Students in a Mathematical Community of Inquiry: What do They Think?

Jodie Hunter
Massey University
jodiehunter@slingshot.co.nz

This study examines the views held by students from an inquiry classroom. Individual interviews were used to explore views held by a group of 9-11 year old students towards explanation and justification of their mathematical reasoning. Responses indicated that the students viewed explanations as thinking tools. Challenge, questioning and debate were viewed as opportunities to re-construct mathematical thinking. The study emphasises the need for a safe, power sharing environment to support student construction of positive mathematical attitudes and dispositions.

In the past decade language and communication has taken an increasingly important role in mathematics education. The New Zealand Mathematics curriculum document maintains that as students solve problems they should "express ideas, and listen and respond to the ideas of others" (Ministry of Education, 1992, p.9). Internationally there has also been increased attention paid to the role of communication (National Council of Teachers of Mathematics, 2000). Peressini and Knuth (2000) argue that communication is a way to make implicit mathematical ideas explicit to all students. These researchers promote a need for classrooms in which students make conjectures and justify their claims and in this way public discourse unfolds. But developing such classrooms is challenging—particularly in regard to the beliefs which the participants within them may hold. Both the teachers and students may not have previously experienced mathematics classrooms in which communicating explanatory reasoning has such a central role. The research reported in this paper examines the views of mathematical learning held by students in an inquiry classroom in which they engaged in public explanation, inquiry and challenge. The focus of the paper explores from the students' perspective how they consider making and listening to explanations and how this affects their construction of mathematical understandings.

Developing productive discourse is an important aspect of communicating mathematical reasoning. Explanations are a key component of this discourse (Knuth & Peressini, 2001). Student provided explanations offer an opportunity to learn from peers in that they promote sense-making and deep understanding (Blunk, 1998). Explanations are communicative acts which make visible and clarify aspects and reasoning which may not be obvious to the explainer themselves or to other participants (Yackel & Cobb, 1996). Blunk maintains that to ensure clear communication of explanatory reasoning in classrooms, norms must be established which ensure that not only student explanations and justifications make sense and are accessible to all but also they are valued by the student community. This researcher identifies a need for recognition and value placed on listening, sense-making and evaluation of other students' reasoning. This is because these are valuable sources of learning. Moreover, the move towards co-construction of strong mathematical arguments by students necessitates students understanding each other's ideas and identifying the inter-connection between their ideas (Zack, 1999).

Through the practice of making sense of explanations, students develop mathematical dispositions of interpretation and sense-making. The students are also able to gain more conceptual understanding of mathematical concepts through using both teachers' and their peers' statements as thinking devices (Knuth & Peressini, 2001). However achieving student understanding of the value of such activity is not easily achieved as Young-Loveridge, Taylor and Hawera's (2005) study demonstrated. In this study, the findings showed how students often focus on explaining their own thinking and discount the importance of knowing about other people's mathematical understanding. These students viewed mathematics learning as private and not a public activity. However, in an alternative study by Young-Loveridge (2005) her findings reveal a contrasting picture. The students in this study were in a school where an inquiry environment had been explicitly enacted. Student autonomy had been addressed through the construction of a safe, power sharing climate. Teachers described a learning partnership with the students within a collaborative learning environment. In contrast with the findings from Young-Loveridge, Taylor, and Hawera's study, the interviews with many of the students from this school revealed that they recognised how important it was to explain their strategy use and be able to build on the thinking of others.

Enactment of a safe learning climate enables students to take academic risks, make mistakes and critique other students' solution methods (Stein, 2001). However, students often have negative feelings towards making errors in the mathematics classroom (Anthony, 1998). Stein proposes that errors need to be viewed by students as an opportunity to engage in mathematical argumentation. In engaging in such activity the students are given an opportunity to take different positions and convince others of their claims through the use of mathematical evidence. In addition, they are also provided with opportunities to formulate a critique when either agreeing or disagreeing.

The theoretical stance of this study is within a socio-cultural perspective (Lerman, 2001). In this perspective, the learning of mathematics is considered not only an individual process but also a social process. How students see themselves as learners and construct a mathematical disposition is based on their active participation in the mathematical practices of the classrooms they are in. These include making explanations and justifying their mathematical reasoning publicly.

