Results of a Teaching Experiment to Foster the Conceptual Understanding of Multiplication Based on Children's Literature

Amanda Worlley
Griffith University
<aworl3@eq.edu.au>

Romina Jamieson-Proctor
Griffith University
<r.proctor@griffith.edu.au>

The importance of conceptual understanding in mathematics has been well documented (Hiebert & Carpenter, 1992). This paper reports selected results from a research study that used a conjecture-driven teaching experiment (Confrey & Lachance, 1999) to enhance Year 3 students' conceptual understanding of multiplication. The teaching experiment employed children's literature as a mediational tool for students to explore and engage in multiplication activities and dialogue. The SOLO taxonomy (Biggs & Collis, 1989) was used to both frame the novel teaching and learning activities, as well as assess the level of students' conceptual understanding of multiplication.

The world is changing rapidly and future members of the workforce will need to reason mathematically to use technologically sophisticated equipment and resources (English, 2002). As future members of this workforce students require access to a quality mathematics education otherwise their personal and economic success may be limited (Tate & Rousseau, 2002). DETYA (2000) highlighted a growing number of early numeracy programs that embrace the "research in children's learning of mathematical understandings and concepts" (p. 25) to guide teaching and learning practice. Most modern curriculum documents encourage teachers to engage students in mathematical activities that develop depth of conceptual understanding, affording them the ability to both think and communicate mathematically (Board of Studies, 1995; Education Queensland, 2002; Queensland Schools Curriculum Council, 2001). The key intention of these mathematics curricular documents is to enable students to function within a society that commonly uses mathematical models and calculations for decision making and to develop the logical functioning capacity to enable them to use mathematics in a variety of contexts. However, Willis (1990) observed that "many people become numerate almost in spite of school mathematics, rather than because of it" (p. 7). Other researchers (Mousley, 1999; Willis, 1990; Wood, Cobb, & Yackel, 1995) have suggested that the *implemented* mathematics curriculum can be quite different from the intended curriculum, with a large proportion of classrooms engaged in rote or procedural teaching and learning that focuses on methods, skills, rules and algorithms. Perhaps one could ask why teachers are not embracing what the research and policy documents are recommending?

Confrey and Lachance (1999) highlighted the need to establish a stronger connection between educational research and the practice of teaching by moving research from 'out-of-school' conditions and embedding it in the constraints of the classroom. They suggested that one of the main purposes of educational research is to "invent, develop and test novel ways of teaching mathematics" (p. 231). Teachers are consumers of research and rather than have research stand apart from practice, placing these investigations within the context of instructional settings would further the relationship between theory and practice (Cobb, 1999; Confrey & Lachance, 1999). However, Kennedy (1997) stated that simply placing research findings within the physical reach of teachers will not guarantee they will be encouraged to examine their practices. It is only when research is placed within the conceptual reach of teachers that change may be imminent and a significant purpose of educational research is to inform and enhance educational practice (Lester & Wiliam, 2002).

The purpose of this paper is to report selected results obtained from a transformative teaching experiment based on a conjecture-driven research design (Confrey & Lachance, 1999) that aimed to enhance Year 3 students' conceptual understanding of multiplication. This teaching experiment further aimed to develop a strong conceptual connection between educational research and the practice of teaching and assessing mathematics (Confrey & Lachance, 1999; Schoenfeld, 2002).

Design and Methods

The theoretical development of the teaching experiment used in this study, based on Confrey and Lachance's (1999) model, has been reported elsewhere (see Worlley & Proctor, 2005). Suffice to say Confrey and Lachance (1999) describe mathematics as a human construction and that limited access to, and significant gaps in achievement are caused in part by how mathematics is taught. Their research model, as depicted in Figure 1, has the teaching experiment embedded within it. Central to the model is a theoretical framework so that activities and methodologies can be both structured and interpreted.

