Historically psychology broke from philosophy, then education splintered from psychology, now mathematics education is separate. With the notion of 'making connections', is this useful? When I was at teachers' college the foci were mathematics and education, now they are more complex. Theories about knowledge, learning, teaching, curriculum, and assessment are not always consistent; lecturers espouse theories as if they are only relevant to their subjects; prospective teachers do not seem to construct generally applicable theories nor develop personal theories. "What's the Use of Theory" was asked by Thomas (1997), and deserves consideration. For me a theory is a model, it provides a way of looking at something, it is not reality, it is a metaphor, a representation or mapping of reality, and like all metaphors, it highlights some things and ignores others.

While wondering about the need for theories, I find myself wanting them. This may be because of my background and my lack of experience with experiential education and mindful awareness that has been written about in phenomenology (Merleau-Ponty, 1962), in education (Varela, Thompson and Rosch, 1991), and in mathematics education (Davis, 1996). Such ways of knowing were highlighted in the sixties when Eastern philosophies were gaining recognition. More recently Belenky, Clinchy, Goldberger, and Tarule (1986) stressed gender differences with separated and connected knowledge (that also emerge with some non-Western people), and Mason and Spence (1999) have discussed knowing-to as distinct from knowing -that, -how, and -why. I think that these notions need consideration.

The word education comes from the Latin *educere* which means 'to lead forth'. Do we know where we are leading our students? and are we leading them or telling them? We can only theorise about learning, teaching, curriculum, and assessment when we know what we (and the community and politicians) mean by education. We need to work towards what we believe in and be explicit in terms of our assumptions so those concerned understand where we are coming from even when they disagree with us.

When looking for theories we are influenced by the accessible Anglo-American ones, but there is much elsewhere. When behaviourism was 'in' we took little notice of Gestalt education which was written about in German. With constructivism Vygotsky's work was not translated for years. In the same way much has been thought about in the East that we ignore. As Engler (1984) wrote regarding Buddhist psychology, when talking about the "I":

... the self, as it has been understood from the perspective of newer developments in ego, object relations and self-psychology, is now seen to be exactly what Buddhist psychology and practice have always said it was: namely, a mental representation or construct, not an entity.

Related to ways of thinking is language. Looking at some differences between languages in the East and West highlights some of the things that reinforce the dualism of Descartes. Some comments from Kelman (1958) illustrate this:

What characterises the East is the subjectifying attitude, the West, the objectifying one. Eastern cognition is interested in consciousness itself. Western cognition is interested in the objects of cognition. ...

East and West differ in their attitudes towards time. ... The West attempts to define it explicitly, to oppose and dominate it, only to become its victim. The east shows pronounced discretion towards time.

Our language is noun-oriented. We make propositions about things. Languages which are verb-oriented make propositions about events. They are more suitable for communicating immediacies. Our language, subject-predicate in form, creates a dualism ... Process languages facilitate experiencing. ...

The East has much to offer and in understanding their contributions, perhaps we will gain an understanding of our international students who come steeped in different traditions. There is considerable research on learning and teaching, but little on curriculum. Teachers see curriculum as beyond their control—nothing could be further from the truth. Curriculum depends on teachers. They implement it by modifying official versions to get teachable versions. They operationalise it by finding what works, they put ideas that advance it into school versions and into the next rounds of official curriculum writing. I would like curriculum reconceptualised as ‘process’ rather than as ‘product’, a shift from specific objectives to more general ideas and their interrelationships, and for flexibility to be built in so that innovation and local needs are encouraged. I would prefer official documents to be used for checking to ensure coverage rather than for planning. Associated with this is assessment—while learning and curriculum have moved past behaviourism, summative assessment has not. In some ways it is used for motivation with its emphasis on learning for qualifications rather than for delight. I feel that learning, teaching, curriculum and assessment need to be much more integrated.

