Mathematical Methods Computer Algebra System (CAS) 2004 Pilot Examinations and Links to a Broader Research Agenda

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Analysis and commentary on the 2002 and 2003 Mathematical Methods (CAS) pilot examinations in Victoria, in particular with respect to common items with the standard course, were reported at the 2002 and 2003 MERGA conferences. This paper extends analysis to student performance on aspects of the 2004 examinations, with consideration of some emerging trends over the three years. Several possible areas for a broader research agenda with respect to implementation of a CAS enabled senior mathematics curriculum are proposed.

Mathematical Methods (CAS) Units 1 - 4 is an accredited pilot study of the Victorian Curriculum and Assessment Authority January 2001 – December 2005 (VCAA, 2004). Following the review of the VCE mathematics study 2002 - 2004 and its reaccreditation in January 2005, Mathematical Methods (CAS) Units 1 - 4 will be available to all schools from 2006, as a parallel and alternative subject to Mathematical Methods Units 1 - 4. Investigation of examination results with respect to student performance on common questions with the corresponding standard Mathematical Methods course, have been previously reported in Evans, Leigh - Lancaster and Norton (2003; 2004).

Related research into student response construction to questions and problems can be found in Ball and Stacey (2004), assessment principles in Flynn (2003) and change in teacher practice with classroom access to CAS in Garner & Leigh-Lancaster (2003). Artigue (2002) and Ruthven (2002) address issues relating to theoretical and practical frameworks for investigation of CAS instrumentation and functionality, while Burrill et alia (2002) provide a meta-analysis of research findings and implication for classroom practice for the use of handheld technology, including CAS. Brown (2003) and Böhm et alia (2004) have recently undertaken qualitative analysis of CAS active examinations items across several systems and jurisdictions (Denmark, Victoria, Belgium, Austria and Switzerland). This paper reports on aspects of interest from the November 2004 examinations for the standard Mathematical Methods cohort of around 17 500 students, and the Mathematical Methods (CAS) pilot cohort of around 400 students, and, for the first time incorporates some qualitative consideration of emerging trends over the three years of these examinations. Several possible areas for further investigation as part of a broader research agenda are also proposed.

Australian states and territories, and many other systems and jurisdictions around the world, permit or assume the use of graphics calculators in some aspects/components of final examinations. As these systems and jurisdictions move to incorporate use of CAS technology in at least some aspects of their curriculum, pedagogy and assessment, related investigations and research are significant as part of broader consideration of the impact of the use of technology, in particular CAS, in secondary mathematics education (see, for example, Leigh-Lancaster, 2004). Indeed, such investigations are pertinent to any system or jurisdiction that permits the use of graphics calculators in examinations and does *not* require clearing of the memory of these calculators (an increasingly difficult process to ensure). As noted in Brown and Leigh-Lancaster (2004a; 2004b) recent models of graphics calculators now have substantial memory capacity and can store and implement a wide range of programs including various *quasi*-CAS programs with symbolic manipulation functionality. At a technical level, the convergence of graphics calculator plus supplementary program functionality (such as *Factor* 9 and *Symbolic*) and CAS functionality is likely to continue apace. Indeed, recent models of such calculators have memory capacities in the order of 0.5 - 3 Mb, which amply exceed the memory requirements for early versions of the computer platform CAS *Derive*, and also enable these calculators to run spreadsheet and dynamic geometry applications, study notes and the like.

The use of CAS is now well established in components of the US College Board's *Advanced Placement Calculus* examinations since 1995 (hand-held graphics calculator/CAS calculator in a common question, technology active, but graphics calculator/CAS neutral examination); the French *Baccalaureate Générale Mathematics* examination since 1999 (hand-held graphics calculators or CAS calculators in a pure mathematically oriented technology neutral examination); and the Danish *Bacclaureat Mathematics* examination since 1997 (graphics calculator or hand-held/computer based CAS in an open book, technology active, format with common questions and some questions with non-CAS/CAS alternative versions). In Austria and Switzerland, teachers set their own examination (subject to audit for conformance with curriculum and assessment requirements) and may choose to incorporate student access to CAS if they wish. The International Baccalaureate Organisation (IBO) commenced a CAS pilot program in conjunction with its Higher-Level mathematics course in September 2004. New Zealand authorities are seriously investigating a CAS active curriculum from Year 9.

