Making Connections: Promoting Connectedness in Early Mathematics Education

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Prominent policy guidelines suggest that effective mathematics pedagogy assists students to "make connections" between various types of mathematical knowledge and between mathematical knowledge and real-life phenomena. This paper describes the perspectives and practices of two early years teachers involved in a reform that encourages teachers to help students make connections between forms of disciplinary knowledge and between disciplinary knowledge and real-life experience. The purpose of this description is to reveal strategies that assist students learn to make mathematical connections.

Helping students learn to make connections between various forms of mathematical knowledge, as well as between mathematics and real-life experience, is increasingly recognised as integral to effective mathematics learning and teaching. Internationally, the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) state that mathematics instruction should enable students to:

- recognise and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognise and apply mathematics in contexts outside of mathematics.

The *Connected Mathematics Project*, a prominent North American mathematics curriculum based on the Principles and Standards, includes among its fundamental themes an emphasis on "significant connections, meaningful to students, among various mathematical topics and between mathematics and problems in other disciplines" (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002, p. 1). In Australia also, the requirement that effective mathematics pedagogy should help students learn to make connections between various forms of disciplinary knowledge and between disciplinary knowledge and real-life experience is embedded in policy guidelines. For example, the Queensland Government's Framework for Action 2007-2010, *Numeracy: Lifelong confidence with mathematics,* states that teachers of mathematics should support students to develop "the confidence to choose and use mathematics skills they learn at school in everyday life" (Education Queensland, 2007, p.1). In other words, there is broad consensus that in order to become numerate, students must become competent in perceiving the connections between mathematics and other forms of knowledge and between mathematics and their lived experience, as well as competent in applying the mathematical knowledge necessary to maximise the productivity of such connections.

The consensus that students should learn how to make mathematical connections does not extend to agreement about the ways in which students should be supported to do this. While current policy guidelines and reform-oriented curricula advocate the use of problem solving and discussion-oriented pedagogies, the ability of such pedagogies to support all students to become numerate is contested (Lerman & Zevenbergen, 2004). Lerman and Zevenbergen (2004) describe a small but important body of research (e.g., Cooper & Dunne, 2000; Lubienski, 2000) that suggests working class students tend to experience greater difficulty completing real-life mathematical tasks than their middle class peers. Various styles of questioning have also been shown to mediate student participation in mathematics education and ultimate access to mathematical knowledge in line with social class (Lubienski, 2004; Zevenbergen, 2000). Rather than dismiss the worth of problem solving and discussion-oriented pedagogies to students from working class backgrounds, more work is needed to establish how teachers can enact such pedagogies so that students from a range of backgrounds can both learn mathematics and learn how to connect mathematics learned with learning in other subject areas and with their real lives.

A recent Queensland education reform, the *New Basics* (Education Queensland, 2000) advocates the utilisation of a set of 'Productive Pedagogies' designed to help students make connections between various forms of disciplinary knowledge and between disciplinary knowledge and real life. These 'Productive Pedagogies' are presented as necessary enablers underpinning students' effective participation in curriculum and assessment oriented towards the solution of complex real-life problems. Four of the model's 20 Productive Pedagogies, those grouped under the category of "connectedness", are specifically concerned with helping students learn to make connections (Department of Education, 2002, p. 20).

The New Basics trial concluded in 2003. However, most of the 38 schools involved in the trial proper as well as another 21 schools involved in a second phase of the model's implementation continue to espouse a New Basics philosophy. Investigation of the practices of teachers in these schools has the potential to reveal how students can be supported to make connections between mathematics and other forms of disciplinary knowledge and between mathematical knowledge and real-life experience when engaged in curricula oriented towards problem solving. This paper describes research examining the perspectives and practices of two teachers in a New Basics school, drawing on concepts from Bernstein's (2000) theorisation of pedagogic discourse to reveal how each teacher conceptualises the importance of learning to make connections within mathematics education, as well as the ways in which they assist students to make connections via their teaching of mathematics.