Methodology

This study reports on the views of one class of children. The participants in the study consisted of twenty-six Year Five and Six students. Most students were achieving at a level above average for their age grouping, however there was a small group of middle and low achievers. In reporting the findings the achievement level and gender are indicated to illustrate the beliefs held by students of different ability levels and gender. The study was conducted at a New Zealand urban primary school. The students came from a predominantly middle socio-economic home environment. The students were primarily from a European New Zealand ethnic grouping with two students of Tongan ethnic grouping and two students of an Indian-European ethnic grouping.

The students had been members of an inquiry classroom for a year in which explanation and justification of mathematical reasoning were the norm. Data gathered included focus group interviews in the third term of the school year and individual interviews at the conclusion of the school year. Interviews were conducted by an independent interviewer. The students were informed that the interviewer was interested in their views about learning mathematics. The open-ended interview questions were comprised of:

1. Do you like explaining your ideas?
2. Is it important for you to be able to explain to other people how you worked out how to solve a problem?
3. Does explaining your ideas to other people help you?
4. Do you like listening to other peoples' explanations in mathematics?
5. Do you think it is important for you to know how other people solve problems?
6. Does listening to other people's explanations help you?

Interviews were audio-taped and wholly transcribed. Data analysis used a grounded approach in which codes, categories, patterns and themes were developed. To maintain trustworthiness, emerging views from the individual interviews were compared with those expressed during the focus group interviews and verified or refuted.

Results and Discussion

This paper presents the views held by students from an inquiry classroom. These students had experienced a year long mathematical programme which focussed on active engagement in listening, explaining and justifying mathematical reasoning.

Making Explanations

Analysis of the data illustrates that consistently the students considered that it was important to be able to explain to other people how they worked out a solution to a problem. They also regarded explaining their ideas to others as a way in which they were able to support their own learning of mathematics. A range of views about why it was important and helpful to make mathematical explanations was expressed.

A number of students stated that explaining mathematical thinking provided them with a reflective tool with which to analyse and clarify the mathematical reasoning they had used:

> It helps me think of what I did and it helps me understand what I did. (Girl, high)

> It can make you really think about what you've done. (Girl, middle)

> It makes me clarify my own thinking because I really have to think it over and make sure I understand every little bit. (Boy, high)

> It usually helps you with what you're thinking and it clarifies your thinking because you have to think why you did it and make sense of it. (Girl, high)

> Explaining it to other people could make you understand it a bit easier. (Boy, low)

The awareness that explanations are provided to a listening audience was a factor considered by some students. Moreover, students viewed questioning from the listening audience as a tool to clarify thinking:

> It helps me because people ask questions which helps clarify my thinking. (Girl, middle)

Many students described that through the expectation that they would explain their reasoning, they were provided with opportunities to explore and test their conjectures. Furthermore through the process of rehearsing their explanations they were able to re-construct their mathematical thinking when needed:

> You can learn that sometimes your ideas don't actually work. (Girl, high)

> Sometimes you have to rethink your strategy and it helps you to find out if you mucked up where you mucked up. (Girl, low)

Analysis by peers of the reasoning used and peer support were also acknowledged as a positive learning tool which advanced construction and reconstruction of mathematical reasoning. The students recognized that through working together in a collaborative partnership more was achieved:

> If I'm working with a buddy and I explain my strategy then if it doesn't work they can build onto it to make it work better. (Girl, high)

> If you're not sure people can help you understand it and they'll teach you ways and help you understand what you've done wrong. (Girl, middle)

> If I got it wrong they can always help me out and tell me what I need to do to re-work it. (Boy, middle)

Close examination of the data also revealed that the students recognised that questioning and challenge to their explanations by peers caused a reflective reconsideration of the reasoning they were using and its reconstruction:

> People might challenge your thinking and you might find out something that you didn't know. (Boy, low)

> They challenge my thinking and give me other ideas. (Boy, high)

Furthermore they described a responsibility to make their reasoning clear and understood, or they were obliged to re-explain in other ways until it was understood:

> It challenges you because if someone doesn't understand you have to find another way to explain it. (Girl, high)