The final year of the VCAA Mathematical Methods (CAS) expanded pilot in 2005 involves several hundred students from a range of metropolitan and regional co-educational and single-sex schools across the sectors, using a variety of approved hand-held (CASIO *Algebra* 2.0+, TI-89, TI-92/*Voyage* 200) and computer based CAS (*Derive*, *Mathematica*).

The Victorian Tertiary Admissions Centre (VTAC) scales VCAA study scores (which are derived from a truncated normal distribution on a scale of 0 - 50 with mean 30 and standard deviation 7) to take into account differences in relative difficulties of studies (based on analysis of how students perform across VCE studies). VTAC scaling data 2002 - 2005 for Mathematical Methods, Mathematical Methods (CAS) and Specialist Mathematics (VTAC, 2004) shows a slightly higher (re-scaled) pilot mean score with a slightly smaller standard deviation, with respect to the standard course, and with a common scale difference in means for both cohorts with respect to Specialist Mathematics. Thus, although the expanded schools are not a stratified random sample, they represent a broad range of school and student backgrounds, and their overall performance on examinations is similar to that of the larger cohort. Thus, Mathematical Methods and Mathematical Methods (CAS) are parallel and alternative courses, for two distinct, but like, populations.

Mathematical Methods (CAS) Examination 1 – 2004

In 2004, as in previous years, the papers for Mathematical Methods (CAS) pilot study Examination 1 and the corresponding Mathematical Methods Examination 1 both comprised 27 multiple choice questions, each worth one mark, and 6 short answer questions worth a total of 23 marks. Twenty-one of the multiple choice questions (about 78 % of this component) were common. No short answer questions were common to both papers, but three questions and the first part of a fourth were very similar (about 65 % of the short answer component).

Discussion of Multiple Choice Questions

As in previous years, the multiple choice component was well done by the pilot cohort, with a mean of 18 marks out of a possible 27. In general, the Mathematical Methods (CAS) cohort performed comparably, or better, than the Mathematical Methods cohort on common multiple choice questions. Table 1 summarises the difference in percentage of correct responses, where a positive difference indicates that a higher proportion of CAS pilot students selected the correct response. The questions have again been classified as technology independent (I); technology of assistance but neutral with respect to graphics calculators or CAS (N); or use of CAS likely to be advantageous (C). Those items for which technology is of assistance, but that are likely to be answered efficiently by conceptual understanding, pattern recognition or mental and/or by hand approaches have been indicated with an asterisk (*). This classification scheme has now been used 2002 - 2004 and has been found by the authors to be simple and robust. It is similar in respects to other schemes used and/or discussed by Brown (2003), Flynn and McCrae (2001) and Kokol-Voljc (2000). Several of these earlier schemes were based on analysis of the *anticipated* impact of access to CAS on items from existing (non-CAS) examinations. This particular scheme has been applied in an operational examination context and has the distinctive feature of identification of items with * classification.

Table 1
Summary of Differences between Percentages of Correct Responses to Common Examination 1 Multiple Choice Items: question numbers

Item type	Negative difference	No difference	Positive difference	
	Up to 4%	Same	Up to 5%	6% to 10%
I	5, 17	10, 26	1, 3, 13, 14, 16, 21, 22, 23, 25	
N	6*, 20*		7*, 18*	12
C			4*, 9*	8

On 15 of the 21 common multiple choice questions, a higher percentage of the CAS cohort obtained the correct answer. For questions which are technology *independent*, the CAS cohort performed better on nine out of the thirteen questions for which there was a difference (with no difference on two questions). The CAS pilot cohort performed better on the three questions where CAS use was classified *a priori* as likely to be *advantageous*.

While access to CAS certainly offers high levels of reliability for items such as differentiation of $\log_e(\cos(2x))$, it is also evident that caution needs to be exercised in characterisation of certain mathematical tasks as being 'trivialised' by access to CAS such as the following question from the 2004 paper (CAS cohort 61% correct, non-CAS 59%):

Question 9

Which one of the following is a **complete** set of **linear** factors of the third degree polynomial $ax^3 - bx$, where a and b are positive real numbers?