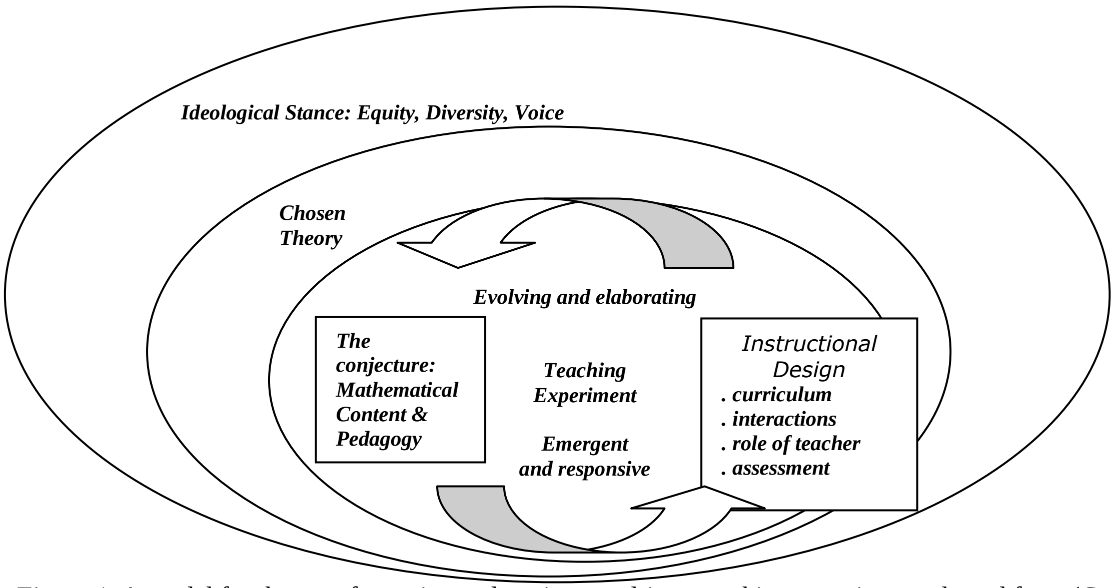

Figure 1. A model for the transformative and conjecture-driven teaching experiment adapted from (Confrey & Lachance, 1999).

The teaching experiment comprises two dimensions, the *conjecture* and the *instructional design*, which both evolve and influence each other. This is a cyclical process where the conjecture guides the instruction and the instruction provides a catalyst for the conjecture to evolve. Whilst the conjecture is stated at the outset, the significant features of a conjecture are its flexibility, scalability and ability to be revised throughout the research period unlike an hypothesis (Confrey & Lachance, 1999). These features add to its appeal in dynamic classroom contexts where learning is never static. Further, the conjecture should contain two main parts: one is *mathematical content*; the other is the *pedagogical approach* that is strategically linked to the content to be taught.

The teaching experiment that forms the basis of this study conjectured that a student's conceptual understanding of multiplication (*mathematics content*) may be advanced, using children's literature as a mediational tool (*pedagogical approach*). The conjecture also proposed the use of a novel assessment item and an instructional design based *firstly* on components of Bruner's seminal Theory of Learning (1964) that requires learning tasks to progressively move from concrete to abstract; *secondly* links were made to the different representations of concrete and pictorial, real world and symbolic (Hiebert & Carpenter, 1992); and *thirdly* the framework of the SOLO taxonomy (Biggs & Collis, 1982, 1989) was used to guide the choice of teaching activities and assessment tasks.

As the evolving nature of the conjecture is of pivotal importance Confrey and Lachance (1999) recommended a team of researchers might be needed over a significant period of time due to the need for discussion, refinement and elaboration of the conjecture. In this study however the first author was the teacher/researcher and she had only a short period of three weeks in which to conduct the teaching experiment. The two researchers conferred daily, discussing field notes and the evolving conjecture and instructional design process.

There were four components to the design of the instruction as recommended by Confrey and Lachance namely: the curriculum; the classroom interactions; the role of the teacher; and assessment. These will be addressed briefly in this paper but a fuller description can be found in Worlley and Proctor (2005).

The Curriculum in this Teaching Experiment

In this teaching experiment the curriculum content was built around the conceptual understanding of multiplication, as opposed to the procedural knowledge which is often realised in classrooms as reciting the 'times tables'. This conceptual understanding was fostered by helping students connect between concrete, real world and symbolic representations of multiplication (Hiebert & Carpenter, 1992).

Specifically, the curriculum was structured around a number of daily lessons across the three week period that centred around reading and discussing as a whole class and in small groups multiplication stories, commonly referred to as Big Books, for example, *The Squirrels Store* (Irons & Gardner, 1999); *Polly the Packer* (Irons & Roberts, 1999); and *Shirts and Skirts* (Irons & Reynolds, 1999). These books, and others used, were purposely written for exploration and discussion of the multiplication concept with students. Each was used over a number of lessons to enable children to revisit, fully explore and understand the multiplication concept the books were depicting. During the lessons, students were encouraged to tell, retell, discuss, explain and model the multiplication stories and multiplication concept depicted.

