Designing Constructivist Computer Games for Teaching about Decimal Numbers

Janine McIntosh
University of Melbourne
<j.mcintosh@edfac.unimelb.edu.au>

Kaye Stacey
University of Melbourne
<k.stacey @edfac.unimelb.edu.au>

Calvin Tromp
University of Melbourne

Darren Lightfoot
University of Melbourne

This paper reports trials of two computer games designed to enhance learning about decimals. In two exploratory studies, students used especially designed computer games that focussed on aspects of decimal understanding. These games were effective in challenging children's misconceptions about decimals. Students developed strategies for dealing with decimals, assisted by teachers. This article considers the nature of teaching assistance that could be programmed into the games to further strengthen their usefulness as a tool for increasing understanding about decimal numbers.

Solid understanding of decimal number concepts is important in many aspects of everyday life, such as when dealing with money and measurement. Learning about decimals is also important for successful academic performance in mathematics and other subject areas at high school and beyond. The Victorian Curriculum and Standards Framework suggests that decimal numbers should be taught in late primary school, from Grade 4 onwards. Around 25% of grade 4 students quickly develop a good understanding of decimals. However, only about 70% of year 10 students in schools in our region have achieved mastery of the decimals concept (Steinle and Stacey, 1998).

International and other Australian research shows that this is a widespread problem (Stacey & Steinle, 1998; Wearne & Hiebert, 1988). This research has established that there is a wide variety of misconceptions that interfere with new learning. Stacey and Steinle (1998) present a substantial listing of misconceptions and Stacey and Steinle (1999) demonstrate that for many children, misconceptions persist through years of schooling.

This paper reports on the development of two computer games designed to assist children with misconceptions about decimals. The games are intended to be used by children at home or in class to supplement normal instruction. Our unpublished work shows that the incidence of misconceptions about decimals can certainly be reduced by even a little targeted teaching. However, not all children get this and some of them do not get it when they need it. In this context, we decided that computer games may be useful addition to the pedagogical armoury.

This paper reports two exploratory studies to guide further development of two computer games. The principal aims were to find out what children learned from playing the games and to design appropriate teaching to be programmed into the games. In addition, the studies field-tested features of the games, such as the attractiveness of the "storyline", the clarity of instructions, technical issues such as the timing of display of feedback and so on. These results are not reported here, but they confirmed the general directions taken and assisted in fine-tuning the presentations of the games.

Computer Games as Teaching Tools

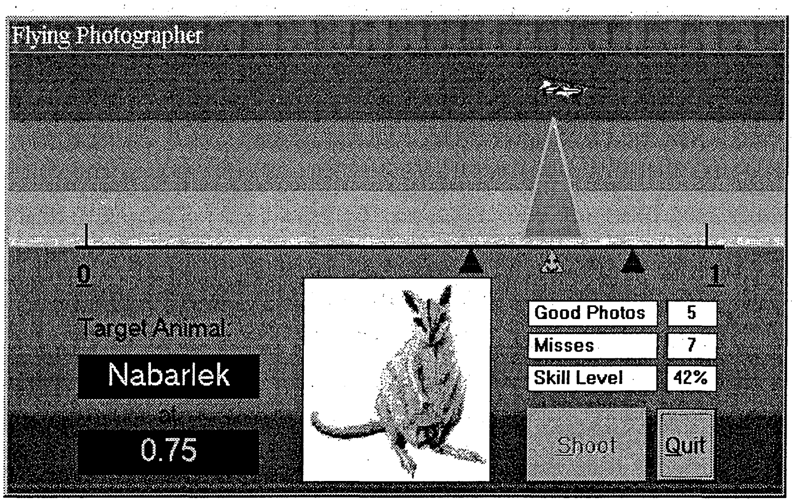
The impact of computers in education has been much discussed and many claims and recommendations have been made. Studies have shown that computer use in mathematics can heighten student interest and cater for a range of abilities, encourage students to pose and solve "What if...?" questions, allow for multiple representations of concepts and encourage the use of global strategies to solve a broad range of problems (Heid, 1997). Heid reminds us that it is important for games software designers to find out how different students use the games and suggests that the games' effectiveness is reliant upon how visible the mathematics is to the person using the game. Mathematics games software research stresses the need for games: to fill a gap in the current curriculum; to focus on the mathematics; to avoid making mathematics a hurdle requirement (i.e., having to perform a calculation before progressing to something exciting); and to lead the user to progressively deeper mathematical thinking. (Murray, Mokros, & Rubin, 1999).