A. $x,\ ax^2 - b$

B. $x,\ ax - b,\ ax + b$

C. $x,\ \sqrt{ax - b}$

D. $x,\ \sqrt{a}\,x - b,\ \sqrt{a}\,x + b$

E. $x,\ \sqrt{a}\,x - \sqrt{b},\ \sqrt{a}\,x + \sqrt{b}$

As noted in Evans, Norton and Leigh-Lancaster (2004) with respect to a similar question (2002, Question 9 for the expression $x^4 + x^3 - 3x^2 - 3x$) *interpretation* of the question, such as selection of the relevant *field* over which the factorisation is to take place (for both cohorts) and *facility* with the appropriate CAS *functionality* (for the CAS cohort) will be relevant to whether CAS is of assistance, or not, given that students could also use a mental or by hand approach (available to either cohort) to factorise the expression as $ax^3 - bx = x(ax^2 - b) = x(\sqrt{a}\,x + \sqrt{b})(\sqrt{a}\,x - \sqrt{b})$. This also appears to be the case for problems involving integration, where both the *formulation* of a suitable definite integral, and correct use of the appropriate CAS functionality are important (a similar issue applies with respect to the use of graphics calculators for problems involving numerical integration). The question where there was the largest difference in performance was on a *technology active* but graphics calculator/CAS *neutral* question, where students had to solve the equation $\log_e(x + 1) = 1 - x$ correct to two decimal places. This was answered correctly by 90% of the CAS cohort and 80% of the Mathematical Methods cohort. This is similar to the results for part of Question 4 in the 2003 Examination 1 Part II paper, where 87% of the CAS cohort were able to find (use of numerical functionality required) the x-coordinate of the intersection of two curves, but only 62% of the Mathematical Methods cohort, and may support the hypothesis expressed previously that CAS operational requirement for careful entry of *symbolic* expressions encourages similar attention to detail more broadly with other functionality. These have now been noted for each of the three years of examinations.

Discussion of Short Answer Questions

Questions that involved parameters or arbitrary constants again presented more challenge to students, and student responses reinforced earlier observations about the complexity of such problems, in particular where contextual interpretation in terms of these parameters is required. For example, one short answer question asked students to find the relationship between *two* parameters m and k such that the line with equation $y = mx$ intersects the curve with equation $y = x^2(x - k)$ at the point (0, 0) and one other point only. While CAS could readily be used to solve the relevant equation, students had difficulty in relating the corresponding solutions to the different *graphical* situations. Clearly the use of two parameters and consideration of the relationship between them added an extra layer of complexity, even though their separate effects are easily interpreted in context and the algebraic form of the relationship is relatively simple.

Mathematical Methods (CAS) Written Examination 2 - 2004

The 2004 examination papers for both pilot and standard cohorts comprised four extended answer analysis questions, with around 55% common material. Question 1 was a pure mathematical functions and calculus question involving a polynomial function where $f(x) = (x - 1)^2(x - 2) + 1$. Transformations, including dilations and translations, of the function were investigated, with specific values for the standard cohort, and generalisation with a parameter for the CAS cohort. Parts of the question included: determination of transformations of the function which yield a given number of positive x axis intercepts; determination of the area of a region of an area after a dilation of factor k (specific value $k = 2$ for standard cohort) from the y axis; description of a sequence of transformations from the graph of the original function to the graph of a new function; determination of set of values of a real number p such that $f(x) = p$ has exactly one solution; and, for the family of functions defined by the parameter, determination of which of them had a minimum point lying on a given straight line. Table 2 shows the mean score from Mathematical Methods and Mathematical Methods(CAS) cohorts on parts of Question 1 (common parts indicated by #).

Table 2

Mean Scores of Performance on Question 1

Part marks	1a 1	1b 2#	1c 2#	1di 2	1dii 2	1diii 2	1e 2#	1f 3	Total 16
MM(CAS) mean	0.91	1.54	0.45	1.28	1.21	0.59	0.27	1.06	7.31

Part marks	1a 2	1b 2#	1c 2#	1d 3	1ei 2	1eii 1	1eii 1	1f 2#	Total 15
MM mean	1.51	1.01	0.19	0.96	1.14	0.25	0.14	0.15	5.35

Parts 1a and 1b involved finding the derivative of the function and determining the coordinates of the turning point, while 1c required students to find the real values of p for which the equation $f(x) = p$ has exactly one solution. This part of the question was conceptual, and technology independent. Students from both cohorts found this graphical interpretation required difficult, however the performance of CAS students was better with a much higher proportion achieving success. Question 1d on the CAS paper involved a single real parameter *k* and was presented in three parts:

For the following, k is a positive real number.

i. Describe a sequence of transformations which maps the graph of $y = f(x)$ onto the graph of $y = f\left(\frac{x}{k}\right) - 1$.

