

# Designing Assessment Using the MATH Taxonomy

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The MATH taxonomy is designed to help teachers plan assessment to test a range of skills and concepts. The communication aspects of mathematics and multiple representations are included. This paper describes the analysis of a pre-test and post-test given to 172 senior secondary students studying a new subject, General Mathematics for the NSW Higher School Certificate. The study highlights the types of questions students find difficult and reasons why.

This study investigates the way students interpret and answer examination style questions. The study forms part of a larger study previously reported (D'Souza & Wood, 2001; 2002). We have used a taxonomy to categorise the questions. We investigated the area of Financial Mathematics, one of the five strands in the NSW General Mathematics (GM) syllabus. The General Mathematics Stage 6 Syllabus replaced the 1981 Mathematics in Society and the 1989 Mathematics in Practice syllabuses. GM takes on an information-processing approach toward learning mathematics, characterised by collecting, organising, interpreting and analysing data. Students use a variety of mathematical tools to solve problems and model real-life situations (Yen, 2000). The importance of language and the interpretation of graphs and tables is promoted throughout this new course. Approximately 25 000 students sit for the General Mathematics subject in the NSW Higher School Certificate each year. Although, this is the lowest level mathematics subject offered and does not include calculus, the subject is challenging and covers a range of mathematical applications.

The features of the syllabus are (Board of Studies, 2001): (1) General Mathematics approaches specific mathematical skills through a range of applications that clearly demonstrate the need for and the use of these skills, (2) General Mathematics puts emphasis on the particular application of mathematics to finance and data analysis and reflects the uses of mathematics that are prevalent in modern society, and (3) the needs of individual students may be catered for through the wider range of applications. This syllabus focuses on skills and techniques that have direct application to everyday life rather than the more abstract approach taken by the higher level mathematics courses.

As part of the larger study, students were asked to complete a test of their prior knowledge. Based on previous research (Wood & Smith, 2002), we were interested in the links between questions and the types of questions that students find difficult. This paper describes an analysis of the students' answers. The study is important in that it highlights the way students respond to examination style questions depending on the type of questions, in other words, questions of procedural versus conceptual nature.

## The MATH Taxonomy

The MATH taxonomy (see Table 1) has been used to analyse examination papers and design assessment for university level mathematics (Smith, Wood, Coupland, Stephenson, Crawford, & Ball, 1996; Ball, Stephenson, Smith, Wood, Coupland, & Crawford, 1998). The taxonomy is adapted from Bloom's taxonomy (1956). While this suggests a behaviourist approach to teaching and learning, this is not the case; the

taxonomy is simply used as a tool to assist with design of examinations. It is still the situation that formal examinations are the norm in mathematics at senior secondary school and undergraduate mathematics. We will not argue the merits or otherwise of this situation here except to say that examination performance is important and the design of good examinations that test a wide range of skills and concepts is critical for mathematics learning.

Table 1  
*The MATH taxonomy*

Group A	Group B	Group C
Factual knowledge (A1)	Information transfer (B1)	Justifying and interpreting (C1)
Comprehension (A2)	Applications in new situations (B2)	Implications, conjectures and comparisons (C2)
Routine use of procedures (A3)		Evaluation (C3)

*Factual knowledge* is remembering a specific formula or definition. Examples of *comprehension* are: understanding the significance of symbols in a formula, recognizing examples and counterexamples of a mathematical object or concept. *Routine use of procedures* covers algorithms that students would have practiced in class as drill exercises, such as changing the subject of a formula. *Information transfer* shows the ability to transform information from one form to another – from verbal to numerical, numerical to graphical and so on. It includes taking a general formula and applying it in a specific situation (that goes beyond routine procedures). *Applications in new situations* tests the ability to choose and apply appropriate methods or information in new situations. Group C categories cover justifying a result, comparisons and implications with justification and evaluation and judgments.