Theoretical Framework

The concept of pedagogic discourse is at the heart of the theoretical model developed by Bernstein (1975; 1990) to explain how schools are implicated in the reproduction of inequitable social relations. Pedagogic discourse is the principle, or "rule", by which knowledge is translated into a form useable for transmission by a teacher to a learner (Bernstein, 1990, p. 183). Such translation or 'recontextualisation' is achieved via pedagogic discourse's two constituent parts: Regulative discourse and instructional discourse (Bernstein, 2000). Regulative discourse establishes the social order within the classroom, by setting norms of conduct, character and manner (Bernstein, 2000). Instructional discourse sets the parameters for particular knowledge discourses by determining the selection, sequencing, pacing and evaluation of what will be considered legitimate knowledge (Bernstein, 2000). Regulative discourse is the dominant of the two discourses, as messages about what constitutes legitimate knowledge are always carried on contained within messages about what constitutes appropriate behaviour. Utilisation of the concept of pedagogic discourse, particularly its reliance on regulative discourse to transmit instructional discourse, allowed the research described here to examine how teachers went about setting norms for what would be considered legitimate knowledge in their classrooms, and how these norms were communicated through the social practices of their classes.

More in-depth description of the features of instructional and regulative discourses circulating within the classes studied is supported by the utilisation of Bernstein's (1975, 1990) concepts of classification and framing. Classification refers to the degree to which boundaries between knowledge discourses are maintained (Bernstein, 1990, 2000). When describing the degree to which a discourse is separated from others, Bernstein (1990) refers to the 'strength' of its classification. A discourse that is strongly classified is one well insulated from others, and therefore viewed as highly specialised. A weakly classified discourse is one whose boundaries are fuzzier and less clearly defined. Framing is the principle through which classificatory relations between knowledge discourses are communicated to students (Bernstein, 1990, 2000). Framing determines both the social and discursive information that is communicated within a particular pedagogy. Like degrees of classification, the nature of the framing within a particular discourse is also typed according to its strength (Bernstein, 1990, 2000). A strongly framed pedagogy is one in which the selection, sequencing, pacing, and evaluation of instructional material is kept under strict control by the teacher. A more weakly framed pedagogy is marked by more relaxed communicative practices. Selection, sequencing, pacing and evaluation are negotiated with learners in an attitude of exchange. The concepts of instructional and regulative discourse, along with classification and framing, are used here to facilitate interrogation of research data and facilitate explanation of how observed classroom practices supported students to make mathematical connections.

Research Design and Approach

Research data described in this paper were collected during an explanatory case study (Yin, 2003) of two Year One/Two classes in a school that identifies itself as a New Basics school, Mirabelle State School. Mirabelle is a small inner-city school with approximately 200 students from mostly middle class and low SES students, with a considerable number of students from culturally and linguistically diverse backgrounds. The two teacher participants had formed a co-operative arrangement, whereby on two days each week their classes were divided. All Year Ones worked with Mrs Kelly on those days, while all Year Twos worked with Mrs Roberts. This had the effect of ensuring that all students in Years One and Two worked extensively with both teachers over a two year period. During the study, both teachers' mathematics teaching was observed and recorded over a period of 14 weeks, first with ethnographic field notes and later using video-tape. Teachers were observed while teaching their own Year One/Two classes as well as when they taught the single year level group made up of students from both classes. Class teachers were also individually interviewed on three or four occasions for at least one hour. During each individual interview, teacher participants responded to a key question related to their perspectives on the purpose and/or appropriate conduct of mathematics education. The main ideas from the teachers' responses were recorded on chart paper to form concept maps (Leiken, Chazan, & Yerushalmy, 2001; Novak, 1998). The ultimate content and design of each concept map was negotiated with teacher participants until they felt that maps produced accurately represented their views. The concept mapping technique enabled the interviewer to prompt and probe the teachers' responses to produce rich conversations which were transcribed as interview data and analysed along with the concept maps. The scope of this paper is limited to certain preliminary conclusions drawn from the inductive analysis of interview and observational data, which revealed patterns in how the teacher participants, Mrs Kelly and Mrs Roberts, helped students learn to make mathematical connections.

Making Connections: Teachers' Perspectives

Both of the teacher participants in the study viewed helping students learn to make connections, or connectedness, as fundamental to mathematics education. When asked "What should children learn in maths at school?" Mrs Kelly explicitly named connectedness as the primary purpose of mathematics education, situating her belief in the importance of connectedness within her commitment to the Productive Pedagogies in general.

Researcher: If you were explaining to a student teacher, what is it that children have to learn in maths at school, what would you say?

Mrs Kelly: Well, the first thing would be, going back to the Productive Pedagogies, is the connectedness to the world. So it's um, they um, they should learn that maths is um…not something isolated, but something, um, that is a part of everyday living. And then, learning how to use maths as part of their everyday living.

The second teacher, Mrs Roberts, also talked about how mathematics education should equip students to see and understand how mathematics is related to everyday tasks.

