Defining Moments in Determining a Complete Graph in a Graphing Calculator Teaching and Learning Environment

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The cognitive, mathematical, and technological processes undertaken by senior secondary students as they searched for a complete graph of a relatively difficult cubic function were investigated. Particular circumstances: that became defining moments in the solution were identified. Those related to students' responses to particular views of the function are presented. A variety of processes were used, however, a mental image of possible forms of a complete graph, understanding the use of graphing calculator features such as Zoom Fit, and the effect of each value in the WINDOW settings facilitated the students' solutions.

Barrett and Goebel (1990) predicted that "the presence of graphing calculators in high school mathematics classrooms [would] have a significant impact on the teaching and learning of secondary school mathematics in the 1990s" (p. 205). In particular, the introduction of graphing calculators impacted on the understanding senior secondary students have of what constitutes a complete graph of a function. Whilst the types of technology potentially available in mathematics classrooms around the world has expanded to include computer algebra systems (CAS), as recently as 2001, Zbiek pointed out that "secondary mathematics teachers … may find the lure of graphing capabilities is more compelling" (p. 3). For this reason, and to add to the knowledge base against which the use of other technologies can be evaluated, it is imperative research continues to be conducted into how students use graphing calculators in their learning of functions.

Rationale

In its statement on calculator use the Australian Association of Mathematics Teachers (1996) signalled the need for technology to be used by all students. In addition, attention is drawn to the fact that, simply having the technology available for use is not sufficient, be it CAS or graphing calculators, there is a corresponding need for teachers to actively seek to change teaching and learning experiences to take advantage of the technology. Employing graphing calculators can move the mathematics curriculum from the passive transfer of knowledge to students being in the position of taking an active approach to their learning (van der Kooij, 2001). Mitchelmore and Cavanagh (2000) echo the call of Penglase and Arnold (1996) for research to explore changes to learning functions when using graphing calculators. Clinical interviews, undertaken by Cavanagh, with twenty-five Year 10 and twenty-five Year 11 high achieving students found that "many difficulties in using a GC—even among high achieving students—may be due to inadequate understanding of some fundamental mathematical ideas including … the link between different representations of functions" (p. 265). Mitchelmore and Cavanagh suggest these difficulties are related to shortcomings in the curriculum that exist in both a graphing calculator and a non-graphing calculator environment. According to Dick (1996), graphing calculators provide more than an alternative way of doing the same old things when teaching functions; rather they allow us to do new things including actively observing "local linearity" (p. 35), allowing “manipulation of the viewing window” (p. 32), and “provid[ing] the power of zooming” (p. 44).

Positioning the Researcher

The context of this researcher’s experiences in teaching and learning is an important and possibly confounding factor in the study described. This researcher’s teaching of functions occurred simultaneously with her introduction to, and use of, graphing applications that included ANUGraph and spreadsheets. Hence, her thinking and teaching of functions is from a graphical (and visual) rather than from an analytical perspective.

With regard to being a teacher in the study, the view of this researcher concurs with that of Eisner (1997, p. 267), that as a teacher researcher in my own classroom I bring a richness of experience and expertise to the experimental/observational setting. I bring “insider knowledge” (Eisner, p. 265) to the research process. “Stake has argued that the best understandings of educational phenomena are likely to be held by those closest to the educational process” (MacDonald, 1980, p. 26).

The Study

“Qualitative methods can be used to obtain the intricate details about phenomena such as … thought processes … that are difficult to extract or learn about through more conventional research methods” (Strauss & Corbin, 1998, p. 11). They are particularly useful where the research is focussed on areas that cannot be described by directly measurable data, where the task is to “document, to grasp and to explain individual processes of mathematical thinking and acting” (vom Hofe, 2001, p. 109) as is the intent of this study. A qualitative case study (Merriam, 2002, p. 8) was therefore used as it was considered to be the most appropriate methodology to achieve the goals of the study, namely, to explore the nature of student interaction with a graphing calculator when working with a partner, the specific focus being student understanding of the graphical representation of functions, particularly of cubic functions.

According to Huberman and Miles (2002), and the interpretation used in this study, case study is a research approach focussing “on understanding the dynamics present within single settings” (p. 8). This case study describes the results of practice in the classroom of the researcher and a colleague as evidenced by a snapshot of the students’ responses to a problem task. Case studies are relevant to the researcher and others, including the participants, as only by increasing understanding of what occurs in classrooms are teachers able to become more effective. They also allow interpretation and evaluation of current practice, enhancing understanding and providing a meaningful guide to action (Merriam, 2002).