> When I'm explaining to other people I have to find a clear way and sometimes people don't understand what you're actually saying so you have to do it in a different way which challenges you. (Girl, middle)

Errors were viewed positively as tools which supported the construction and reconstruction of mathematical thinking. In a climate in which risk-taking with reasoning was supported and encouraged the students recognised that errors were a chance to re-conceptualise thinking:

> If I think I've gone wrong somewhere if I've explained it then I might be able to work out where I've gone wrong. (Girl, high)

> If it's incorrect they'll help you and tell you why and then they can prove their way and then you can work on yours and you might change your thinking. (Boy, high)

However, some of the lower achieving students recognised the risks to their self-efficacy in providing public explanations. Although these students expressed a positive attitude to explaining, at the same time they were conscious of loss of face if they were incorrect:

> Sometimes I don't really like it because my ideas are wrong and they just don't seem right to all the others and other times I like sharing them because they are right and it's fun to share. (Boy, low)

> Sometimes you think your idea is really good and then it turns out that it's not really good. (Girl, low)

Analysis of the data also revealed a gender difference. The students who simply wanted other members of the class to understand their thinking were predominantly male:

> It makes other people know how I think and what my way of maths is. (Boy, high)
>
> You get a chance to show them what you have come up with. (Boy, middle)
>
> Others can know what I choose for the problem. (Boy, low)

In contrast, many female students viewed the provision of explanations as a way of supporting other students in the class and helping them learn:

> It helps other people to think of another way to talk about the problem. (Girl, high)
>
> You want to show them about it so they can understand it. (Girl, high)
>
> It can help other people increase their knowledge. (Girl, low)
>
> You can help other people in your group to understand it better. (Girl, middle)
>
> To help each other's learning. (Girl, high)

Listening to Explanations

Analysis of the data shows that all the students valued listening to other students' explanations as important for their own learning. They indicated they knew they were constructing or deepening their mathematical understanding when listening to explanations:

> It's interesting to learn off them and the ideas that they have and clarify them. (Girl, middle)
>
> You can learn things like new strategies and new ways. (Boy, low)
>
> I learn new strategies from it and I learn what other people are doing as well as what I am doing. (Girl, high)
>
> If you listen to other people, you learn new ideas as well and you find a new strategy. (Boy, high)
>
> I can learn off them and take in what they said, like if they did something like a new way of figuring out something then I could take that and use it again. (Girl, high)

The students also described how listening to other students' explanations supported them to broaden their mathematical knowledge. They outlined how listening to explanations provided them with a wider range of strategies to use to solve problems:

> It teaches you lots of different ways to work out what you've been doing. (Boy, middle)
>
> You can learn different strategies off them which you can use to make your maths better. (Boy, low)
>
> It gives you a range of ways of solving problems, not just sticking with one strategy (Girl, middle).
>
> They could have a better way of doing it and I could know their way and know lots of different ways of working it out. (Girl, high)
>
> Then you know more strategies to use. (Girl, low)

Furthermore listening to explanations also provided the students with an opportunity to analyse strategies. Many of the students used listening to explanations as a tool to recognise more efficient strategies which could be used in future problem-solving:

> You can use new strategies if they show you how to do it and you can learn faster strategies. I was sharing one of my strategies and someone had a more efficient strategy and now I use the strategy

they used to solve problems. (Boy, low)

It could be a quicker way or easier way to do it. (Girl, middle)

You can learn new strategies that are quicker. (Girl, high)

The students revealed cognisance of their obligation to actively engage in listening to explanations. They also described how they were obliged to take a position and challenge the reasoning used in the explanation unless convinced of the correctness of it. However, if they agreed with the reasoning used then they were obliged to position themselves as supporters of the explainers in convincing others:

If they're wrong, I can help prove to them that they are wrong and if they're right and other people are unsure, I can help them try to prove to the other person that they are right. (Girl, middle)

If I had an idea, I can always listen to other people and see if I got the same answer and I can either disagree or agree. If it looks any different I can always say how did you get that or I disagree with that can you please explain it to me in a better way. (Boy, middle)