In contrast to many "drill and practice" computer games, underlying the design of our games is a constructivist view of the teaching of mathematics. They have been designed within the literature on teaching that addresses misconceptions and challenges student's thinking. Bell's (1993) principles for the design of teaching have been useful. The games are designed around a variety of tasks which can highlight the errors that students make and present them with cognitive conflict to be resolved. Bell notes the importance of timely feedback, and this occurs within all our games. A degree of intensity of experience is provided by games - students can play as many times as they wish, with constant feedback, and the element of fun that games engender can make the experience more memorable. Bell notes the importance of adjusting the degree of challenge of the tasks that learners are faced with - in a computer games potentially this can be done in very sophisticated ways and a Bayesian network is being developed to do this (Dettman, Nicholson, Sonenberg, Stacey, & Steinle, 1999). Finally, Bell notes that the student should be exposed to a broad range of situations where the concept is illustrated in many ways and that this can be done by making changes of element, structure and context. This is achieved across the set of games, not in any one game.

In Bell's diagnostic teaching, cognitive conflict is resolved through children working together on tasks followed by teacher-led discussion, which also encourages reflection and review of learning. This cannot be guaranteed when children are playing a computer game, although students working in pairs can help. The computer game genre is often not one of thoughtful consideration and so it is a particular challenge to program useful teaching into the games. Useful literature on providing teaching within a computer game is scant. Most of the literature is concerned with on-line assistance for teaching the user to use large applications such as word processors (e.g., Duffy, Palmer & Mehlenbacher, 1992). The studies described below therefore wished to investigate whether the planned programmed teaching was indeed useful, whether children would ask for it when it was needed or whether it should be provided automatically when the student appeared to be making errors.

Study 1: Flying Photographer

The aim of this study was to observe what children learn from playing the game Flying Photographer and to investigate what teaching may enhance it. Flying Photographer (see Figure 1) was designed to develop student's appreciation of the size of decimal numbers as reflected in their positions on a number line. The student is in an aeroplane flying over a national park and has to photograph animals whose location is given as a decimal number on a number line. For example, locations in the (relevant part of) the national park may be given by numbers on the interval [1.2, 1.3] and the photographer may be seeking a potaroo at 1.254. The game is about estimation, not precision, so it moves fast and there is an (adjustable) tolerance interval. Flying Photographer gives immediate feedback, rewarding correct responses with a dialogue box that reads "Well Done!", a picture of the target animal and the correct location of the decimal on the number line is shown with a green triangle with a happy face on it. Flying Photographer indicates incorrect responses with a red triangle with a sad face in the photographed position. After three unsuccessful attempts to locate the number, the correct position is shown with a green triangle and a dialogue box directs the student to it with the comment "Wrong. Look at the green triangle."



Note. The endpoints are 0 and 1. The player has successfully located a Nabarlek at 0.75 on a third attempt, indicated by the triangle with a happy face. This is the twelfth item in this game; the player has located five num-bers successfully and made seven incorrect responses, giving a skill level of 42%.

Figure 1. A screen shot from Flying Photographer.

Method

The Decimal Comparison Test (DCT), was administered to one year 5/6 class in an inner city primary school. This 10 minute paper and pencil test requires children to choose the larger of pairs of decimal number. The pairs of numbers are chosen so that misconceptions can be quite precisely identified, using methods outlined by Stacey and Steinle (1998). Four students who all have the same misconception about decimals were selected by the teacher to trial the game. All of these students see the decimal point as separating two whole numbers, rather like a time written in minutes and seconds. As a result, they believe longer decimals are larger. When confronted with a pair of numbers like 0.36 and 0.5 they choose 0.36 as the larger because thirty-six is bigger than five. Students with the same misconception were chosen so that the questions which they are most likely to get right and wrong were the same (and thus could be controlled by the experimenter). This particular misconception was selected because it is the most common misconception amongst year 5 and 6 children in Australian and international studies (Steinle & Stacey, 1998).

A task-based interview was used by a student teacher well known to the students. The children played two games of Flying Photographer to familiarise themselves with it with the researcher's help. After the initial two games, he also provided the teaching, either when requested by the child or when the child made three errors in a row. We distinguish these as solicited and unsolicited teaching. In both cases, the teaching consisted of placing a clear plastic overlay over the number-line on the screen, dividing it into finer gradations. If, for example, the endpoints of the number line were 1.2 and 1.3 , then the gradations marked 1.21, 1.22, 1.23, and so on. If the endpoints were 0 and 1, then the overlay marked 0.1, 0.2, 0.3, and so on. The overlay was in place for only one item then removed. The researcher observed and took extensive notes, observing any difficulties students had or comments they made. For analysis, these were incorporated with the computer file showing all the moves made by the child. On occasion, the researcher asked the child questions for clarification.