The case studied is the community of students studying functions in a graphing calculator learning environment as part of their study in Mathematical Methods in their final two years at a particular Victorian secondary school. This case was selected “because it [exhibited] characteristics of interest to the researcher” (Merriam, 2002, p. 179). Within this case student pairs were *purposively selected* (Merriam, 2002, p. 20). Given the limited number that can be studied in depth, pairs were selected so the process of interest was "transparently observable" (Huberman & Miles, 2002, p. 13) so as to maximise what could be learned. It must be acknowledged, however, that the way in which the students were prepared to participate in the study and expound their ideas may be a result of the situational context established by the teachers, a context in which “knowing occurs, and ... the learner is participating” (Barab & Kirshner, 2001, p. 5) in activities and an environment where students’ voices are expected to be heard. The two mathematics teachers of the students in the study worked closely together using a variety of methods emphasising understanding through exploration, discussion, and collaboration. Students in their classes tended to be accomplished users of graphing calculators and were expected to use them when directed by the teacher and at their own instigation.

Research questions. Specifically, the research questions to be addressed are

(a) What processes—cognitive, mathematical, and technological—are undertaken by senior secondary students as they search for a complete graph of a particular function using a graphing calculator?
(b) What mathematical knowledge and features of the graphing calculator are used by students in their response to the problem?, and
(c) Which actions of the students best facilitate the solution process?

Participants. Five pairs of students at an inner city Victorian state secondary college, were selected on the basis of being pairs of students who often worked together, and who could be expected to solve the problem and articulate their ideas as they proceeded with this task. Two pairs (Pairs 1 and 2) were from one Year 12 class and three pairs from a Year 11 class (Pairs 3, 4, and 5). Participants’ behaviour in experimental settings can be shaped by many factors. In this study, the safeguards taken included the purposive (Merriam, 2002) selection of pairs who (a) worked and talked together in class, so providing an experimental setting paralleling the classroom situation, and (b) were confident in mathematics. Moreover, the use of two person protocols helped reduce the pressure on the individual and was more likely to capture students’ typical thinking, thus reducing some of the limitations of verbal data (Goos & Galbraith, 1996; Schoenfeld, 1985).

The problem task. The task set was that of graphing completely a specific cubic function, namely, $y = x^3 - 19x^2 - 1992x - 92$. Binder (1995) had used this particular function previously to investigate a graphical approach to solving an algebraic problem. The task was selected on the basis that no part of the function is visible in the standard viewing window, being $-10 \le x \le 10$, $-10 \le y \le 10$, on a Texas Instruments (TI) TI-83 graphing calculator.

Administration of the task. Each task solving session was audiotaped, with students asked to articulate their ideas as much as possible. Students used a graphing calculator, set to the standard window, attached to a view screen and overhead projector. The calculator screen output was then videotaped via the overhead projector accurately recording the students’ actions. In addition, the researcher took observational notes. Students were provided with the written task and plain paper to record working out, or notes, in order to complete a hand-sketched solution to the problem.

Case record. Raw data in the form of tape recordings of the pairs of students undertaking the problem task and videotapes of the graphing calculator screen output allowed a permanent record of the students' interactions with the graphing calculator to be made. A protocol of each pair's efforts was produced by matching the combined recordings and student scripts. These were supplemented by observational notes where necessary. The protocols collectively formed the case record that was used to determine the behaviour of the students. The novel method of recording all graphing calculator screens allowed the researcher to assemble a more complete record than merely ordinary videotaping using two students would have allowed. The screens appear to capture more of the students' immediate thought processes than the words being uttered and recorded. Thus, not only do graphing calculators increase the learning opportunities for students, but also they provide the opportunity for teachers and researchers to witness more closely the understandings students have as inferred by the actions of students represented by the graphing calculator screens.

The Analysis

The protocols were coded and then divided into "macroscopic chunks" (Schoenfeld, 1992, p. 189) that were then classified according to the particular behaviours of interest associated with working with functions in a graphing calculator environment. These behaviours are referred to as *episodes* and "represent periods of time during which the problem solvers are engaged in a particular activity" (p. 189). In addition, time-line diagrams, detailing the length and type of episode undertaken, and the representations being used at any given point in time, were constructed. These were an adaptation by the researcher of Schoenfeld's time-line diagrams (p. 190) used to study problem solving.

Microscopic Analysis. The analysis in this study goes beyond the macroscopic analysis of Schoenfeld and was used as a way of looking for *defining moments* in the solution process that were then explored using microscopic analysis to search for explanations. The use of the graphing calculator screen data to supplement and enhance the dialogue data in the current study allowed a more finely grained classification than was possible by Schoenfeld's (1985) scheme. The protocols were coded using codes devised by this researcher to classify actions according to distinct behaviours specific to solving the particular problem task used. The codes referred to the following categories of activities: reading, organising or planning, selecting a viewing window, searching for or identifying a local feature, searching for a global view, adjusting scale marks, evaluating, and recording.