Mrs Roberts: Can you see what I'm saying? They've got to see that counting out the forks and knives, and putting one-to-one correspondence and stuff, to see that that is maths. Because, you know, they'll come back and say "But that's not maths, that's just setting the table."

Researcher: So they've got to see that maths is useful and they've got to see the maths …

Mrs Roberts: … in everyday things. Yes. Yes … …That it's not a separate entity. That's it's not something you just pack away in the corner. Oh, we've got to learn [it] at school but it's got no function.

Concept maps produced during individual interviews illustrated the teacher participants' view that learning to make connections is fundamental to mathematics education, as evidenced by the example of Mrs Roberts' concept map provided in Figure 1.

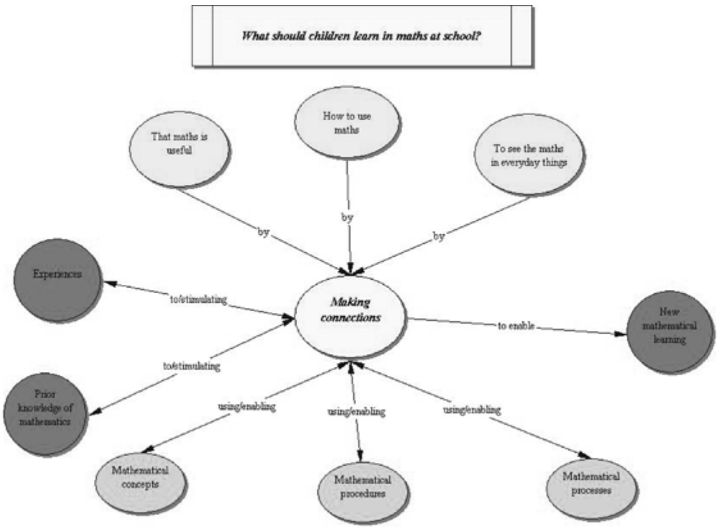

Figure 1. Mrs Roberts' concept map: What should children learn in maths at school?

Making Connections: Teachers' Practices

Observation and analysis of Mrs Kelly's and Mrs Roberts' teaching revealed a number of ways in which they supported students learn to make mathematical connections. In this excerpt from a lesson transcript, Mrs Kelly sets students the task of creating a symmetrical representation of a bat. Her presentation of the task and the way in which she responds to student suggestions creates the conditions for students to identify and capitalise upon connections between the mathematics task at hand and learning in other subject areas.

Mrs Kelly: Now, think back to what symmetry is. I want you to draw or cut out a bat and make sure it is symmetrical. How do you think you might be able to do that?

Robin: When you do one side, and you're up to the other side, you'd just copy.

Mrs Kelly: So, you'd do one side at a time.

Robbie: I would fold it in half so I could cut out half a bat … .We did it in art before.

Michael: I would draw an ear, then the other ear.

Mrs Kelly: So, you'd go from side to side. Anyone know how you could use your grid book to make sure your bat was symmetrical?

Will: Drawing a wing, count how much squares.

Mrs Kelly: Oh, looking at the area covered? Your challenge now is how are you going to go about producing a bat that is symmetrical?

This transcript shows that one student, Robbie, made a connection between the mathematics task at hand and a skill learned in Art. Soon after this exchange, Robbie again discussed his plan to utilise learning from another subject area to enhance his ability to complete a mathematical task:

Mrs Kelly: Can you do it in your grid book first, then on black paper? Some people sometimes like to try two different things to see which one works for them.

Robbie: I'm going to do it like in Art.

Mrs Kelly: Can you see how other subjects sometimes help us? What we've learned in Art is now helping us in maths.

By endorsing Robbie's plan to draw on a skill learned in Art to help complete a mathematics task, Mrs Kelly made it clear to students that they could profitably work across disciplinary boundaries without compromising the boundary strength (classification) of each discipline, and that such inter-disciplinary work could enhance their effectiveness as mathematicians. The opportunity for this learning arose because Mrs Kelly had not dictated the procedures by which students should create their bat, but indicated that they must select an appropriate procedure themselves. This is an example of a weakly framed pedagogy, in that the teacher has relaxed selection of available procedures and also relaxed the lesson's pacing to allow students time to select and test a number of procedures. However, it was certainly not a case of 'anything goes'. At the beginning of this lesson, Mrs Kelly told students that they would later be required to explain and justify their choice of method to others in the context of a sharing circle. By asking students to suggest possible ways of approaching the task early in the lesson, and then pointing out the mathematical aspects of each method, Mrs Kelly modelled how methods selected could be justified within the specialised language of mathematics. When the sharing circle was convened later in the lesson, students' ways of working were evaluated by the teacher and others against mathematical criteria (e.g., "So, is your bat symmetrical?"), acting to reinforce the boundary strength of mathematics. The regulative discourse circulating within this class was one which valued choice, reflection, and discussion. As a consequence, the instructional discourse conveyed the message that mathematical problem-solving can proceed through hypothesis testing, developing justifications and proofs that draw on a wide range of ideas - provided those ideas are re-framed for communication within the specialised language of mathematics.