Results

Table 1 shows that the students all followed a general trend toward improvement in their ability to place decimal numbers on the number line as they played more games with the same endpoints. When the game presented new endpoints, students had to learn again. For example, Table 1 shows Student A was able (after at most three trials) to locate 55% of the specified numbers on the interval [0,1] in the first game. This improved to 100% by the fourth game. (The dashed line under *skill level* for game three shows that the player has prematurely quit that game, possibly accidentally.) When the interval was changed to [1.2, 1.3], the skill level dropped as new tasks were presented.

The shaded boxes indicate where Student A was shown the teaching screen. The results in Table 1 indicate that, when it was provided, the teaching screen was effective. Student C did not know what a number line was, so this was explained by the researcher and whole-number endpoints were used first. Student C had the most to learn, was given the most teaching and made good progress.

Table 1.
Results from Games of Flying Photographer

	Student A		Student B		Student C		Student D	
Game	Endpoints	skill level %	Endpoints	skill level %	Endpoints	skill level %	Endpoints	skill level %
1	0,1	55	0,1	70	Whole	45	0,1	85
2	0,1	83	0,1	87	Whole	75	0,1	85
3	0,1	--	0,1	88	0,1	32	0,1	94
4	0,1	100	0,1	100	0,1	88	1.2,1.3	54
5	1.2,1.3	--	1.2,1.3	37	0,1	93	1.2,1.3	54
6	1.2,1.3	89	1.2,1.3	17				
7	0,1	88	1.2,1.3	54				
8	0,1	100	1.2,1.3	-				
9	1.2-1.3	89	1.2,1.3	89				
10	Mixed	60*	1.2,1.3	100				

Note. * Average of three games

A DCT taken two months later showed that students B and D retained the same misconception. Student A however showed some progress, still thinking of the decimal part as like a whole number, but adding the understanding that 0 in the tenths column meant that the number was quite small and "close to zero". Student C made sporadic errors, indicating movement away from whole number thinking, but not yet solid understanding.

Strategies

As they played Flying Photographer, children developed four strategies, which do not show explicit understanding of place value, but rather show associated knowledge.

Benchmarks. Sometimes students successfully used known positions on the number-line to assist in placement of the number. For example, 0.25 was "a quarter of the way along" for student B. However, it was not helpful when he wanted to place 1.25 between endpoints of 1.2 and 1.3. He was expecting 1.25 to be quarter of the way along from 1.2 and was surprised to see the green arrow at the midpoint of the number-line and commented, "In the middle. I don't know. This is wrong, man."

Sections theory. When the teaching screen was used, student A saw the tenths divisions as naming sections of the number line. In a game with endpoints of 0 and 1, when presented with 0.95, he commented "that's in the 9 section". Similarly he observed that 0.66 is "in the 6 section and 0.098 would be "in the zero section". Student C also developed this strategy. He had placed 0.25 between 0.8 and 0.9, but upon seeing the green triangle commented "you go to the 2 and then five more little ones".

Zeroes in the tenths column. Student B observed that "zeroes in the tenths column make the number smaller". Student D also noted "I noticed there is a zero in it" after correctly placing 0.09. This is a significant step for students who regard decimal numbers as whole numbers would ordinarily regard 0.9 and 0.09 as the same number.

Ignoring initial digits that remain constant. When they moved away from the interval [0,1] three of the four students made comments that indicated that knew they could ignore the start of the number because it remained constant. Student A said, " if you pretend 1.2 is a zero then it is like the other game". Student B observed " 1 and 2 is at the start ... you don't count that". Student C made the comment, "you have to look at the first number and match zero with the number line". These students are beginning to reason in a way similar to putting words in alphabetical order. For example, once we have located *cats* after *catalogue* in a dictionary, we no longer look at the prefix *cat* to place *catnip*; we look only to the fourth letter of the word. Interestingly this strategy was not useful to student A in a game with mixed endpoints as can be noted from Table 1.

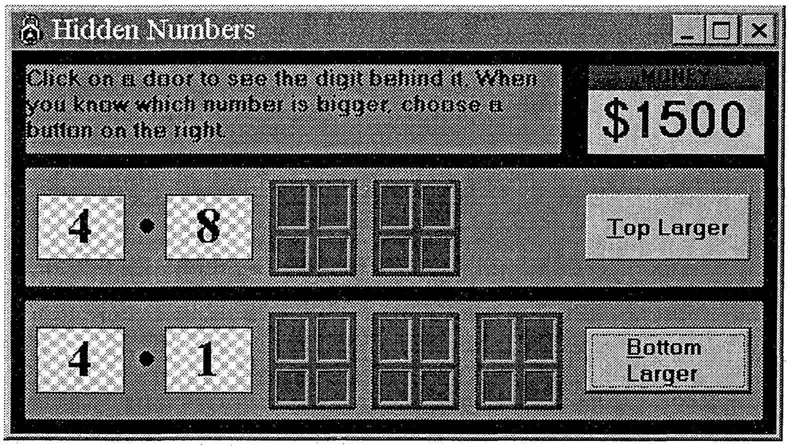
Was the Teaching Effective?