The work on learning theories has been divergent. We have a range of constructivist theories according to one’s emphasis on individual or social, and ones view of knowledge as being objectivist or relativist. If one considers enactivism (Davis, 1996; Sumara and Davis, 1997), which is an emerging theory that moves past constructivism and takes a more holistic view of knowledge, these differences can be extended to include cognitive and non-cognitive knowing. Some influences on enactivism are from:

- phenomenology (Merleau-Ponty, 1962);
- ideas about Cartesian dichotomies (Damasio, 1994);
- non-cognitive knowing (Maturana and Varela, 1987), which includes ‘mindful awareness’ from Theravadin Buddhism (Nhat Hanh, 1987), from Zen (Batchelor, 1999) and from other Eastern thinkers (Krishnamurti, 1956);
- neural biological work which emphasises evolutionary or Darwinian notions (Edelman, 1987; Plotkin, 1998; Sacks, 1995; Varela, Thompson and Rosch, 1991);
- systems theory (Bertalanffy, 1968); and
- notions of embodiment (Varela, Thompson and Rosch, 1991; Lakoff and Johnson, 1999).

I can only give a hint of what enactivism means for me and will do this by providing some ideas from Dawson (1999), but I suggest that we need to familiarise ourselves with this theory. Enactivism involves becoming aware of what you are doing without judging it, and it moves *from a culture based on judgement to one based on possibility*. It requires us to see knowledge not as independent of individuals and their environments, or as something that can be tested/matched against external standards, but rather as embodied action with all of us being responsible for our actions. Enactivism and embodied action mean we maintain the *yin-yang* of self and world, and of ideas ‘out there and ‘in here’. From this perspective the aim in teaching is not to link learners’ experiences to some external curriculum, *but to view the curriculum as being occasioned by the learners’ experiences in their school environment*. Such ecological perspectives are located within a complex web of relations with all *decisions and actions being both constrained by and influencing all nodes of the web*. The enactivist classroom could be construed as a dynamic system with *the teacher listening not to check or*

*model the students, but to participate* with them (Kieren, 1995). Whether one accepts enactivism is not the issue, consideration of alternatives shows how ideas are added to the field we started to explore two decades ago with constructivism.

I have three other concerns. With lecturing and supervision, little has changed; and some on-line alternatives seem no better. Secondly the affective domain—McLeod and Ortega (1993) noted the role of emotions in learning mathematics before Goleman (1996) wrote on emotional intelligence, but are we doing enough? I used to think that those of us drawn to mathematics rather than the humanities were the cold, rational and unemotional types, but looking around this room I do not have evidence for that hypothesis. Thirdly, having enjoyed working in an alternative school, I am heartened to see people addressing the challenges from critical mathematics education (Kietel, 1994), liberatory pedagogy in mathematics and teacher education. (Price and Ball, 1998), and alternative education (Uhrmacher, 1997). However, my concern is that most teachers and policy makers do not heed these challenges.

### *Research*

In discussing research in mathematics education I will mention some concerns I have, these are from the past but need attention in the future.

Firstly where do we see research sitting with respect to theory and practice? I used to think of these aspects of our work as vertices of a triangle with the links between them being the sides of the triangle. The lengths of each side varied according to the relative importance I gave the links, but this image distanced the aspects from each other. I prefer sides of zero length and to think of the aspects as constantly interacting parts of a whole.

This leads to my second point, how broadly do we conceive of research? For some there is little acknowledgment of scholarship, and does scholarship include creative work? The New Zealand Qualifications Authority (1995) believes it is and that we also need to recognize consultancy and professional practice as often equivalent to research. I think we should take this more inclusive view of research in our discipline.

My third worry relates to scholarship. At conferences speakers talk of people like Gadamer, Heidegger, and Merleau-Ponty. Some attendees do not link these with our discipline but instead, they quickly back off. This seems to me to be an odd reaction when there are new ideas to engage with. Have we learnt the caution that perhaps we teach our students, are we afraid to take risks? How can we encourage others to inquire if we do not? In my case I know I am often unfamiliar with such people's work, but a conference of peers, like an alternative school, is a safe place to make mistakes. Why should we worry if our questions are naïve? They are often the questions others wanted to ask!

