Professional Knowledge Required when Teaching Mathematics for Numeracy in the Multiplicative Domain

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This paper presents findings as part of a wider study that investigated the professional knowledge of teachers when teaching mathematics for numeracy in the primary school classroom. This paper focuses on teachers in action as they taught two lessons on multiplication. It outlines the specific pedagogical categories the teachers used and the impact their knowledge had on student learning.

Capturing the essence of teaching by studying what it is a teacher does, why they do it, and what effect it might have on student learning is an on-going topic of research and discussion (Barton, 2009). As Barton (2009) explained, we do not currently have the theories, or research, to inform teachers why it is that some highly mathematically qualified and highly motivated teachers are unsuccessful, and why it is that the students of some mathematically unqualified teachers receive top results. The role of the teacher and the professional knowledge currently required is more complex and sophisticated, and has changed in response to the major societal, economic, cultural, and political changes, which have taken place (Hattie, 2003).

Concern over the mathematical knowledge of primary school teachers, has been expressed for many years (Ball, Thames & Phelps, 2008; Ma, 2010). Linking the professional knowledge of teachers, to the relationship between classroom practice, and student understandings as a result of those practices, has thus been a focus of researchers in recent times (Ball et al., 2008; Chick, Baker, Pham, & Cheng, 2006; Schoenfeld, 2011). Much of the recent research has been founded on the work of Shulman (1986, 1987), who was one of the first researchers to identify the complexities associated with different categories of knowledge teachers require for students' mathematics learning. Shulman introduced the term pedagogical content knowledge (PCK) as being of particular interest to teachers, as it contains a special type of knowledge which distinguishes teaching from other professions.

Recent years have seen more use of the term *numeracy* in education (Perso, 2006). Often the terms mathematics, and numeracy, are used interchangeably and yet some argue that there is a difference in meaning (Coben, 2000; Perso, 2006). Mathematics is about the exploration and use of patterns and relationships in quantities, space and time, about representing and symbolising these ideas, and eventually learning to abstract and generalise (Bobis, Mulligan, & Lowrie, 2013; Ministry of Education, 2007). The development and conceptualisation of the term *numeracy* has been an important influence on the teaching of mathematics, and was first attributed to the United Kingdom's Crowther report in 1959, where numeracy was described as the mirror image of literacy (Tout & Motteram, 2006). Perso (2006) argued that prior to the 1950s school mathematics focussed on computation and it was with introduction of computational tools, and the associated need for higher-order thinking skills, that the need for people to be able to transfer their mathematics understandings to everyday life became greater. Perso (2006) questioned whether in the current cultural and social context of schooling, educators are primarily teachers of mathematics, or teachers of mathematics for numeracy? She argued that there

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needs to be a shift in focus from pure mathematics, to a focus on mathematics as the fundamental prerequisite for numeracy for all children throughout their schooling, as they prepare for life skills requirements beyond the classroom.

The teaching of mathematics in schools throughout the twentieth century saw six identifiable phases, each with its unique emphasis: drill and practice, meaningful arithmetic, new maths, back to basics, problem solving, and standards and accountability (Lambdin & Walcott, 2007). Each of these phases introduced what was seen as new and innovative practices, for that particular period of time. In more recent times education reforms emphasised that learners of any age will not succeed at mathematics unless they are taught in ways which enable them to bring their intelligence, rather than rote learning, into use when learning their mathematics (Skemp, 1989).

One contributing factor often cited as part of the reason for poor mathematics proficiency, is the focus that was previously on developing procedural knowledge, at the expense of conceptual understanding (Skemp, 2006). Thus, the current standards-based education system supports a curriculum that emphasises concepts and meanings, rather than rote learning, and promotes integrated, rather than piecemeal usage of mathematical ideas (Howley, Larsen, Solange, Rhodes, & Howley, 2007). Ma (2010) asserted that in order to facilitate conceptual learning, teachers need to emphasise and promote the connections between, and among ideas that for non-teachers are implied. Ma described this as well-developed, interconnected, knowledge packages, made up of a thorough understanding of mathematics, having breadth, depth, connectedness, and thoroughness. She referred to this as *profound understanding of fundamental mathematics* (PUFM). She noted that "although the term 'profound' is often considered to mean intellectual depth, its three connotations, deep, vast and thorough are interconnected" (Ma, 2010, p. 120).