### Theoretical Basis for Research

The researchers support the constructivist theory of learning to enhance mathematical learning, according to which learning occurs in direct relationship to what is already known and how that prior knowledge is organised into mental models and beliefs that are used to interpret new objects and events (Gasiorowski, 1998). In other words, learners, in a constructivist environment, interact with their surroundings as they create and internalise their own interpretations of reality according to their own experience, beliefs, and knowledge. Constructivist theories of knowledge are based on a fundamental assumption that knowledge is constructed in the mind of the learner. Glasersfeld (1989) describes constructivism as a “*theory of knowledge with roots in philosophy, psychology and cybernetics*” (p.162). In the constructivist paradigm, learning emphasises the process and not the product. The retrieval of an ‘objectively true solution’ is not as important as how one arrives at a particular answer. Learning is a process of constructing meaningful representations. The notion of doing something ‘right’ or ‘correctly’ is to do something that fits with “*an order one has established oneself*” (Glasersfeld, 1987, p. 15). However, the assessment of learning in mathematics is done by written, timed examinations that can create conflict with the idea of constructivism. Attempts to modify the examination system with extended assessment outside of examinations have met with little success, such as the VCE tasks in Victoria.

What is needed is for the examination system to be wide enough to allow students to demonstrate deep understanding.

## Methodology and Design of Study

The students came from 10 classes across six high schools in Sydney, NSW. Although participation in the study was voluntary, most students attempted the test (150 out of 172 attempted the pre-test and 116 out of 172 attempted the post-test). The pre-test was conducted at the beginning of Term 3 2001 and the post-test towards the end of the term after they had studied Financial Mathematics. The pre-test (see Appendix A1) and post-test (see Appendix A2), each consisted of 12 questions that tested students' knowledge of mathematics in the context of the Financial Mathematics stream of the General Mathematics syllabus.

## Results and Analysis

The results were interesting. Tables 2 and 3 summarise the numbers of students that scored a range of marks as well as the Taxonomy categorisation for each of the 12 questions in both the pre-test and post-test. It is clear that majority of students found questions 7 and 11 to be rather difficult (for both tests) with only one student scoring full marks for question 7 in the pre-test and 2 students in the post-test while 2 students in each of the pre-test and post-test scored full marks for question 11. Recall that we categorised these questions as an *application in a new situation* (B2). Question 4 (categorised as an *application in a new situation* (B2)) also posed some problems. Approximately one-third of the students in the pre-test and over one-fifth of students in the post-test that scored 1 out of the 2 possible marks had not realised that they needed to change the number of periods (expressed in months) into years, even though some understood the concept of *final balance* as being the sum of the interest earned and the principal. Questions 2 and 3 (in both tests) was well answered by the majority of students. It was originally thought that the two extra columns of data namely, *Principal* and *Interest Rate* would create added complications, but this was not the case.

Table 2  
Scores for Pre-Test Questions 1-12 (N=150)

Marks	Taxonomy category	0	1	2	3	4	5	6	Non-response
Question 1 (out of 6)	A2	6	20	26	7	11	31	32	17
Question 2 (out of 2)	A3	19	26	90					15
Question 3 (out of 4)	B1	5	3	7	0	98			37
Question 4 (out of 2)	B2	59	51	20					20
Question 5 (out of 6)	A2/B1	38	4	4	5	7	2	7	83
Question 6 (out of 3)	B1	26	4	4	40				76
Question 7 (out of 3)	B2	85	0	0	1				64
Question 8 (out of 2)	B1	33	0	36					81
Question 9 (out of 2)	A3	41	9	18					82
Question 10 (out of 2)	A3	31	10	52					57
Question 11 (out of 3)	B2	61	1	1	2				85
Question 12 (out of 5)	A2/B1/B2	17	5	5	17	3	4		99

Perhaps this is due to the fact that the question explicitly stated what two variables had to graphed and therefore students understood that the other two columns of data were to be ignored and were supplied as *extra information*. Recall that question 2 was categorised as *routine use of procedures* (A3). Students would have been comfortable answering a question of this nature out of familiarity. It is interesting to note that the degree of non-response increased quite significantly towards the second half of both the pre-test and post-test with the highest non-response recorded for question 12 (66% in the pre-test and 64% in the post-test). Perhaps students found the second half of the test harder or simply chose not to answer due to the voluntary nature of the study.