Classroom Interactions

Classroom interactions are influenced by the underpinning theoretical framework, chosen by the researcher, for the teaching experiment. In this teaching experiment the chosen theoretical framework supported the assumptions of constructivism, namely, conceptual knowledge will not be passively acquired by a student as it is passed by language from the teacher, but rather, it must be actively constructed within the social context of a classroom community (Doig, McCrae, & Rowe, 2003; von Glasersfeld, 1995). Social interaction in this teaching experiment was achieved through the class sharing of the purpose-written multiplication stories (Big Books); collaborative whole class and small group activities based around these multiplication stories; and collaborative small group project work to create their own multiplication stories as a culminating activity.

Mathematical concepts cannot be developed in the absence of language (Australian Education Council, 1991) and students need to be afforded opportunities to talk about, share solutions and strategies, explain and clarify their own thinking (Australian Education Council, 1991; Doig et al., 2003; von Glasersfeld, 1991; Wood et al., 1995).

The aim was to encourage students to discuss and reflect on, the multiplication stories depicted in the Big Books in order to make sense of the multiplication concept and the various representations presented. All students were encouraged to contribute their views in order to arrive at a consensus of opinion about the multiplication concept. Students were asked to explain and support their ideas with evidence. These interactions encouraged the teacher/researcher and the students to be co-participants in the dialogic inquiry aimed at transforming their shared understanding of multiplication.

Role of the Teacher

The teacher/researcher facilitated language, reflection and discussion about multiplication using the shared literature. She also facilitated student-to-student interactions by grouping students into small groups of three that were then scaffolded in the creation of their own multiplication stories to share with the class. These created stories, along with other diagnostic testing, were then analysed.

Assessment

In this study the range of assessment strategies used included (1) pre and post assessment items to assess entering and exiting levels of conceptual knowledge about multiplication; (2) open-ended tasks in which students demonstrated individually and in small groups their level of understanding of the concept of multiplication by creating their own multiplication stories, which were assessed using the SOLO Taxonomy (Biggs & Collis, 1982, 1989); and (3) interviews with students individually to further clarify their level of conceptual understanding.

Measurement Instruments and Procedures

A mixed methodology was employed in this study with a range of both qualitative (audio & video taping, field notes, interviews, artefact collection) and quantitative (pre and post tests) data collection methods. The pre and post-tests which were identical in format but with changed numerical values, were conducted individually with each student and recorded the student's responses to tasks and interview questions to test the student's conceptual knowledge of multiplication. The tests were audio taped to enable the researcher to review and recall data from these sessions and enable member checking between the participants, the regular classroom teacher and the researchers.

A number of analytical tools were used to evaluate the data including the SOLO taxonomy (Biggs & Collis, 1982, 1989) and SPSS Version 11. Students' responses to pre and post-testing, the collaborative group product and individual stories were assessed and coded using the SOLO model to determine the SOLO level that they displayed using a four point scale (1 = pre-structural, 2 = uni-structural, 3 = multi-structural, 4 = relational). Coding was conducted on two separate occasions by researchers to ensure consistency. The study by Mulligan and Watson (1998) provided a valuable referent in this study as they provided an analysis of young students' development of multiplication and division concepts based on a multi-modal SOLO model. They classified responses to multiplication problems based on the emergence of composite structure and the increasing sophistication of the corresponding calculation strategy using SOLO. Prior to this teaching experiment, a number of examples of possible responses to the test activities and their proposed classification according to the SOLO taxonomy were constructed in order to ensure consistency in coding.

The coded pre and post data were then entered into SPSSv11 for analysis. The means from the pre and post-tests were compared using dependent *t* tests for each question individually and for the whole pre and post-test total scores.

Results

Pre and Post-test Total Score Results

The pre-test and post-test interview questions and tasks were designed in order to assess the impact of the teaching experiment on students ability to connect concrete and pictorial; real world; and symbolic representations (Hiebert & Carpenter, 1992).

Using examples from Huinker's (1993) study as a guide, the interview format developed for this teaching experiment asked students individually to represent the following situations involving multiplication:

1. Represent real world with concrete/pictorial by using concrete materials or pictures to represent a real world word problem. (3 questions)
2. Represent real world with symbolic by writing an algorithm or equation (number story) to represent a word problem. (1 question)
3. Represent concrete with real world by constructing a word problem to represent concrete materials displayed. (1 question)
4. Represent concrete with symbolic by writing an algorithm or equation that represents concrete material. (4 questions)
5. Represent symbolic with concrete by representing an algorithm with concrete materials. (1 question)
6. Represent symbolic with real world by representing an algorithm with a real world word problem. (1 question)

Three additional questions were also asked to reveal the student's operational sense:

7. Tell me what you think multiplication means?
8. What makes multiplication like or different from addition?
9. How do you know when you should multiply?