The emphasis on teaching concepts and meanings positions mathematical knowledge as a social process, whereby children construct mathematical ideas from their understanding and experiences, of the world in which they live (Ross 2005). The 'drill and practice' of basic facts and taught routines, will not prepare children for a technological world. Current teaching focuses on the structure underlying numbers and number operations (Anghileri, 2006; Mulligan & Mitchelmore, 2009). The single most influential factor on student learning is the teacher (Hattie, 2003).

Methodology

Aim of the Study

This main purpose of this study was to identify the strengths and weaknesses in the professional knowledge of primary school teachers, when teaching mathematics for numeracy in the multiplicative domain, and the impact these have on student learning. These might then be addressed in professional learning sessions, to assist in teacher development.

Research Design

A multiple-case study design was used. Multiple-case study design refers to the investigation of more than one participant, where the focus is both within and across the cases (Creswell, 2008). The ability to conduct a number of case studies may then bring with it a need to form some type of generalisability, which was required in this research.

The qualitative and quantitative data collected merged as the various data sets from the four case study teachers, were analysed and interpreted.

Research Setting and Participants

Two teachers from two schools were the four case-study teachers, who along with the children in their classes formed the basis of this study. School A was a full primary (Years 1 to 8) inner city school, while School B was an urban primary school (Years 1 to 6). The case-study research was based around the senior classes of each school: the teacher of the Year 5 and 6 class (Andy), and the teacher of the Year 7 and 8 class (Anna) from School A, along with two teachers of Year 5 and 6 classes from School B (Beth and Bob). The teachers at School A taught their own class for maths, while School B grouped their classes by ability. Bob's class was third in ranking (one being the top class out of the six), and Beth's class fourth class in ranking.

Research Approach

A mixed-methods approach was employed to collect data. Mixed-methods research is often described as research in which the investigator collects and analyses data, integrates the findings, and draws inferences using both qualitative and quantitative approaches and methods, in a single study or programme of inquiry (Cohen, Manion & Morrison, 2000;). The rationale behind mixed-methods research is that more can be learned about a research topic if the strengths of qualitative research, are combined with the strengths of quantitative research, while compensating at the same time for the weaknesses of each (Cohen et al., 2000).

Data Sources

Data collection came from multiple sources including questionnaires, assessments, observations, and interviews. Classroom observations were the key part of data collection which focused primarily on the professional knowledge of teachers in action. In order to validate the observations of the lessons, field notes were written, photos taken, and lessons both audio-taped and video-recorded. This meant that the researcher could return to the details of the lessons and cross-check details at a later date.

Questionnaires were administered to the teachers and their students at various times throughout the study. Questionnaire data were later compared to in class observations. An initial questionnaire was given to the teachers containing three sections: (1) teachers' views about mathematics; (2) multi-choice questions around aspects of subject matter knowledge and (3) scenarios about the teaching of mathematics, where judgements were required in relation to mathematical understanding.

This research related to the teaching of multiplication and division. Pre-unit and post-unit assessment tasks designed by the researcher were administered to the students. The tasks were based on key aspects of knowledge students at Years 5 and Years 6 are expected to implement according to Level three of the New Zealand Curriculum (Ministry of Education, 2007) and the National Mathematics Standards (Ministry of Education, 2009).

The two lessons from each of the case-study teachers were subsequently coded for detailed analysis. Following transcription of lessons the qualitative data was exported into the computer programme NVivo 10 which was used for the coding. Coding stripes were used to group information about particular themes together. The basis for the coding used in this research, were the categories identified on the PCK framework developed by Chick, Baker, Pham and Cheng, (2006), which became one of the key features in determining the professional knowledge of teachers of primary school mathematics.

Results

Pre-Unit Assessment

Prior to teaching the multiplication and division unit of work the students were given nine assessment tasks to ascertain current knowledge (Figure 1). The students were asked to solve each problem, explain how they worked it out, and where possible draw a diagram to show their thinking Most of the tasks were at Levels 2 and 3 in The New Zealand Curriculum, and the majority of children should have been capable of correctly solving these (Ministry of Education, 2007; 2009).