Table 3  
Scores for Post-Test Questions 1-12 (N=116)

Marks	Taxonomy category	0	1	2	3	4	5	6	Non-response
Question 1 (out of 6)	A2	7	5	9	11	8	35	31	10
Question 2 (out of 2)	A3	17	3	92					4
Question 3 (out of 4)	B1	14	3	6	1	79			13
Question 4 (out of 2)	B2	41	38	25					12
Question 5 (out of 6)	A2/B1	20	2	3	5	6	5	12	63
Question 6 (out of 3)	B1	28	5	3	55				25
Question 7 (out of 3)	B2	76	0	0	2				38
Question 8 (out of 2)	B1	34	0	17					65
Question 9 (out of 2)	A3	32	7	17					60
Question 10 (out of 2)	A3	32	3	42					39
Question 11 (out of 3)	B2	59	0	0	2				55
Question 12 (out of 5)	A2/B1/B2	20	4	8	7	0	3		74

### Concluding Remarks

This study has highlighted the fact that students feel more comfortable answering questions that test routine skills and the ability to reproduce factual knowledge (Group A tasks) as opposed to questions of a more conceptual nature (Group B tasks) which students find relatively harder and many are unable to answer. Examination situations often test a narrow range of skills that encourage students to take a surface approach to learning. In this study we investigated the effects of having questions that tested a broader range of skills, like multiple representations for instance. Although many students might be able to 'pass' a test or examination, the question we need to ask ourselves is - *have students be able to show that they understand what they have learnt?* One way we can ensure this is by asking simple yet probing questions that test students' grasp of a particular content area. In conclusion, we can say that students have the ability to retain vast amount of information, but many seem to forget much of it and do not appear to make good use of what they do remember. Less mathematically inclined students may cling to a more surface approach to learning and need to be encouraged to extend their mathematical approach. We believe that students have the capability to take on a deeper approach to learning if they see it as necessary in order to succeed. It is possible to improve students' learning by paying more attention to assessment methods that test a broader range of skills.

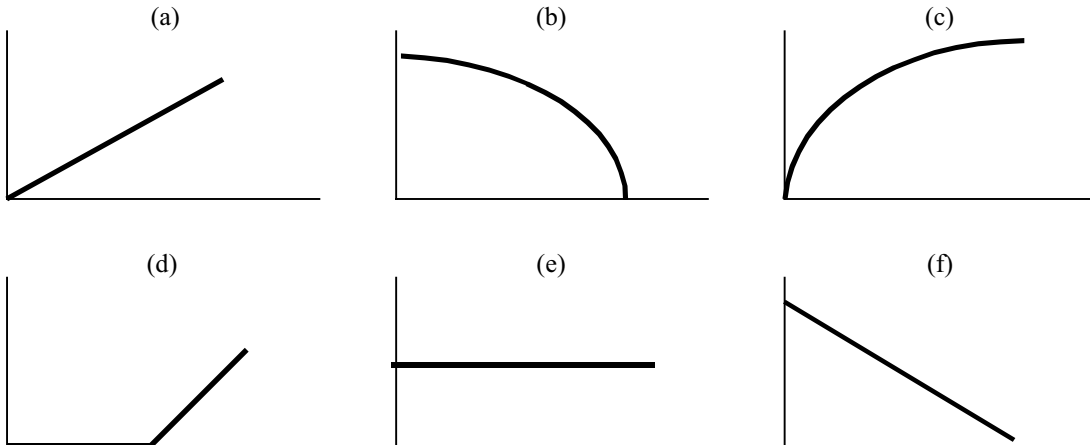
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## Appendices

### Appendix A1 - Pre-test

1. Identify which of the following graphs represented below describe a *linear* relationship and which graphs describe a *non-linear* relationship.