Defining Moments

After close analysis of the episode and time-line diagrams and scrutiny of the case record where this facilitated macroscopic analysis, a number of defining moments became apparent in the solution processes of the pairs of students. The term, defining moments, is being used to refer to important or momentous events rather than a particular instant in time. A defining moment was interpreted as being a *circumstance* where some action, cognitive or physical, or a decision (i.e., a metacognitive action), or a series of these may have had the potential to facilitate or impede the solution process. Defining moments, therefore, occurred at critical points in the solution process. Similar circumstances may have occurred in the solution processes of other pairs but it was the responses of the pairs to these circumstances that determined whether or not they became defining moments for a particular pair's solution. Each circumstance can be described in terms of the *situation,* the *condition* and the *response action*. For every circumstance the question that must be answered is: What situation gave rise to this condition that, in turn, led to this response?

A situation was the context in which the students found themselves, usually it was a specific stage in the solution process; however, it could be a general situation. The *ongoing search for a global view of the function*, for example, was a situation some pairs found themselves in for a substantial amount of time throughout their attempt at the problem task, whereas, the situation, *global view found,* occurred at particular points in the solution process, albeit during the solution process or at its end. One situation could lead to a number of conditions. A condition was the particular state that one or more pairs was observed experiencing or considering such as the sighting of an apparent no view of the function. Differing situations were observed in conjunction with the same condition. In turn, several conditions gave rise to the same response action. The same combination of situation and condition may have occurred in conjunction with differing response actions from different pairs of students. Questions asked of the data in identifying defining moments included:

- What circumstance or set of circumstances established this response action as a defining moment for this particular pair's solution?
- What condition elicited this response action?
- What situation led to this condition?

Findings and Discussion

The defining moments observed during the macroscopic analysis to be discussed in this paper are those related to how students responded to particular views of the function including their non-acceptance of an other than global view of the function.

Defining Moments Related to Students' Responses to Particular Views

This set of defining moments relates to how students responded to particular views of the function during their initial and ongoing *search for a global view*. The views seen included those of *apparent no view*, *apparent vertical lines*, a section of *the function coincident with the y axis*, and other *unusual or unexpected views*. Three response actions were observed in this set of circumstances, namely, the *use of Zoom Fit*, the *use of Zoom Out*, and the making of *adjustments to the WINDOW settings*. The sets of situation, conditions, and response actions are shown in Figure 1. During their initial search for a global view students responded by using Zoom Fit, Zoom Out, and adjusting the viewing window but only the latter two actions were used by the students in their ongoing search for a global view.

Response Actions: Use of Zoom Fit or Use of Zoom Out.

The response action of either *use of Zoom Fit* or *use of Zoom Out* in particular situations and circumstances were defining moments in the solution process as these actions determined initial success in accessing the problem task, allowed the students to avoid repetition of the vertical line view of the function or avoid it altogether, and appeared immediately after the initial selection of the standard window by Pair 1 and when Pair 2 set the WINDOW informed by the algebraic representation. Both of these actions were followed by use of Zoom Fit.

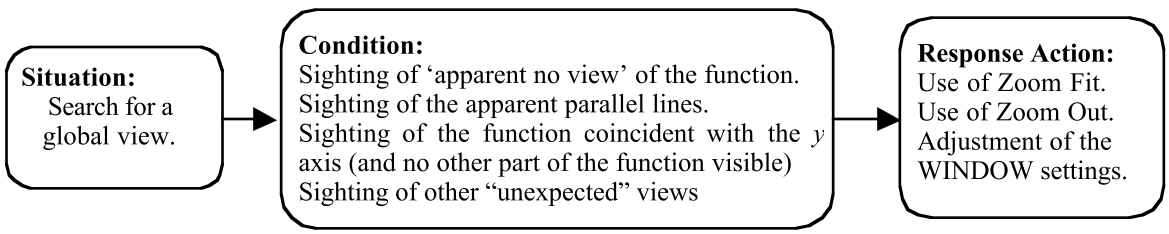

Figure 1. The circumstances relating to the defining moments.

Another instance occurred when Zoom Out was used by Pair 5 immediately after the selection of the standard window in conjunction with mathematical knowledge. The unsuccessful paths initially followed by Pairs 3 and 4 included no use of either Zoom Out or Zoom Fit. In contrast, the mathematical and graphing calculator knowledge and choices of Pair 1 enabled them to select a sequence of actions that ensured that their solution process quickly became routine. None of the other pairs of students consistently applied their mathematical and graphing calculator knowledge in conjunction with sensible choices of actions so their solution pathways were less efficient.