Task 1 Mult as repeated addition 4+4+4+4+4+4=24 How would you write this as a multiplication fact?	Task 2 Draw a Diagram of 3 x 5 =	Task 3 Division Partitive 20÷4	Task 4 Division Quotitive 20÷4	Tasks 5 & 6 Commutative Property 2 x 5	Task 7 Using x5 Basic Facts: I have 6 groups of 5 cubes and know to write this as 6 x 5 = 30. How could I use this to work out 6 x 4 = ?	Task 8 Using known Basic Facts: I know that 4 x 7 = 28. How can I use this to work out 4 x 14 = ?	Task 9 Division with remainders: 30 apples into 4 equal sized bags

Figure 1: Pre-Unit assessment task types

The pre-assessment results showed that there were only two tasks where greater than fifty percent of the children in any class, were able to give a correct answer. Task 7 saw 55% of Beth's class give a correct answer, while 75% of Anna's solved Task 8 correctly. Task 4 saw the poorest result with one child correct in two classes, two correct in one class, while no-one solved the problem correctly in the other class. The correct responses on the other tasks, ranged from 5% of Beth's class on Task 3 (partitive division), to 40% of Bob's class on Tasks 5 and 6 (understanding of commutativity).

Multiplication Lessons Observed

Two lessons were observed for data collection: one at the start of a six week unit on multiplication and division, and one at the end of the unit. All teachers began the first lesson by establishing the meaning of the multiplication ('×') symbol. In mainstream New Zealand classes the first number of a multiplication expression represents the multiplier and the second number the multiplicand. The lesson focused on use of the commutative property of multiplication unpacking the difference in representation between the two different equations, for example 5 × 3 and 3 × 5. Two teachers (Anna and Bob) became confused themselves when explaining the difference and this led to confusion among the students in their classes. The final lesson differed for each teacher, according to the progress the students had made throughout the unit.

Clearly PCK

One of the greatest weaknesses in relation to the teaching of multiplication of all the teachers was their curriculum knowledge. The teachers were unclear as to exactly what they should be teaching students at Level 3 (in Anna's case level 4) of the curriculum. Stages 6 and 7 of the Number Framework (Ministry of Education, 2008a), directly align to

Curriculum Levels 3 and 4, and the teachers did not immediately recognise what strategies and knowledge the students were expected to utilise. The lesson expectations were consistently below national expectation, and the teachers made little attempt to probe the students and push them along.

The teachers struggled to identify cognitive demands of the tasks and aspects that affected their complexity, from the viewpoint of their students. The main problem was the students' difficulty in understanding the multiplier as the first number in the equation, and the multiplicand as the second number, and the significance of acquisition of this knowledge for the students as they moved on to more complex problems with double digits.

All teachers identified a learning intention for their lesson, which began with 'We are learning to…' (referred to as the WALT). While the WALT provided a focus for each lesson, it also became a hindrance, as many opportune moments were missed for the students to bring their own thinking to their problem solving. Observations suggested there was a two-fold reason why the teachers maintained focus on the WALT: management of the children; and apprehension of coping with something mathematical that may arise, to which the teacher may not know the answer. So long as the WALT was at the forefront of the lesson, they were prepared to answer any questions that may be asked, during the lesson.

The nature of the lesson depended on whether the teachers recognised the misconceptions the students currently held. The initial lesson taken by Andy and Bob was very teacher directed. The children were given little opportunity to discuss ideas together and responses to questions were directed at specific students. These students generally had raised their hands because they knew the answer to the given question, and while incorrect responses were sometimes given, it was generally due to inaccurate computation rather than misunderstandings, or misconceptions. Beth's students all had manipulatives available to them, which allowed her to visually see many of the misconceptions the students had. The models the students had constructed along with the discussion as the students explained their thinking, allowed her to recognise misunderstandings the students may have had.

Content Knowledge in a Pedagogical Context

The frequency with which the teachers were required to deconstruct content also aligned to the nature of the lesson. In the first lesson the teachers were very much involved in the problem solving with the lessons being teacher directed throughout. This meant the teachers were able to clarify uncertainties immediately, as they were 'right on the spot' to do so. In the latter lesson the teachers posed problems, and the students were left to solve them on their own more. Thus the teachers were not always in a position to be aware of students' difficulties until discussions were held later in the lessons. As they deconstructed content the teachers discussed the relationship between repeated addition and multiplication, the link between repeated addition and the array model of multiplication, the importance of recognising patterns in mathematics, and the need to have some basic facts as instant recall to assist in working out other facts.

There was little evidence of what was originally referred to by Ma (2010) as *Profound Understanding of Fundamental Mathematics* by any of the teachers. Lessons appeared to be planned and procedurally implemented and as students struggled with understandings, the teachers lacked the depth and breadth of knowledge required to reframe questions and offer explanations in alternative ways. Seldom were connections made between or within ideas. While the teachers could solve problems themselves, their number sense was weak, and of concern.