Coupled with notions of challenge and debate, the students explained how they used listening to explanations as a means to analyse and justify their own thinking. Listening to an alternative strategy provided them with a reflective thinking tool with which to examine their own strategy:

It challenges my thinking to challenge their thinking. I like trying to challenge their thinking to explain their strategy so it's easier to understand. (Boy, middle)

I was in my group for maths clinic once and Henry was challenging all of our thinking and proving to us what he's done and how it works. (Boy, middle)

The students considered questioning and challenging explanations as scaffolding both their own and other students' learning. Questioning was used by students as a reciprocal tool both to benefit themselves and other students within the classroom:

You can learn from them and if you don't understand it you can ask questions and they can explain it to you. (Boy, low)

You can make them think harder about it because if I didn't get something that they said, I would say why did you do that? Or ask them a question and so it would make them think harder about what they're doing and if it was wrong or something they might change their thinking about it. (Boy, high)

Close examination of the data also illustrated how in the act of listening to explanations the students engaged in meta-cognitive activity. They described how active participation in listening was tied to exploration of their own reasoning. They reviewed their thinking and made comparisons and connections with their own thinking and the thinking of others. In this way the students recognised that explanations were reflective tools:

When I was working in a group with Rob and we were talking about what's in between 3.6 and 3.7 and he was saying that there is an infinite amount of numbers that is in between them because there's no end to it and that made me think because I wasn't really thinking about that — I was just thinking about the obvious ones. (Girl, high)

You can figure out if you've gone wrong and if you don't understand the question you can figure out where you've gone wrong and how you've gone wrong. You can question their thinking to see what they're thinking and how they worked it out. (Girl, high)

When we're in maths clinic, others give me ideas and I feed off that and then if someone else has an idea I feed off that and then I put them altogether with my own ideas and try and make one huge explanation. (Boy, high)

> If I've said something and they've said something I put their idea into my idea and then line them together and see if they could work together. When you come into a group and you're sitting down and then someone says something in your group like they say "oh I just figured this out" and then someone shares something in the group and you're kind of sitting there and you're thinking oh so that's what you do and you combine your way and their way and then it kind of clicks. (Girl, middle)

The students described how listening to others explain their reasoning provided a scaffold for sense-making when working independently. They described how ideas proffered publicly were applied to their mathematical activity when working independently:

> At the clinic I got a strategy and I used it later when I was working in an independent learning time. (Boy, high)

> When I've been trying to work out a problem, when I'm doing individual maths I could use their way to solve it. I've used Billy's because he had this good strategy that I learnt in maths clinic so I used it to work out a problem. (Girl, high)

> Sometimes when I'm doing my maths homework I might try an idea or another person's thinking. (Boy, high)

Conclusions and Implications

The use of student responses to questions which explored their views on making and listening to mathematical explanations provides evidence that communicating in mathematics classrooms affects their construction of mathematical understandings. The students reported a wide range of positive outcomes as a result of engaging in listening and explaining mathematical reasoning. Many of the student views supported those outlined by Stein (2001). They recognised the value of inquiry and debate as a tool to analyse and reconstruct reasoning. Evidence is provided in this study which supports Stein's contention that within an intellectual climate where risk taking is valued, errors are viewed as learning opportunities. Furthermore, the views expressed by the students support Knuth and Peressini's (2001) assertion—that when students publicly make explanations and justifications, these have the potential to become reflective thinking devices.

The findings in this study also concur with those outlined by Young-Loveridge's (2005) study in which she proposed that a safe, power sharing climate supported student construction of positive mathematical attitudes and disposition. This research reports on how the students' view of their learning in mathematics' classrooms supported their development of a positive mathematical disposition.

Also illustrated is the importance of taking cognisance of students' individual perspectives. The many divergent students' views showed how they considered explaining and listening to mathematical reasoning as important.

However, given the limited sample of twenty-six students participating in this study, further research is needed to examine the consistency of these results. In addition examination needs to be made to establish the specific factors in the classroom discourse which supported these results. Also further research is required to examine the factors which affect how the less able students view their involvement in an inquiry mathematics classroom. The potential risks to these students of explaining their thinking would benefit with further examination to ascertain what factors support their mathematical learning and participation in discussion rich classrooms.

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