The programmed feedback of red and green triangles was extremely effective and used by all students. It aroused cognitive conflict in all students. One example of this occurred when student D was attempting to place 0.008 on the interval [0,1]. He expressed surprise at seeing the number near the beginning of the number-line. In a later game this same student had changed understanding and said, "I noticed that there's a zero in it" when placing 0.09 correctly near the beginning of the [0,1] interval. The researcher observed that less teaching was offered to the students than had been expected and that this was probably due to the success of the programmed feedback.

Only student A asked for teaching assistance and after game 5, he requested that the teaching screen not be shown, so he could work out the answers for himself. One other student was given the teaching screens. The teaching screen was helpful to those students and was probably the impetus for the development of the *sections* theory.

Study 2: Hidden Numbers

This study has the same aims as the first study, but has investigated a second game, Hidden Numbers. This is designed to combat misconceptions which judge the size of the decimal part of a number solely by its length, both longer-is-larger or smaller-is-larger. It also encourages the use of an expert *left-to-right* method of comparison of the digits in the columns in a decimal number. The common strategy of adding zeroes to the shorter decimal to equalise length is not a useful one for this game. This game requires the player to select which of two decimal numbers is the larger. Each digit of the decimal is behind a door, and students need to pay to open the doors, one by one. They can guess which number is the larger at any time. Hidden Numbers gives right/wrong *feedback* immediately a choice is made. The bank balance on the screen (see Figure 2) measures success.



Note. The player has uncovered the whole numbers and tenths. This player has a bank balance of $1500. A whole number thinker would incorrectly choose the bottom number as larger even after observing the greater number of tenths in the top number.

Figure 2. The Hidden Numbers screen.

Method

The DCT was administered to each child in a year 5 class from a Government primary school in the eastern suburbs of Melbourne. Fourteen of the 26 students in the class had the whole number misconception as discussed above, and 5 of these were selected by the classroom teacher to participate. The researcher was a student teacher, well known to the students. The game was programmed to present pairs of random decimal numbers, challenging to these children. An individual task-based interview was conducted as before. The results were logged in the output file and observations were recorded in real time.

Two forms of teaching were used. These had not been programmed but were provided on paper or simulated by the researcher on request or when a student had made a series of errors.

LAB demonstration. Linear arithmetic blocks (LAB - see Archer and Condon, 1999) were used to give the student a visual, linear model of the uncovered digits to date. So, for example if the student had uncovered a 2 in the tenths column of one number and 8 in the hundredths of another, the relative lengths of these represented in LAB were shown.

Unsolicited teaching - the pop-up screen. This was a simulated pop-up screen that read "Save money, open less doors" which was shown by the researcher when the player was regularly opening more doors than was necessary.

Results

Because the children had not seen LAB before, a short teaching session was conducted by the researcher before the game was played. Unfortunately for the research, this session may have been highly effective, because all students displayed a high degree of proficiency with the game, answering between 85% and 100% of the questions correctly. It is unclear if these results are attributable to the game or the pre-game LAB session.

None of the students requested teaching. Two of them received the pop-up screen and the LAB demonstrations. When they were asked after the game, all students reported that the LAB pre-game session was helpful and thought that a screen version of LAB teaching would be helpful.

Strategies

The students seemed to learn two strategies whilst playing this game. As with the students in the study above, students playing Hidden Numbers developed a strategy that is linked to understanding the effect of a zero in the tenths column. Students used this strategy to take what the researcher called an "expert punt". If one number had a zero in the tenths column, then there was a good chance that the second number was bigger. Additionally, all students showed some refining of the left-to-right strategy.

Discussion

Both studies found that these games were useful, although students developed knowledge associated with decimal numbers rather than direct place value knowledge. Interestingly, both games helped students to develop knowledge about zeroes in the tenths column to make judgements about the size of a decimal. Sackur-Grisvard and Leonard (1985) suggest that this adding of an extra separate knowledge of decimals comes next in a developmental sequence after whole number thinking. The *sections* theory was an unexpected outcome, which is part of the complex web of relationships that understanding the decimal concept entails. The experiments showed that teaching assistance should be programmed to automatically appear - students did not request it

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