Pedagogical Knowledge in a Content Context

Classroom techniques, or generic classroom practices, of raising hands to ask/answer questions, using manipulatives to explain thinking, and using a modelling book during group work, were implemented by all of the teachers. The teachers asked the students to share ideas with others, and discuss problems together, but this seldom occurred. The students 'talked' together, but rarely 'discussed' ideas or justified findings.

Knowledge of assessment was limited by all of the teachers. Prior assessment data was under-utilised. The pre-unit assessment data was not used to identify gaps and weaknesses, which could then be incorportaed into the planning of lessons. Similarly it appeared that the results of other matheamtics assessment tools had not been used.

Questioning was very much of the supportive nature and seldom did the teachers extend the thinking of the students. The teachers readily accepted answers given by students, and when a problem was answered correctly, they acknowledged the response and continued with the lesson. The teachers did not ask for justification of responses, and seldom pushed the students to the next level with questions such as, "What would happen if we changed…" or "If we changed this number (for example the multiplier), what affect would it have on this number (the multiplicand)?".

Post-Unit Assessment

At the conclusion of teaching the multiplication and division unit of work, the students were given nine assessment tasks (note: Tasks 5 & 6 were combined and shown as one task to report data) similar to those of the pre-unit assessment (Figure 2). Of the four classes and eight tasks (32 counts in total) there was a percentage decline in the number of students who solved the problem correctly on 14 occasions, an increase of correct responses on 15, while 3 remained the same. The results showed that more than 50% of Bob and Beth's students were correct on Task 1, with more than 50% of Anna's students correct on Task 7. All other tasks saw less than 50% of the children correct with a range of zero on task 3 from Anna's class, and Beth's class on tasks 4 and 8, through to 48% correct on tasks 5 and 6 from Andy's class.

Task 1	Task 2	Task 3	Task 4	Tasks 5 & 6	Task 7	Task 8	Task 9
Mult as repeated addition 5+5+5+5=20 How would you write this as a multiplication fact?	Draw a Diagram of 3 x 6 =	Division Partitive 12÷3	Division Quotitive 12÷3	Commutative Property 3 x 5	Using x5 Basic Facts: I have 6 groups of 5 cubes and know to write this as 6 x 5 = 30. How could I use this to work out 6 x 6 = ?	Using known Basic Facts: I know that 3 x 10 = 30. How can I use this to work out ☐ x 5 = 30?	Division with remainders: 26 apples into 4 equal sized bags

Figure 2: Post-Unit assessment task types

Discussion

Overall, the results were of both considerable interest and concern. The pre-unit assessment results showed that generally the students were below, and in many instances well below, their expected levels (Ministry of Education, 2009). This should have been an indication to the teachers that there was a great deal of knowledge teaching required for the students to understand the concepts associated with multiplication and division. The post-unit assessment showed that little progress had taken place throughout the six weeks, with less than half of the tasks showing an increase in the number of students obtaining correct responses. The teachers had taught many of these ideas in class, and questions must be asked as to why the expected improvement did not occur. Some of these can be attributed to the students themselves, while close analysis of the teaching also highlighted gaps in teachers' professional knowledge.

The teachers' lack of curriculum knowledge and uncertainty of exactly what is required of them in their teaching is of concern. Teachers must understand the requirements of the Curriculum Levels (Ministry of Education, 2007) and align these to the Number Framework Stages (Ministry of Education, 2008), and the Mathematics Standards expectations (Ministry of Education, 2009). The alignment needs to be instantly recognisable if effective decision making during a lesson is to be made. What questions to ask, what problems are given, how far to extend the students in their thinking, are all dependent on having at their fingertips an understanding of the progressions of learning.

There were times when both the teachers and children displayed misconceptions. The term 'misconception' suggests wrong understanding of concepts. Rather than wrong understanding it would be more pertinent to suggest it was often a muddled, or confused, understanding. The teachers seldom exhibited a deep and thorough conceptual understanding of aspects of the mathematics they were teaching (Chick et al., 2006), referred to by Ma (2101) as Profound Understanding of Fundamental Mathematics (PUFM). This contributed to their confusion within the key mathematical concepts they were teaching, and the significance of consistently using correct mathematical language. With current teaching focusing on the structure underlying numbers and number operations (Anghileri, 2006; Mulligan & Mitchelmore, 2009), the teachers PUFM could be narrowed down to the need for a stronger understanding of number and number sense (SUN).