Mrs Roberts also occasionally enacted a weakly framed pedagogy to allow students to explore connections identified between mathematical learning and real life. Here, in a lesson during which Mrs Roberts had planned to cover how times that are "half past" the hour are represented, Mrs Roberts relaxed her pacing to pursue an alternate mathematical topic introduced by students related to their real life experience.

Mrs Roberts: How does that sound? So, it's halfway. So, if my big hand was here. Where must that big hand be, to have a half past time, Alan?

Alan: On the six.

Mrs Roberts: Which numbers tell you that it's halfway?

Lex: Something, then the number thirty.

Mrs Roberts: Yeah, but why's the 30 underneath the six? (Mrs Roberts holds up her arm so that the students can see her watch) I don't have…I do have numbers, but they're special numbers.

Kristy: From a different country, or from a Grandpa Clock.

Alan: My dad has those on his watch.

… Mrs Roberts stands and goes to the chalkboard. She writes the series of Roman numerals from I to X vertically on the board.

As she writes the children count aloud in unison from one to ten.

Mrs Roberts points to the I and asks the children what that numeral says. The children reply "One" . Mrs Roberts asks what makes them think that numeral means "One".

Fiona: 'Cause we're just counting on. 'Cause it's only got one line.

Mrs Roberts: So, what do you think this one means? (pointing to the V symbol).

Fiona: Five … …

Mrs Roberts: So, nobody knows what they are? (No one responds) No, I'm going to get you to go home and ask your mum and dad. Sometimes on a clock, you have these types of numbers and do you know what they are? We got a bit side-tracked from our halfway discussion.

While Mrs Roberts frequently relaxed the pacing of lessons to elaborate on connections made by students to prior learning or to real-life experience, she retained strong control over the selection and evaluation of material. Mrs Roberts' teaching included numerous episodes of 'drill and practice'. This reflects her view as represented in her concept map (Figure 1) that children must learn mathematical processes, procedures and concepts in school to enable them to make connections to real life experience. She sought to help students learn a repertoire of mathematical skills so that when confronted with a problem situation, they would have a number of ways of working 'at their fingertips' from which to choose. The regulative discourse circulating in this class valued conforming to established protocols, which carried the message (instructional discourse) that mathematicians used a number of established and accepted procedures to respond to problem situations.

Conclusion

The conviction expressed by Mrs Kelly and Mrs Roberts that making connections was fundamental to mathematics education influenced their teaching practice in a number of important ways. The effectiveness of their practice was evidenced by examples of students in classes taught by these teachers who demonstrated the ability to make connections between mathematical knowledge and other forms of disciplinary knowledge, and between mathematical knowledge and real life. The data presented in this paper suggest a number of strategies that, while not forming a complete practice guide, can be used by teachers to help students learn to make mathematical connections when engaged in problem solving:

1. Assist students to become competent in using a range of mathematical procedures.
2. Require students to select the knowledge and procedures that will assist with the solution of mathematical problems. Don't always tell students what procedures to use.
3. Expect students to explain and justify methods selected for working out problems.
4. Encourage students to draw on ideas from other disciplines or from their own experience when solving problems or recording their thinking.
5. Assist students to re-frame ideas or information from other disciplines or their own experience so that they are expressed using the specialised language of mathematics. Model how to do this when evaluating students' suggestions and chosen methods.
6. Respond positively when students themselves identify connections between diverse bodies of disciplinary knowledge, or between mathematical knowledge and real life.

Not all of these strategies were practised by both teachers to the same degree. While Mrs Kelly and Mrs Roberts utilised different styles of pedagogy, the effect of their co-operative arrangement was that students benefited from receiving explicit instruction about the boundaries and contents of mathematics as a result of Mrs Roberts' strongly classified and framed pedagogy, and were able to utilise this knowledge in the more weakly framed pedagogy utilised in Mrs Kelly's room.

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