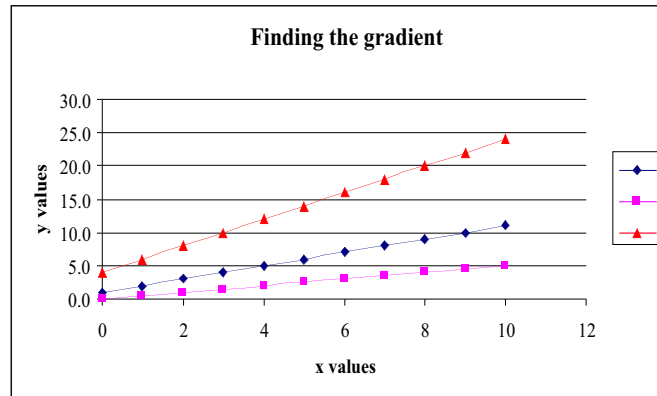


2. If  $I = Prn$ , where  $I$  = interest earned,  $P$  = principal and  $n$  = number of periods, and  $r$  = rate of interest (%), then calculate  $I$ , given that  $P = \$2600$ ,  $n = 5$  years and  $r = 4.25\%$  per annum.
3. Sketch a graph of simple interest against the number of years, for the following table of values:

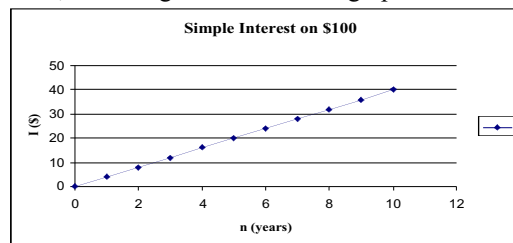
P (\$)	r (%)	n (years)	I (\$)
1000	6	0	0
1000	6	1	60
1000	6	2	120
1000	6	3	180
1000	6	4	240
1000	6	5	300

4. If  $I = Prn$ , where  $I$  = interest earned,  $P$  = principal and  $n$  = number of periods (in months), and  $r$  = rate of interest (%), then calculate  $I$ , given that  $P = \$1800$ ,  $n = 8$  months and  $r = 5.25\%$  per annum. What is your new closing balance?
5. Find the gradient of the following graphs shown below. What are the equations that have been plotted?

x	y <sub>1</sub>	y <sub>2</sub>	y <sub>3</sub>
0	1.0	0.0	4.0
1	2.0	0.5	6.0
2	3.0	1.0	8.0
3	4.0	1.5	10.0
4	5.0	2.0	12.0
5	6.0	2.5	14.0
6	7.0	3.0	16.0
7	8.0	3.5	18.0
8	9.0	4.0	20.0
9	10.0	4.5	22.0
10	11.0	5.0	24.0



6. Create a table of values to find  $I$ , given  $P = \$10$ ,  $r = 6\%$  per annum, and  $n = 1, 2, \dots, 10$ .
7. You wish to purchase a car for your trip to Tasmania this Christmas. The car will cost you \$10,200. The Commonwealth Bank of Australia is offering a 4.10% flat rate of interest. What is the amount that would need to be deposited each month in order to be able to buy the car for your trip in 6 months time?
8. Given the formula  $I = Prn$ , find the gradient from the graph as shown below:



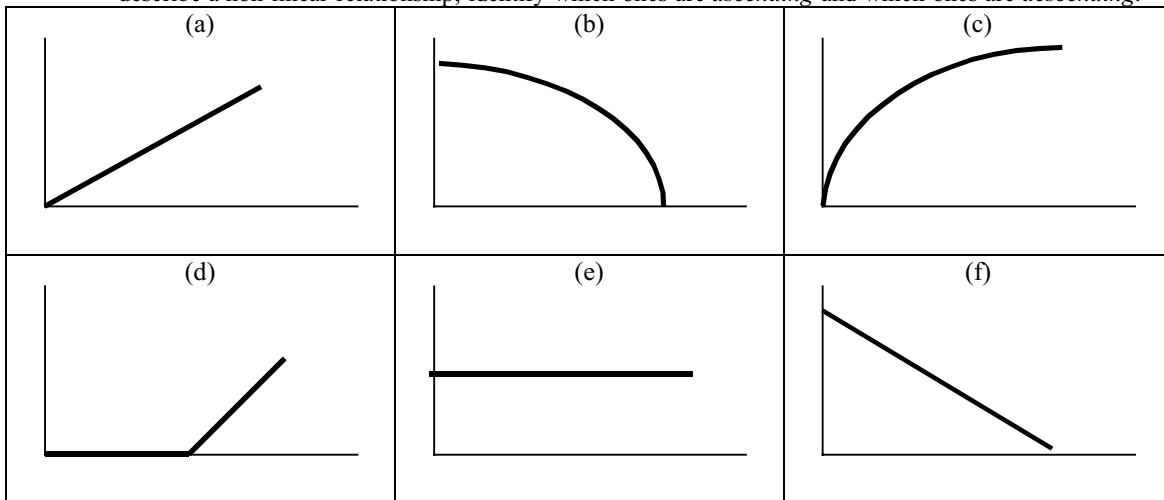
9. Make  $r$  the subject of the formula  $I = Prn$ .
10. Given that  $A = P(1 + r)^n$ , find  $A$  if  $P = \$6200$ ,  $r = 6.25\%$  and  $n = 7$  years.
11. Over your school holidays, you wish to go to Byron Bay. How much money will you need to invest now at 4% per annum compounded annually, if your trip is going to cost you \$4400 in 2 years time?

Periods	1%	5%	10%
1	10.10000	10.50000	11.00000
2	10.20100	11.02500	12.10000
3	10.30301	11.57625	13.31000
4	10.40604	12.15506	14.64100
5	10.51010	12.76282	16.10510
6	10.61520	13.40096	17.71561
7	10.72135	14.07100	19.48717
8	10.82857	14.77455	21.43589
9	10.93865	15.51328	23.57948
10	11.04622	16.28895	25.93742

12. Above is a table of compounded values of \$10. Calculate the future value of \$5000 invested for 8 years compounded annually at 10% if  $FV = PV(1 + r)^n$ , where  $FV$  = future value and  $PV$  = present value. How would you change the above formula if you wanted to calculate the value  $PV$ ?

*Appendix A2 - Post-test*

1. Identify which of the following graphs represented below describe a *linear* relationship and which graphs describe a *non-linear* relationship. Of those that describe a linear relationship, identify which ones have a *positive gradient* and which ones have a *negative gradient*. Similarly, of those that describe a non-linear relationship, identify which ones are *ascending* and which ones are *descending*.

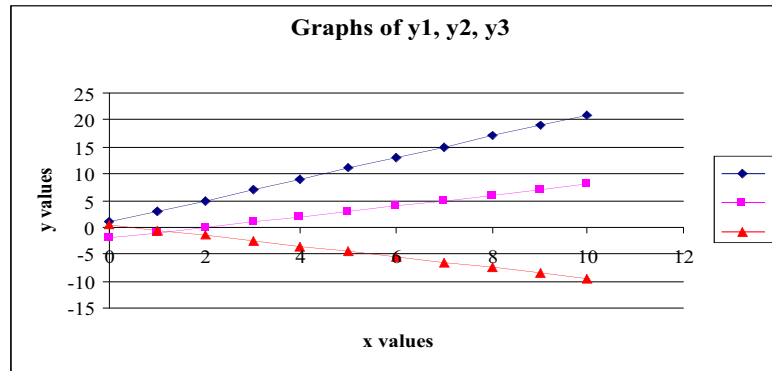


2. If  $I = Prn$ , where  $I$  = interest earned,  $P$  = principal and  $n$  = number of periods, and  $r$  = rate of interest (%), then calculate  $I$ , given that  $P = \$1600$ ,  $r = 6.5\%$  per annum and  $n = 8$  years.
3. Sketch a graph of simple interest against the number of years, for the following table of values:

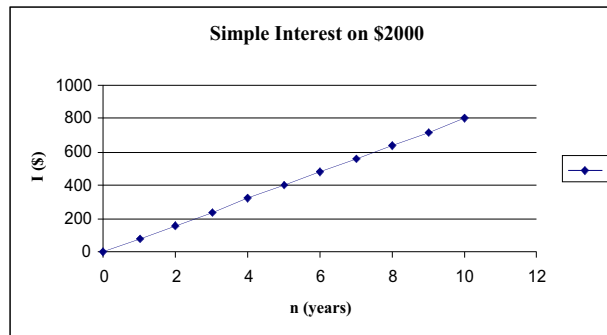
P (\$)	r (%)	n (years)	I (\$)
100	4	0	0
100	4	1	4
100	4	2	8
100	4	3	12
100	4	4	16
100	4	5	20
100	4	6	24
100	4	7	28
100	4	8	32
100	4	9	36
100	4	10	40

4. If  $I = Prn$ , where  $I$  = interest earned,  $P$  = principal and  $n$  = number of periods (in months), and  $r$  = rate of interest (%), then calculate  $I$ , given that  $P = \$2200$ ,  $n = 10$  months and  $r = 4.75\%$  per annum. What is your new closing balance?
5. Find the gradient of the following graphs shown below. What are the equations that have been plotted?

x	y <sub>1</sub>	y <sub>2</sub>	y <sub>3</sub>
0	1	-2	0.5
1	3	-1	-0.5
2	5	0	-1.5
3	7	1	-2.5
4	9	2	-3.5
5	11	3	-4.5
6	13	4	-5.5
7	15	5	-6.5
8	17	6	-7.5
9	19	7	-8.5
10	21	8	-9.5



- Create a table of values to find  $I$ , given  $P = \$500$ ,  $r = 3\%$  per annum, and  $n = 1, 2, \dots, 10$ .
- You wish to purchase a car for your trip to Brisbane next year. The car will cost you \$8800. ANZ is offering a 4.25% per annum flat rate of interest. What is the amount that would need to be deposited each week in order to be able to buy the car for your trip in 12 months time?
- Given the formula  $I = Prn$ , find the gradient from the graph as shown below:



- Make  $n$  the subject of the formula  $I = Prn$ .
- Given that  $A = P(1 + r)^n$ , find  $A$  if  $P = \$7400$ ,  $r = 3.25\%$  and  $n = 10$  years.
- You wish to go to Christmas Island. How much money will you need to invest now at 5% per year compounded monthly, if your trip is going to cost you \$2400 in 2 years' time?
- Below is a table of compounded values of \$10. Calculate the future value of \$10000 invested for 2 years compounded annually at 5% if  $FV = PV(1 + r)^n$ , where  $FV$  = future value and  $PV$  = present value. How would you change the above formula if you wanted to calculate the present value  $PV$ ?

P	=	\$10	
Periods	1%	5%	10%
1	10.10000	10.50000	11.00000
2	10.20100	11.02500	12.10000
3	10.30301	11.57625	13.31000
4	10.40604	12.15506	14.64100
5	10.51010	12.76282	16.10510
6	10.61520	13.40096	17.71561
7	10.72135	14.07100	19.48717
8	10.82857	14.77455	21.43589
9	10.93665	15.51328	23.57948
10	11.04622	16.28895	25.93742