While it is essential that students are aware of the learning intention of each lesson, teachers must take care not to let the focus over-ride the opportunity for new learning to occur. Opportune and teachable moments must not be overlooked, as addressing an issue when it arises will often mean the student will make more sense of the solution and retain the newfound knowledge. This does not mean taking each lesson in a totally different direction from the planned purpose, but if students are to remember key ideas from the lesson, then the learning experience must be meaningful to them.

Problem solving and the associated skills of discussion and justification are now an accepted part of classroom practice. This study showed that while the students were given problems to solve together, they often worked as individuals within their groups, and struggled with the notion of challenging each other's thinking. The students seldom participated in 'friendly argumentation'. Similarly, while the teachers supported the students in their solution methods, there is a definite need for them to extend given ideas by questioning the students thinking more. This would also assist in the students progressing through the Number Framework stages and curriculum levels.

Conclusion

The mathematics classroom of today places a significant emphasis on conceptual understanding, and the importance of making mathematics meaningful beyond the classroom. This suggests that teachers are now teachers of mathematics for numeracy, challenging them to consider the mathematical concepts being taught as well as the contexts within which they are taught. The professional knowledge required by teachers is complex and multi-layered, requiring ongoing attention to the many aspects of PCK originally mooted by Shulman, if students are to achieve, and move beyond, their expected levels.

References

Anghileri, J. (2006). *Teaching number sense.* (2nd ed.). London, UK: Continuum.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides* (Proceedings of the 32nd annual conference on Mathematics Education Research Group of Australasia, pp. 3-9), Palmerston North, NZ: MERGA.

Bobis, J., Mulligan, J., & Lowrie, T. (2013). *Mathematics for children: Teaching children to think mathematically.* Frenchs Forest, NSW: Pearson Australia.

Chick, H., Baker, M., Pham, T., & Cheng, H. (2006). Aspects of teachers' pedagogical content knowledge for decimals. In J. Novotná, H. Moraová, M. Krátká, & N. Stehliková (Eds.), *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 297-304. Prague, The Czech Republic: PME.

Coben, D. (2000). Numeracy, mathematics and adult learning. In I. Gal (Ed.), *Adult numeracy development: Theory, research, practice.* Cresskill, NJ: Hampton Press Inc.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education (5th Ed).* London, UK: Routledge Falmer.

Creswell, J. W. (2008). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research.* Upper Saddle River, NJ: Pearson Education.

Hattie, J. (2003). Teachers make a difference: What is the research evidence? Presented at *Australian Council for Educational Research Annual Conference on: Building teacher quality.* Retrieved from http://www.education.auckland.ac.nz/uoa/fms/default/education/staff/Prof

Howley, A., Larsen, W., Solange, A., Rhodes, A. M., & Howley, M. (2007). Standards-based reform of mathematics education in rural high schools. *Journal of Research in Rural Education, 22*(2). pp. 1-11. Retrieved from http://jrre.psu.edu/article/22-2.pdf .

Lambdin, D., & Walcott, C. (2007). Changes through the years: Connections between psychological learning and theories and the school mathematics curriculum. In W. G. Martin, M.E. Strutchens, & P. C. Elliott (Eds.). *The Learning of Mathematics: sixty-ninth Yearbook.* Reston, VA: NCTM.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Ministry of Education (2007). *The New Zealand curriculum*. Wellington: Author.

Ministry of Education (2008). *Book 1: The Number Framework: Revised edition 2007.* Wellington, NZ: Author.

Ministry of Education (2009). *The New Zealand Curriculum mathematics standards for years 1– 8.* Wellington, NZ: Author.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21*(2), 33-49.

Perso, T. (2006). Teachers of mathematics or numeracy? *Australian Mathematics Teacher, 62*(2), 36-40.

Ross, V. (2005). Pointing the way: Possible avenues of development in the field of mathematics education. *Curriculum Inquiry, 35*(2), 235-246.

Schoenfeld, A. H. (2011). *How we think: A theory of goal-oriented decision making and its educational applications.* New York, NY: Routledge.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Skemp, R. R. (1989). *Mathematics in the primary school.* London, Great Britain: Routledge.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School, 12*, 88-95.

Tout, D., & Motteram, G. (2006). *Foundation numeracy in context.* Camberwell, Vic: ACER Press.