A fourth concern is to do with analysis and synthesis. In schools we analyse to get our objectives instead of synthesising for general ideas and their interrelationships. Research seems similar. Bishop (1998) made this point when commenting on research at an ICME conference. We seem to become so concerned about details that we ignore the total situation—we concentrate on immediate aims and forget about long-term goals. This emphasis on analysis may reflect Western ways of thinking and may make it harder for non-Western people to value our research.

My fifth notion is to do with language. Phrases like “the researcher found” are often used, but people do research. A person selects a topic, decides what data will be relevant, makes decisions about the sample, decides what will be asked or observed, selects from the data, and interprets it with this interpretation being based on prior experiences. Research is personal, we need to be honest and use “I” in our reports.

My sixth idea also relates to writing up research. The usual form of a thesis or report follows a traditional format. It was exciting for me to hear O'Reilly (1998) share with us how

she had written her doctoral thesis using 'plateaus'. Later I read some of Deleuze and Guattari (1988) for a background on this term. Instead of focussing on findings, plateaus focus on the process and the changes that occur in the researcher's thinking while involved. I think this alternative is to be commended and we need to consider other possible formats.

My seventh concern is about what we measure and why. In economics the easy thing to measure is financial costs and we avoid social, personal and environmental costs that can not easily be put into dollar terms. The situation is similar in research. As Giorgi (1970) said, the problem is that for something to be measured only its tangible aspects can be apprehended, and, as a result, the indices associated with these aspects of the phenomenon become more important than the phenomenon. In the search for the measurable element one loses sight of the significance of the other aspects of the phenomenon. The non-measurable aspects tend to be discarded, and only the measurable elements are included.

My eighth worry is to do with time scales. I have been 'guilty' of collecting data and quickly interpreting it. The alternative is to take time living with it for a while, getting the feel of it, and interrogating it. Merleau-Ponty (1962) talks of the need to not merely describe what one perceives in a superficial way, but to become more mindfully aware of what one perceives. This for me relates to time for research and different ways of knowing.

My ninth point is why do research when we know that educational policies are not research-based? 'Policy' shares its origin with 'politic' and educational policies are political (and economic). Mason (1998) suggested that the fundamental purpose of research is the growth of the researcher. I believe it is this and more, research is finding out for ones self, which is the purpose of education. Our emphasis should be on this process rather than on outcomes. Currently primary school students do lots of (research) projects, high school students seem to do fewer, and undergraduates even fewer—they should all be doing lots.

Three final comments on research: I know we want independent researchers, but sink or swim is not the way to achieve this; we need to be more concerned about research supervision. Next, we have much in common with researchers in other disciplines and we need to have more contact with them. I would enjoy joint conferences where mathematics educators and people concerned with other subjects discuss research together. My last comment is a generalized finding from science to keep in mind, it is Newton's third law for researchers, "for every researcher there is an equal and opposite researcher".

### *Community*

Group, Australasia, incorporated, and association imply collaboration, collegiality, and cooperation but these can not be assumed to be valuable. Collaboration can mean *working traitorously with the enemy*. Hargreaves (1994) talks of 'contrived collegiality'. Ball (1999) spoke of fabricating cooperation by putting more names on papers to increase research outputs for the bean counters (and using the salami methods for getting more papers off a project by cutting the slices very thinly). Politicians would argue for competition.

While we see ourselves as a community, we are also part of a larger community of educators, and of the society that we live in. Developing community requires us to listen to all the voices from this society because, although we might think that they are uninformed or conservative, it is their children's education that we are influencing.

Is the word community a verb or a noun? For me a noun suggests a set of people (peers, students, supervisors), but a verb implies an interacting between the people.

One final comment—incorporated means brought together into a larger body, and often this is to avoid (financial) risks. This suggests that we either fear taking risks, or want to take risks but avoid dire financial consequences. I hope we are in the second category.

## Conclusion

Perhaps this sounds like a sermon with me telling you things you know already or would rather not know. This was intended. My professional responsibility is to keep stirring; although I wonder if this sermon will make any more impact than others you have heard. Our work often seems confined by boundaries and limits, but I believe we impose many of these on ourselves. I see taking risks not as putting our heads on the chopping block, but rather as flying free. I hope that in some small way I might have 'opened a door' or boosted someone's confidence to do something differently. Kia ora katoa.

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