The SOLO codes for the 14 questions on the pre and post-tests were entered into SPSSv11 for each student. These data were then analysed using paired samples *t* tests to compare the pre and post-test means for each question individually and the overall mean of each test. The comparison of the overall mean scores for the pre and post-tests produced a statistically significant difference between the pre-test mean (M = 2.4135) and post-test mean (M = 3.1692), t (18) = -7.969, p = 0.000 indicating that overall the students seemed to improve their conceptual understanding of multiplication as a result of this short teaching experiment. The analyses of each individual question (pre to post) is beyond the scope of this paper and will be reported elsewhere.

Individual Multiplication Stories

At the end of the study, prior to the post-testing, students were given an opportunity to write their own short multiplication story. Due to absences through illness only 15 students, out of 19, participated in this task.

There are no comparative data to investigate the impact of the teaching experiment on the students' ability to write their own multiplication stories (pre to post). However, these data were used to provide additional evidence to validate the pre and post-test results and to substantiate the results of the group story-writing project (reported elsewhere).

Two students out of 15 provided what was deemed to be a pre-structural response:

[Student 21]: There were nine snakes and three snakes went away. How many snakes = 6
[Student 17]: Once there were two large geckos on a leaf and two more joined the leaf. How many altogether?

Another student who had focused on the relevant domain but had only picked up one multiplication feature offered what was deemed to be a uni-structural response:

[Student 6]: There were two dogs in each home.

Four students out of 15 offered multi-structural responses, indicating that they had picked up more relevant features but had not integrated them. They explored a composite unit in a repeated format. An example of these stories is:

[Student 3]: One day there were twelve lady beetles in a group and then twelve cockroaches and twelve butterflies came along and sat down in the garden. Then the twelve lady beetles came and sat with the butterflies and cockroaches. That is up to thirty-six.

The remaining eight students offered relational responses, indicating they had integrated composite units, including:

[Student 11]: There were six children. They had three pencils each. How many [pencils] altogether?
[Student 7]: There were three bugs and they each had four donuts each. How many [donuts] altogether?

All students who offered a relational response used the multiplication model of equal groups in their story. Thus, approximately 53% of students demonstrated a relational understanding of multiplication at the conclusion of the teaching experiment as evidenced by their individual multiplication stories.

Conclusion

This study began with the question "Can students' conceptual understanding of multiplication be fostered through a teaching experiment based on children's literature?" The study in addition had two major aims. The first was to strengthen the connection between research and practice and to make research of value to the classroom teacher. The second was to investigate alternate authentic assessment opportunities (pre and post interviews; and individual and group multiplication story creation) because of the criticisms that traditional forms of assessment have not changed classroom practice (Pfannkuch, 2000) to mirror the changing beliefs about learning mathematics highlighted by research (Leder, 1992).

Empirical research on curriculum content (Hiebert & Carpenter, 1992; Mulligan & Watson, 1998), pedagogy (Bruner, 1964; Irons & Irons, 1989) and assessment (Biggs & Collis, 1982, 1989) guided the design of the teaching experiment (Confrey & Lachance, 1999; Worlley & Proctor, 2005). Children's literature (Irons & Reynolds, 1999; Irons & Roberts, 1999; Irons & Gardner, 1999) was engaged as both a meditational tool and also provided an assessment opportunity for students to demonstrate what they understand about the concept of multiplication.

The results indicate that structuring and sequencing the curriculum (Bruner, 1964) and teaching using stage relevant language and learning activities (Irons & Irons, 1989) to move students along the continuum of concrete to abstract; making links between the different representations of concrete/pictorial, real world and symbolic (Hiebert & Carpenter, 1992); and, using the framework of the SOLO taxonomy (Biggs & Collis, 1982, 1989) to guide both teaching activities and assessment increased the conceptual understanding of multiplication for this group of Year 3 students.

These results obtained after only a short time frame are worthy of further investigation to examine if teaching and learning experiences in mathematics, based on this empirical research model and designed using this composition of activities and procedures could be used to increase the conceptual understanding of multiplication in other children, or for that matter the understanding of other mathematical concepts. The unfolding conjecture and teaching experiment could and should be tested against a matched control sample to determine its impact on student understanding of mathematical concepts in relation to other teaching methods. Also another research cycle could aim to determine the specific features of the Big Books (by Irons et al) that lead to enhanced conceptual understanding in this study. Certainly this study has provided interesting suggestions for future research.

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