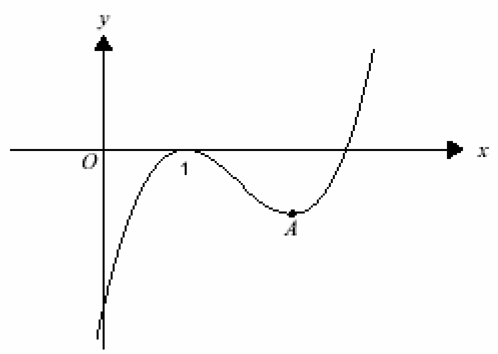
ii. Find the x-axis intercepts of the graph of $y = f\left(\frac{x}{k}\right) - 1$ in terms of k.

iii. Find the area of the region bounded by the graph of $y = f\left(\frac{x}{k}\right) - 1$ and the x-axis in terms of k.

The corresponding question (1e parts i, ii and ii respectively) of the non-CAS paper was identical, but with $k = 2$, the last part requiring an answer correct to two decimal places. While the two cohorts were similarly able to describe a suitable sequence of transformations with similar levels of success, the CAS cohort were able to tackle parts ii and iii more successfully notwithstanding the use of a parameter rather than a specific value. Question 1e on the CAS paper and the 1f on the non-CAS paper: "Find the real values of h for which only one of the solutions of the equation $f(x + h) = 1$ is positive." were common and required conceptual and graphical understanding of transformations.

Question 1f (CAS paper only) again required a solution in terms of a single real parameter:

The graph of $y = (x - 1)^2(x - a)$ where $a > 1$ is shown below.



Find the exact value of a such that the local minimum at point A lies on the line with equation $y = -4x$.

This question was completed successfully by 20% of the students with another 9% of students losing only one mark. This was a good proportion of the cohort given the complexity of the task, and many students demonstrated efficiently and effectively use of CAS. While students seemed to be comfortable with the use of a single parameter, marks were typically lost through not presenting suitable working and use of approximation rather than exact forms. More detailed statistical information about student performance for both cohorts (including proportions of students attaining marks on each part question) and related commentary, can be accessed from the VCAA website. What appears to be the case from a general scrutiny of CAS examination results over three years is that CAS does ‘scaffold’ and enable students to engage, and continue to engage, in extended response analysis questions, with comparatively good level of success.

Some Proposals for Links to a Possible Broader Research Agenda

The relationship between curriculum, pedagogy and assessment within a given policy framework and implementation context, and mathematics education research is dynamic and recursive. Practical applications within a given system, jurisdiction or situation stimulate the genesis of research questions for investigation by various methodologies, while related research can inform consideration of issues and practical developments in context. In the current context, research could address some questions in detail, for example, the nature of the relationship between access to CAS functionality and the capacity of students to comprehend and use *parameters*; while other research could be broader in scope, for example, addressing the question of ‘*effective pedagogy*’ in a CAS active environment. The following are some proposals for a broader research agenda:

- professional development to support effective *teacher* use of CAS (*discipline* content knowledge, *technical* knowledge and *pedagogical* knowledge);
- CAS in the *middle school* mathematics curriculum;
- the impact of CAS on the *teaching and learning* of particular topics (content, concepts, skills and processes)for example, as indicated in the preceding commentary;
- the impact of technology, including CAS, on *curriculum design* and *assessment* (for example, on-line modes, formative and summative, of assessment);
- the *relationship* between *mental*, *by hand* and *technology* assisted approaches in mathematical inquiry: investigations, modelling, problem-posing and problem-solving (the *praxis* of working mathematically);
- the impact of CAS on the *integration* of algebraic, graphical and numeric forms of mathematical information and their use;
- the impact of CAS on student *interpretation* of features of families of functions;
- *meta-cognition* in mathematical inquiry when students use CAS;
- comparison of the performance of students with and without CAS in the completion of tasks requiring *higher order thinking*; and
- the *relationship* between mental and by hand skills and efficient and effective use of CAS.

A range of research methodologies would be relevant to such investigations, both in their own right and in their natural combination, such as: *empirical* studies (investigation of the efficacy of different technical approaches to solving particular classes of problems using CAS); *case studies* and *autobiographical narrative* (change in teacher practice); *philosophical inquiry* (historical and cultural developments and their impact on values, beliefs and choices), and *meta-analysis* of the existing literature (for example, the utility of various question classification schemes) to mention just a few.

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