The students' actions on seeing an apparent no view of the function suggest that the use of Zoom Fit or Zoom Out was successful where success is defined as seeing a portion of the graph visible in the resulting viewing window. Some resulting views, for instance a section of the graph, particularly a view of the function easily locatable in the students' mental image of the function, were more helpful to students than others in their continued pursuit of a complete graph of the function.

All students encountered views of the function other than what were usual. It cannot always be inferred, and nor should it be, that these views were either unexpected or confronting for all students. The use of Zoom Fit by both Year 12 pairs, for example, on seeing unusual views of the function immediately provided a familiar view for these students suggesting greater graphing calculator knowledge reduces the number of images seen by students that may cause conflict with their mathematical knowledge.

Response Actions: Adjustment of the WINDOW Settings

All of the pairs adjusted the WINDOW settings in their initial search for a global view of the graphical representation of the function. When combined with their mathematical knowledge, the use of other features of the graphing calculator, previous views of the function seen in the viewing window, and opportunistic planning, this response action led to the solution process becoming routine or potentially routine. Any one of these factors may have contributed to whether or not the circumstance became a defining moment or not. All of the pairs undertook actions that demonstrated they had a clear mental image of the function for which they were searching, however, the actions of some pairs on some occasions suggested that they did not have a clear understanding of which section of the function was currently the focus for the viewing window and consequently opportunities for particular conditions leading to defining moments were lost.

Adjustment of the WINDOW settings produced different outcomes on different occasions. On seeing no apparent view of the function, such a response may or may not have led to success. The initial window, the number of values altered in the WINDOW settings, the effect of each of these, the linking of mathematical knowledge, or previous views of the function with this adjustment contributed to whether or not this particular action was successful and in turn, when success occurred, how helpful this view was in the solution process.

Both Year 12 pairs on observing unusual views selected Zoom Fit. In contrast, all the Year 11 pairs adjusted the WINDOW settings, however, this had very different consequences for different pairs some of which furthered the solution process. This suggests that of itself this was not an inappropriate action to take. Mathematical knowledge and differences in the graphical view of the function visible may have been confounding factors in differences between the results of the actions of the Year 11 pairs.

Conclusions, Implications and Further Research

Students in this study used a variety of processes (cognitive, meta-cognitive, mathematical, and technological) as they searched for a complete graph, and these may have been undertaken in conjunction with a range of mathematical and graphing calculator knowledge. *Adjustment of the WINDOW settings*, for example, was effective in facilitating the solution process when used in conjunction with mathematical knowledge, reflection on previous views, and/or opportunistic planning. The clear mental image of the function demonstrated by all pairs, and hence their lack of acceptance of their initial view or a less than global view of the function, underpinned all actions facilitating the solution process.

Whilst the *use of Zoom Fit* and *use of Zoom Out* were defining actions for only some pairs of students, the defining action *adjustment of the WINDOW settings* was undertaken by all pairs at some stage during their solution. However, depending on other factors including the situation, context, application of mathematical knowledge, and the specific settings altered, this may or may not have become a defining moment. The mathematical and graphing calculator knowledge facilitating this defining action included a mental image of the function and an understanding of the effects of altering WINDOW setting values.

Although the visual images provided by the graphing calculator were confusing to students at times, these images did not mislead them into inappropriate acceptance of the view presented being the global view of the cubic function under consideration. These findings are in contrast with those of Steele (1995) who suggested students might have accepted these initial views of the function as the complete graph. This behaviour was not displayed by any of the students in this study. The mathematical knowledge of the students allowed them to consider the only possible shapes of the graphs of cubic functions and hold onto this view even when their ideas were challenged by unexpected views of the function on the calculator screen.

Given the defining action, *use of Zoom Fit*, appeared to have a major effect on ensuring the solution process became routine, questions need to be raised about whether, and when, the use of a broad range of calculator functions should be explicitly taught. In this study it was used only by the Year 12 students who had either discovered this function themselves or been alerted to its use by their peers (including one such instance during the study).

This small study undertaken with the teacher as researcher raises many questions that need further research, including:

- What effect does the type of graphing calculator have?
- Would the findings be similar for other students studying the same level of mathematics?
- What was specifically taught to the students in this study including mathematical knowledge, graphing calculator knowledge, how one learns, and what constitutes understanding, that impacted on the findings?
- What understandings do students, and teachers, have of the implied meaning of a complete graph of a function?
- Would a researcher who was not the teacher of the students gain the same understandings of the educational phenomena as observed by this researcher?

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